

## Principal Stresses:

This document talks about the principal stresses in a gross sense than its theoretical explanations. And, here I go...

Most of the times we do wonder what use are these principal stresses of. But, to see in real, these vectors are calculated to have a measure of the magnitude of kind of stress at each of the locations in the component (By magnitude, I mean Tensile or Compressive) under study.

The principal stresses are calculated at each point in the component through the cubic equation below

$$\begin{vmatrix} \sigma_x - \sigma_0 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y - \sigma_0 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z - \sigma_0 \end{vmatrix} = 0$$

where:

$\sigma_0$  = principal stress (3 values)

$$\Rightarrow (\sigma_x - \sigma_0) * [(\sigma_y - \sigma_0) * (\sigma_z - \sigma_0) - \sigma_{yz} * \sigma_{yz}] - \sigma_{xy} * [\sigma_{xy} * (\sigma_z - \sigma_0) - \sigma_{yz} * \sigma_{xz}] + \sigma_{xz} * [\sigma_{xy} * \sigma_{yz} - \sigma_{xz} * (\sigma_y - \sigma_0)] = 0$$

$$\Rightarrow (\sigma_x - \sigma_0) * [(\sigma_y \sigma_z - \sigma_y \sigma_0 - \sigma_0 \sigma_z + \sigma_0^2 - \sigma_{yz}^2) - \sigma_{xy} * [\sigma_{xy} \sigma_z - \sigma_{xy} \sigma_0 - \sigma_{yz} \sigma_{xz}] + \sigma_{xz} * [\sigma_{xy} \sigma_{yz} - \sigma_{xz} \sigma_y + \sigma_{xz} \sigma_0]] = 0$$

$$\Rightarrow \sigma_x \sigma_y \sigma_z - \sigma_0 \sigma_x \sigma_y - \sigma_0 \sigma_x \sigma_z + \sigma_0^2 \sigma_x - \sigma_x \sigma_{yz}^2 - \sigma_0 \sigma_y \sigma_z + \sigma_0^2 \sigma_y + \sigma_0^2 \sigma_z - \sigma_0^3 + \sigma_0 \sigma_{yz}^2 - \sigma_{xy}^2 \sigma_z + \sigma_0 \sigma_{xy}^2 + \sigma_{xy} \sigma_{yz} \sigma_{xz} + \sigma_{xy} \sigma_{yz} \sigma_{xz} - \sigma_{xz}^2 \sigma_y + \sigma_0 \sigma_{xz}^2 = 0$$

$$\Rightarrow -\sigma_0^3 + \sigma_0^2 (\sigma_y + \sigma_z - \sigma_x) - \sigma_0 (\sigma_x \sigma_y + \sigma_x \sigma_z + \sigma_y \sigma_z - \sigma_{yz}^2 + \sigma_{xz}^2 + \sigma_{xy}^2) + \sigma_x \sigma_y \sigma_z - \sigma_x \sigma_{yz}^2 - \sigma_{xy}^2 \sigma_z - \sigma_{xz}^2 \sigma_y + 2 * \sigma_{xy} \sigma_{yz} \sigma_{xz} = 0$$

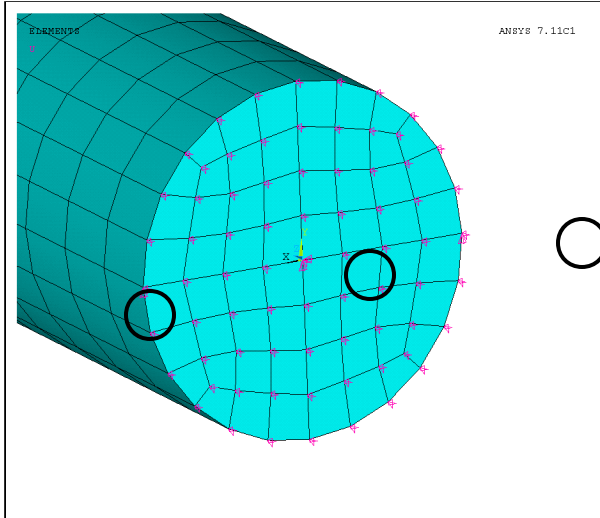
By expanding the determinant above, we get an equation of the cubic order as above. As known, we get three roots from this equation as solution. The roots are then arranged in an order so as to get the most positive, most negative and the last one left and assigned as  $\sigma_1$ ,  $\sigma_3$  and  $\sigma_2$  (First, Third and Second Principal Stresses) respectively. The second principal stress is not of much significance.

So, in our analysis of the components, these principal stresses would help us determine where the maximum tensile and compressive stresses are induced and how these tensile and compressive stresses are distributed in the component.

Let's try to understand the same with some examples...

