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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE RESTRICTED REPORT

THE STRENGTH AND STIFFNESS OF SHEAR WEBS

WITH AND WITHOUT LIGHTENING HOLES

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SUMMARY

Nearly 200 tests were made on the strength of shear webs of 24S-T aluminum alloy, with and without lightening holes. The tests were made in a jig of the single-specimen type, in which the specimen is free to collapse completely without developing diagonal tension. The lightening holes were circular and had either flanged edges or beaded edges, the specimens with flanged edges constituting by far the largest test group. The following equations were found for the shear stresses  $\tau$  causing collapse, all stresses being given in kips per square inch:

- (a) Solid webs:  $\tau_{coll} = (37 - 0.283 h/t)$  if  $h/t < 60$  and  $\tau_{coll} = 1200 t/h$  if  $h/t > 60$ . The second formula applies only to sheet 0.036 inch thick; for other thicknesses, the collapsing stress may be obtained from a graph

- (b) Webs with flanged holes:

$$\tau_{coll}(\text{net}) = k [\tau_{cr} + (\tau_{ult} - \tau_{cr}) D/b]$$

where the shear stress is based on the net section

- (c) Webs with beaded holes:  $\tau_{coll} = 440 (t/h)^{3/4}$  where the shear stress is based on the gross section. Within the rather narrow test range, the size and the spacing of the holes has a practically negligible effect on the strength of webs with beaded holes.

In these equations,  $h$  is the width of the sheet;  $t$ , the thickness;  $D$ , the hole diameter;  $b$ , the hole spacing;  $k$ , a correction factor (not differing greatly from unity), which depends

on the sheet thickness;  $T_{cr}$ , the buckling stress; and  $T_{ult}$ , the ultimate shear strength of the material.

Simple empirical formulas are given for the shear stiffness appropriate to various groups of specimens. For webs with flanged holes, design charts are presented; these charts make it possible to determine by inspection the proportions of the lightest web for a given set of design conditions.

## INTRODUCTION

The shear webs employed in aircraft structures are frequently perforated with regularly spaced holes to lighten the web or to provide access to the interior of the structure. Round holes with flanged edges were used in airship girders before the metal monocoque structure came into general use for airplanes, and they continue to be the most common type of lightening hole.

The problem of computing the strength of a web with lightening holes by theoretical means offers formidable mathematical difficulties. There appears to be no published record of any attempt at a purely theoretical solution, the nearest approach being a general, but extremely laborious, method of computing the stresses in a web with plain holes. It has been necessary, therefore, to rely on tests for proving the strength of perforated webs. Individual tests are sufficient for the immediate purpose of proving the strength of a given design, but they furnish no information on the optimum design proportions. A sufficiently extensive series of systematic tests would furnish information on the optimum design proportions and would eliminate the need for many individual tests. Unfortunately, so many parameters are involved that a very large number of specimens would be necessary to cover the range of proportions; this obvious fact has acted as an effective deterrent for many years.

A fairly extensive series of tests was published by Schüssler (reference 1), but his results have not been fully accepted by aeronautical engineers. A number of aircraft manufacturers have been interested for some time in obtaining additional data; it was finally agreed that these manufacturers would furnish the test specimens and the NACA would do the testing. Each manufacturer was to use his standard dies for flanging but to provide a sufficient number of specimens to cover the range of variables as far as practicable. The specimens tested in the present investigation were furnished by the Bell Aircraft Corporation. Special acknowledgment is due this company for their willingness to cooperate by making a

large number of test specimens at a time when unprecedented demands are being made on all production facilities.

The extensive test work involved was performed by Mr. S. H. Diskin of the NACA staff.

### TEST PROCEDURE

In its most general form, the problem of shear webs with lightening holes involves the following variables:

- (1) Material of sheet
- (2) Thickness of sheet,  $t$
- (3) Width of sheet,  $h$
- (4) Type of edge support of sheet
- (5) Size of holes
- (6) Shape of holes
- (7) Spacing of holes,  $b$
- (8) Shape of flanges or beads around holes

It is obvious that systematic tests covering the entire range of all variables would require a prohibitive number of specimens. Any given investigation, then, can cover only a limited range of designs and, if it becomes apparent that a different range of designs offers promise of being better in some respect, a new series of tests will become necessary. The fact that additional tests are certain to be required makes it desirable to discuss in some detail the test procedure used and the difficulties encountered in these tests, in order that later investigations may benefit from the experience gained.

Test specimens. - The specimens furnished by the Bell Aircraft Corporation consisted of the following: 125 specimens with flanged holes, including 52 duplicates; 27 specimens with beaded holes, including 4 duplicates; 8 specimens with plain holes, including 4 duplicates; and 4 specimens without holes. Typical cross sections of the flanges and of the beads are shown in figure 1. All specimens were made of 24S-T aluminum alloy, as were 28 specimens without holes prepared by the NACA.

The perforated specimens ranged in thickness from 0.032 to 0.064 inch. Three standard widths of specimens with holes were furnished: 6, 5, and  $4\frac{1}{2}$  inches, measured between center lines of bolt rows. The nominal hole diameters (clear diameters) were 0.8, 1.1, and 1.6 inches. All specimens were about 33 inches long; the exact length  $L$  was determined in each case by the hole spacing, the end being taken halfway between holes. The free ends of the specimens were reinforced by  $90^\circ$  flanges having a width of 1 inch.

The specimens without holes ranged in thickness from 0.015 to 0.065 inch. They were about 33 inches long, with the exception of one specimen ( $t = 0.065$  in.,  $h/t = 210$ ) that was 77.5 inches long. The widths of specimens without holes ranged from 1 to 13 inches.

Inspection of the specimens before the tests disclosed that a number of the flanged specimens had cracks in the flanges, sometimes radial and sometimes circumferential. Even in an extreme case, however, where every flange in the specimen was cracked circumferentially, the static strength of the specimen was evidently unimpaired.

Test jig.- Shear tests on sheets with or without holes have commonly been made in the type of jig shown schematically in figure 2(a). (See references 1, 2, and 3, p. 603.) This type of jig is very suitable for tests concerned with buckling loads; for tests concerned with ultimate loads, however, the jig is objectionable because the rigid fixation of the outer bars enables the shear webs to develop diagonal tension and, consequently, to develop higher loads than they could develop in the actual structure.

For the present investigation, the single test jig shown schematically in figure 2(b) was chosen. In this type of jig, the specimen is free to collapse completely when the buckles become deep enough to cause yielding of the material at the crests. Figure 3 is a scale drawing of the actual jig, and figure 4 shows the jig in use.

For a few tests, the jig was modified by joining the fixed bar and the movable bar by links to produce a parallelogram; in such a parallelogram jig, the conditions are between those in a single jig and those in a double jig. The tests, which are not included in the paper, indicated an increase in strength of about 10 percent over the single-jig results.

Very heavy bars were used to hold the specimen along the outer, or free, edge in order to insure as uniform as possible a distribution of the shear stress along the length of the specimen. The

importance of this consideration was first pointed out by Mathar. (See reference 2.) The bars that receive the concentrated test load (bar C in fig. 2(a); bar B in fig. 2(b)) are subjected to longitudinal stresses and strains; as a result, the displacement of the loading bars - and with it the shear strain in the specimens - is a maximum at the point of load application and decreases from there toward the end, or ends, of the bars. The introduction of the load at the middle of the bar (fig. 2(b)) instead of at the end (fig. 2(a)) offers two advantages: The maximum amount of nonuniformity of shear strain is reduced to one-fourth; and the maximum shear strain occurs in the middle of the specimen instead of at the ends, where conditions are already uncertain. The size of the bars was chosen such that, theoretically, the maximum shear strain in the specimen exceeded the average shear strain by less than 2 percent in the worst case when perforated specimens were being tested.

As shown in figures 3 and 4, two dial gages reading to 1/10000 inch were used to measure the shear deformation of the specimens.

The load was applied by a portable hydraulic testing machine; the accuracy of load measurement was one-half of 1 percent.

Attachment of specimens.- The large thickness of the loading bars made it impossible to use rivets for attaching the specimens to them; half-inch bolts were used for this purpose. The bolt holes were at first drilled through the specimen with a special lip-cutting drill. The shear deformation measured on the first specimen without lightening holes agreed with the calculated value within the accuracy of measurement, and the first tests with perforated specimens gave very smooth load-deformation curves. It was therefore believed that the method of drilling the holes was sufficiently accurate, particularly since the emphasis in these tests was on strength, not on stiffness. After two groups of specimens had been tested, however, it was found that, under average conditions in continued testing, the original accuracy of the holes could not be maintained; in all the rest of the specimens the holes were therefore drilled undersize and line-reamed. The reamed holes gave better results than the drilled holes at the expense of doubling the time required for testing; with drilled holes it had been possible to make four tests a day; with reamed holes the average dropped to two tests a day. For extensive test series, it would be desirable to use tapered holes in the test jig to provide for taking up the wear caused by repeated reaming operations.

Edge support.- The specimens were at first clamped directly between the loading bars (fig. 5(a)). A comparable degree of edge restraint is not likely to exist in an actual structure. A number of tests were therefore made with a practical substitute for

supported edges. The conversion into the second type of support was made as shown in figure 5(b). The loading bars were separated and drill rods were placed between the bars and the specimens along the inner edges of the bolt holes. The first type of support will be referred to as "bar support," the second type as "rod support." For the largest values of  $h/t$  tested, the bar support may be considered to give clamped edges, the rod support to give supported edges. At small values of  $h/t$ , the clamping effect of the bar support is apparently not sufficient to produce the equivalent of rigidly clamped edges. The rod support, on the other hand, has some restraining effect that becomes more noticeable at lower values of  $h/t$ ; it is caused by the restraining action of the bolts on the parts of the specimen that overhang the rods.

The specimens with flanged holes were divided into two interlocking groups; one group was tested with bar supports and the other group, with rod supports. The test points obtained with rod supports appeared to show less scatter than the test points obtained with bar supports, and the edge restraint provided by the rod support was more nearly representative of actual conditions. Rod supports were therefore used for most of the specimens with beaded holes and for the specimens with plain holes. Both types of support were used for specimens without holes.

Loading procedure.- In the main group of tests, each specimen was preloaded once or several times to about 20 percent of the maximum load and was adjusted until the two dial gages gave approximately equal readings. The load was then applied in increments of 500 or 1000 pounds until the specimen completely collapsed and the load dropped off. Dial-gage readings were taken at each load increment.

After the strength tests had been completed, a small number of duplicate specimens were tested in the following manner: Each specimen was preloaded and adjusted to give approximately equal readings on the two dial gages. The load was then increased by the usual increments to two-thirds of the estimated maximum value and decreased again to zero. A second run to two-thirds load and back to zero load was then made, and finally the specimen was loaded to destruction. These tests were intended chiefly to obtain some data on permanent set; incidentally, they served the usual purposes of repeat tests.



## TEST RESULTS

## The Strength of Shear Webs

The strength of solid shear webs.- The dimensions of the shear webs without holes and the maximum loads carried by them are given in table 1. The experimental shear stresses causing the webs to collapse  $\tau_{coll}$  were calculated, from the test load causing the specimen to collapse  $P_{coll}$ , by the formula

$$\tau_{coll} = \frac{P_{coll}}{L_e t} \quad (1)$$

the effective length  $L_e$  being taken as (see fig. 5)

$$L_e = L - \frac{1}{2}h_1 \quad (2)$$

for bar supports as well as for rod supports. This correction for ineffectiveness at the free ends was also used by Schüssler (reference 1) and is based on photoelastic tests reported in reference 3 (p. 605). Strain measurements made on the upper half of one specimen with bar supports showed stresses equal to 79 and 99 percent of the calculated stress at distances of  $0.2h_1$  and  $0.4h_1$ , respectively, from the end; the measured stress at the middle of the specimen was 105 percent of the calculated stress. This excess at the middle is explained qualitatively by the fact that the load is applied in concentrated form, as mentioned in the discussion of the test jig. The fact that a 5-percent excess was measured instead of a 2-percent excess, as estimated, may be due to experimental error, inadequacy of the simple formula used for making the estimate, local overstressing due to oversized holes, and finally to the high load carried in the solid specimen.

The experimental values of  $\tau_{coll}$  are shown in figure 6. The evidence is not so complete as might be desired but appears to warrant the conclusion that the method of edge support does not affect the collapsing load. For values of  $h/t < 60$ , the data can be represented by the empirical formula

$$\tau_{coll} = (37 - 0.283 h/t) \text{ kips per square inch} \quad (3)$$

At values of  $h/t > 60$ , the curves separate for different thicknesses, the thinner sheets develop higher stresses than the thicker sheets. For a thickness of 0.036 inch, the experimental curve for  $h/t > 60$  can be expressed by the empirical formula

$$\tau_{coll} = 1,200 \quad t/h \quad \text{kips per square inch} \quad (4)$$

No attempt was made to express the curves for other thicknesses in analytical form.

For comparison, figure 6 also shows the well-known theoretical curves for the critical shear stresses  $\tau_{cr}$ . These curves are valid only as long as the stress in the material has not passed the limit of proportionality; beyond this point, corrections must be made analogous to the case of column curves at low slenderness ratios. There is no established method of making such corrections in the case of critical shear stress, but an upper limit for  $\tau_{cr}$  may obviously be obtained by using  $\tau_{coll}$  whenever it is lower than  $\tau_{cr}$ .

The strength of shear webs with flanged holes.— Because webs with round flanged lightening holes are widely used, an effort was made to develop an empirical strength formula of such a form that it could be used for extrapolation beyond the test range<sup>1</sup> with a reasonable degree of accuracy. The formula developed is

$$\tau_{coll} \text{ (net)} = k \left[ \tau_{cr} + (\tau_{ult} - \tau_{cr}) D/b \right] \quad (5)$$

where

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<sup>1</sup> Formula (5) for the strength of shear webs with flanged lightening holes, as given in this report, was based on a fairly large number of tests (119 tests). The range of some of the variables was, however, quite limited; in particular, there were practically no tests with a diameter-to-depth ratio greater than 0.5. Additional tests have been started to extend the range of variables; only a few of these tests have now been completed (Sept. 1942), but they appear to indicate definitely that the formula becomes unconservative outside the test range. Pending the completion of these tests, it is recommended that the application of formula (5) be strictly confined to webs falling within the test range, which may be defined as follows:

$$D/h < 0.5; \quad h < 5.5 \text{ inches}; \quad t > 0.32 \text{ inch}$$

$\tau_{coll}$  (net) shear stress that causes collapse, based on the net section. The net section per inch run is taken as  $t(1 - D/b)$

$\tau_{cr}$  critical stress at which the sheet would buckle if it had no holes

$\tau_{ult}$  ultimate shear stress of material

$D$  clear diameter of holes

$b$  center-to-center spacing of holes

$$k = 0.675 + 7.5 t \quad (t \leq 0.050 \text{ in.})$$

$$k = 1.050 \quad (t \geq 0.050 \text{ in.})$$

(6)

It will be seen that formula (5) involves the properties of the material; namely,  $\tau_{ult}$  and  $E$  (in  $\tau_{cr}$ ). The formula gives either approximately correct values or conservative values for all possible limiting cases as follows:

When the holes are so closely spaced that the flanges of adjacent holes touch each other ( $D/b \rightarrow 1$ ), the shear stress developed over the net section may be expected to equal the ultimate shear stress of the material as long as the sheet is thick enough to prevent buckling of the narrow net section. Formula (5) reduces for the case of  $D/b \rightarrow 1$  to  $\tau_{coll} \text{ (net)} = k\tau_{ult}$ , which indicates a net shear stress lower than  $\tau_{ult}$  for thin sheet, increasing to a net shear stress somewhat larger than  $\tau_{ult}$  for thick sheet. This excess, which has a maximum value of 5 percent according to formula (6), can probably be explained by the fact that the value of  $\tau_{ult}$  as obtained from reference 4 is somewhat conservative.

When the holes become vanishingly small but a finite spacing is still maintained or when the spacing becomes very large for any arbitrary size of holes ( $D/b \rightarrow 0$ ), formula (5) reduces to  $\tau_{coll} \text{ (net)} = k\tau_{cr}$ . This value is conservative for large ratios of  $h/t$  and approximately correct for low ratios of  $h/t$  provided that the  $\tau_{coll}$  curve is used as a cut-off curve for  $\tau_{cr}$ , as suggested in the discussion of the strength of solid webs.

