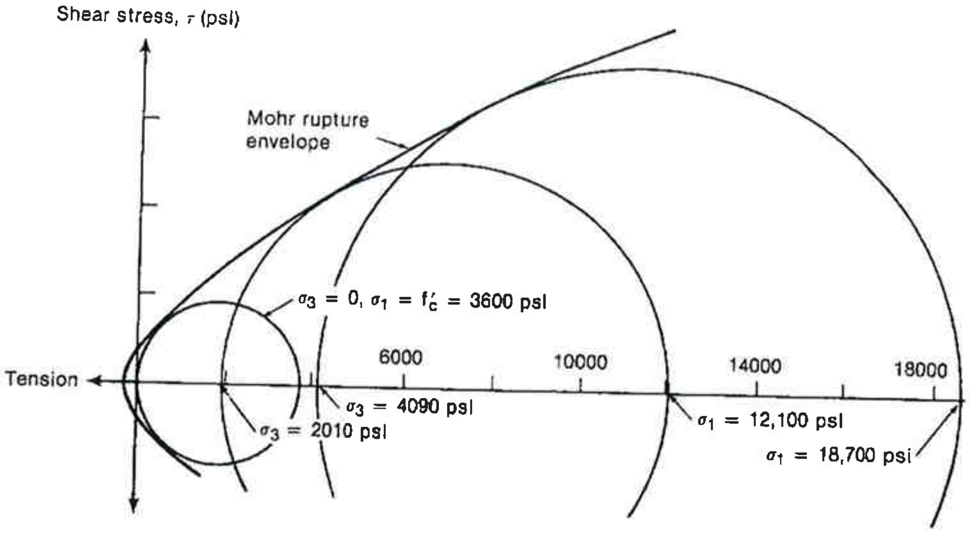



Fig. 3–16
Mohr rupture envelope for concrete tests from Fig. 3–15

Tests of lightweight and high-strength concretes,[3–26, 3–27] suggest that their compressive strengths are less influenced by the confining pressure, with the result that the coefficient 4.1 in Eq.3–15 drops to about 2.0.

The strength of concrete under combined stresses can also be expressed using a *Mohr rupture envelope*. The Mohr's circles plotted in Fig. 3–16 correspond to three of the cases plotted in Fig. 3–15. The Mohr's circles are tangent to the Mohr rupture envelope shown with the outer line.

In concrete columns or in beam–column joints, concrete in compression is sometimes enclosed by closely spaced hoops or spirals. When the width of the concrete element increases due to Poisson's ratio and microcracking, these hoops or spirals are stressed in tension, causing an offsetting compressive stress in the enclosed concrete. The resulting triaxial state of stress in the concrete enclosed or *confined* by the hoops or spirals increases the ductility and strength of the confined concrete. This is discussed in Chap. 11.

## 3–3 MECHANICAL PROPERTIES OF CONCRETE

The behavior and strength of reinforced concrete members is controlled by the size and shape of the members and the stress–strain properties of the concrete and the reinforcement. The stress–strain behavior discussed in this section will be used in subsequent chapters to develop relationships for the strength and behavior of reinforced concrete beams and columns.

### Stress–Strain Curve for Normal-Weight Concrete in Compression

Typical stress–strain curves for concretes of various strengths are shown in Fig. 3–17. These curves were obtained in tests lasting about 15 minutes on specimens resembling the compression zone of a beam.

The stress–strain curves in Fig. 3–17 all rise to a maximum stress, reached at a strain of between 0.0015 and 0.003, followed by a descending branch. The shape of this curve results from the gradual formation of microcracks within the structure of the concrete, as discussed earlier in this chapter.

The length of the descending branch of the curve is strongly affected by the test conditions. Frequently, an axially loaded concrete test cylinder will fail explosively at the point of maximum stress. This will occur in axially flexible testing machines if the strain energy released by the testing machine as the load drops exceeds the energy that the specimen can absorb. If a member loaded in bending, or bending plus axial load, the descending branch will exist since, as the stress drops in the most highly strained fibers, other less highly strained fibers can resist the load, thus delaying the failure of the highly strained fibers.

The stress–strain curves in Fig. 3–17 show five properties used in establishing the mathematical models shown in Fig. 3–18 for stress–strain curve of concrete in compression:

1. The initial slope of the curves (initial tangent modulus of elasticity) increases with an increase in compressive strength.

The modulus of elasticity of the concrete, $E_c$, is affected by the modulus of elasticity of the cement paste and that of the aggregate. An increase in the water–cement ratio in-

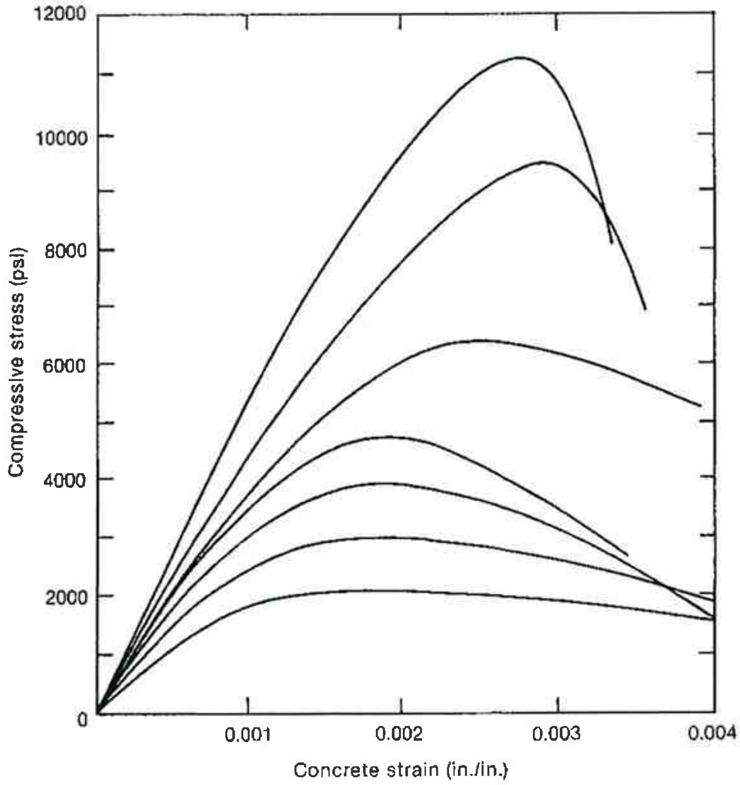
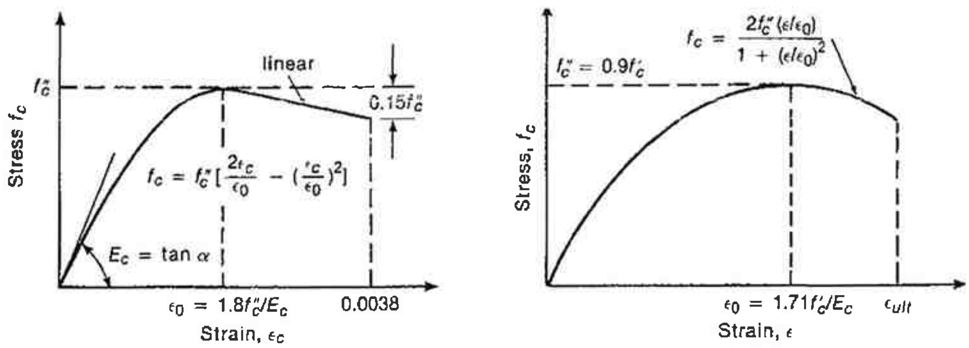



Fig. 3–17
Typical concrete stress–strain curves in compression. (From Refs. 3–28 and 3–29.)