### Example 1:

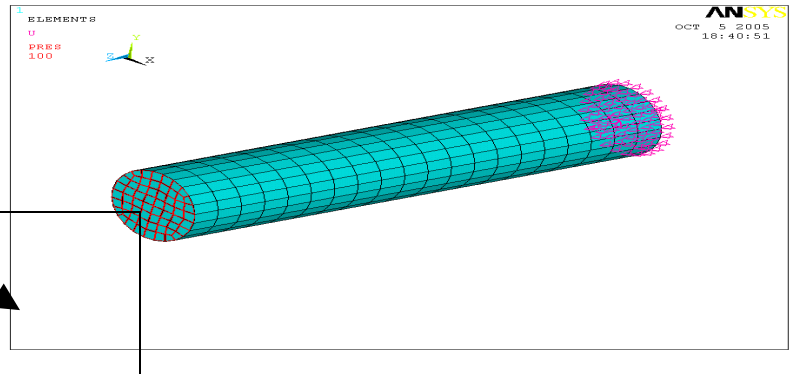
A Cylinder of diameter of 20 mm and length 100 mm is subjected to a pressure of 100 MPa Pressure on one side with the other side constrained to move in the direction of pressure as shown in the picture below.



All nodes on one side (as said in the above statement), are constrained to move in the direction of pressure and one node at the center is arrested in other directions too. And two nodes at the ends marked by red circle are constrained to move in lateral direction. Just to achieve convergence with ease and to suppress rigid body motion. (It should be noted that as long as the imposed extra constraints do not produce stresses of significant magnitude, they can be considered non-affecting.)

Two cases of pressure application are studied here.

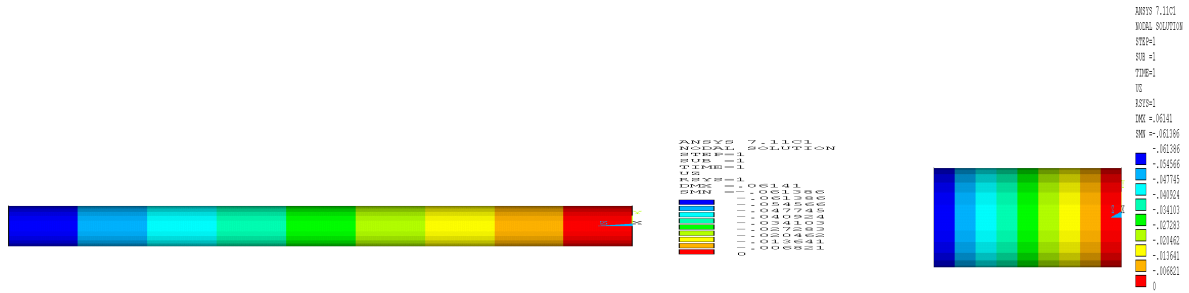
Positive pressure of 100 MPa simulating a pressing of the model against the constraints.  
Negative pressure of 100 MPa simulating a pulling of the model against the constraints.



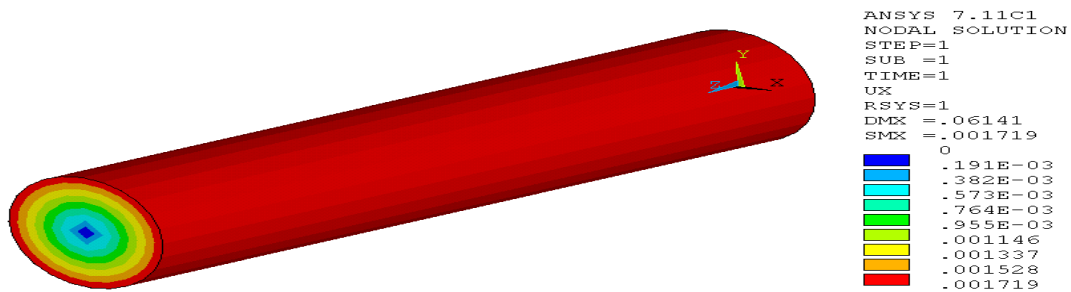
Young's Modulus,  $E = 162900$  MPa  
Poissons Ratio,  $\nu = 0.28$

## Results of Case 1

Displacement Plot in Direction of pressure:

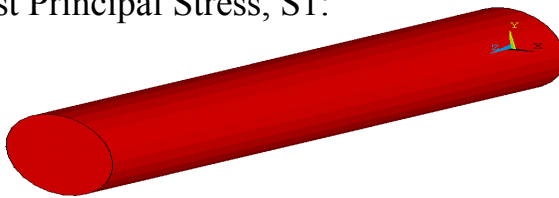


### Displacement Plot in Cylinder Radial Direction:



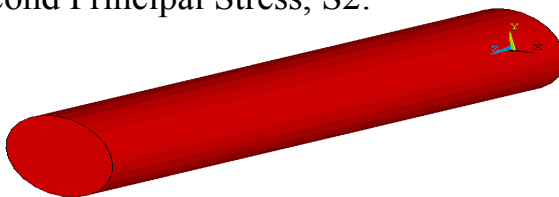
Principal Stress Plots:

First Principal Stress, S1:



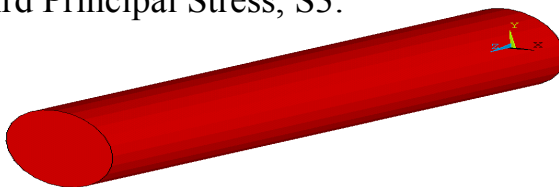
```
ANSYS 7.11c1  
NODAL SOLUTION  
STEP=1  
SUB =1  
TIME=1  
S1 (AVG)  
DMX =.06141
```