The linear dependence of  $\tau_{coll}$  on  $D/b$  was established empirically; a sample test plot is shown in figure 7. It was first

believed that the change of  $\tau_{coll}$  should depend on a function of  $D$ ,  $b$ , and  $h$ , the most obvious one being  $D^2/bh$  that expresses the amount of lightening (ratio of area removed to original area) except for inessential constants. It was found, however, that much closer correlation could be obtained with the parameter  $D/b$  than with  $D^2/bh$ .

Tables 2, 3, and 4 give the dimensions of the test specimens, the test loads, the experimental values of  $\tau_{coll}$ , and the calculated values of  $\tau_{coll}$  for the shear webs with flanged lightening holes. The experimental values of  $\tau_{coll}$  were calculated by the formula

$$\tau_{coll} \text{ (net)} = P_{coll}/A_e \quad (7)$$

where the effective net cross-sectional area  $A_e$  was taken as

$$A_e = (n - 1) (b - D)t \quad (8)$$

$n$  being the number of holes in the specimen.

The correction for ineffectiveness at the ends included in formula (8) is based on the assumption that the material outboard of the last hole on each end carries no stress. Qualitatively, this correction seems more appropriate for perforated specimens than the correction used for solid specimens, and it does not differ greatly from the correction for solid specimens within the test range. Quantitatively, however, the correction is not verified and constitutes the largest item of uncertainty in the evaluation of the test data. The error due to this uncertainty is estimated to be, in most cases, less than 5 percent.

The calculated values of  $\tau_{coll}$  were obtained by using formulas (5) and (6). The values of  $\tau_{cr}$  needed for use with formula (5) were taken from the curves shown in figure 8. These curves were obtained by drawing tentative straight lines on all test plots, analogous to the plot shown in figure 7. The tentative values for  $\tau_{cr}$  obtained in this manner were then plotted against  $h/t$  and faired. The modulus  $E$  was taken as 10,600 kips per square inch and the ultimate strength as  $\tau_{ult} = 37$  kips per square inch, according to reference 4.

It will be noted that, for the two main groups of tests (with reamed bolt holes), formula (5) represents the test data quite well.

Errors in excess of 10 percent are shown for 11 percent of all tests, and the maximum errors are 22 percent on the conservative side and 10 percent are shown for 11 percent of all tests, and the maximum errors are 22 percent on the conservative side and 10 percent are shown by 16 percent of all tests; the maximum error on the conservative side is 24 percent and the maximum error on the unconservative side, 20 percent. Compared with the formulas of reference 1, formula (5) has, therefore, the twofold advantage of somewhat better accuracy and of much greater usefulness for extrapolating beyond the test range. The test group with drilled bolt holes averages 10 percent low, presumably reflecting the influence of uneven load distribution caused by irregular oversized holes.

The strength of shear webs with beaded holes.- The results of the tests on webs with beaded holes are given in table 5. Application of the formula developed for webs with flanged holes showed large irregular scatter, indicating that the behavior of the webs with beaded holes differs considerably from the behavior of the webs with flanged holes. The beads stiffen a fairly large portion of the sheet and, as a result, the webs with beaded holes appear to act more nearly as uniformly stiffened sheets. The collapsing stress of webs with beaded holes is therefore based on the gross, not on the net, section and is calculated by the formulas used for webs without holes, namely, formulas (1) and (2). In order to emphasize this point, the shear stress thus calculated will be designated  $\tau_{coll}$  (gross).

The experimental values of  $\tau_{coll}$  (gross) are plotted in figure 9 against the ratio  $h/t$ . Curve A is plotted from the equation

$$\tau_{coll} = 440 (t/h)^{3/4} \text{ kips per square inch} \quad (9)$$

This formula represents all the test data for beaded holes with about the same degree of accuracy as formula (5) represents the test data on webs with flanged holes. On the webs having a hole diameter  $D = 1.05$  inches, the influence of hole spacing is sufficiently definite to justify the fairing of individual curves for different hole spacings  $b$ . Curve B in figure 9 is faired through the test points for webs with  $b = 4$  inches, curve C through the test points for webs with  $b = 3$  inches. The curve for  $b = 3.5$  inches was omitted to simplify the figure. For the webs having a hole diameter  $D = 1.60$  inches, the tests indicate no relation between the allowable stress and the hole spacing. The number of tests is not sufficient to draw more definite conclusions on the influence of hole size and hole spacing.

Three beaded-hole specimens were tested with bar supports. It will be noted that the test points fall practically on the same curves as points for tests with rod supports. The conclusion that the method of edge support does not influence the strength of webs with beaded holes is in agreement with the conclusion first stated that this type of web fails in the same general manner as a uniform sheet, because the tests on solid webs indicated no influence of the method of edge support on the strength.

The strength of webs with plain holes.- Since only four different sizes of webs with plain holes (without flanges) were tested, it is impossible to draw any general conclusions. The test results are given in table 6.

#### The Stiffness of Shear Webs

The shear displacement  $\delta$  of a solid web is given by the elementary formula

$$\delta = \frac{Th_1}{G} = \frac{Ph_1}{L_e tG} \quad (10)$$

as long as the sheet does not buckle and the limit of proportionality of the material is not exceeded. The depth  $h_1$  of the web between the center lines of the bolt rows is used in all cases when deformations are being calculated.

The displacement of a perforated web may be calculated by the same formula if the product  $tG$  in formula (10) is multiplied by an efficiency factor  $\eta$ . This factor will be denoted by  $\eta_0$  when it applies to the initial straight-line part of the load deformation curve. For many webs, this initial straight-line part is so short as to be of little practical significance. The factor  $\eta$  (without subscript) recommended for general use is, therefore, based on the measured displacement  $\delta$  at two-thirds of the collapsing load; this load was chosen because, under present design requirements, the limit load is two-thirds of the ultimate design load.

A simple formula for the shear-stiffness factor may be obtained by assuming that the material between the holes and the edges is entirely ineffective, leaving as effective material rectangular strips having a length  $(b-D)$ ; the formula is evidently

$$\eta = 1 - D/b \quad (11)$$

If this formula is modified by introducing an exponent  $m$

$$\eta = 1 - (D/b)^m \quad (11a)$$

it may be adjusted to fit individual groups of test data as well as the scatter of the data will permit.

The experimental displacement curves often exhibited marked irregularities; some of these irregularities were probably caused by loose fit of the bolts, some by buckling between the lightening holes. No attempt was made, therefore, to derive formulas of general validity to represent the experimental shear-stiffness factors.

Only the results for webs with rod supports are given. It is believed that the restraining influence exerted by the bar supports on the shear displacements is never approached in a practical structure, and the results obtained with bar supports are, consequently, of no practical interest.

The stiffness of solid webs.- By definition, the shear-stiffness factor  $\eta_0$  equals unity for solid webs.

If buckling begins at a load less than  $2/3 P_{coll}$ , the value of  $\eta$  will depend on the amount of buckling. The condition is similar to that in diagonal-tension fields but is complicated by the fact that a web free to collapse is more sensitive to initial buckles than a diagonal-tension web. There were additional experimental difficulties in some cases, such as the small magnitude of the displacements caused by  $h_1$  being very small, and uncertainties concerning the fit of the bolts. As a result, the usable data obtained are too isolated to warrant publication.

The stiffness of webs with flanged holes.- The basic formula (11) was found to represent quite well the experimental values of  $\eta_0$  obtained for webs with flanged holes having thicknesses from 0.040 to 0.064 inch (fig. 10). For webs having a thickness of 0.032 inch, the values of  $\eta_0$  were appreciably lower (fig. 11). The factor  $\eta$  for the stiffness at  $2/3 P_{coll}$  is shown in figure 12; all thicknesses of sheet are included in this plot because there was no discernible influence of the thickness on the stiffness factor. Figure 13 shows the factors  $\eta$  obtained on the specimens used for permanent-set tests. These specimens had been loaded twice to  $2/3 P_{coll}$ ; it may be assumed, therefore, that the play in the bolt holes was fairly well eliminated, and the results average correspondingly higher than the results shown in figure 12.

It may be concluded from figures 12 and 13 that the stiffness factor may be taken as

$$\eta = 1 - (D/b)^{2/3} \quad (12)$$

for webs with flanged holes when the joint along the loaded edge has no play; since a well-riveted joint has no play, the formula should be applicable to webs with riveted joints.

The stiffness of webs with beaded holes. - The basic formula (11) represents fairly well the experimental values of  $\eta_0$  for webs with beaded holes having a thickness of 0.064 inch (fig. 14). For smaller thicknesses, the values of  $\eta_0$  are lower (fig. 15).

The shear-stiffness factor  $\eta$  of webs with beaded holes at high loads exhibits the same characteristic as the strength of these webs; namely, that the influence of hole size and hole spacing is negligible within the test range (fig. 16). The thickness, however, has some influence and the experimental averages can be expressed by the empirical formula

$$\eta = 0.1 + 4.5t \quad (t \leq 0.064 \text{ in.}) \quad (13)$$

#### Permanent-Set Tests

The permanent set of shear web may be thought of as caused by two distinct phenomena: (1) permanent set of the specimen itself and (2) permanent set in the joints - riveted or bolted - along the edges.

The magnitude of the permanent set suffered by the specimen itself depends on the magnitude of the maximum stress and on the extent of the region experiencing high stresses. In perforated webs, the maximum stress covers only a very narrow band in the net section. There may be some concentration of stress, but this concentration would be too localized to affect appreciably the permanent set of the entire specimen. There may exist a buckle over the net section, adding local bending stresses to the basic shear stresses; in the range covered by the tests, however, these buckles were always very small if at all perceptible, and they disappeared completely upon removal of the load.

At the two-thirds load chosen as standard for defining the permanent set, the maximum stress in a perforated shear web may,



therefore, be taken as approximately equal to  $\frac{2}{3} \tau_{coll}$ . Since  $\tau_{coll}$  is always less than  $\tau_{ult}$ , the maximum stress is always less than  $\frac{2}{3} \tau_{ult}$ . In 24S-T aluminum alloy, the yield stress is roughly equal to  $\frac{2}{3} \tau_{ult}$ . Consequently, there is little likelihood of an appreciable amount of permanent set occurring in the net section of a perforated web loaded to  $\frac{2}{3} P_{coll}$ .

Permanent set in riveted joints is caused by bearing failures of the sheet or the rivets and by deformation of the rivets. This subject forms a separate field of study and need not be considered here. Permanent set in bolted joints is caused chiefly by bearing failures of the sheet and by slippage in oversized holes.

The results of the permanent-set tests are given in table 7. It will be seen that the permanent sets of specimens with flanged holes tested with bar supports range roughly from 5 to 10 percent of the displacement under load. The net shear stresses are below the yield stress of the material, and the sets recorded are, therefore, believed to be mostly caused by slippage in the bolt holes.

The permanent set recorded for webs with flanged holes tested with rod supports are about ten times as large as those with bar supports. Since the net shear stresses are of the same order of magnitude for both groups of tests, it must be concluded that the slippage in the bolt holes and the bearing failures of the sheet were much more pronounced in the tests with rod supports than in those with bar supports. The difference presumably arises from the fact that the bar supports transmit an appreciable part of the load by friction, thus relieving the bearing pressures and delaying the occurrence of slip. In addition, the bolts are subjected to a certain amount of bending when the loading bars are separated by the rods.

The belief that the recorded set is largely caused by slippage is supported by a study of the load-displacement curves discussed in the appendix. These curves suggest strongly that large amounts of slippage take place at loads between 4 and 8 kips when the rod supports are used. The possibility of large amounts of slippage despite the use of reamed holes is explained by wear in the test jig. An index to the relative amount of wear in the jig is furnished by the test numbers, which are given in tables 1 to 4; it may be noted that the set tests on specimens with flanged holes carry test numbers 161 to 175. The irregular shape of the worn holes and the large thickness of the loading bars made it impossible to measure the actual amount of wear in the holes; it is estimated, however, that the wear in many holes amounted to at least 0.002 to 0.004 inch when the set tests were being made.

$$S = \tau_{coll} h t (1 - D/b) \quad (18)$$

The result was a series of points from which the strength curve for the assumed values of  $h$  and  $t$  could be plotted against  $D/h$ . In this manner, strength curves were calculated for various standard values of  $t$  and for two values of  $h$  delimiting the test range. The strength curves are shown in figures 17(a) and 17(b) as lines sloping down to the right. The number at each point gives the value of  $R$  determining the optimum hole spacing.

For each web calculated as described, the weight was then calculated. The weights obtained were used to construct curves of equal weight, shown as lines sloping up to the right. In order to facilitate comparisons, the equal weight curves are not numbered in terms of actual weights but in terms of the thickness  $t_s$  of the corresponding solid sheets.

There were no tests available with  $D/h < 0.14$ . The strength curves and the equal weight curves were therefore stopped at  $D/h = 0.15$ , and straight guide lines were drawn to the values of  $D/h = 0$ , which are based on the tests on sheets without holes. Individual judgment must be used should it be necessary to design webs falling within this region.

It will be noted that the strength curves, when extended to  $D/h = 0$ , pass near the points derived from tests on sheets without holes for a certain range but not over the entire range of the two charts. Theoretically, there is no reason why the strength curves should pass through these points, because the theoretical case of a web with vanishingly small holes is not identical with the case of a sheet without holes. The strength curves assume that the optimum hole spacing is used in each case, which means that there is a finite reduction of section along the center line of the web even when the holes became vanishingly small. On the other hand, the validity of equation (5) is assured only if there is a flange of a certain depth around each hole. In the case of very small holes, there must exist, then, a ridge of closely spaced flanges along the center line of the web, and this ridge would exert a stiffening influence. It should be realized, however, that this reasoning is theoretical and qualitative only. Caution should be used in designing perforated webs in the region where the strength of the solid sheet is appreciably lower than the strength of the perforated sheet until full experimental verification is obtained for this region.

For webs having a depth of either 4 or 8 inches, the answer to any design problem may be obtained from figure 17 by inspection.

For the design of webs with intermediate depths, figure 18 was prepared, using figure 17 as a basis. The ratio  $D/h = 0.8$  is about the maximum value that can be used in practice without having undue interference between the flanges and the rivet rows; figure 18(b) will therefore be used to obtain the most efficient designs, because inspection of figure 17 indicates that the most efficient design is always obtained by using as large a hole as possible. If it should be necessary to use smaller holes, the allowable value of the running shear  $S/h$  may be obtained by interpolating between the curves of figures 18(a) and 18(b).

The design charts are based on the assumption that the optimum hole spacing is used. Larger hole spacing will increase the strength but will lower the strength-weight ratio. Smaller hole spacing will lower the strength as well as the strength-weight ratio. The influence of the hole spacing is illustrated by the three test groups shown in figures 19(a), 19(b), and 19(c). The figures illustrate the value of the formula for finding the optimum hole spacing when the optimum falls outside the test range.

Examples for use of design charts.- Example A: A web 4 inches deep is required to carry a transverse shear load of 1550 pounds. Find the design proportions giving the best strength-weight ratio, assuming that practical considerations limit the value of  $D/h$  to 0.8.

By inspection of figure 17(a), it is found that a web 0.040 inch thick will just carry the required load. The hole diameter is  $0.8 \times 4 = 3.2$  inches. The chart gives  $R = 0.57$ ; the optimum hole spacing is, therefore,  $b = 3.2/0.57 = 5.6$  inches. The weight of this web is slightly more than that of a solid web 0.025 inch thick.

Example B: A web 6 inches deep is required to carry a transverse shear load of 2280 pounds. Find the design proportions giving the best strength-weight ratio, assuming that practical design considerations limit the value of  $D/h$  to 0.8.

The required running shear is  $S/h = 2280/6 = 380$  pounds per inch. Figure 18(b) shows that to carry this running shear with a depth of 6 inches, a thickness of 0.040 inch is required. By interpolation, the value of  $R$  is 0.591. The hole diameter is  $0.8 \times 6 = 4.8$  inches; the optimum hole spacing is therefore  $4.8/0.591 = 8.12$  inches.

Comparison of three types of web.- Comparisons between solid webs with flanged holes may be made conveniently by inspection of figure 17. It will be seen that the perforated webs may be stronger or weaker than solid webs of the same thickness. For a given strength, however, the most efficient web is always a perforated web, never a solid web.

Comparative calculations for webs with flanged holes and webs with beaded holes are shown in table 8. The ratio  $D/h$  for flanged holes was limited to 0.8, because larger ratios may cause interference between the flanges and the rivets. The strength of the webs with beaded holes was based on formula (9). The hole diameter was taken as 1.6 inches, and the hole spacing as 3 inches, which is about the closest spacing possible. This close spacing, although beyond the test range, was chosen in order to make the comparison more favorable for the beaded holes. As table 8 shows, however, the webs with flanged holes require a smaller volume of material and, consequently, are more efficient than the webs with beaded holes unless the webs have a very low  $h/t$  ratio.