(a) Modified Hognestad. (From Ref. 3-31)

(b) Todeschini. (From Ref. 3-32.)

Fig. 3–18
Analytical approximations to the compressive stress–strain curve for concrete.

creases the porosity of the paste, reducing its modulus of elasticity and strength. This is accounted for in design by expressing $E_c$ as a function of $f'_c$.

Of equal importance is the modulus of elasticity of the aggregate. Normal-weight aggregates have modulus of elasticity values ranging from 1.5 to 5 times that of the cement paste. Because of this, the fraction of the total mix that is aggregate also affects $E_c$. Lightweight aggregates have modulus of elasticity values comparable to the paste, and hence the aggregate fraction has little effect on $E_c$ for lightweight concrete.

The modulus of elasticity of concrete is frequently taken as given in ACI Sec. 8.5.1:

$$E_c = 33(w^{1.5})\sqrt{f'_c}\text{ psi} \tag{3–16}$$

where $w$ is the weight of the concrete in lb/ft$^3$. This equation was derived from short-time tests on concretes with densities ranging from 90 to 155 lb/ft$^3$ and corresponds to the secant modulus of elasticity at approximately $0.50f'_c$.[3-30] The initial tangent modulus is about 10% greater. Because this equation ignores the type of aggregate, the scatter of data is very wide. Equation 3–16 systematically overestimates $E_c$ in regions where low-modulus aggregates are prevalent. If deflections or vibration characteristics are critical in a design, $E_c$ should be measured for the concrete to be used.

For normal-weight concrete with a density of 145 lb/ft$^3$, ACI Sec. 8.5.1 gives the modulus of elasticity as

$$E_c = 57{,}000\sqrt{f'_c}\text{psi} \tag{3–17}$$

**2.** The rising portion of the stress–strain curve resembles a parabola with its vertex at the maximum stress.

For computational purposes the rising portion of the curves is frequently approximated by a parabola.[3-31] This curve tends to become straighter as the concrete strength increases.[3-29]

**3.** The strain, $\epsilon_0$, at maximum stress increases as the concrete strength increases.

**4.** The slope of the descending branch of the stress–strain curve tends to be less than that of the ascending branch for moderate strength concretes. This slope increases with an increase in compressive strength.

**5.** The maximum strain reached, $\epsilon_{cu}$, decreases with an increase in concrete strength.

The descending portion of the stress–strain curve after the maximum stress has been reached is highly variable and is strongly dependent on the testing procedure. Similarly, the maximum or limiting strain, $\epsilon_{cu}$, is very strongly dependent on the type of specimen, type of loading, and rate of testing. The limiting strain tends to be higher if there is a possibility of load redistribution at high loads. In flexural tests values from 0.0025 to 0.006 have been measured (see Sec. 4–1).

The two most common representations of the stress–strain curve consist of a parabola followed by the sloping line shown in Fig. 3–18a, terminating at a limiting strain of 0.0038, or a parabola followed by a horizontal line terminating at a limiting strain of 0.003 or 0.0035, which is widely used in Europe.[3-6] The stress–strain diagram in Fig. 3–18a is referred to as a modified Hognestad stress–strain curve.[3-31]

The stress–strain curve shown in Fig. 3–18b is convenient for use in analytical studies because it is a continuous function. The highest point in the curve, $f''_c$, is taken to equal $0.9f'_c$ to give stress block properties similar to the rectangular stress block of Sec. 4–2 when $\epsilon_{ult} = 0.003$ for $f'_c$ up to 5000 psi. The strain $\epsilon_0$, corresponding to maximum stress, is taken as $1.71f'_c/E_c$. For any given strain $\epsilon$, $x = \epsilon/\epsilon_0$. The stress corresponding to that strain is

$$f_c = \frac{2f''_c x}{1 + x^2} \tag{3–18}$$

The average stress under the stress block from $\epsilon = 0$ to $\epsilon$ is $\beta_1 f''_c$, where

$$\beta_1 = \frac{\ln(1 + x^2)}{x} \tag{3–19}$$

The center of gravity of the area of the stress–strain curve between $\epsilon = 0$ and $\epsilon$ is at $k_2\epsilon$ from $\epsilon$, where

$$k_2 = 1 - \frac{2(x - \tan^{-1}x)}{x^2\beta_1} \tag{3–20}$$

where $x$ is in radians when computing $\tan^{-1}x$. The stress–strain curve is satisfactory for concretes with stress–strain curves that display a gradually descending stress–strain curve at strains greater than $\epsilon_0$. Hence it is applicable to $f'_c$ up to about 5000 psi for normal-weight concrete and about 4000 psi for lightweight concrete.

As shown in Fig. 3–15, a lateral confining pressure causes an increase in the compressive strength of concrete, and a large increase in the strains at failure. The additional strength and ductility of confined concrete are utilized in hinging regions in structures in seismic regions. Stress–strain curves for confined concrete are described in Ref. 3–33.

When a compression specimen is loaded, unloaded, and reloaded it has the stress–strain response shown in Fig. 3–19. The envelope to this curve is very close to the stress–strain curve for a monotonic test. This, and the large residual strains that remain after unloading, suggest that the inelastic response is due to damage to the internal structure of the concrete as suggested by the microcracking theory presented earlier.

## Stress–Strain Curve for Normal-Weight Concrete in Tension

The stress–strain response of concrete loaded in axial tension can be divided into two phases. Prior to the maximum stress, the stress–strain relationship is slightly curved. The diagram is linear to roughly 50% of the tensile strength. The strain at peak stress is about 0.0001 in pure tension and 0.00014 to 0.0002 in flexure. The rising part of the stress–strain curve may be approximated either as a straight line with slope $E_c$ and a maximum stress equal to the tensile strength, $f'_t$, or as a parabola with a maximum strain, $\epsilon'_t = 1.8f'_t/E_c$, and a maximum stress, $f'_t$. The latter curve is illustrated in Fig. 3–20a with $f'_t$ and $E_c$ based on Eqs. 3–11 and 3–17.

After the tensile strength is reached, microcracking occurs in a *fracture process zone* adjacent to the point of highest tensile stress, and the tensile capacity of this concrete drops very rapidly with increasing elongation. In this stage of behavior, elongations are concen-

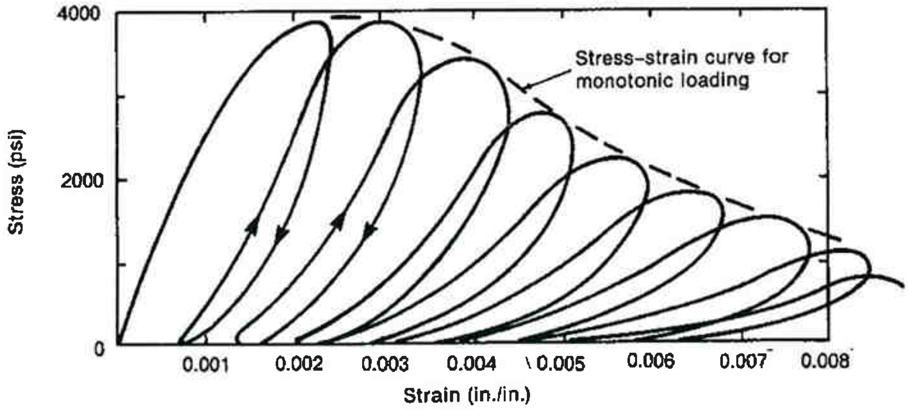



Fig. 3–19
Compressive stress–strain curves for cyclic loads. (From Ref. 3–34.)