Second Principal Stress, S2:



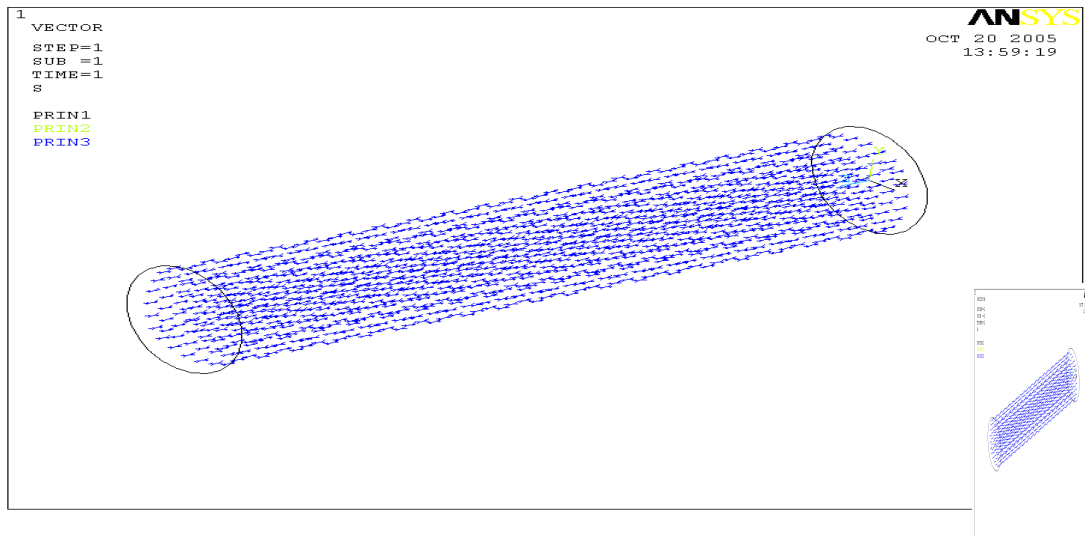
```
ANSYS 7.11c1  
NODAL SOLUTION  
STEP=1  
SUB =1  
TIME=1  
S2 (AVG)  
DMX =.06141
```

Third Principal Stress, S3:



```
ANSYS 7.11c1  
NODAL SOLUTION  
STEP=1  
SUB =1  
TIME=1  
S3 (AVG)  
RSYS=0  
DMX =.06141  
SMN =-100  
SMX =-100  
-100  
-100
```



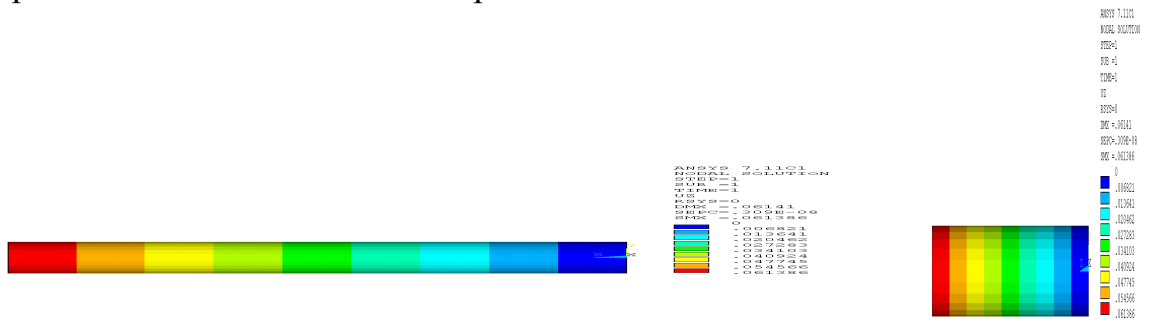


As you can see from the above plots, the Principal stresses S1 indicates no stress indicating that no tensile stresses are induced in the component. Whereas, Principal stress S3 shows a value of -100 in the complete model meaning that stresses induced in the component are completely compressive.

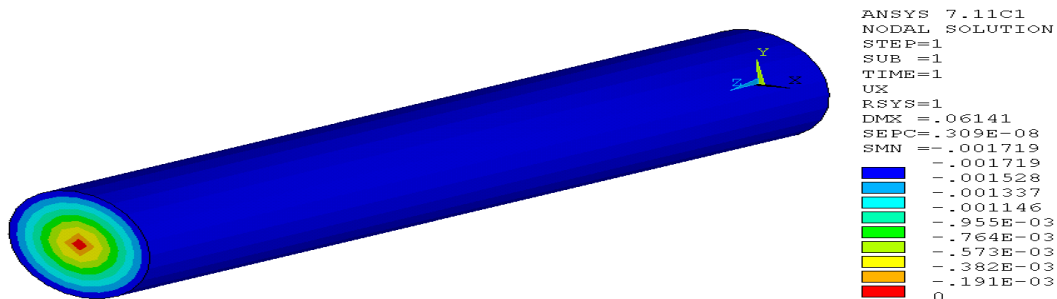
The above vector plot gives a clear picture of compressive stresses being completely induced in the component showing no traces of S1 or S2.