Comparisons not included here show that for the same thickness and hole diameter, the web with beaded holes will carry more load, or at least the same load, as the web with flanged holes. The web with flanged holes can be made more efficient, however, by using larger holes, while the size of the bead effectively limits the size of the hole.

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## APPENDIX

## LOAD-DISPLACEMENT CURVES OF SHEAR WEBS

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While a discussion of load-displacement curves is only of slight interest to the designer and to the stress analyst, it is of interest to the engineer confronted with the task of devising a test procedure. Increasing attention is being paid to questions of stiffness and, consequently, there will be an increasing demand for information that can be obtained only by tests. A discussion of points brought out in the present investigation will therefore be in order in preparation for future tests.

It is impractical to present all the data; only samples are shown for the most important test groups. In order to avoid personal bias in the choice of the samples, the choice was made by arbitrarily designating test numbers without referring to the curves.

The sample curves for solid webs are shown in figure 20. It will be seen that the initial tangent agrees quite well with the calculated straight line, but the initial straight-line part of the curve may be quite short.

In references 1 and 2 it is stated that the typical load-displacement diagram starts as a straight line, then bends through a knee into a second straight line with smaller slope, and finally rounds over into a curve approaching the horizontal. The knee between the two straight-line parts was interpreted in these two references as indicating the buckling load.

The curve shown in figure 20 for specimen 1 answers this general description, and the knee of the curve lies in the region of the critical load calculated on the assumption of supported edges. On the curve for specimen 4D, however, there is obviously no relation between the location of the knee of the curve and the critical load.

On the perforated webs with bar supports and drilled bolt holes (fig. 21) the curves do not show a knee that might be considered as indicating a buckling load. On the same type of specimens with reamed holes, a knee might be identified on three of the four curves shown (fig. 22).

On the perforated webs with rod supports (fig. 23), all the curves show a more or less pronounced irregularity. The displacement curve indicates a sudden reduction of shear stiffness, followed by a sudden

increase of a smaller amount. It would be very difficult to explain this action as being due to buckling, when the specimen is free to collapse. On the other hand, it is easy to explain this action on the assumption that the bolt holes were oversize and that the sudden apparent loss of stiffness is, in fact, caused by slippage.

If the displacement curves obtained with bar supports are re-examined in the light of this conclusion, it will be seen that they show similar tendencies, only much less pronounced. Since the bar supports give a much larger contact area on the specimens than the rod supports, slippage probably occurs more gradually and is thus effectively masked.

It is stated in reference 1 that the knee of the load-displacement curve was used as prime evidence of buckling but that corroborative evidence was obtained by observing reflections on the surface of the specimen between lightening holes. This method is quite sensitive for detecting the instant at which a plane surface begins to curve slightly, but it is difficult to detect changes of curvature by this method. In the specimens used for the present investigation, it was generally found that the flanging operation had left the sheet slightly curved between the holes, so that it was difficult to detect buckles at an early stage of development by observing reflections. In general, clearly visible buckles began to appear at about  $2/3 P_{coll}$ . Earlier buckling was noted on some solid sheets and on a number of specimens with bar supports and reamed holes, but the buckles were often so shallow that their existence remained doubtful over a large range of loading, sometimes over a range equal to one-third of the collapsing load.

The observations made lead to the conclusion that the load-displacement curves obtained in these tests are falsified by slippage in the bolt holes, to a moderate extent when bar supports were used and to a marked extent when rod supports were used. It may also be concluded that whenever there is any possibility of such slippage, a knee in the load-deformation curve cannot be regarded as a reliable indication that buckling occurs in the specimen.

TABLE 1  
SOLID SHEAR WEBS

Specimen	Test	$L_e$ (in.)	$t$ (in.)	$h$ (in.)	$P_{coll}$ (kips)	Exp. $\tau_{coll}$ (kips/sq in.)	Calc. $\tau_{coll}$ (kips/sq in.)	Exp. Calc.
Bar supports, reamed bolt holes								
2B	37	32.28	0.0374	3.94	15.70	13.00	11.38	1.14
2C	44	32.38	.0394	3.94	15.90	12.46	12.00	1.04
2D	42	32.63	.0385	2.94	20.75	16.52	15.70	1.05
2E	43	32.78	.0396	2.94	21.30	16.41	16.18	1.01
4B	38	32.34	.0643	4.03	40.00	19.23	19.00	1.01
4C	39	32.02	.0642	3.97	41.50	20.14	19.30	1.04
4D	40	32.44	.0634	2.94	47.90	23.29	23.88	.98
4E	41	32.69	.0637	3.00	49.00	23.56	23.56	1.00
137A	176	27.64	.0420	9.97	5.50	4.74	4.90	.97
137B	177	27.52	.0419	9.97	6.50	5.64	4.88	1.16
138A	178	29.52	.0232	6.22	3.48	5.08	5.30	.96
138B	179	29.57	.0233	6.25	3.60	5.24	5.30	.99
139A	184	32.11	.0426	1.03	40.90	29.90	30.14	.99
139B	185	32.11	.0424	1.03	41.90	30.79	30.12	1.02
140	190	32.13	.0148	1.00	8.76	18.42	17.90	1.03
142	191	28.97	.0631	7.00	18.00	9.88	10.65	.93
143	192	69.94	.0619	13.00	21.20	4.90	5.00	.98
						Average of ratios above unity (10 tests) = 1.05		
						Average of ratios below unity (7 tests) = .97		
						Average of all ratios (17 tests) = 1.02		
Rod supports, reamed bolt holes								
2F	85	31.94	0.0377	5.31	13.00	10.80	8.45	1.28
4F	86	32.06	.0641	5.31	30.30	14.80	14.50	1.02
133A	131	31.22	.0142	3.99	3.30	7.45	7.60	.98
133B	132	31.24	.0139	3.96	3.43	7.90	7.58	1.04
134A	129	28.08	.0401	10.02	5.30	4.71	4.64	1.02
134B	130	28.27	.0392	10.02	4.64	4.19	4.53	.92
135A	181	31.99	.0144	2.71	4.50	9.77	8.30	1.18
135B	182	31.95	.0144	2.71	3.75	8.15	8.30	.98
136A	146	29.59	.0393	7.18	8.96	7.70	6.46	1.19
136B	180	29.56	.0417	7.18	8.96	7.27	6.88	1.06
141	193	32.41	.0148	1.74	5.18	10.80	11.06	.98
						Average of ratios above unity (7 tests) = 1.11		
						Average of ratios below unity (4 tests) = .97		
						Average of all ratios (11 tests) = 1.06		
Bar supports, drilled bolt holes <sup>a</sup>								
1	32	30.75	0.0315	4.00	9.66	9.97	9.50	1.05
2	17	30.72	.0406	4.00	14.15	11.35	12.22	.93
3	12	30.59	.0512	4.19	18.90	12.07	14.65	.82
4	1	30.72	.0656	4.06	36.00	17.87	19.20	.93
						Average of all ratios (4 tests) = 0.93		
<sup>a</sup> Specimens with drilled bolt holes are not shown on plot.								





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TABLE 2  
SHEAR WEBS WITH FLANGED LIGHTENING HOLES  
[Bar supports, reamed bolt holes]

Specimen	Test	L (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	A <sub>e</sub> (sq.in.)	P <sub>coll</sub> (kips)	Exp. T <sub>coll</sub> (net) (kips/sq.in.)	Calc. T <sub>coll</sub> (net) (kips/sq.in.)	Exp. Calc.
Strength tests												
6A	65	31.3	21	0.0311	3.97	0.77	1.50	0.454	9.35	20.59	19.51	1.06
7A	66	33.0	19	.0314	3.97	.77	1.75	.554	10.50	18.96	17.49	1.08
8A	67	33.7	17	.0313	3.97	.77	2.00	.616	10.65	17.29	15.86	1.09
10A	45	31.2	21	.0409	3.97	.77	1.50	.597	15.80	26.46	22.63	1.17
11A	46	33.0	19	.0411	4.00	.77	1.75	.725	16.40	22.62	20.56	1.10
12A	47	33.8	17	.0400	3.94	.74	2.00	.806	14.10	17.49	18.32	.95
14B	80	31.3	21	.0519	3.97	.75	1.50	.779	21.50	27.62	26.27	1.05
15B	77	33.0	19	.0524	3.94	.75	1.75	.943	23.70	25.13	24.73	1.02
16B	81	33.8	17	.0507	3.97	.75	2.00	1.014	24.25	23.92	22.78	1.05
19B	79	33.0	19	.0654	4.03	.77	1.75	1.154	32.00	27.73	28.00	.99
21A	69	35.3	19	.0311	4.00	1.14	1.88	.412	10.00	24.30	22.23	1.09
22A	70	33.4	15	.0312	3.97	1.14	2.25	.485	9.75	20.11	19.36	1.04
23A	71	33.8	13	.0311	4.00	1.14	2.63	.554	10.00	18.04	17.19	1.05
24	68	32.7	11	.0311	4.00	1.14	3.00	.582	9.90	17.00	15.68	1.08
25A	48	35.3	19	.0391	4.00	1.13	1.88	.524	14.05	26.80	24.50	1.09
26A	49	33.5	15	.0392	3.94	1.13	2.25	.615	14.35	23.34	21.76	1.07
27A	51	33.8	13	.0422	3.97	1.14	2.63	.752	16.30	21.68	20.89	1.04
28	50	32.8	11	.0419	3.97	1.15	3.00	.775	15.70	20.25	19.33	1.05
29B	82	35.4	19	.0519	3.94	1.15	1.88	.677	20.70	30.56	29.19	1.05
30B	76	33.5	15	.0522	3.94	1.15	2.25	.804	23.70	29.48	26.72	1.10
31B	83	33.9	13	.0520	3.97	1.15	2.63	.920	23.25	25.26	24.74	1.02
34B	78	33.5	15	.0652	3.97	1.15	2.25	1.004	32.00	31.87	29.56	1.08
37A	72	32.2	13	.0309	3.97	1.63	2.50	.323	6.96	21.57	23.48	.92
38	73	32.7	11	.0313	3.97	1.65	3.00	.423	7.63	18.05	20.65	.87
39	74	34.7	10	.0309	3.97	1.65	3.50	.515	9.40	18.27	18.25	1.00
40	75	35.7	9	.0310	3.97	1.65	4.00	.583	9.60	16.47	16.58	.99
41A	52	32.2	13	.0420	3.94	1.62	2.50	.444	11.80	26.61	26.62	.99
42	53	32.7	11	.0421	4.00	1.65	3.00	.568	13.80	24.28	24.02	1.01
43	54	34.7	10	.0422	3.94	1.65	3.50	.703	15.40	21.92	21.99	1.00
45B	84	33.2	13	.0528	3.97	1.60	2.50	.570	16.20	28.41	29.96	.95
57A	56	32.3	13	.0408	2.97	1.65	2.50	.416	11.00	26.43	28.71	.92
58A	55	32.8	11	.0407	2.97	1.65	3.00	.550	13.55	24.66	26.20	.94
59A	57	34.7	10	.0396	2.97	1.65	3.50	.659	15.50	23.51	23.90	.98
60	58	35.8	9	.0404	2.97	1.65	4.00	.760	16.80	22.12	22.97	.96
71A	63	34.7	10	.0319	2.50	1.62	3.50	.540	11.30	20.94	21.77	.96
72A	64	35.8	9	.0312	2.47	1.62	4.00	.594	11.80	19.87	20.24	.98
73A	59	32.3	13	.0410	2.47	1.65	2.50	.418	11.30	27.02	30.41	.89
74A	60	32.7	11	.0393	2.47	1.65	3.00	.531	13.30	28.48	27.63	.91
75A	61	34.7	10	.0395	2.47	1.65	3.50	.658	16.20	27.05	26.27	.94
76A	62	35.8	9	.0391	2.44	1.65	4.00	.735	17.00	26.00	25.17	.92
Average of ratios above unity (22 tests) = 1.06												
Average of ratios below unity (18 tests) = .95												
Average of all ratios										(40 tests) = 1.01		
Duplicate strength tests												
7B	172	32.9	19	0.0308	4.00	0.77	1.75	0.543	10.00	18.41	17.27	1.07
11B	173	33.0	19	.0398	4.00	.77	1.75	.702	14.80	21.08	20.11	1.05
22B	174	33.4	15	.0302	4.03	1.15	2.25	.465	9.85	21.18	19.14	1.11
26B	175	33.5	15	.0403	4.00	1.15	2.25	.621	14.60	23.53	22.29	1.06
53B	171	32.3	13	.0300	2.53	1.65	2.50	.306	6.89	22.52	25.30	.89
69C	170	32.2	13	.0306	2.53	1.65	2.50	.312	7.67	24.58	25.57	.96
Average of ratios above unity (4 tests) = 1.07												
Average of ratios below unity (2 tests) = .93												
Average of all ratios										(6 tests) = 1.02		

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TABLE 3  
SHEAR WEBS WITH FLANGED LIGHTENING HOLES  
Rod supports, reamed bolt holes

Specimen	Test	L (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	A <sub>e</sub> (sq in.)	P <sub>coll</sub> (kips)	Exp. T <sub>coll</sub> (net) (kips/sq in.)	Calc. T <sub>coll</sub> (net) (kips/sq in.)	Exp. Calc.
Strength tests												
6B	102	31.2	21	0.0310	5.27	0.77	1.50	0.453	8.72	19.27	18.02	1.07
8B	103	33.8	17	.0313	5.27	.77	2.00	.616	9.40	15.26	13.97	1.09
10B	87	31.3	21	.0404	5.31	.77	1.50	.590	12.20	20.68	19.99	1.03
12B	88	33.8	17	.0407	5.31	.77	2.00	.801	15.15	18.91	15.77	1.20
21B	104	35.3	19	.0314	5.27	1.15	1.88	.410	8.40	20.50	21.31	.96
23B	105	33.9	13	.0314	5.27	1.15	2.63	.556	8.92	16.05	15.69	1.02
25B	89	35.3	19	.0402	5.27	1.13	1.88	.539	12.95	24.02	22.94	1.05
27B	90	33.8	13	.0399	5.27	1.15	2.63	.706	12.15	17.20	17.39	.99
33B	100	35.4	19	.0652	5.27	1.15	1.88	.851	28.90	33.96	27.76	1.22
35B	101	33.8	13	.0652	5.27	1.15	2.63	1.154	29.90	25.91	22.74	1.14
37B	106	32.2	13	.0311	5.27	1.65	2.50	.317	6.90	21.75	22.72	.96
41B	91	32.3	13	.0394	5.31	1.60	2.50	.426	9.25	21.74	23.98	.91
57B	115	32.3	13	.0402	4.27	1.65	2.50	.410	10.00	24.39	25.46	.96
58B	116	32.8	11	.0403	4.24	1.65	3.00	.544	12.20	22.42	22.06	1.02
59B	117	34.8	10	.0412	4.27	1.65	3.50	.686	13.70	19.97	19.83	1.01
61A	118	32.3	13	.0504	4.18	1.60	2.50	.544	14.60	26.82	28.30	.95
62	119	32.7	11	.0525	4.24	1.65	3.00	.709	18.90	26.66	25.97	1.03
63	120	34.7	10	.0529	4.24	1.65	3.50	.881	21.80	24.75	23.80	1.04
64	121	35.7	9	.0529	4.27	1.65	4.00	.995	23.80	23.93	22.03	1.09
65B	122	32.3	13	.0665	4.27	1.65	2.50	.694	21.70	31.26	31.35	1.00
67	124	34.7	10	.0649	4.37	1.63	3.50	1.092	29.45	26.97	26.49	1.02
68	123	35.8	9	.0635	4.37	1.63	4.00	1.204	30.00	24.92	24.71	1.01
69B	107	32.3	13	.0308	3.77	1.65	2.50	.314	6.90	21.96	23.19	.95
70B	108	32.8	11	.0310	3.74	1.65	3.00	.419	8.20	19.59	19.94	.98
71B	109	34.8	10	.0305	3.74	1.65	3.50	.508	9.00	17.72	17.40	1.02
72B	110	35.8	9	.0308	3.77	1.63	4.00	.584	9.55	16.35	15.51	1.05
73B	92	32.2	13	.0405	3.81	1.65	2.50	.413	10.80	26.14	26.10	1.00
74B	93	32.7	11	.0417	3.81	1.65	3.00	.563	13.20	23.45	23.27	1.01
75B	94	34.7	10	.0410	3.81	1.65	3.50	.683	15.00	21.97	20.68	1.06
76B	95	35.7	9	.0409	3.81	1.65	4.00	.769	14.60	18.99	18.89	1.01
77A	111	32.2	13	.0498	3.74	1.60	2.50	.538	14.65	27.24	29.24	.93
78A	112	32.8	11	.0532	3.74	1.60	3.00	.745	19.20	25.78	27.37	.94
79A	113	34.8	10	.0524	3.74	1.65	3.50	.873	22.30	25.56	25.60	1.00
80A	114	35.8	9	.0496	3.77	1.65	4.00	.933	21.85	23.43	22.94	1.02
81B	96	32.2	13	.0649	3.77	1.65	2.50	.662	21.50	32.48	32.70	.99
82B	97	32.8	11	.0635	3.81	1.65	3.00	.857	24.60	28.69	30.23	.95
83B	98	34.8	10	.0652	3.77	1.60	3.50	1.115	30.70	27.53	29.11	.95
84B	99	35.8	9	.0651	3.77	1.65	4.00	1.224	33.30	27.21	28.28	.96
84A	183	35.7	9	.0651	4.38	1.65	4.00	1.224	30.70	25.08	25.24	.99
										Average of ratios above unity (22 tests)=1.05		
										Average of ratios below unity (17 tests)=.96		
										Average of all ratios (39 tests)=1.01		
Duplicate strength tests												
61B	165	32.3	13	.0523	4.31	1.60	2.50	.565	14.70	26.03	28.33	0.92
71C	169	34.7	10	.0303	3.84	1.65	3.50	.505	8.70	17.24	17.26	1.00
77B	161	32.2	13	.0522	3.81	1.60	2.50	.564	15.35	27.23	29.59	.92
78B	162	32.7	11	.0500	3.81	1.60	3.00	.700	16.55	23.64	26.26	.90
79B	163	34.7	10	.0500	3.81	1.65	3.50	.833	20.50	24.62	24.59	1.00
80B	164	35.7	9	.0524	3.81	1.65	4.00	.985	23.50	23.86	23.83	1.00
81A	166	32.2	13	.0630	3.81	1.65	2.50	.643	20.20	31.43	32.25	.97
82A	167	32.7	11	.0842	3.73	1.60	3.00	.899	25.95	28.87	30.43	.95
83A	168	34.7	10	.0651	3.88	1.65	3.50	1.084	29.60	27.31	28.80	.95
										Average of ratios above unity (2 tests)=1.00		
										Average of ratios below unity (7 tests)=.94		
										Average of all ratios (9 tests)=.96		