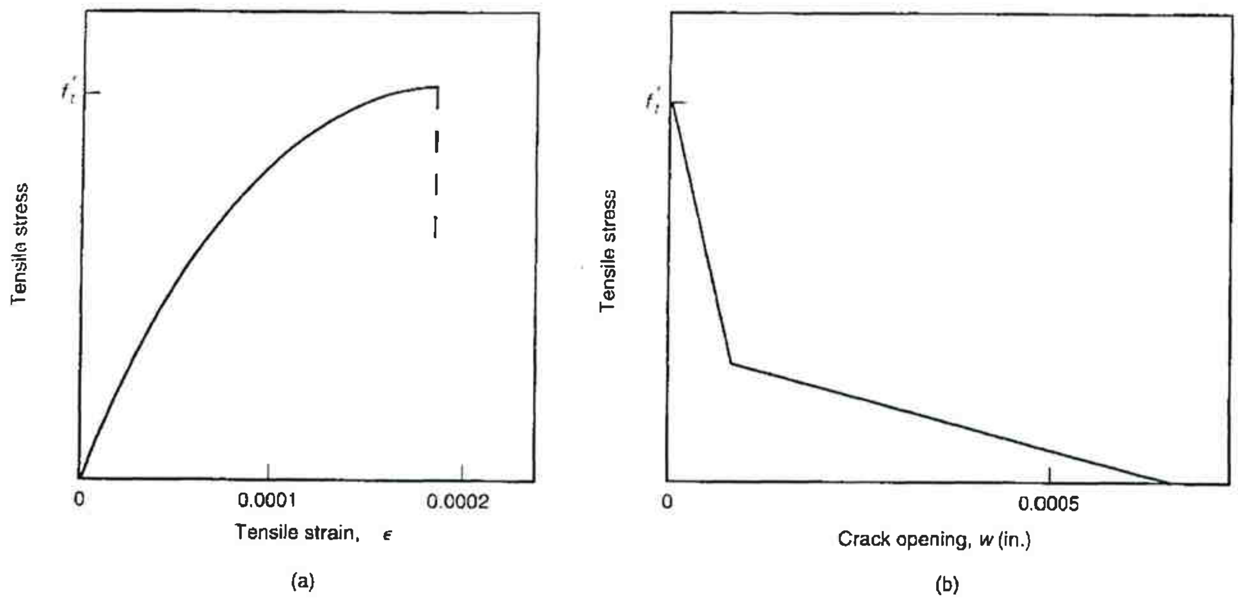



Fig. 3–20
Stress–strain curve and stress–crack opening curves for concrete loaded in tension.

trated in the fracture process zone while the rest of the concrete is unloading elastically. The unloading response is best described by a *stress versus crack opening diagram* as shown in Fig. 3–20b. The crack widths shown in this figure are of the right magnitude. The actual values depend on the situation. The tensile capacity drops to zero when the crack is completely formed. This occurs at a very small crack width. A more detailed discussion is given in Ref. 3–35.

### *Poisson's Ratio*

At stresses below the critical stress (see Fig. 3–1) Poisson's ratio for concrete varies from about 0.11 to 0.21 and usually falls in the range 0.15 to 0.20. Based on tests of biaxially loaded concrete, Kupfer et al.[3–23] report values of Poisson's ratio for 0.20 for concrete loaded in compression in one or two directions, 0.18 for concrete loaded in tension in one or two directions, and 0.18 to 0.20 for concrete loaded in tension and compression. Poisson's ratio remains approximately constant under sustained loads.

## 3–4 TIME-DEPENDENT VOLUME CHANGES

Concrete undergoes three main types of volume change which may cause stresses, cracking, or deflections which affect the in-service behavior of reinforced concrete structures. These are shrinkage, creep, and thermal expansion.

### Shrinkage

*Shrinkage* is the shortening of concrete during hardening and drying under constant temperature. The amount of shrinkage increases with time as shown in Fig. 3–21a.

The primary type of shrinkage is called *drying shrinkage* or simply *shrinkage*. It occurs due to the loss of a layer of adsorbed water from the surface of the gel particles. This

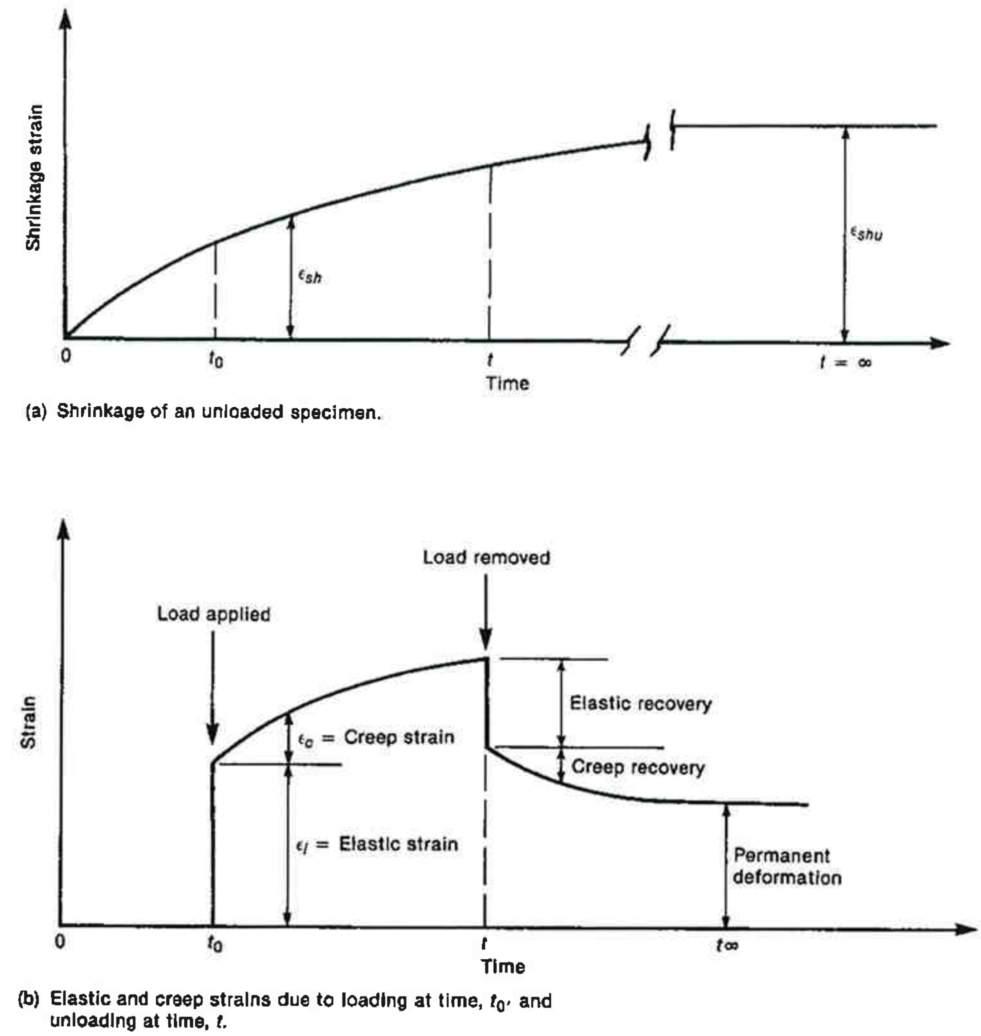



Fig. 3–21
Time-dependent strains.

layer is roughly one water molecule thick or about 1% of the size of the gel particles. The loss of free unadsorbed water has little effect on the magnitude of the shrinkage.