## Results of Case 2

Displacement Plot in Direction of pressure:

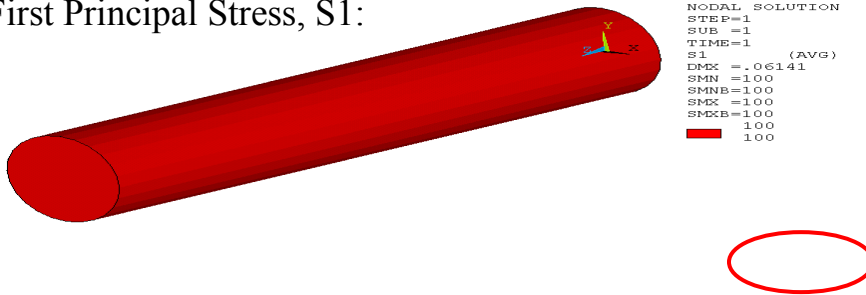


Displacement Plot in Cylinder Radial Direction:

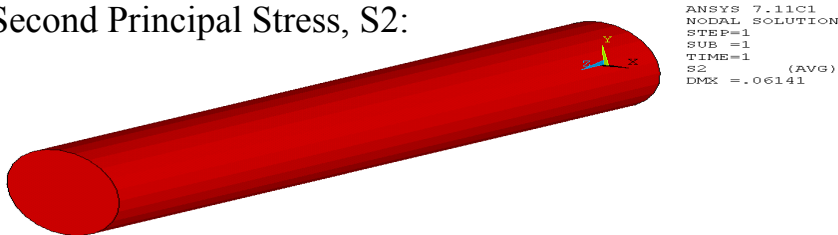


Principal Stress Plots:

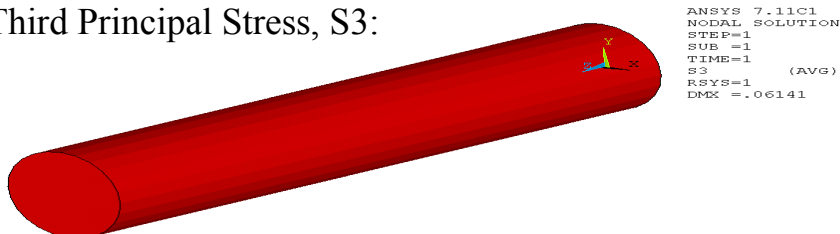
First Principal Stress, S1:

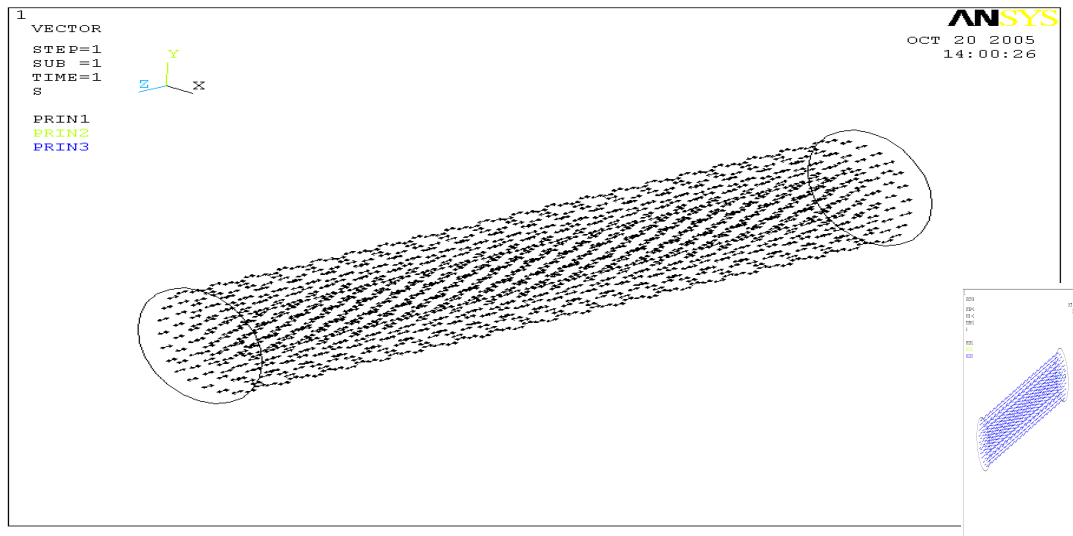


Second Principal Stress, S2:



Third Principal Stress, S3:





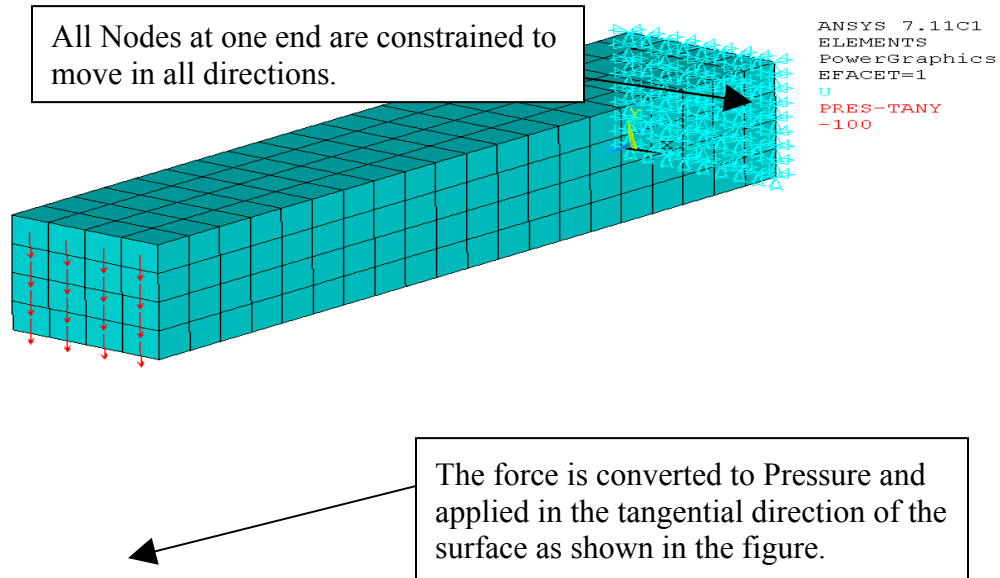
And here, you can see from the above plots, the Principal stresses S3 indicates no stress indicating that no compressive stresses are induced in the component. Whereas, Principal stress S1 shows a value of 100 in the complete model meaning that stresses induced in the component are completely tensile.

The above vector plot gives a clear picture of tensile stresses being completely induced in the component showing no traces of S2 or S3.



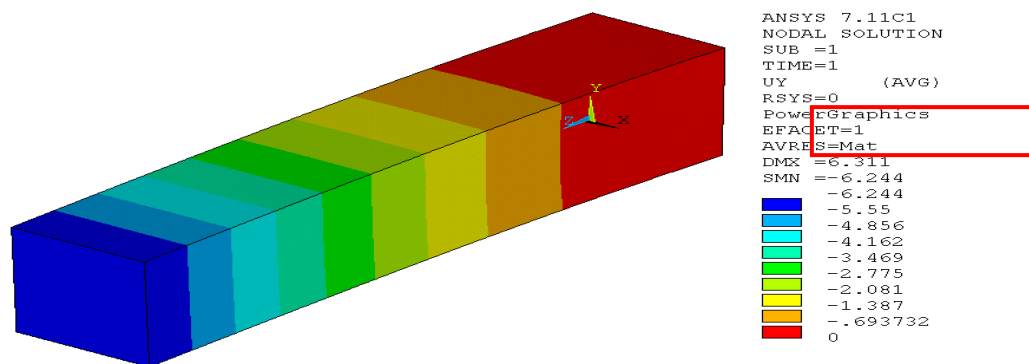
## Example 2:

A cuboid of dimension 20 X 20 X 100 mm which is subjected to a force of 4000 N on one side in the lateral direction (Similar to Cantilever Beam) with the other side constrained to move in all the directions as shown in the picture below.



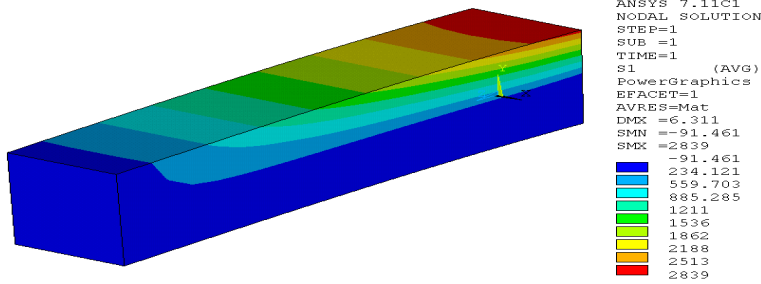
## Results

Displacement of the component in the direction of applied pressure:

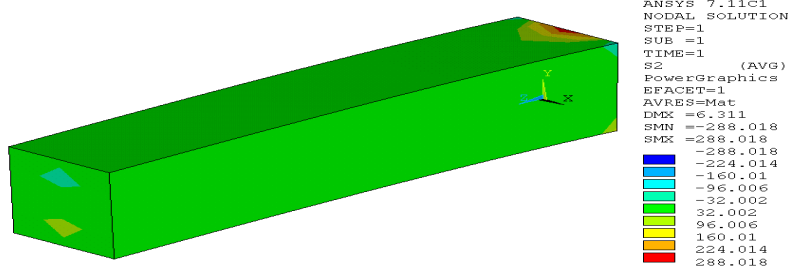


Principal Stress Plots:

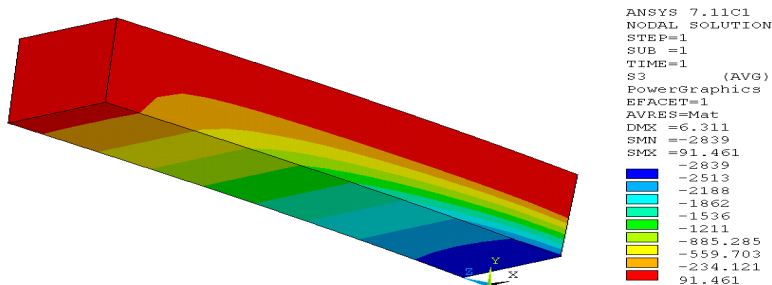
First Principal Stress, S1:



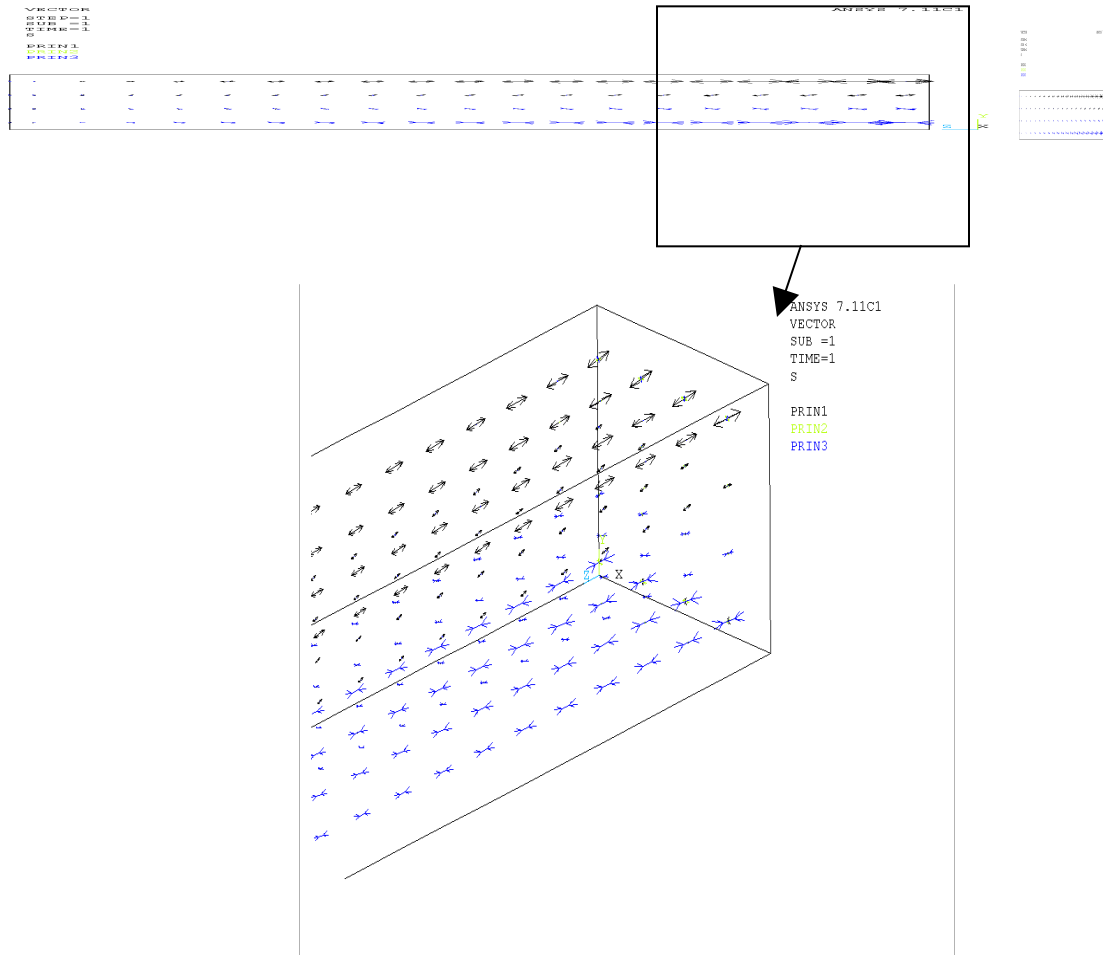
Second Principal Stress, S2:



Third Principal Stress, S3:



Vector Plot of Principal Stresses:

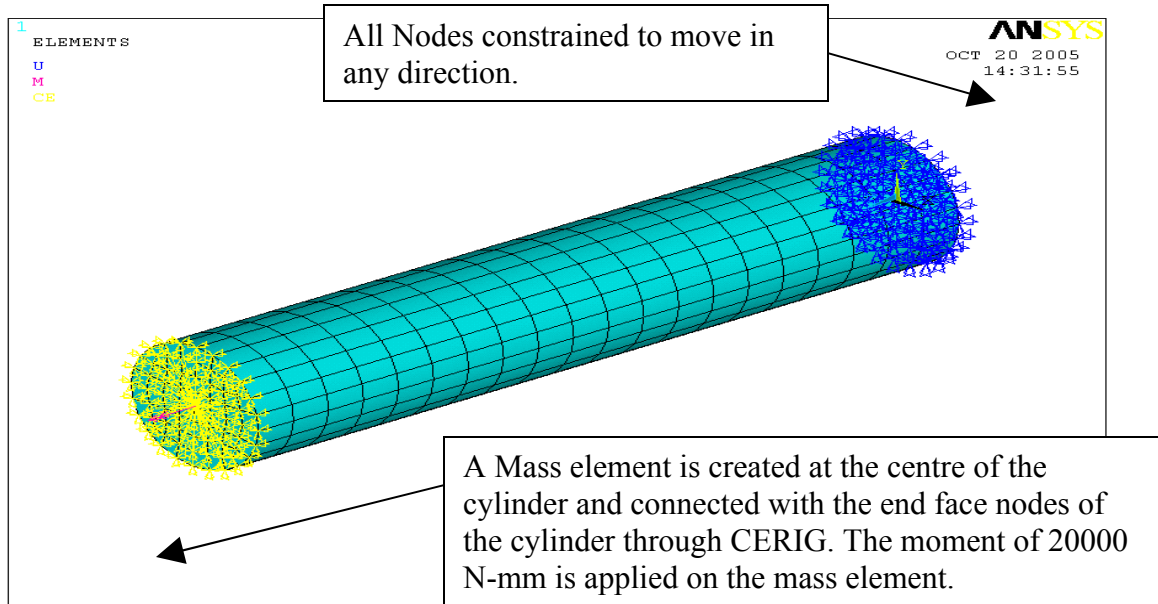


As we can see from the above figures, the principal stress plots give a clear indication of the tensile and compressive stress distribution in the component. The top layer of the cantilever is tensed while the bottom layer is compressed. The magnitudes of the tension and compression are exactly the same.

The above vector plot graphically shows the magnitude of all the three vectors.

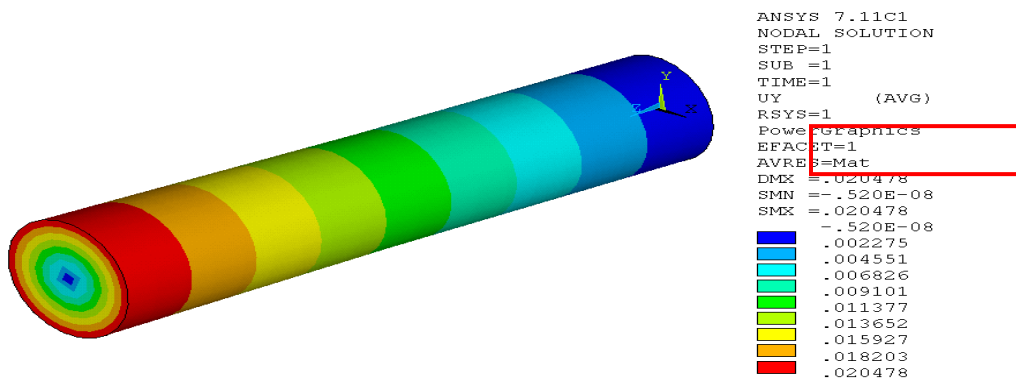
### Example 3:

A cylinder of dimension 20 X 20 X 100 mm which is subjected to torsion of 20000 N-mm on one side with the other side constrained to move in all the directions as shown in the picture below.



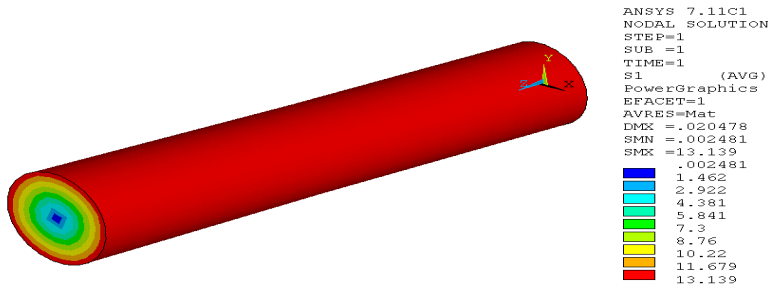
### Results

Displacement of the component in the direction of applied torsion:

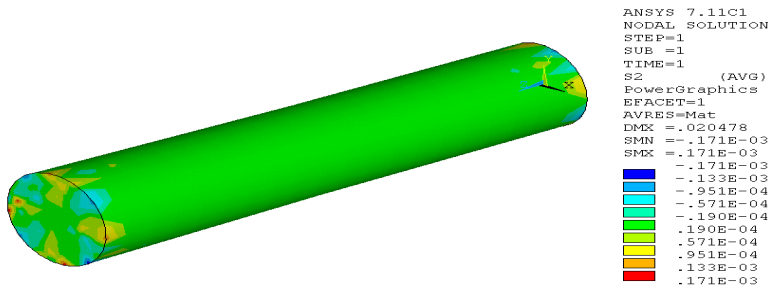


Principal Stress Plots:

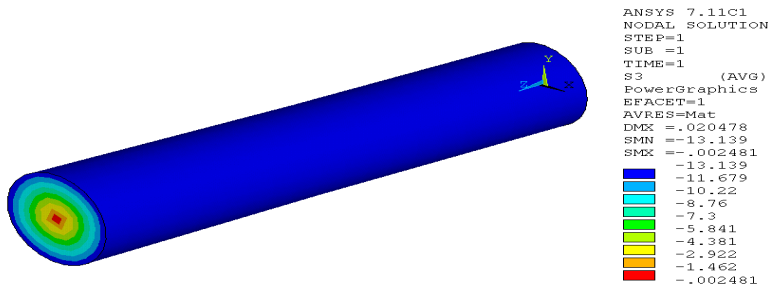
First Principal Stress, S1:



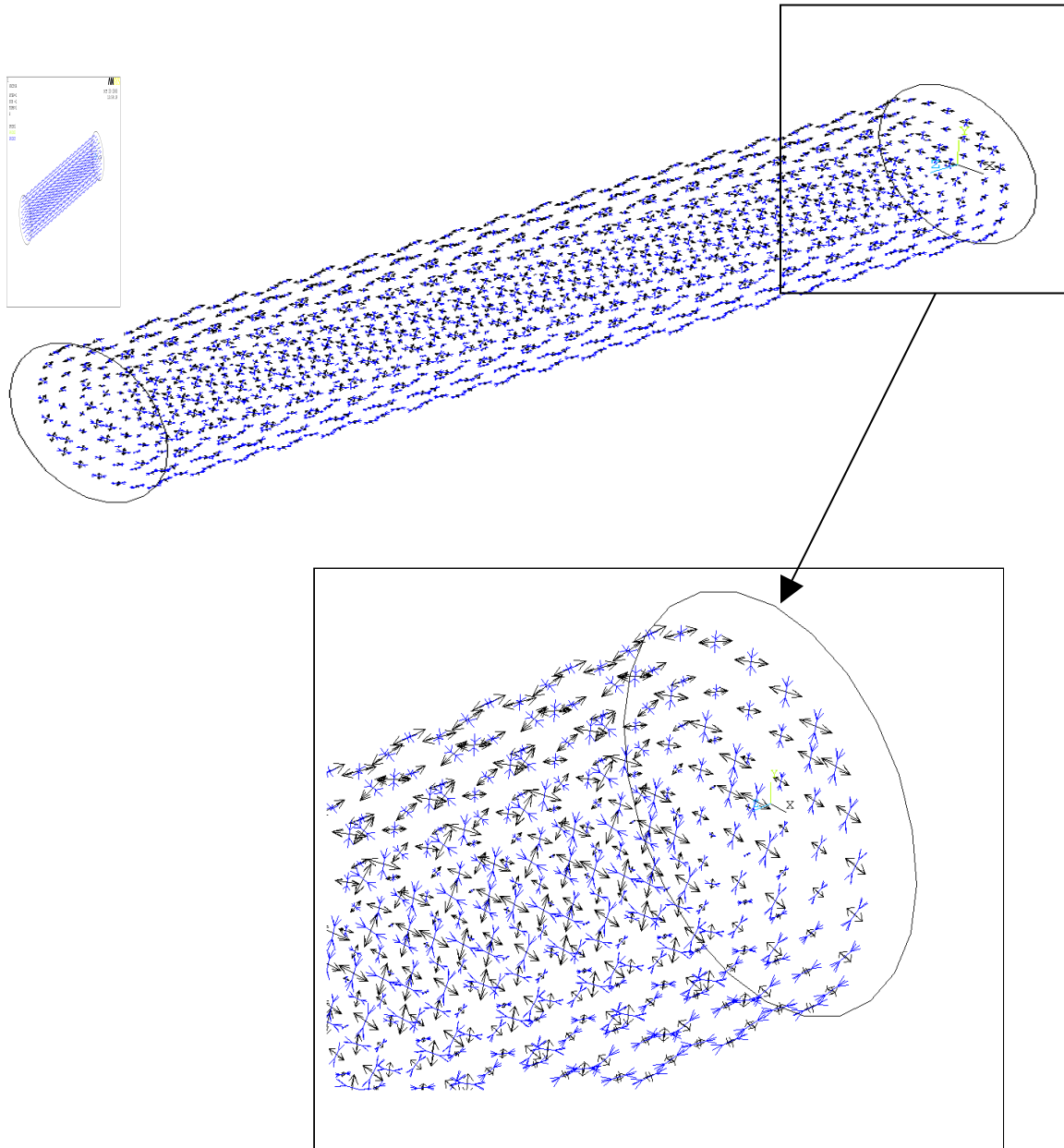
Second Principal Stress, S2:



Third Principal Stress, S3:



Vector Plot of Principal Stresses:



As we can see from the above figures, the principal stress plots give a clear indication of the tensile and compressive stress distribution in the component. The stresses start increasing towards the outer layer of the cylinder. The Principal stresses  $S_1$  and  $S_3$  are of the same magnitude and take their peak values at the outer most layer. The above vector plot gives an idea on the orientation of the principal stresses.