TABLE 4  
SHEAR WEBS WITH FLANGED LIGHTENING HOLES  
[Bar supports, drilled bolt holes]

Specimen	Test	L (in.)	Number of holes	t (in.)	h (in.)	D (in.)	b (in.)	A <sub>e</sub> (sq.in.)	P <sub>coll</sub> (kips)	Exp. $\tau$ (net) (kips/sq.in.)	Calc. $\tau$ (net) (kips/sq.in.)	Exp. Calc.
14A	14	31.3	21	.0495	4.06	0.75	1.50	0.743	14.70	19.80	25.33	0.78
15A	15	32.9	19	.0510	4.06	.75	1.75	.918	19.20	20.92	23.90	.88
16A	16	33.7	17	.0507	4.03	.75	2.00	1.014	18.50	18.24	22.53	.81
19A	3	32.9	19	.0658	4.00	.77	1.75	1.161	28.00	24.12	28.22	.85
29A	18	35.3	19	.0509	4.03	1.15	1.88	.664	20.25	30.49	28.79	1.06
30A	19	33.4	15	.0522	4.06	1.17	2.25	.789	18.40	23.31	26.57	.88
31A	20	33.7	13	.0527	4.06	1.15	2.63	.933	21.40	22.94	24.61	.93
32	21	32.7	11	.0527	4.06	1.15	3.00	.975	19.60	20.10	23.23	.87
33A	27	35.4	19	.0651	4.03	1.15	1.88	.850	26.20	30.84	31.31	.99
34A	28	33.5	15	.0649	4.06	1.15	2.25	1.000	28.45	28.46	29.18	.98
35A	13	33.8	13	.0651	4.03	1.15	2.63	1.152	27.80	24.13	27.90	.86
36	29	32.7	11	.0654	4.06	1.15	3.00	1.210	29.00	23.97	26.79	.89
45A	22	32.3	13	.0521	4.13	1.60	2.50	.563	14.95	26.57	29.48	.90
46	23	32.7	11	.0518	4.03	1.60	3.00	.725	17.40	23.99	26.91	.89
47	24	34.7	10	.0516	4.06	1.58	3.50	.892	20.10	22.54	24.65	.91
48	26	35.7	9	.0510	4.03	1.63	4.00	.967	21.95	22.70	23.47	.97
49A	6	32.2	13	.0654	4.06	1.63	2.50	.683	19.50	28.56	32.04	.89
50	9	32.7	11	.0648	4.06	1.65	3.00	.875	23.20	26.52	29.92	.89
51	10	34.7	10	.0657	4.03	1.60	3.50	1.123	27.90	24.84	28.42	.87
52	11	35.8	9	.0642	4.00	1.60	4.00	1.233	30.60	24.82	27.05	.92
53A	33	32.2	13	.0311	3.06	1.63	2.50	.325	7.09	21.84	24.46	.89
56	34	35.8	9	.0317	3.06	1.65	4.00	.596	10.25	17.20	18.43	.93
65A	30	32.2	13	.0655	3.19	1.60	2.50	.707	22.00	31.10	33.62	.93
66	31	32.8	11	.0662	3.06	1.63	3.00	.907	26.05	28.72	32.55	.88
70A	35	32.7	11	.0312	2.56	1.62	3.00	.431	8.40	19.51	22.93	.85

Average of ratios above unity (1 test) = 1.06

Average of ratios below unity (24 tests) = .89

Average of all ratios (25 tests) = .90

TABLE 6  
SHEAR WEBS WITH PLAIN LIGHTENING HOLES  
[Rod supports, reamed bolt holes]

Specimen	Test	L <sub>e</sub> (in.)	Number of holes n	t (in.)	h (in.)	D (in.)	b (in.)	P <sub>coll</sub> (kips)	Exp. $\tau_{coll}$ (gross) (kips/sq.in.)	A <sub>e</sub> (sq.in.)	Exp. $\tau_{coll}$ (net) (kips/sq.in.)
Strength tests											
NF1A	156	28.05	25	0.0311	5.26	1.00	1.25	3.61	4.14	0.187	19.35
NF2A	155	27.95	25	.0402	5.27	1.00	1.25	5.67	5.05	.241	23.51
NF3A	154	28.02	25	.0514	5.27	1.00	1.25	8.40	5.83	.308	27.24
NF4A	153	27.89	25	.0649	5.27	1.00	1.25	11.90	6.58	.389	30.56
Duplicate strength tests											
NF1B	189	27.95	25	0.0307	5.34	1.00	1.25	3.60	4.20	0.184	19.54
NF2B	188	27.98	25	.0400	5.27	1.00	1.25	5.84	5.22	.240	24.33
NF3B	187	27.97	25	.0499	5.31	1.00	1.25	8.36	5.99	.299	27.92
NF4B	186	27.99	25	.0643	5.20	1.00	1.25	12.45	6.83	.386	32.27





TABLE 5  
SHEAR WEBS WITH BEADED LIGHTENING HOLES

Specimen	Test	$L_e$ (in.)	Number of holes $n$	$t$ (in.)	$h$ (in.)	$D$ (in.)	$b$ (in.)	$P_{all}$ (kips)	Exp. $T_{coll} (gross)$ (kips)	Calc. $T_{coll} (net)$ (kips)	Exp. Calc.
Strength tests											
Rod supports, reamed bolt holes											
101A	147	29.64	11	0.0316	5.27	1.05	3.00	9.00	9.61	9.46	1.02
102A	141	29.77	11	0.0394	5.27	1.10	3.00	12.70	10.83	11.15	.97
103A	125	29.70	11	0.0508	5.27	1.05	3.00	20.10	13.32	13.58	.98
104A	140	29.74	11	0.0641	5.27	1.03	3.00	30.40	15.95	16.20	.98
105	150	32.23	11	0.0304	4.29	1.05	3.00	10.45	10.67	10.72	.99
106	151	33.17	10	0.0302	4.27	1.08	3.50	11.60	11.58	10.70	1.08
107	152	33.17	9	0.0300	4.27	1.08	4.00	12.35	12.41	10.66	1.16
108	143	30.24	11	0.0400	4.27	1.05	3.00	15.30	12.65	13.30	.95
109	144	32.24	10	0.0400	4.27	1.07	3.50	18.20	14.12	13.30	1.06
110	145	33.24	9	0.0394	4.27	1.07	4.00	19.75	15.08	13.12	1.15
111	128	30.20	11	0.0502	4.27	1.03	3.00	22.00	14.51	15.80	.92
112	133	32.19	10	0.0521	4.31	1.03	3.50	25.90	15.44	16.10	.96
113	134	33.20	9	0.0506	4.27	1.03	4.00	28.50	16.96	15.88	1.07
114	135	30.19	11	0.0640	4.31	1.03	3.00	34.00	17.60	18.76	.94
115	136	32.16	10	0.0628	4.31	1.03	3.50	39.00	19.31	18.50	1.04
116	137	33.17	9	0.0643	4.34	1.03	4.00	41.80	19.60	18.72	1.05
119	148	31.67	10	0.0311	5.27	1.60	3.50	9.95	10.10	9.34	1.08
120	149	32.69	9	0.0300	5.31	1.60	4.00	9.40	9.59	9.04	1.06
123	142	31.67	10	0.0376	5.27	1.60	3.50	14.00	11.76	10.76	1.09
127	126	31.70	10	0.0503	5.27	1.60	3.50	20.90	13.11	13.50	.97
128	127	32.70	9	0.0506	5.27	1.60	4.00	22.25	13.45	13.56	.99
131	138	31.63	10	0.0635	5.31	1.60	3.50	30.70	15.29	16.00	.96
132	139	32.70	9	0.0637	5.27	1.60	4.00	31.90	15.31	16.10	.95
Average of ratios above unity (11 tests) = 1.08											
Average of ratios below unity (12 tests) = .96											
Average of all ratios (23 tests) = 1.02											
Duplicate strength tests											
Rod supports, reamed bolt holes											
104B	157	29.64	11	0.0641	5.27	1.03	3.00	30.70	16.16	16.16	1.00
Strength tests											
Bar supports, reamed bolt holes											
101B	160	29.74	11	0.0303	3.95	1.05	3.00	11.20	12.43	11.38	1.09
102B	159	29.70	11	0.0390	3.97	1.04	3.00	15.80	13.64	13.78	.99
103B	158	29.67	11	0.0511	3.97	1.03	3.00	23.60	15.57	16.90	.92
Average of all ratios (3 tests) = 1.00											



TABLE 7  
PERMANENT-SET TESTS

Specimen	$\delta$ (in.)	Set after run 1 (in.)	Set after run 2 (in.)	P (kips)	$\tau$ (kips/sq.in.)
Webs with flanged holes and bar supports					
7B	179 $\times 10^{-4}$	11 $\times 10^{-4}$	0 $\times 10^{-4}$	6.9	12.7 <sup>a</sup>
11B	193	20	2	10.6	5.1
22B	74	18	2	6.4	3.8
26B	98	16	5	9.8	5.8
53B	195	11	0	4.5	14.7
69C	175	14	0	4.5	14.4
Webs with flanged holes and rod supports					
61B	296	111	10	10.0	17.7 <sup>a</sup>
71C	225	67	13	6.0	11.9
77B	272	24	15	10.0	17.7
78B	254	36	12	12.0	14.3
79B	272	51	17	4.0	16.9
80B	210	104	9	15.0	15.2
81A	352	150	16	14.0	21.8
82A	278	52	13	16.5	16.3
83A	325	25	10	20.5	19.0
84A	282	34	3	20.0	16.4
Webs with beaded holes and bar supports					
101B	212	23	9	6.0	6.7 <sup>b</sup>
102B	210	26	5	10.0	8.6
103B	288	99	13	13.0	8.6
Webs with beaded holes and rod supports					
104B	375	82	5	20.0	10.5 <sup>b</sup>
Webs with plain holes and rod supports					
NF1B	181	12	0	2.4	13.0 <sup>a</sup>
NF2B	153	10	6	3.75	15.6
NF3B	170	22	4	5.5	18.4
NF4B	208	111	5	7.8	20.2
Solid webs with bar supports					
137B	154	4	1	3.6	3.1 <sup>a</sup>
138B	132	0	0	2.3	3.4
139B	242	194	7	27.0	19.8
Solid webs with rod supports					
135B	230	50	2	3.0	6.5 <sup>a</sup>
136B	289	27	2	6.3	5.1

<sup>a</sup> $\tau$  (net).<sup>b</sup> $\tau$  (gross).

TABLE 8

COMPARISONS BETWEEN WEBS WITH  
FLANGED HOLES AND WEBS WITH BEADED HOLES

Flanged holes :  $D/h = 0.8$ Beaded holes :  $D = 1.6$  in.,  $b = 3.0$  in.

h (in.)	t (in.)	S (lb)	$V_{\text{flanged}}$ (in <sup>3</sup> /in.)	$V_{\text{beaded}}$ (in <sup>3</sup> /in.)
4	0.064	3500	0.1811	0.1745
4	.025	790	.0621	.0745
8	.064	4840	.3216	.4170
8	.025	1520	.1225	.2150



L-402

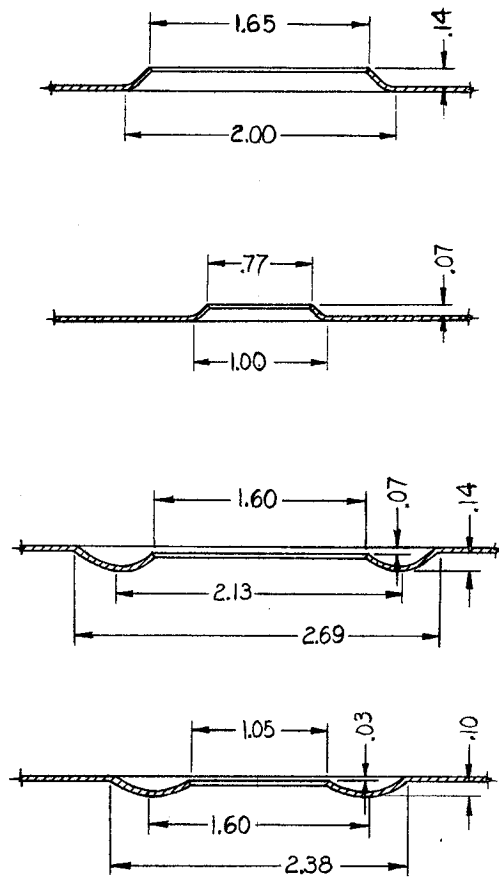
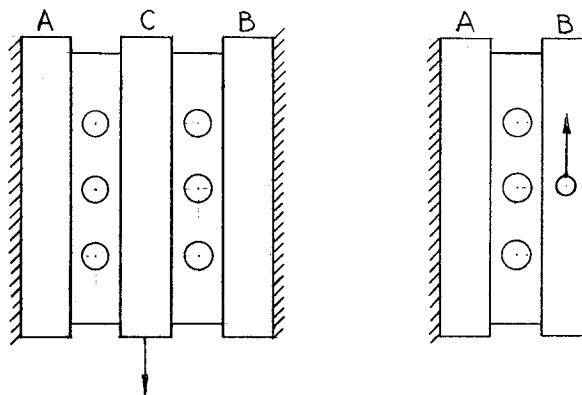


Figure 1.- Typical cross sections of flanges and beads.



(a) Double jig.

(b) Single jig.

Figure 2.- Schematic arrangements of test jigs.