Shrinkage strains are dependent on the relative humidity and are largest for relative humidities of 40% or less. They are partially recoverable on rewetting the concrete, and structures exposed to seasonal changes in humidity may expand and contract slightly due to changes in shrinkage strains.

The magnitude of shrinkage strains also depends on the composition of the concrete. The hardened cement paste shrinks, whereas the aggregate does not. Thus the larger the fraction of the total volume of the concrete that is made up of hydrated cement paste, the greater the shrinkage. The aggregates act to restrain the shrinkage. There is less shrinkage with quartz or granite aggregates than with sandstone aggregate, because the quartz has a higher modulus of elasticity. The water–cement ratio affects the amount of shrinkage because high water content reduces the volume of aggregate, thus reducing the restraint of the shrinkage by the aggregate. The more finely a cement is ground, the more surface area it has, and as a result, there is more adsorbed water to be lost during shrinkage and hence more shrinkage.

Drying shrinkage occurs as the moisture diffuses out of the concrete. As a result, the exterior shrinks more rapidly than the interior. This leads to tensile stresses in the outer skin of the concrete and compressive stresses in the interior. For large members, the ratio of volume to surface area increases, resulting in less shrinkage because there is more moist concrete to restrain the shrinkage. The shrinkage develops more slowly in large members.

A secondary form of shrinkage called *carbonation shrinkage* occurs in carbon dioxide–rich atmospheres such as those found in parking garages. At 50% relative humidity the amount of carbonation shrinkage can equal the drying shrinkage, effectively doubling the total amount of shrinkage. At higher and lower humidities the carbonation shrinkage decreases.

The ultimate drying shrinkage strain, $\epsilon_{shu}$, for a 6 in. × 12 in. cylinder maintained for a very long time at a relative humidity of 40% ranges from 0.000400 to 0.001100 (400 to 1100 $\times 10^{-6}$ strain), with an average of about 0.000800. Thus, in a 25-ft bay in a building, the average shrinkage strain would cause a shortening of about $\frac{1}{4}$ in. in unreinforced concrete. In a structure, however, the shrinkage strains will tend to be less for the same concrete because:

1. The ratio of volume to surface area will generally be larger than for the cylinder, and as a result, drying takes place much more slowly.
2. A structure is built in stages and some of the shrinkage is dissipated before adjacent stages are completed.
3. The reinforcement restrains the development of the shrinkage.

The Euro-International Concrete Committee (CEB)[3-6] and the American Concrete Institute[3-15] have both published procedures for estimating shrinkage strains. The CEB method is more recent than the ACI procedure and accounts for member size in a better fashion. It will be presented here. The equations that follow apply only to the longitudinal shrinkage deformations of plain or lightly reinforced normal-weight concrete elements.

The axial shrinkage strains, $\epsilon_{cs}$, occurring between times $t_s$ at the start of shrinkage and $t$ in plain concrete can be predicted using the formula

$$\epsilon_{cs}(t,t_s) = \epsilon_{cso}\beta_s(t,t_s) \tag{3–21}$$

where $\epsilon_{cso}$ is the basic shrinkage strain for a particular concrete and relative humidity, given by Eq. 3–22, and $\beta_s(t,t_s)$ is a coefficient given by Eq. 3–25 to describe the development of shrinkage between time $t_s$ and $t$ as a function of the effective thickness of the member.

$$\epsilon_{cso} = \epsilon_s(f_{cm})\beta_{\mathrm{RH}} \tag{3–22}$$

where

$$\epsilon_s(f_{cm}) = [160 + \beta_{sc}(9 - f_{cm}/f_{cmo})] \times 10^{-6} \tag{3–23}$$

where $f_{cm}$ is the mean compressive strength at 28 days, psi. This can be taken equal to $f_{cr}$ as given by Eq. 3–3. For concrete with a standard deviation, $s$, equal to $0.15f'_c$, $f_{cm}$ would be $1.20f'_c$. We shall use this value. Shrinkage is not a function of compressive strength per se. It decreases with decreasing water/cement ratio and decreasing cement content. The strength $f_{cm}$ in Eq. 3–23 is used as an empirical measure of these quantities.

$f_{cmo}$ = 1450 psi

$\beta_{sc}$ = coefficient that depends on the type of cement
= 50 for Type I cement and 80 for Type III cement

$\beta_{\mathrm{RH}}$ = coefficient that accounts for the effect of relative humidity on shrinkage

For RH between 40 and 99%;

$$\beta_{\mathrm{RH}} = -1.55\left[1 - \left(\frac{\mathrm{RH}}{\mathrm{RH}_0}\right)^3\right] \tag{3–24}$$

For RH equal to or greater than 99%;

$$\beta_{RH} = +0.25$$

RH relative humidity of the ambient atmosphere in percent

$RH_0 = 100\%$

The effect of relative humidity on the total shrinkage is illustrated in Fig. 3–22. More shrinkage occurs in dry ambient conditions.

The development of shrinkage with time is given by

$$\beta_s(t,t_s) = \left[\frac{(t - t_s)/t_1}{350(h_e/h_0)^2 + (t - t_s)/t_1}\right]^{0.5} \tag{3–25}$$

where $h_e$ is the effective thickness in inches to account for the volume/surface ratio and is given by

$$h_e = 2A_c/u \tag{3–26}$$

and

$A_c$ is the area of the cross section, in.$^2$

$u$ is the perimeter of the cross section exposed to the atmosphere, in.

$h_0 = 4$ in.

$t$ is the age of the concrete, days

$t_s$ is the age of the concrete in days when shrinkage or swelling started, generally taken as the age at the end of moist curing

$t_1 = 1$ day

The development of shrinkage with time predicted using Eq. 3–25 is shom in Fig. 3–23 for effective thicknesses of 4 in. and 24 in. Shrinkage develops much more rapidly in thin members because moisture diffuses out of the concrete more rapidly.

Because $\beta_{RH}$ is negative, the computed shrinkage strain is also negative, implying that the concrete shortens due to shrinkage. In atmospheres with relative humidities greater than 99%, $\beta_{RH}$ is positive, indicating the concrete swells in such environments.

The shrinkage predicted by Eq. 3–21 has a coefficient of variation of about 35%, which means that 10% of the time the shrinkage will be less than 0.55 times the predicted

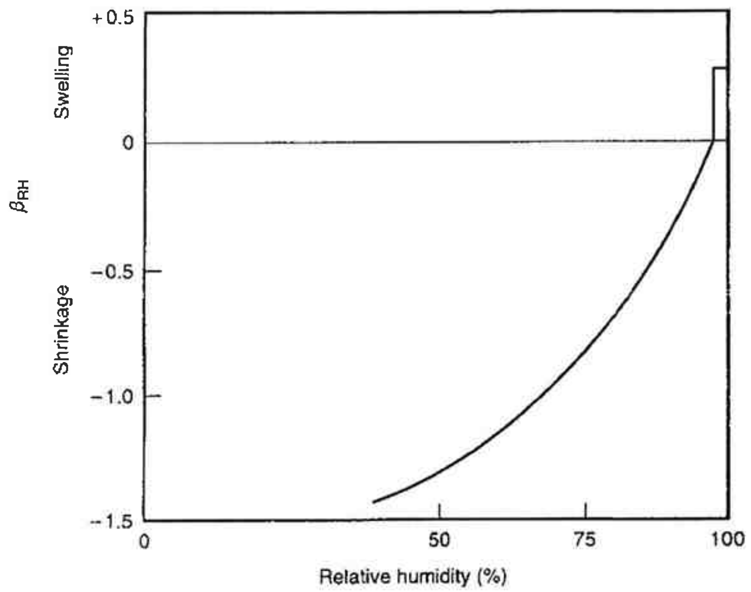



Fig. 3-22
Effect of relative humidity on shrinkage.