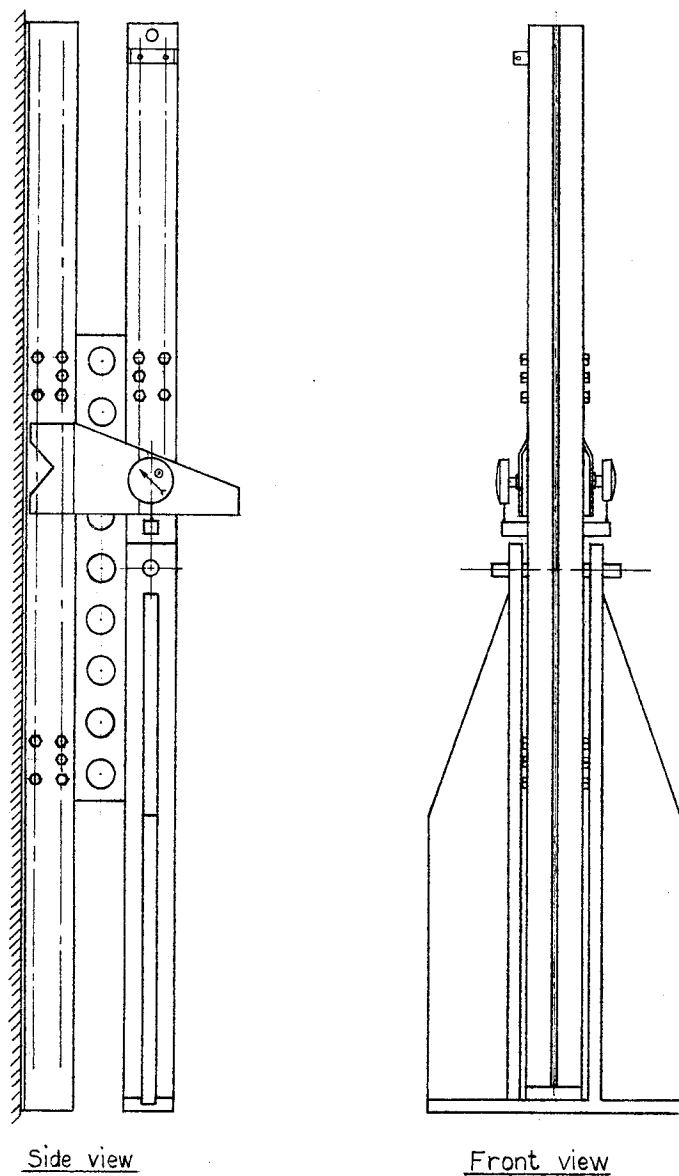
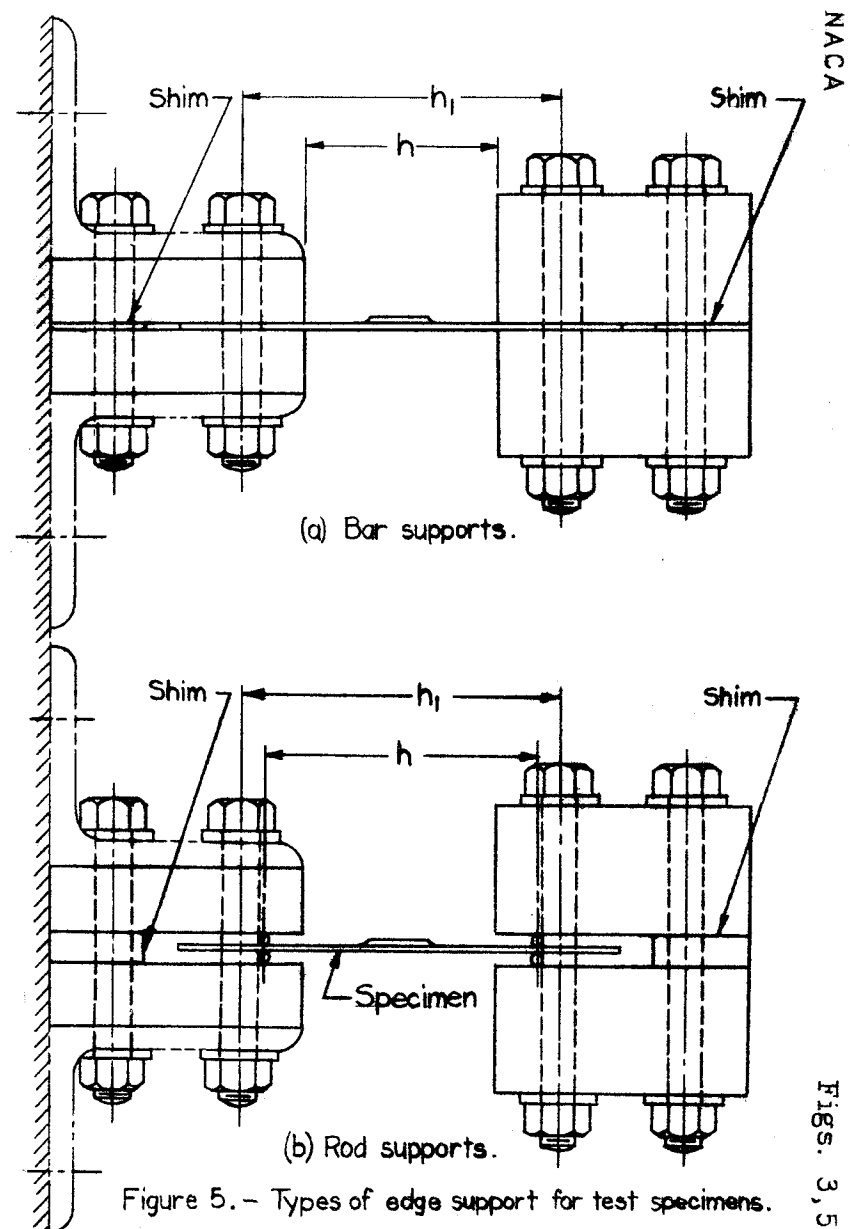


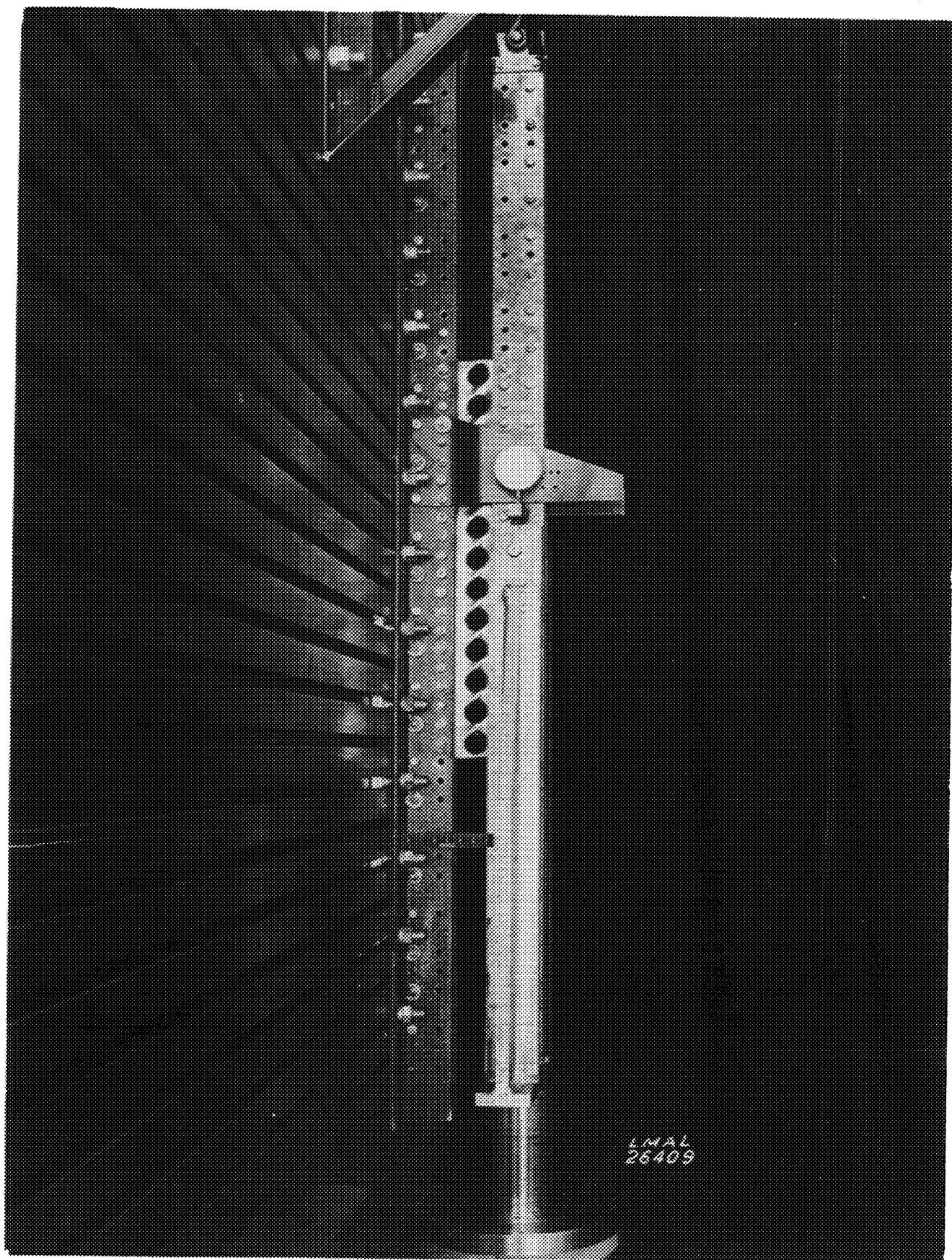
Figure 3. - Test jig.







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NOT REPRODUCIBLE

Figure 4.- Test jig in operation.



L-402

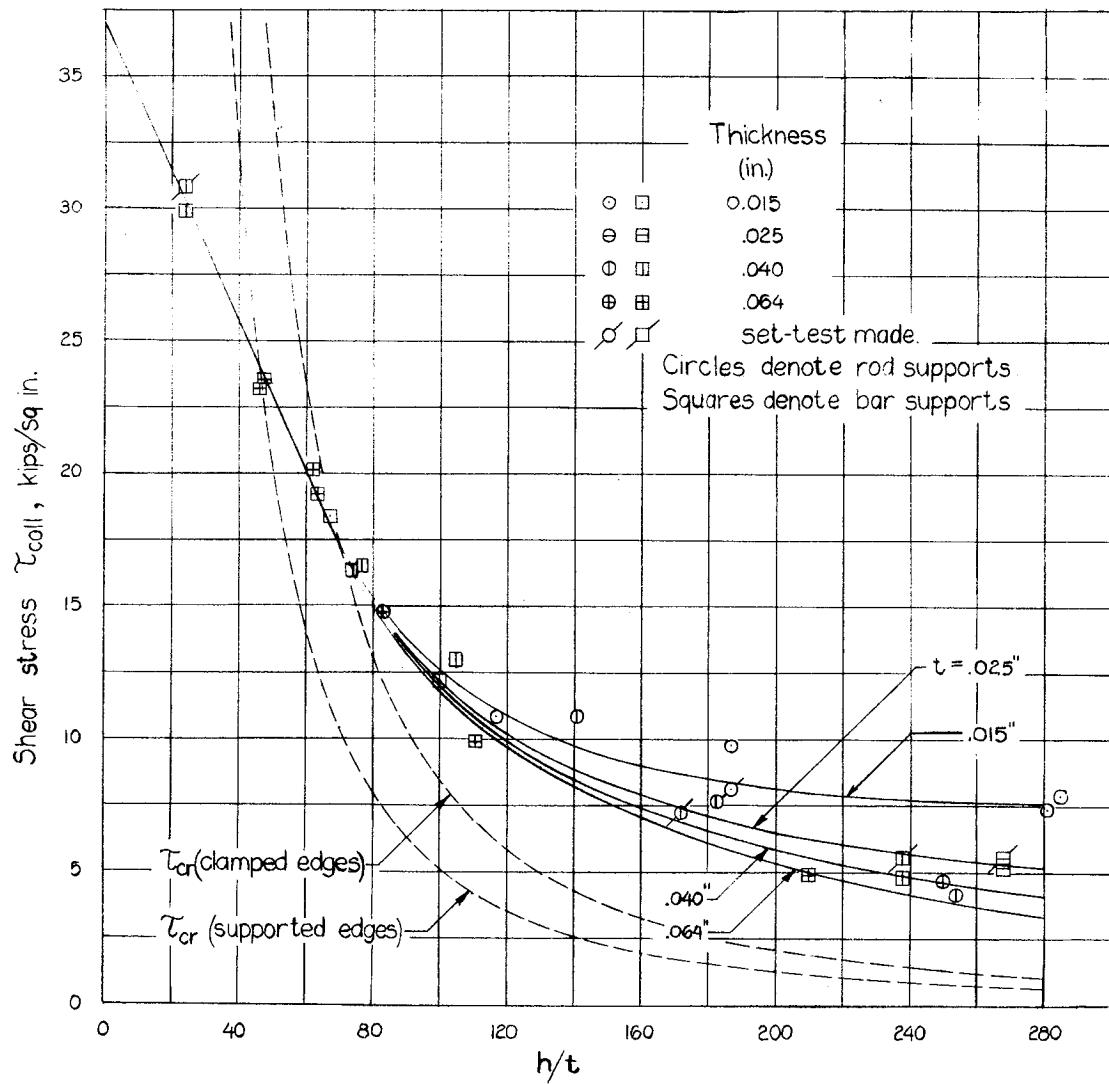


Figure 6 - Experimental shear stresses causing collapse of solid flat sheets.



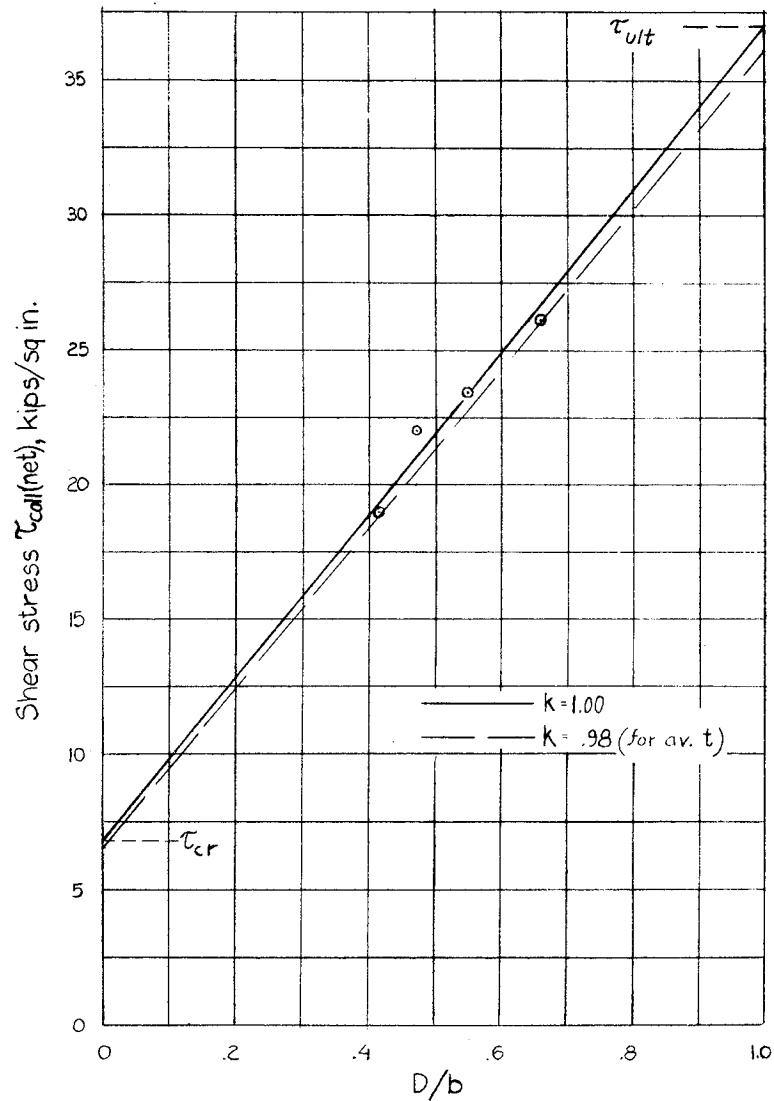
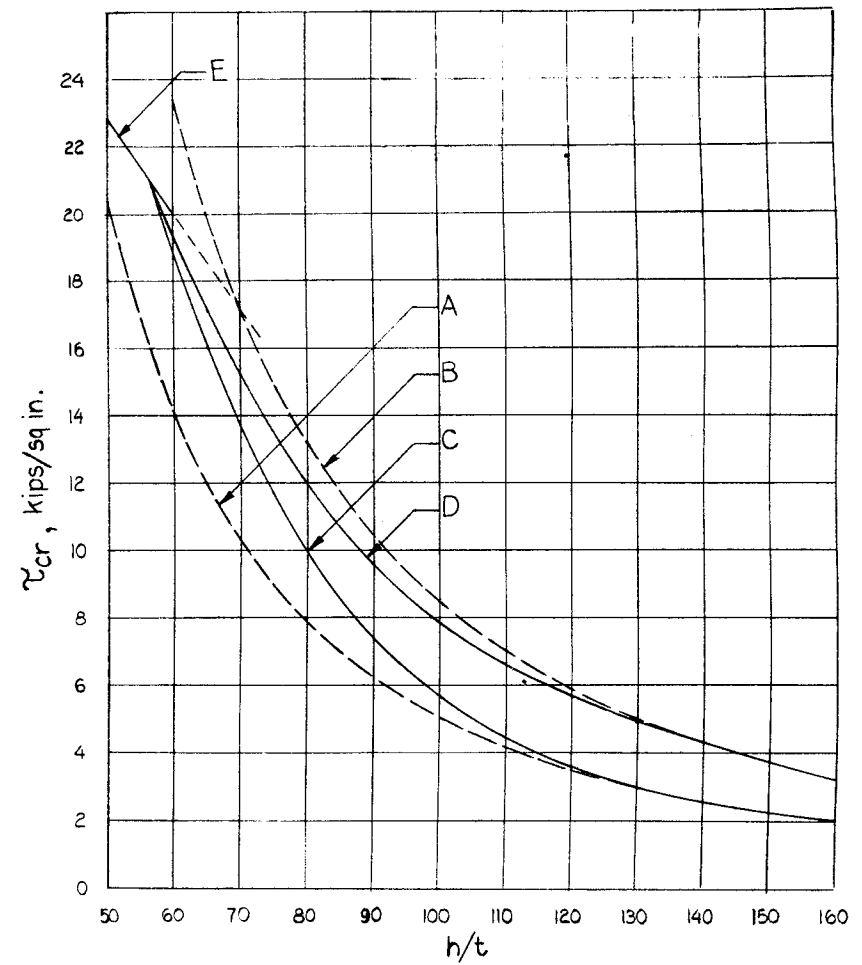


Figure 7. - Experimental shear stresses for flanged-hole webs with rod supports and reamed bolt holes (specimens 73B, 74B, 75B, and 76B).



- Curve A Ideally supported edges
- Curve B Ideally clamped edges
- Curve C Rod supports
- Curve D Bar supports
- Curve E Cut-off ( $\tau_{coll}$  for solid sheet)

Figure 8. - Critical shear stresses used for computing  $\tau_{coll}$ .



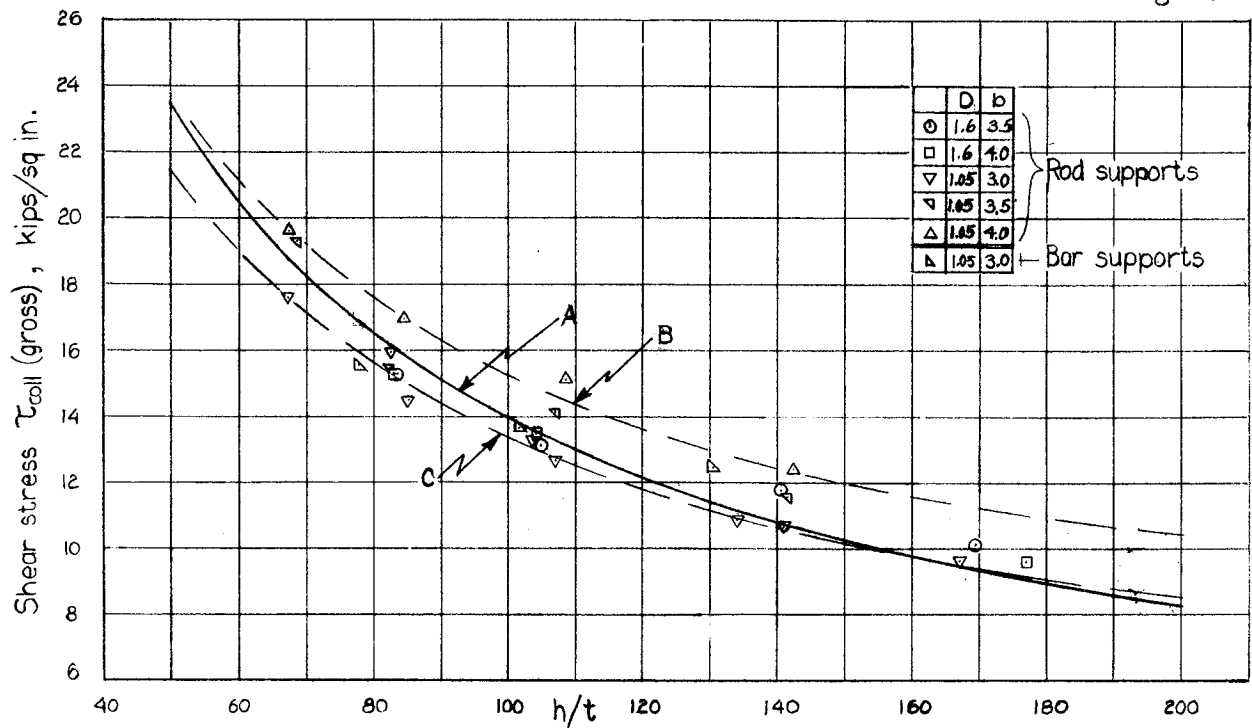


Figure 9.- Experimental shear stresses causing collapse of webs with beaded holes. Curve A from formula (9); curve B for  $b=4$  inches; curve C for  $b=3$  inches.

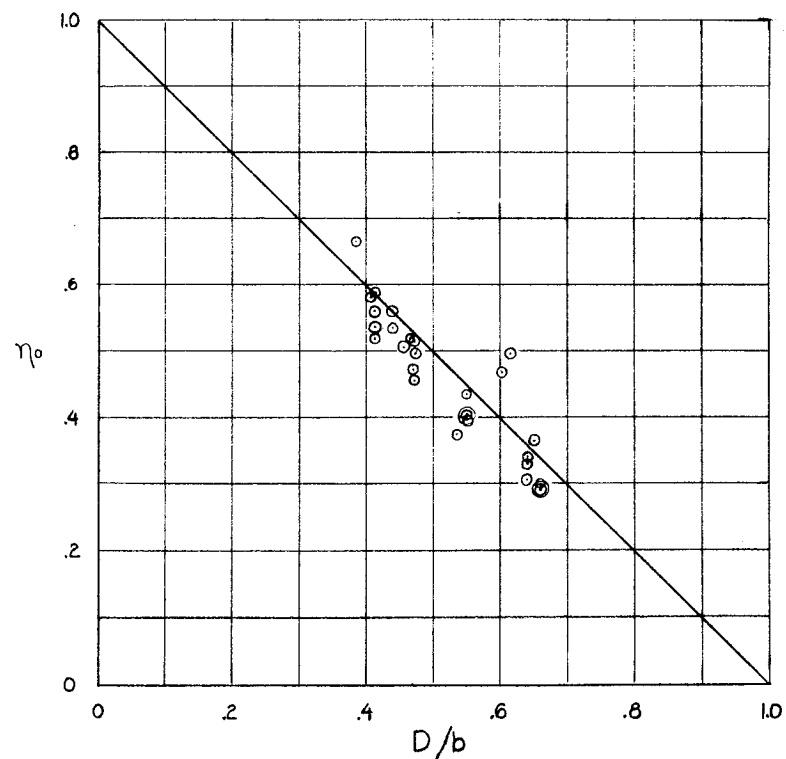


Figure 10.- Shear-stiffness factor  $\eta_0$  for webs with flanged holes 0.040, 0.051, and 0.064 inch thick.





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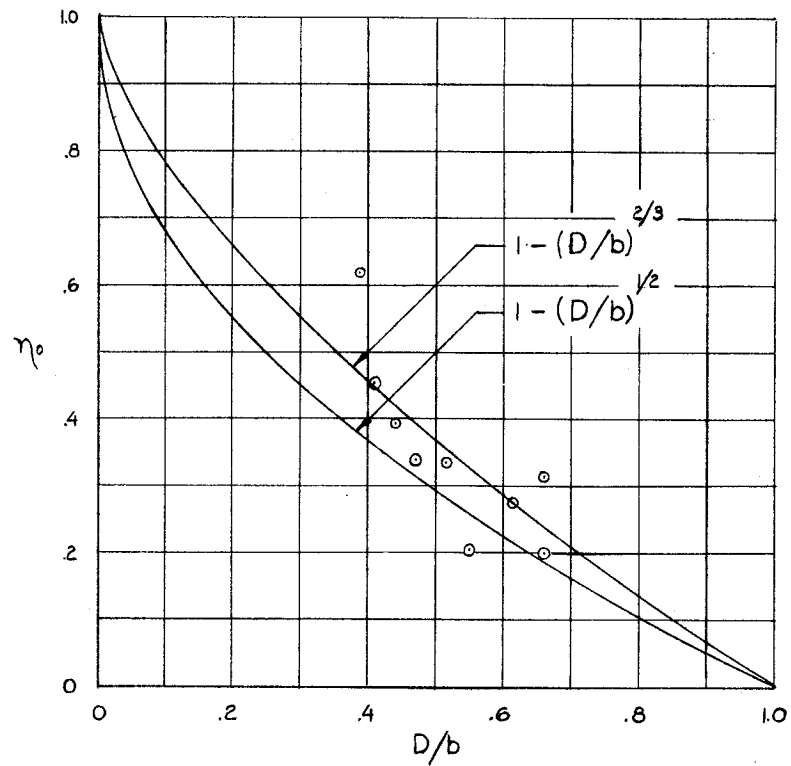


Figure 11.- Shear-stiffness factor  $\eta_0$  for webs with flanged holes 0.032 inch thick.

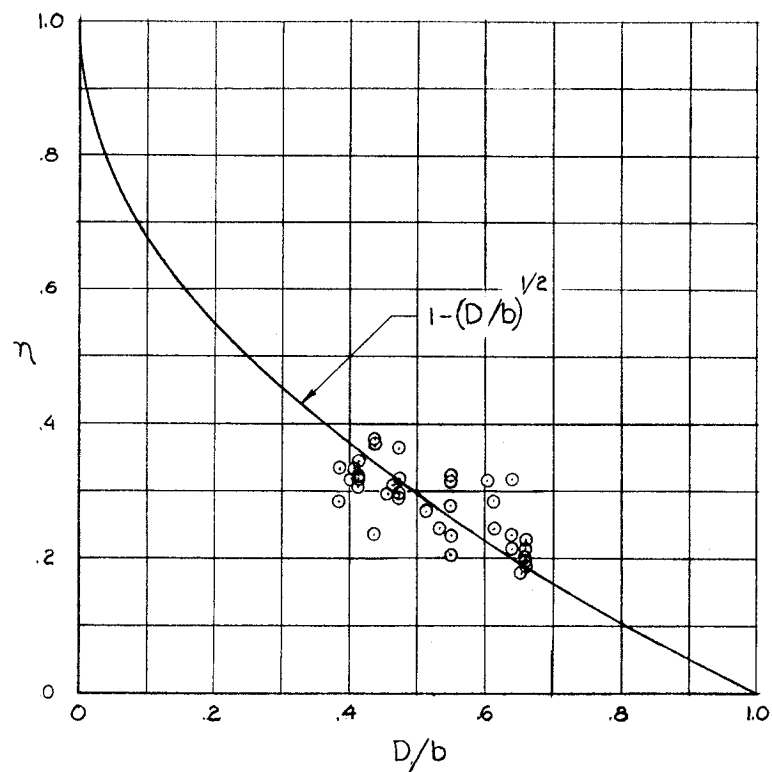


Figure 12.- Shear-stiffness factor  $\eta$  for webs with flanged holes 0.032 to 0.064 inch thick.



NACA

Figs. 13, 14

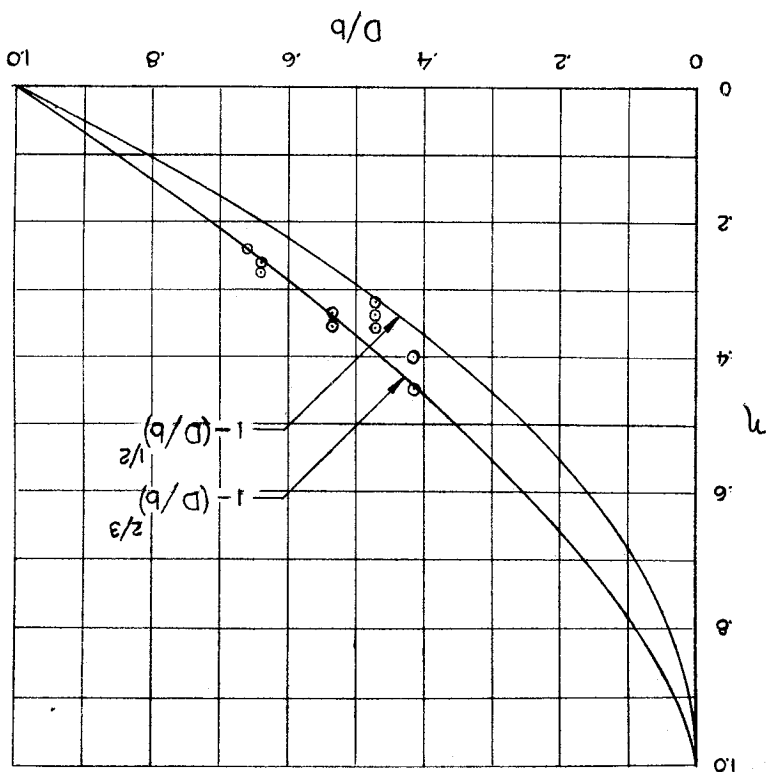


Figure 13.- Shear-stiffness factor  $\eta$  for webs with flanged holes; webs preloaded twice to  $2/3 F_{all}$ .

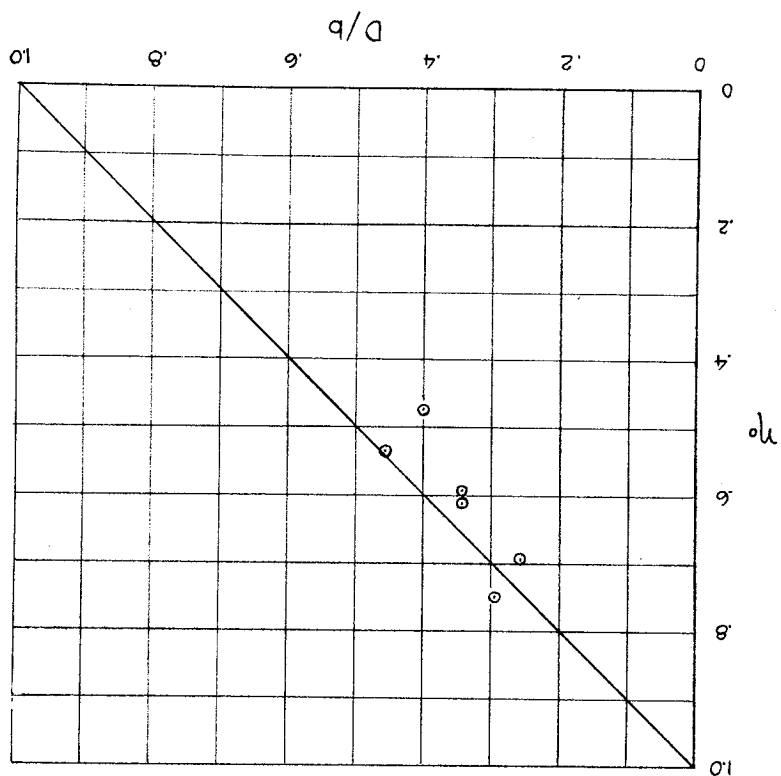


Figure 14.- Shear-stiffness factor  $\eta_0$  for webs with beaded holes 0.064 inch thick.