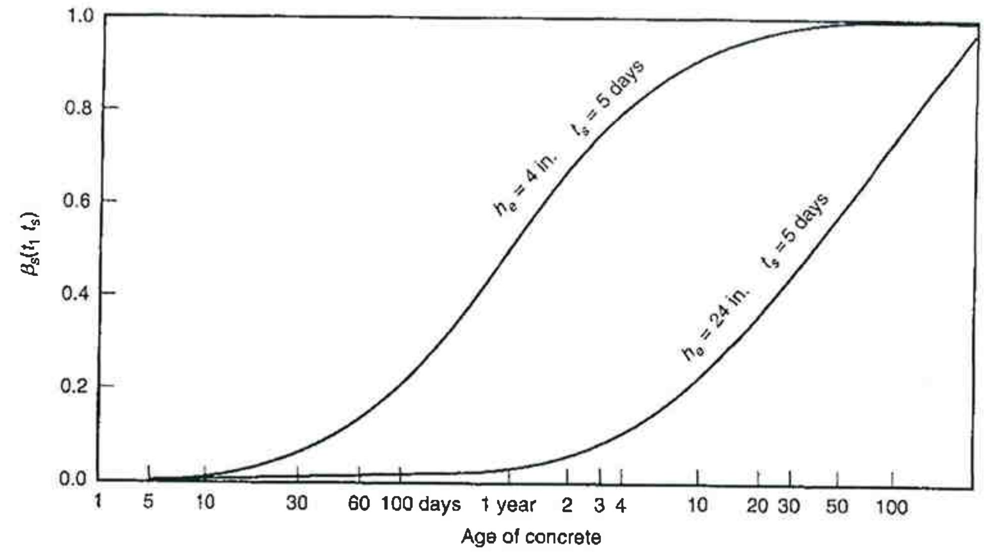



Fig. 3–23
Effect of effective thickness, $h_e$, on the rate of development of shrinkage.

shrinkage and 10% of the time it will exceed 1.45 times the predicted value. This is a very large spread. If shrinkage has a critical effect on a given structure, shrinkage tests should be carried out using the concrete in question.

If a lower level of accuracy is acceptable, the following values are representative of the shrinkage that would occur in 70 years in normal-weight structural concrete having strengths between 3000 and 7500 psi.

Dry atmospheric conditions or inside, RH = 50%:

Effective thickness, $h_e$ = 6 in., $\epsilon_{cs}(70y) = -0.000560$ strain

Effective thickness = 24 in., $\epsilon_{cs}(70y) = -0.000470$ strain

Humid atmospheric conditions, RH = 80%:

Effective thickness = 6 in., $\epsilon_{cs}(70y) = -0.000310$ strain

Effective thickness = 24 in., $\epsilon_{cs}(70y) = -0.000260$ strain

## EXAMPLE 3–2 Calculation of Shrinkage Strains

A lightly reinforced 6-in.-thick floor in an underground parking garage is supported around the outside edge by a 16-in.-thick basement wall. Cracks have developed in the slab perpendicular to the basement wall at roughly 6 ft on centers. The slab is 24 months old and the wall is 26 months old. The concrete is 3000 psi, made from Type I cement, and was moist cured for 5 days in each case. The relative humidity is 50%. Compute the width of these cracks assuming that they result from the restraint of the slab shrinkage parallel to the wall by the basement wall.

**FLOOR SLAB**

1. **Compute the basic shrinkage strain, $\epsilon_{cso}$.**

$$\epsilon_{cso} = \epsilon_s(f_{cm})\beta_{\text{RH}} \tag{3–22}$$

$$\epsilon_s(f_{cm}) = [160 + \beta_{sc}(9 - f_{cm}/f_{cmo})] \times 10^{-6} \tag{3–23}$$

where

$\beta_{sc}$ = 50 for Type I cement

$f_{cm}$ = mean concrete strength $\approx 1.20f'_c$

= 3600 psi

$f_{cmo}$ = 1450 psi



1 psi = 6.8948 kPa

$$\epsilon_s(f_{cm}) = [160 + 50(9 - 3600/1450)] \times 10^{-6}$$
$$= 486 \times 10^{-6} = 0.000486 \text{ strain}$$

$$\beta_{RH} = -1.55\left[1 - \left(\frac{RH}{RH_0}\right)^3\right] \tag{3–24}$$

where RH = 50%, $RH_0$ = 100%, and

$$\beta_{RH} = -1.55[1 - (50/100)^3]$$
$$= -1.356$$

Therefore, the basic shrinkage is:

$$\epsilon_{cso} = 0.000486 \times -1.356$$
$$= -0.000659 \text{ strain}$$

**2. Compute the coefficient for the development of shrinkage with time.**

$$\beta_s(t,t_s) = \left[\frac{(t - t_s)/t_1}{350(h_e/h_0)^2 + (t - t_s)/t_1}\right]^{0.5} \tag{3–25}$$

where $h_e$ is the effective thickness $= 2A_c/u$. Consider a 1-ft-wide strip of slab exposed to the air on the top and bottom:

$$h_e = \frac{2 \times (6 \times 12)}{2 \times 12} = 6 \text{ in.} \qquad h_0 = 4\text{in.}$$

$$t = 730 \text{ days} \qquad t_s = 5\text{days} \qquad t_1 = 1\text{day}$$

$$\beta_s(t,t_s) = \left[\frac{(730 - 5)/1}{350 \times (6/4)^2 + (730 - 5)/1}\right]^{0.5}$$
$$= 0.692$$

This means that after two years 69% of the slab shrinkage will have occurred.

**3. Compute the shrinkage strain, $\epsilon_{cs}(t,t_s)$.**

$$\epsilon_{cs}(t,t_s) = \epsilon_{cso}\beta_s(t,t_s) \tag{3–21}$$
$$= -0.000659 \times 0.692$$
$$\epsilon_{cs}(t,t_s) = -0.000456 \text{ strain}$$

**BASEMENT WALL**

**1. Compute the basic shrinkage strain, $\epsilon_{cso}$.** This will be the same as for the floor slab since $f_{cm}$, $\beta_{sc}$, and RH are the same.

**2. Compute the coefficient for the development of shrinkage with time.** Compute the effective thickness, $h_e = 2A_c/u$. Again considering a 1-ft-wide strip of wall. It is exposed to air only on the inside face.

$$h_e = \frac{2 \times (16 \times 12)}{12} = 32\text{in.} \qquad h_0 = 4\text{in.}$$

$$t = 791\text{days} \qquad t_s = 5\text{days} \qquad t_1 = 1\text{day}$$

$$\beta_s(t,t_s) = \left[\frac{(791 - 5)/1}{350 \times (32/4)^2 + (791 - 5)/1}\right]^{0.5}$$
$$= 0.184$$

3. Compute the shrinkage strain, $\epsilon_{cs}(t,t_s)$

$$\epsilon_{cs}(t,t_s) = -0.000659 \times 0.184$$
$$= -0.000121 \text{ strain}$$

**RELATIVE SHRINKAGE AND CRACK WIDTH**

Thus the relative shrinkage strain between the slab and the wall when the slab is one year old is $-0.000456 - (-0.000121) = -0.000335$. If the average crack spacing is 6 ft, the shortening between cracks will be about $6 \times 12 \times -0.000335 = -0.024$ in. Hence the cracks will be about 0.024 in. wide on average.