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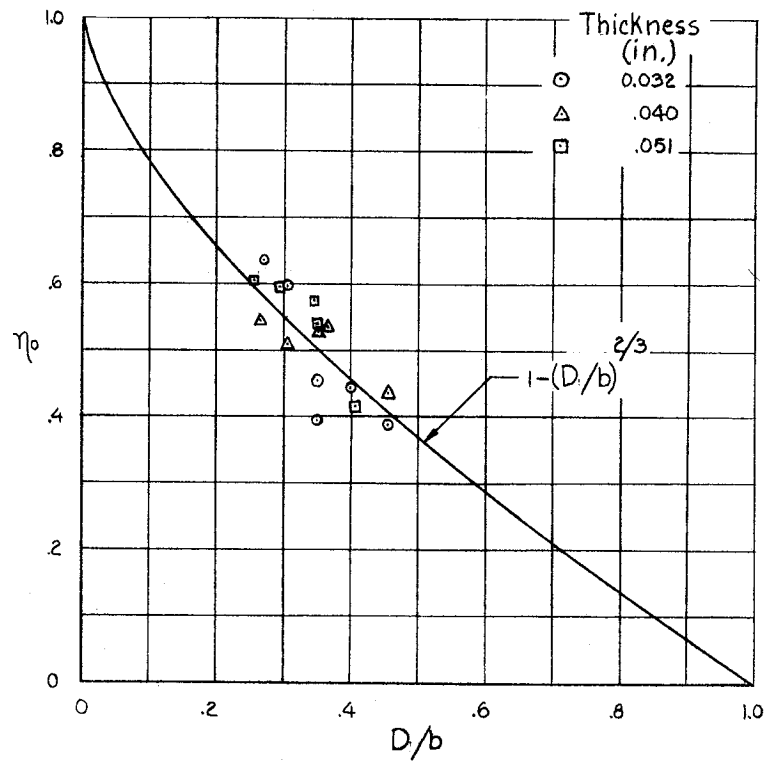


Figure 15.- Shear-stiffness factor  $\eta_0$  for webs with beaded holes 0.032 to 0.051 inch thick.

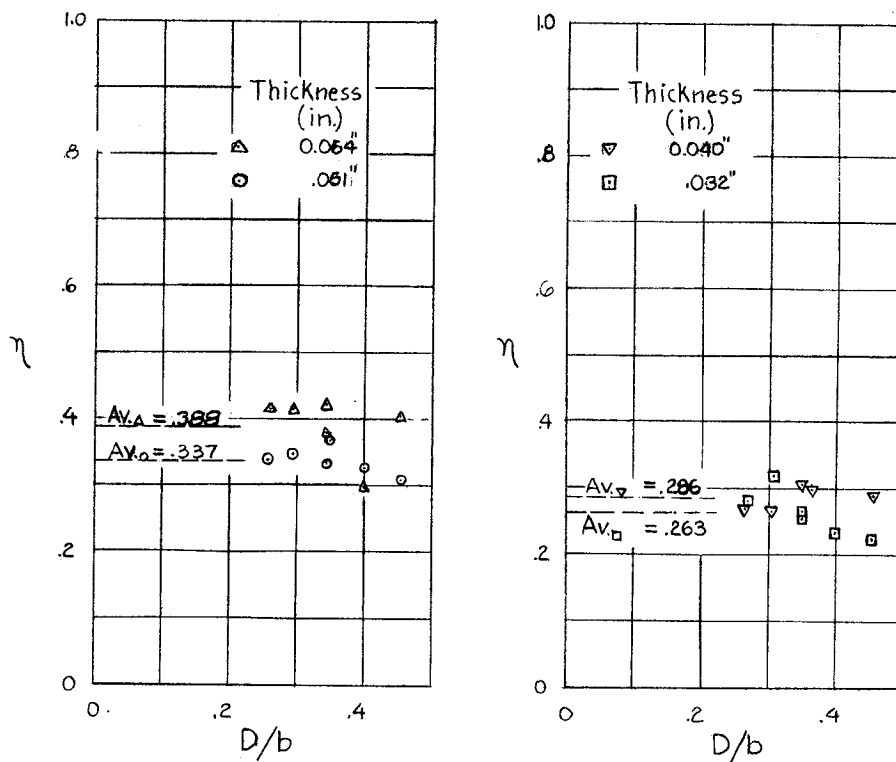


Figure 16.- Shear-stiffness factor  $\eta$  for webs with beaded holes.



Figure 17. - Design chart for shear webs of 24S-T alloy with round flanged holes.

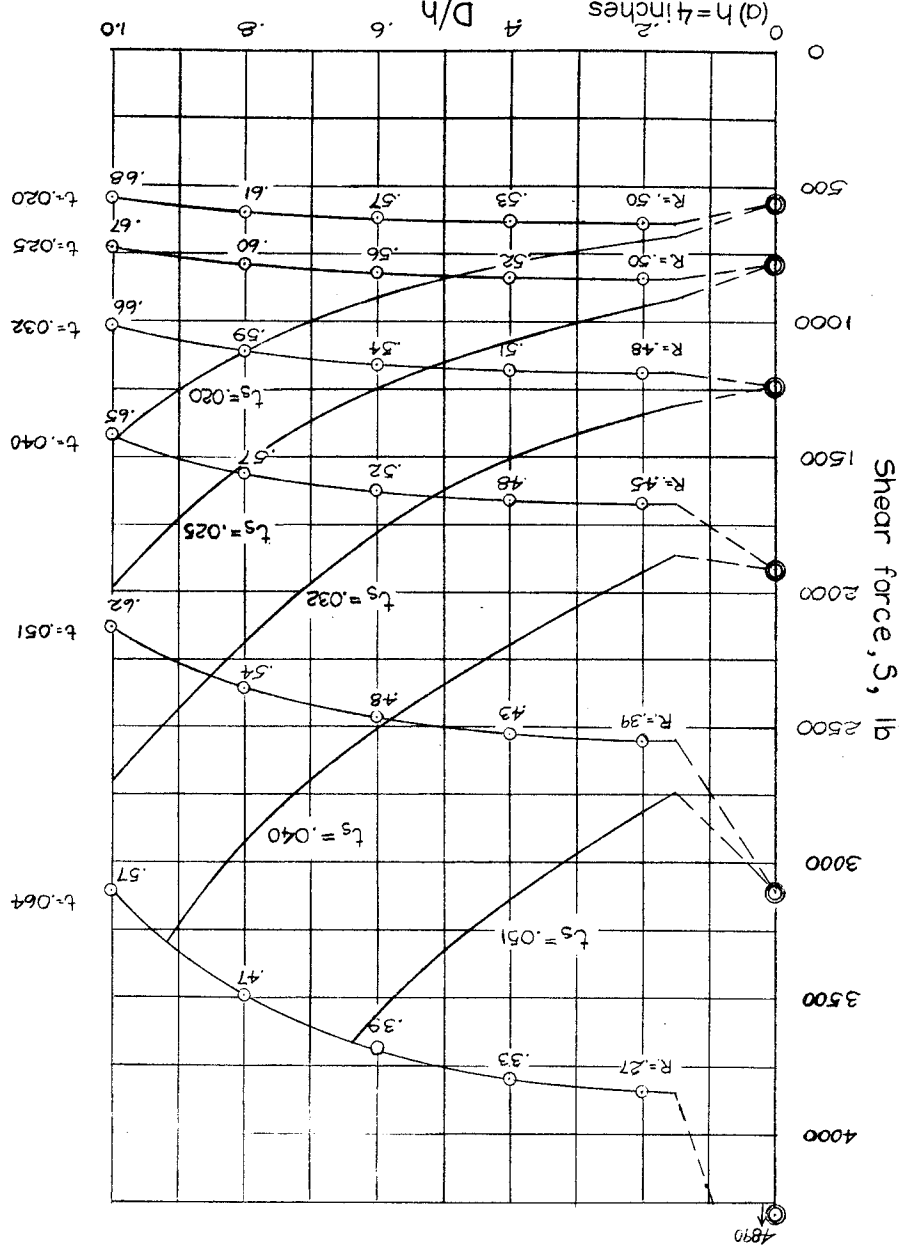


Figure 17. - Concluded.

(b)  $h = 8$  inches. Experimental verification limited to  $h/t < 180$ .

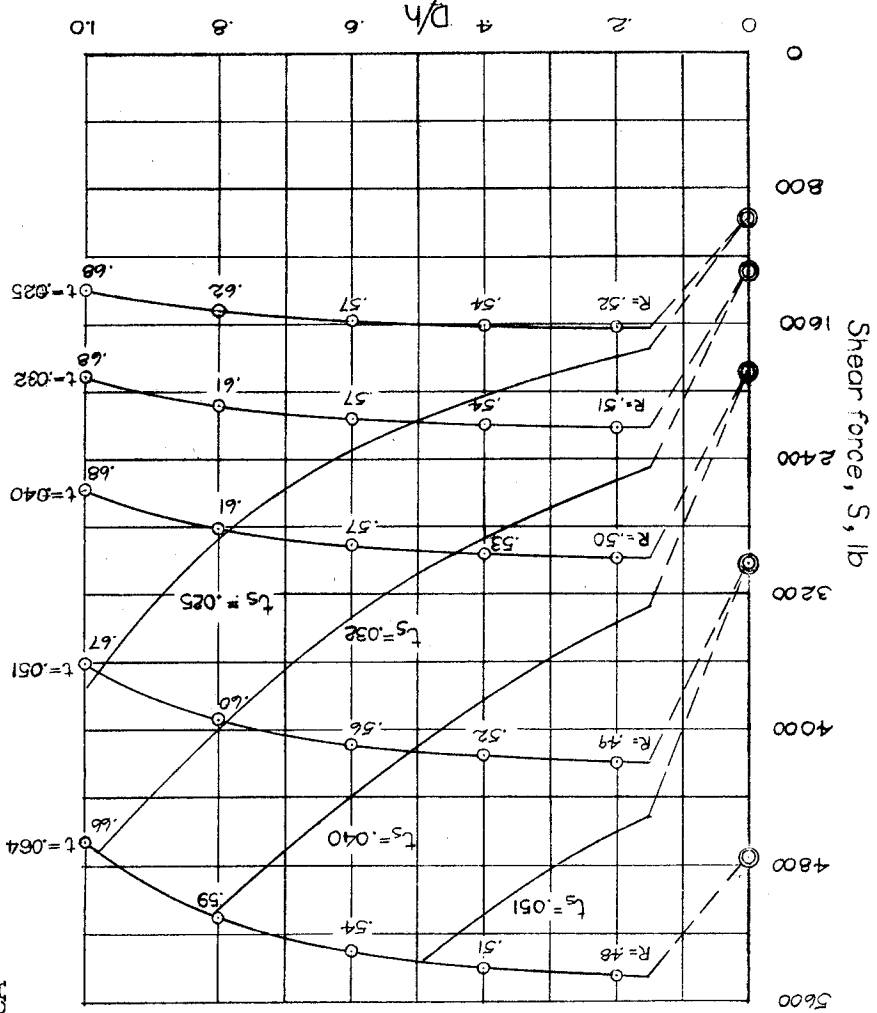
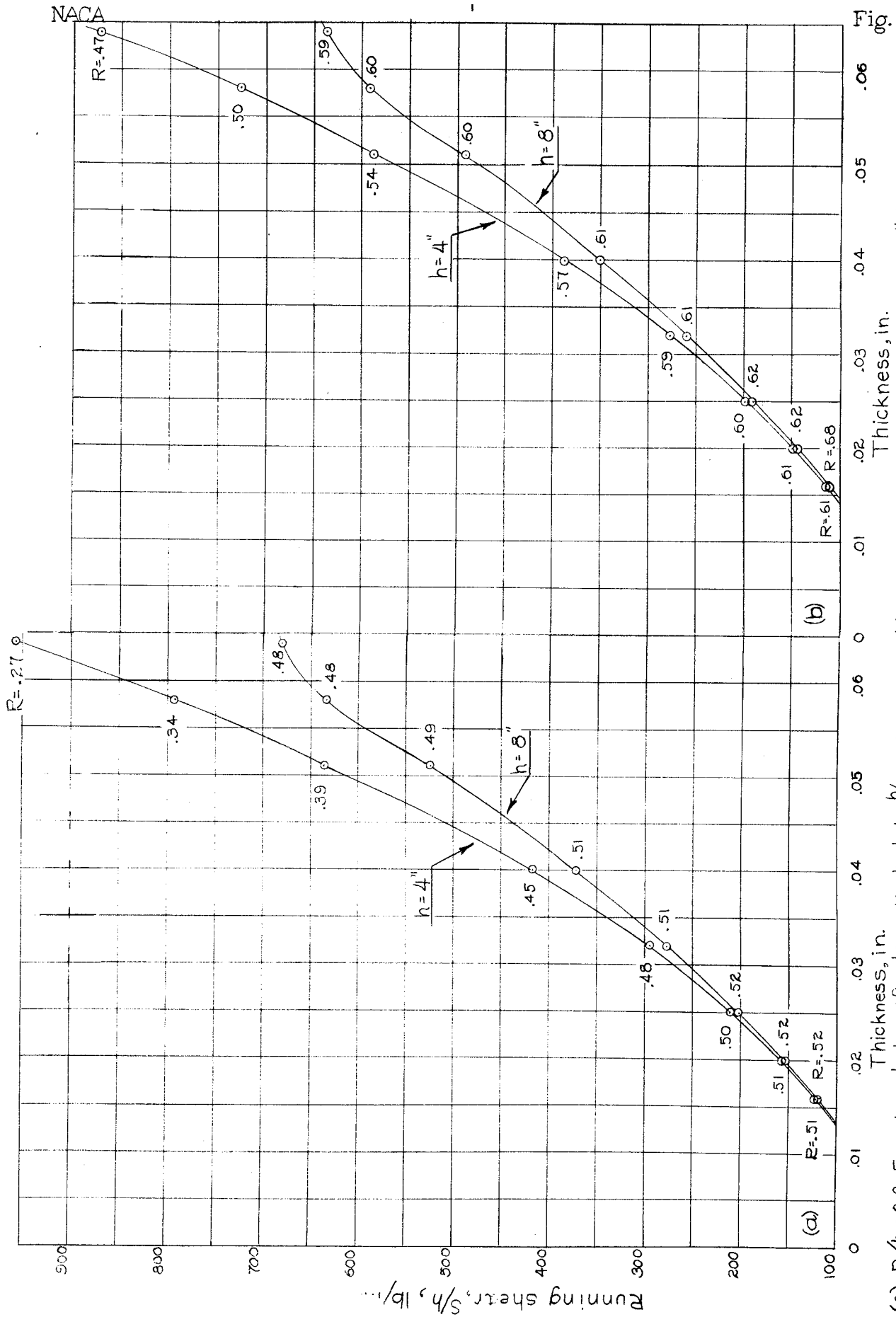