This calculation does not allow for the effect of the reinforcement in restraining the shrinkage strains. The actual shrinkage would be 75 to 100% of the calculated values. ■

## Creep

When concrete is loaded, an instantaneous elastic strain develops as shown in Fig. 3–21b. If this load remains on the member, creep strains develop with time. These occur because the adsorbed water layers tend to become thinner between gel particles which are transmitting compressive stress. This change in thickness occurs rapidly at first, slowing down with time. With time, bonds form between the gel particles in their new position. If the load is eventually removed, a portion of the strain is recovered elastically and another portion by creep, but a residual strain remains, as shown in Fig. 3–21b, due to the bonding of the gel particles in the deformed position.

The creep strains, $\epsilon_c$, are on the order of one to three times the instantaneous elastic strains. Creep strains lead to an increase in deflections with time; may lead to a redistribution of stresses within cross sections; cause a decrease in prestressing forces; and so on.

The ratio of creep strain after a very long time to elastic strain, $\epsilon_c / \epsilon_i$, is called the *creep coefficient*, $\phi$. The magnitude of the creep coefficient is affected by the ratio of the sustained stress to the strength of the concrete, the humidity of the environment, the dimensions of the element, and the composition of the concrete. Creep is greatest in concretes with a high cement paste content. Concretes containing a large aggregate fraction creep less because only the paste creeps, and that creep is restrained by the aggregate. The rate of development of the creep strains is also affected by the temperature, reaching a plateau about 160°F. At the high temperatures encountered in fires, very large creep strains occur. The type of cement (i.e., normal or high early strength cement) and the water–cement ratio are important only in that they affect the strength at the time when the concrete is loaded.

For creep, as for shrinkage, several calculation procedures exist.[3-6, 3-15] The method given here is from the *CEB-FIB Model Code 1990*.[3-6] It is applicable for concretes up to compressive strengths of about 10,000 psi subjected to a compressive loading up to about $0.40f'_c$ at an age $t_0$, exposed to relative humidities of 40% or higher and mean temperatures between 40° and 90°F. For stresses less than $0.40f'_c$, creep is assumed to be linearly related to stress. Beyond this stress, creep strains increase more rapidly and may lead to failure of the member at stresses greater than $0.75f'_c$, as shown in Fig. 3–2a. Similarly, creep increases significantly at mean temperatures in excess of 90°F.

The total strain, $\epsilon_c(t)$, at time $t$ in a concrete member uniaxially loaded with a constant stress $\sigma_c(t_0)$ at time $t_0$ is

$$\epsilon_c(t) = \epsilon_{ci}(t_0) + \epsilon_{cc}(t) + \epsilon_{sc}(t) + \epsilon_{cT}(t) \qquad (3\text{–}27)$$

where

$\epsilon_{ci}(t_0) =$ initial strain at loading $= \sigma_c(t_0)/E_c(t_0)$

$\epsilon_{cc}(t) =$ creep strain at time $t$ where $t$ is greater than $t_0$

$\epsilon_{cs}(t)$ = shrinkage strain at time $t$

$\epsilon_{cT}(t)$ = thermal strain at time $t$

$E_c(t_0)$ = modulus of elasticity at the age of loading

The stress-dependent strain at time $t$ is

$$\epsilon_{c\sigma}(t) = \epsilon_{ci}(t_0) + \epsilon_{cc}(t) \tag{3–28}$$

For a stress $\sigma_c$ applied at time $t_0$ which remains constant until time $t$, the creep stain $\epsilon_{cc}$ between time $t_0$ and $t$ is

$$\epsilon_{cc}(t,t_0) = \frac{\sigma_c(t_0)}{E_c(28)}\phi(t,t_0) \tag{3–29}$$

where $E_c(28)$ is the modulus of elasticity at the age of 28 days, given by Eq. 3–16 or 3–17, and $\phi(t,t_0)$ is the creep coefficient, given by

$$\phi(t,t_0) = \phi_0\beta_c(t,t_0) \tag{3–30}$$

where $\phi_0$ is the basic creep given by Eq. 3–31, and $\beta_c(t,t_0)$ is a coefficient to account for the development of creep with time, given by Eq. 3–35.

$$\phi_0 = \phi_{\mathrm{RH}}\beta(f_{cm})\beta(t_0) \tag{3–31}$$

where

$$\phi_{\mathrm{RH}} = 1 + \frac{1 - \mathrm{RH}/\mathrm{RH}_0}{0.46(h_e/h_0)^{1/3}} \tag{3–32}$$

$$\beta(f_{cm}) = \frac{5.3}{(f_{cm}/f_{cmo})^{0.5}} \tag{3–33}$$

$$\beta(t_0) = \frac{1}{0.1 + (t_0/t_1)^{0.2}} \tag{3–34}$$

where $h_e$, $h_0$, RH, $\mathrm{RH}_0$, $f_{cm}$, $f_{cmo}$, and $t_1$ are as defined in connection with Eqs. 3–21 to 3–26.

The basic creep coefficient, $\phi_0$, is actually a function of the relative humidity, the composition of the concrete, and the degree of hydration at the start of loading. The last two of these are expressed empirically in Eqs. 3–33 and 3–34 as functions of the mean 28-day strength, $f_{cm}$, and the age at loading, $t_0$.

The development of creep with time is given by

$$\beta_c(t,t_0) = \left[\frac{(t - t_0)/t_1}{\beta_H + (t - t_0)/t_1}\right]^{0.3} \tag{3–35}$$

with

$$\beta_H = 150\left[1 + \left(1.2\frac{\mathrm{RH}}{\mathrm{RH}_0}\right)^{18}\right]\frac{h_e}{h_0} + 250 \leq 1500 \tag{3–36}$$

The effects of the effective thickness and age at the time of loading on the creep coefficient $\phi(t,t_0)$ are illustrated in Fig. 3–24. The creep coefficient is about half as big for concrete loaded at one year as it is for concrete loaded at 7 days. The effective thickness has less effect than it did on shrinkage (Fig. 3–23), reducing the value of $\phi(t,t_0)$ by about 20% for the example shown.

When compared to creep test data, the creep coefficient $\phi(t,t_0)$ computed in this way has a coefficient of variation of about 20%.[3–6] Ten percent of the time the actual value of $\phi(t,t_0$ will be less than 75% of the computed value and 10% of the time it will exceed 125% of the computed value. If creep deflections are a serious problem for a particular structure, consideration should be given to carrying out creep tests on the concrete to be used.

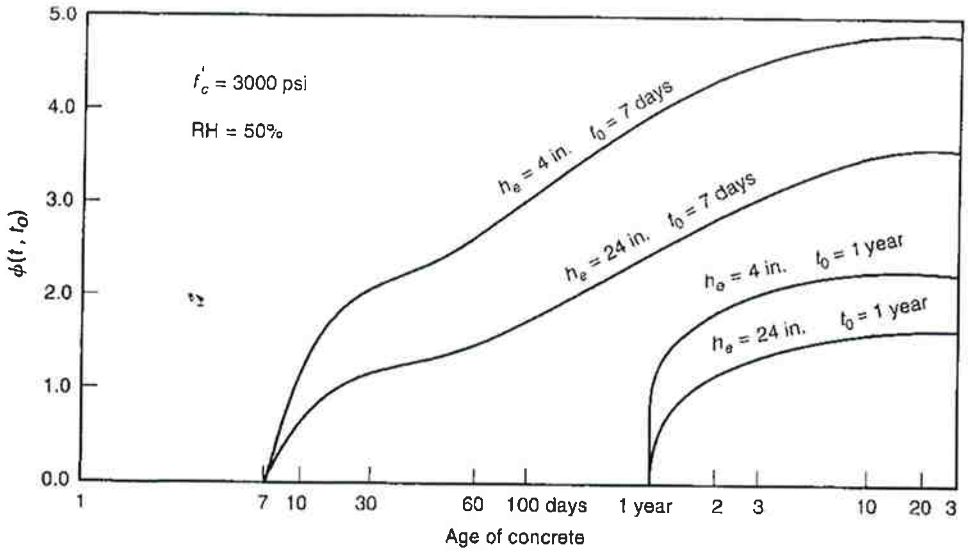



Fig. 3–24
Effect of effective thickness, $h_e$, and age at loading, $t_0$, on creep coefficient, $\phi(t,t_0)$

TABLE 3–1 Creep Coefficient, $\phi(70y,t_0)$ for Normal-Weight Concrete after 70 Years of Loading

| Age at Loading, $t_0$ (days) | Dry Atmospheric Conditions (RH = 50%) | | Humid Atmospheric Conditions (RH = 80%) | |
|---|---|---|---|---|
| | Effective Thickness, $h_e$ (Eq. 3–26) | | | |
| | 6 in. | 24 in. | 6 in. | 24 in. |
| 1 | 4.8 | 3.9 | 3.4 | 3.0 |
| 7 | 3.3 | 2.7 | 2.4 | 2.1 |
| 28 | 2.6 | 2.1 | 1.8 | 1.6 |
| 90 | 2.1 | 1.7 | 1.5 | 1.3 |
| 365 | 1.6 | 1.3 | 1.1 | 1.0 |

Source: Ref. 3–6.

In cases where a lower level of accuracy is acceptable, the creep coefficient at 70 years, $\phi(70y,t_0)$, can be taken from Table 3–1.

The total shortening of a plain concrete member at time $t$ due to elastic and creep strains resulting from a constant stress $\sigma_c$ applied at time $t_0$ can be computed using Eq. 3–28, which becomes

$$\epsilon_{c\sigma}(t) = \sigma_c(t_0)\left[\frac{1}{E_c(t_0)} + \frac{\phi(t,t_0)}{E_c(28)}\right] \tag{3–37}$$

The term in brackets is the *creep compliance function*, $J(t,t_0)$, representing the total stress-dependent strain per unit stress.

## EXAMPLE 3–3 Calculation of Unrestrained Creep Strains

A plain concrete pedestal 24 in. × 24 in. × 10 ft high is subjected to an average stress of 1000 psi. Compute the total shortening in 5 years if the load is applied 2 weeks after the concrete is cast. The properties of the concrete and the exposure are the same as in Example 3–2.

1. **Compute the basic creep coefficient, $\phi_0$.**

$$\phi_0 = \phi_{RH}\beta(f_{cm})\beta(t_0) \tag{3–31}$$

$$\phi_{RH} = 1 + \frac{1 - RH/RH_0}{0.46(h_e/h_0)^{1/3}} \tag{3–32}$$

where

$$RH = 50\% \qquad RH_0 = 100\% \qquad h_0 = 4\text{in.}$$

$$h_e = 2A_c/u = 2 \times 24 \times 24/4 \times 24 = 12\text{in.}$$

$$\phi_{RH} = 1 + \frac{1 - 50/100}{0.46(12/4)^{1/3}} = 1.754$$

$$\beta(f_{cm}) = \frac{5.3}{(f_{cm}/f_{cmo})^{0.5}} \tag{3–33}$$

where

$$f_{cm} = 1.20f'_c = 3600\text{ psi} \quad f_{cmo} = 1450\text{psi}$$

$$\beta(f_{cm}) = \frac{5.3}{(3600/1450)^{0.5}} = 3.364$$

$$\beta(t_0) = \frac{1}{0.1 + (t_0/t_1)^{0.2}} \tag{3–34}$$

where

$$t_0 = 14\text{ days} \quad t_1 = 1\text{ day}$$

$$\beta(t_0) = \frac{1}{0.1 + (14/1)^{0.2}} = 0.557$$

Thus

$$\phi_0 = 1.754 \times 3.364 \times 0.557 = 3.29$$

The basic creep coefficient is 3.29.

2. **Compute the development of creep with time, $\beta_c(t,t_0)$.**

$$\beta_c(t,t_0) = \left[\frac{(t - t_0)/t_1}{\beta_H + (t - t_0)/t_1}\right]^{0.3} \tag{3–35}$$

where:

$$\beta_H = 150\left[1 + \left(1.2\frac{RH}{RH_0}\right)^{18}\right]\frac{h_e}{h_0} + 250 \le 1500 \tag{3–36}$$

$$= 150\left[1 + \left(1.2\frac{50}{100}\right)^{18}\right]\frac{12}{4} + 250 = 700$$

$$t = 5 \times 365 = 1825\text{ days} \qquad t_0 = 14\text{ days} \qquad t_1 = 1\text{ day}$$

$$\beta_c(t,t_0) = \left[\frac{(1825 - 14)/1}{700 + (1825 - 14)/1}\right]^{0.3} = 0.907$$

This indicates that 90.7% of the total creep has occurred at the end of 5 years.

3. **Compute the creep coefficient, $\phi(t,t_0)$.**

$$\phi(t,t_0) = \phi_0\beta_c(t,t_0) = 3.29 \times 0.907 \tag{3–30}$$

$$= 2.98$$

4. **Compute the total stress-dependent strain, $\epsilon_{c\sigma}(t,t_0)$**

$$\epsilon_{c\sigma}(t) = \sigma_c(t_0)\left[\frac{1}{E_c(t_0)} + \frac{\phi(t,t_0)}{E_{ci}}\right] \tag{3–37}$$

where

$$\sigma_c(t_0) = 1000\text{psi}$$
$$E_c = 57{,}000\sqrt{f'_c} \tag{3–17}$$

$E_c(t_0)$ is modulus of elasticity at 14 days, where the concrete strength at 14 days is given by Eq. 3–5 with $t$ = 14 days:

$$f'_c(t) = f'_c(28)\left(\frac{t}{4 + 0.85t}\right)$$
$$= 3000\left(\frac{14}{4 + 0.85 \times 14}\right) = 2830\text{psi}$$

$$E_c(t_0) = 57{,}000 \times \sqrt{2830} = 3{,}032{,}000\text{psi}$$

$E_{ci}$ is the modulus of elasticity at 28 days.

$$E_{ci} = 57{,}000\sqrt{3000} = 3{,}122{,}000\text{psi}$$

$$\epsilon_{c\sigma}(5y) = 1{,}000\left(\frac{1}{3{,}032{,}000} + \frac{2.98}{3{,}122{,}000}\right)$$
$$= 0.000330 + 0.000955 = 0.001285 \text{ strain}$$

The creep strain is almost three times the instantaneous strain.

5. **Compute the total shortening.**

$$\Delta\ell = \ell \times \epsilon_{c\sigma}(5y) = 120 \times 0.00128$$
$$= 0.154 \text{ in.}$$

The pedestal would shorten by 0.154 in. in 5 years. ■

In an axially loaded reinforced concrete column, the creep shortening of the concrete causes compressive strains in the longitudinal reinforcement, increasing the load in the steel and reducing the load, and hence the stress, in the concrete. As a result, a portion of the elastic strain in the concrete is recovered and, in addition, the creep strains are smaller than they would be in a plain concrete column with the same initial concrete stress. A similar redistribution occurs in the compression zone of a beam with compression steel.

This effect can be modeled using an *age-adjusted effective modulus*, $E_{caa}(t,t_0)$, and an *age-adjusted transformed section* in the calculations[3–36 to 3–38] where

$$E_{caa}(t,t_0) = \frac{E_c(t_0)}{1 + \chi(t,t_0)[E_c(t_0)/E_c(28)]\phi(t,t_0)} \tag{3–38}$$

where $\chi(t,t_0)$ is an aging coefficient that can be approximated by[3–39]

$$\chi(t,t_0) = \frac{t_0^{0.5}}{1 + t_0^{0.5}} \tag{3–39}$$

The axial strain at time $t$ in a column loaded at age $t_0$ with a constant load $P$ is

$$\epsilon_c(t,t_0) = \frac{P}{A_{traa} \times E_{caa}(t,t_0)} \tag{3–40}$$

where $A_{traa}$ is the age-adjusted transformed area of the column cross section. The concept of the transformed sections is presented in Sec. 9–2. For more information on the use of the age-adjusted effected modulus, see Ref. 3–37 and 3–38.

## EXAMPLE 3–4 Computation of the Strains and Stresses in an Axially Loaded Reinforced Concrete Column

A concrete column 24 in. × 24 in. × 10 ft high has 8 No. 8 longitudinal bars and is loaded with a load of 630 kips at an age of 2 weeks. Compute the elastic stresses in the concrete and steel at the time of loading and the stresses and strains at an age of 5 years. The properties of the concrete and the exposure are the same as in Examples 3-2 and 3–3.

Steps 1, 2, and 3 are the same as in Example 3–3. The following quantities are computed:

$$f'_c(14) = 2830 \text{ psi} \qquad f'_c(28) = 3000 \text{ psi}$$

$$E_c(14) = 3{,}032{,}000 \text{ psi} \qquad E_c(28) = 3{,}122{,}000 \text{ psi}$$

$$\phi(t,t_0) = 2.98$$

**4. Compute the transformed area at the instant of loading, $A_{tr}$.** (Transformed sections are discussed in Sec. 9–2.)

$$\text{Elastic modular ratio} = n = \frac{E_s}{E_c(14)} = \frac{29{,}000{,}000}{3{,}032{,}000}$$

$$= 9.56$$

The steel will be "transformed" into concrete, giving the transformed area

$$A_{tr} = A_c + (n-1)A_s = 576 \text{ in.}^2 + (9.56 - 1) \times 6.32 \text{ in}^2$$

$$= 630 \text{ in.}^2$$

The stress in the concrete is 630,000 lb/630 in.$^2$ = 1000 psi. The stress in the steel is $n$ times the stress in the concrete = 9.56 × 1000 psi = 9560 psi.

**5. Compute the age adjusted effective modulus, $E_{caa}(t,t_0)$, and the age-adjusted modular ratio, $n_{aa}$.**

$$E_{caa}(t,t_0) = \frac{E_c(t_0)}{1 + \chi(t,t_0)[E_c(t_0)/E_c(28)]\phi(t,t_0)} \tag{3–38}$$

where

$$\chi(t,t_0) = \frac{t_0^{0.5}}{1 + t_0^{0.5}} = \frac{14^{0.5}}{1 + 14^{0.5}} \tag{3–39}$$

$$= 0.789$$

$$E_{caa}(t,t_0) = \frac{3{,}032{,}000}{1 + 0.789 \times \dfrac{3{,}032{,}000}{3{,}122{,}000} \times 2.98}$$

$$= 923{,}400 \text{ psi}$$

$$\text{Age-adjusted modular ratio, } n_{aa} = \frac{E_s}{E_{caa}(t,t_0)} = \frac{29{,}000{,}000}{923{,}400}$$

$$= 31.4$$

**6. Compute the age-adjusted transformed area, $A_{traa}$, stresses in concrete and steel, and shortening.** Again the steel will be transformed to concrete.

3–5

$$A_{trau} = A_c + (n_{ua} - 1)A_s = 576 \text{ in}^2 + (31.4 - 1) \times 6.32 \text{ in.}^2$$
$$= 768 \text{ in.}^2$$

$$\text{Stress in concrete} = f_c = \frac{P}{A_{trau}} = \frac{630{,}000 \text{ lb}}{768 \text{ in.}^2}$$
$$= 820 \text{ psi}$$

$$\text{Stress in steel } n_{ua} \times f_c = 31.4 \times 820 \text{ psi}$$
$$= 25{,}750 \text{ psi}$$

$$\text{Strain} = \frac{f_c}{E_{cua}} = \frac{820}{923{,}400}$$
$$= 0.000888 \text{ strain}$$

$$\text{Shortening } \epsilon \times \ell = 0.000888 \times 120 \text{ in.}$$
$$= 0.107 \text{ in.}$$

The creep has reduced the stress in the concrete from 1000 psi at the time of loading to 820 psi at 5 years. During the same period, the steel stress has increased from 9560 psi to 25,750 psi. A column with less reinforcement would experience a larger increase in the reinforcement stress. To prevent yielding of the steel under sustained loads, ACI Sec. 10.9.1 sets a lower limit of 1% on the reinforcement ratio in columns.

The plain concrete column in Example 3–3, which had a constant concrete stress of 1000 psi throughout the 5-year period, shortened 0.154 in. The column in this example, which had an initial concrete stress of 1000 psi, shortened two-thirds as much. ■

### Thermal Expansion

The coefficient of thermal expansion or contraction, $\alpha$, is affected by such factors as composition of the concrete, moisture content of the concrete, and age of the concrete. Ranges from normal-weight concretes are 5 to 7 $\times$ $10^{-6}$ *strain*/°F for those made with siliceous aggregates, and 3.5 to 5 $\times$ $10^{-6}$/°F for concretes made from limestone or calcareous aggregates. Approximate values for lightweight concrete are 3.6 to 6.2 $\times$ $10^{-6}$/°F. An all-around value of 5.5 $\times$ $10^{-6}$/°F may be used. The coefficient of thermal expansion for reinforcing steel is 6 $\times$ $10^{-6}$/°F. In calculations of thermal effects it is necessary to allow for the time lag between air temperatures and concrete temperatures.

As the temperature rises, so does the coefficient of expansion and at the temperatures experienced in building fires, it may be several times the value at normal operating temperatures.[3-40] The thermal expansion of a floor slab in a fire may be large enough to exert large shear forces on the supporting columns.

## 3–5 HIGH-STRENGTH CONCRETE

Concretes with strengths in excess of 6000 psi are referred to as *high-strength concretes*. Strengths of up to 18,000 psi have been used in buildings. Reference 3–27 presents the state of the art of the production and use of high-strength concrete.

Admixtures such as superplasticizers, silica fume, and supplementary cementing materials such as certain fly ashes improve the dispersion of cement in the mix and produce workable concretes with much lower water–cement ratios than possible previously. The resulting concrete has a lower void ratio and is stronger than normal concretes.