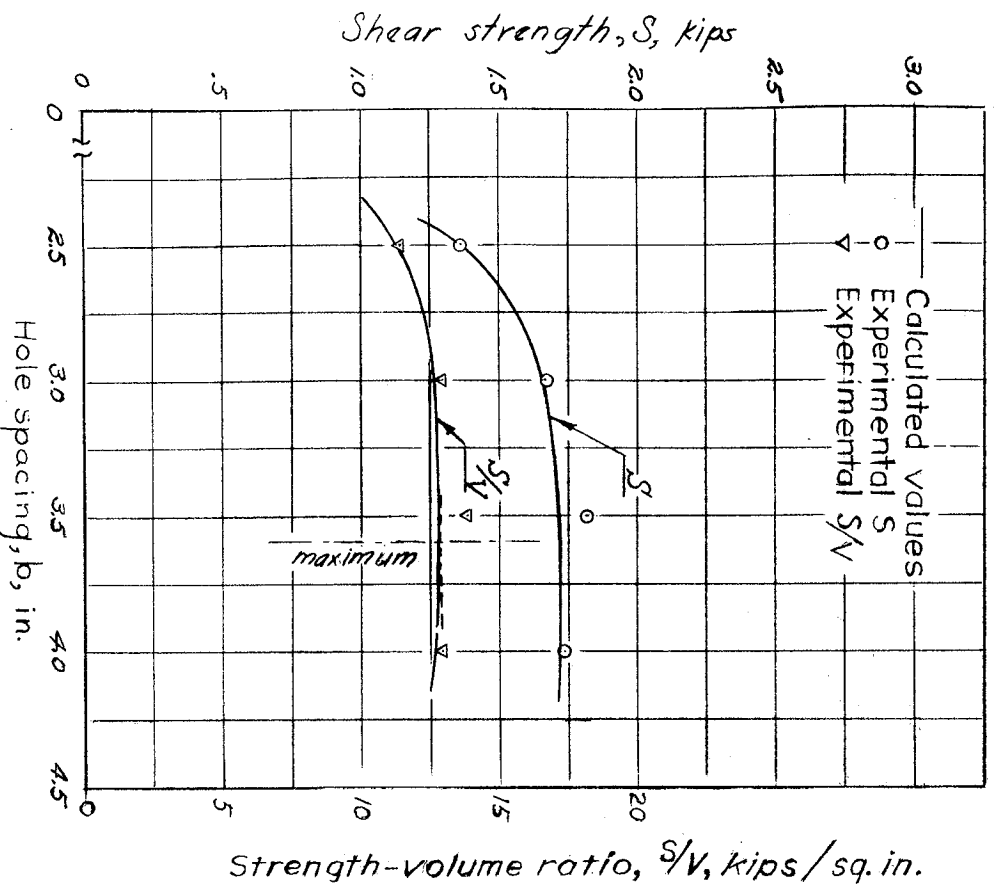
Fig. 17



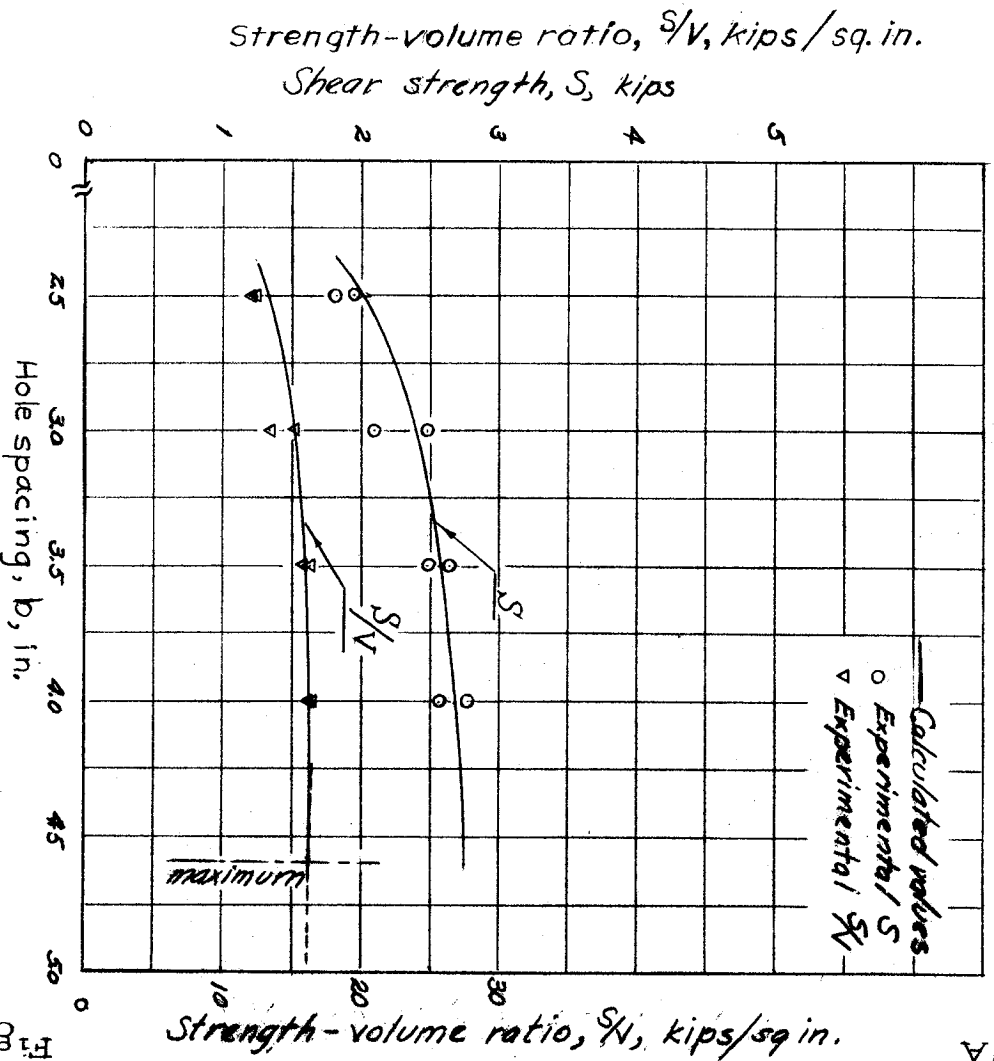






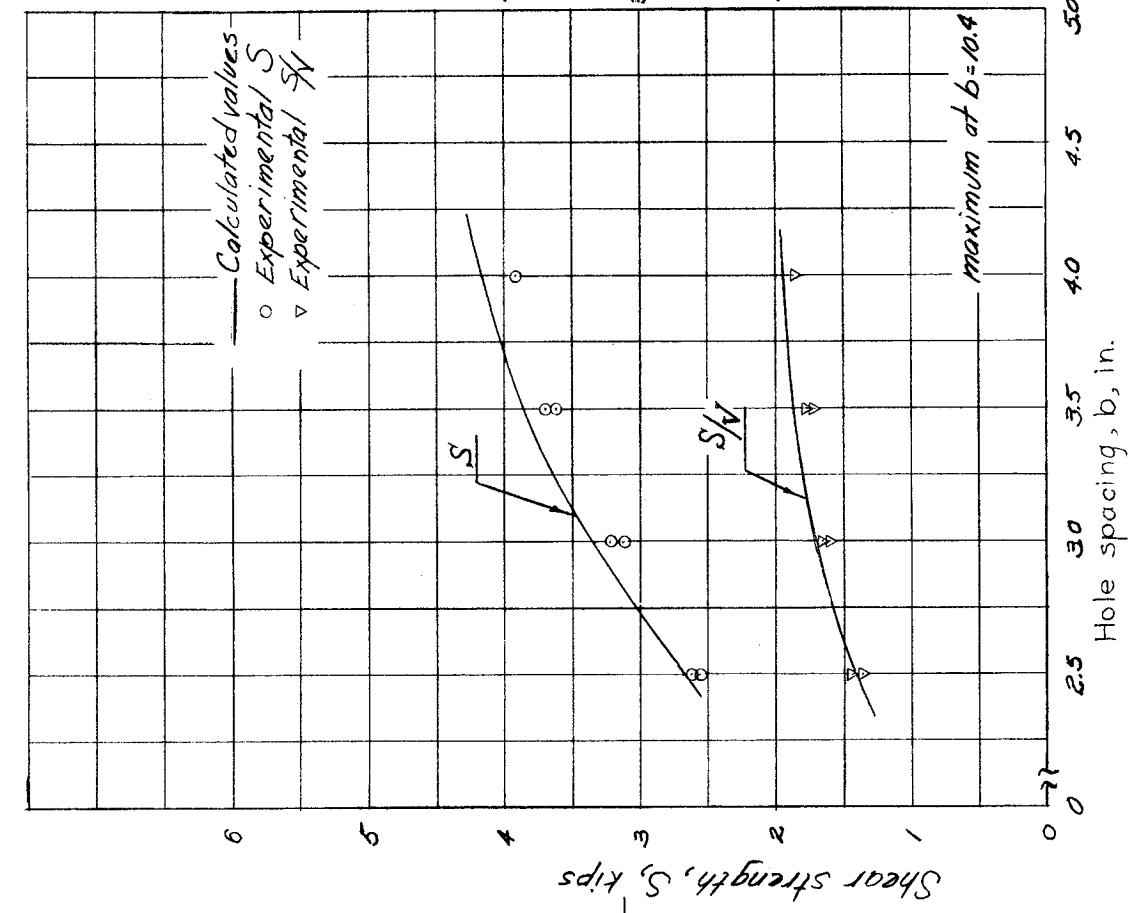


(a) Specimens 73-76.  
 Shear strength and strength-  
 volume ratio.



(b) Specimens 77-80 A and B.  
 Continued.





(c) Specimens 81-84 A and B.  
 Figure 19. - Concluded.

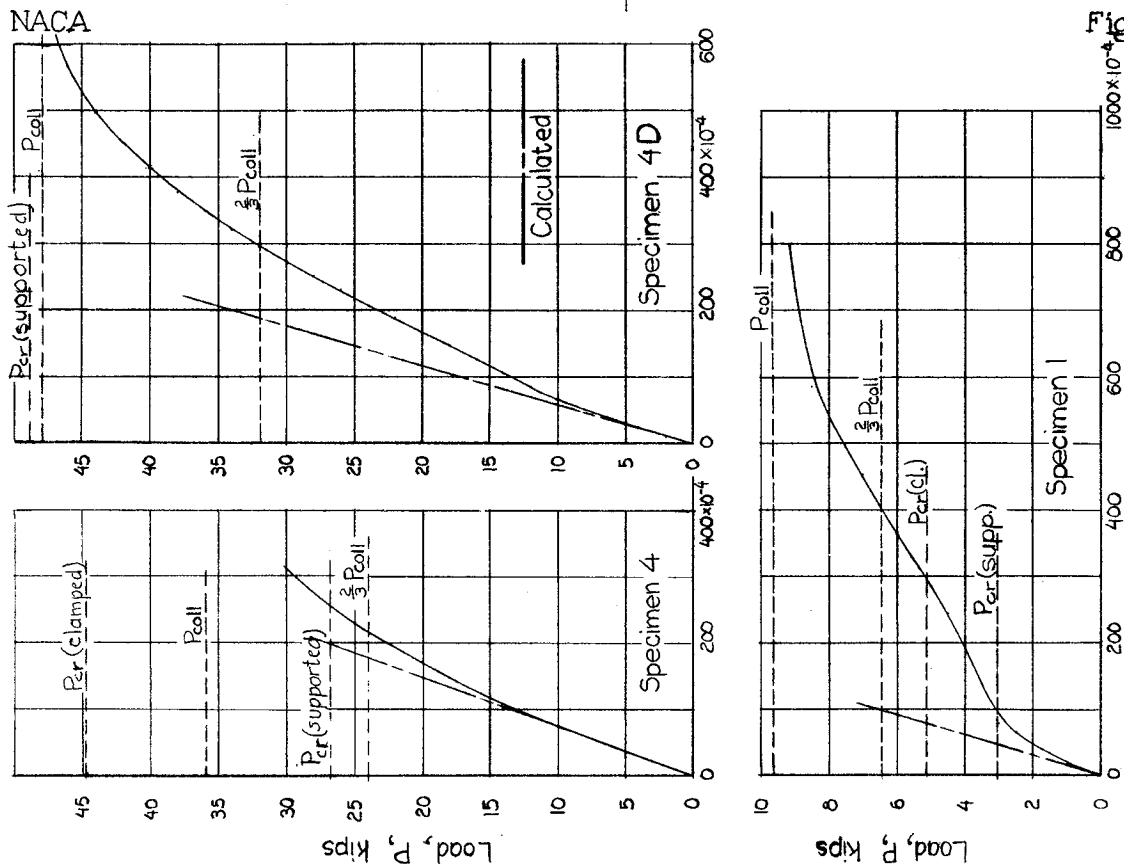


Figure 20. - Average load-displacement curves for solid webs with bar supports.

Fig. 19c, 20



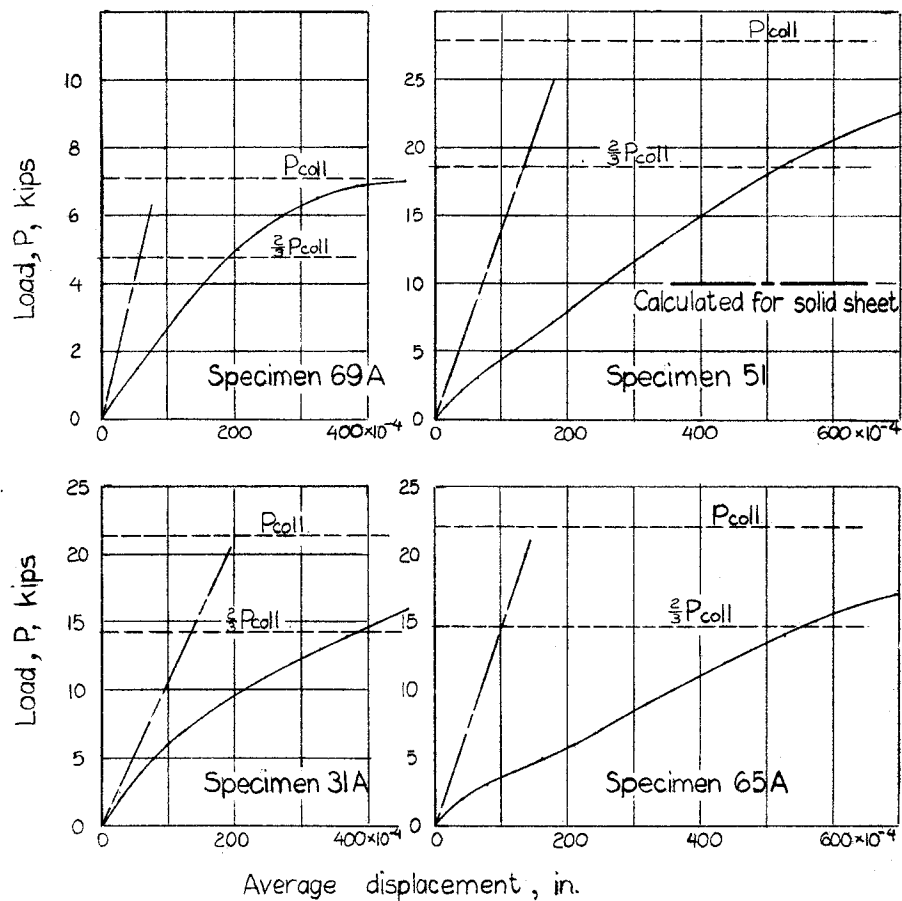


Figure 21.- Load-displacement curves for flanged-hole webs with bar supports and drilled bolt holes.

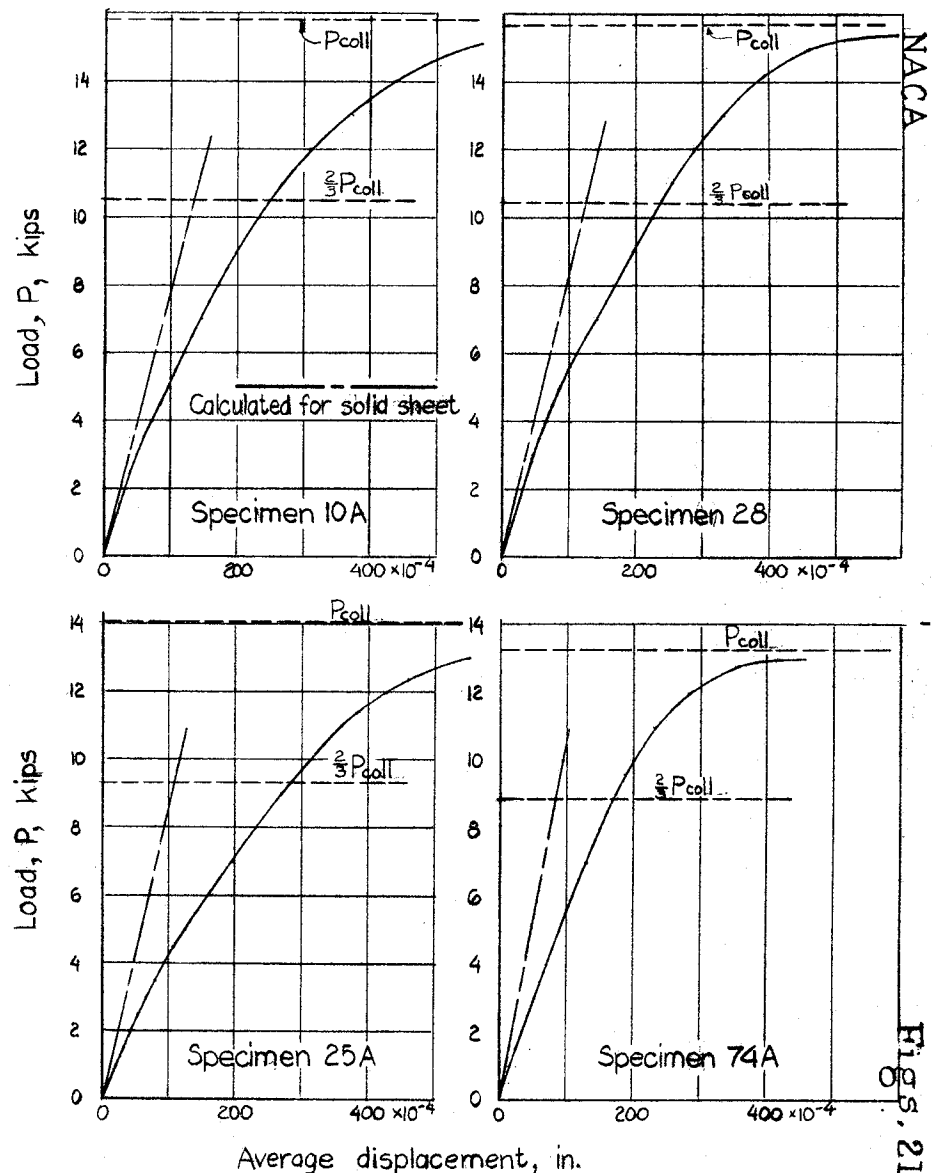


Figure 22.- Load-displacement curves for flanged-hole webs with bar supports and reamed bolt holes.





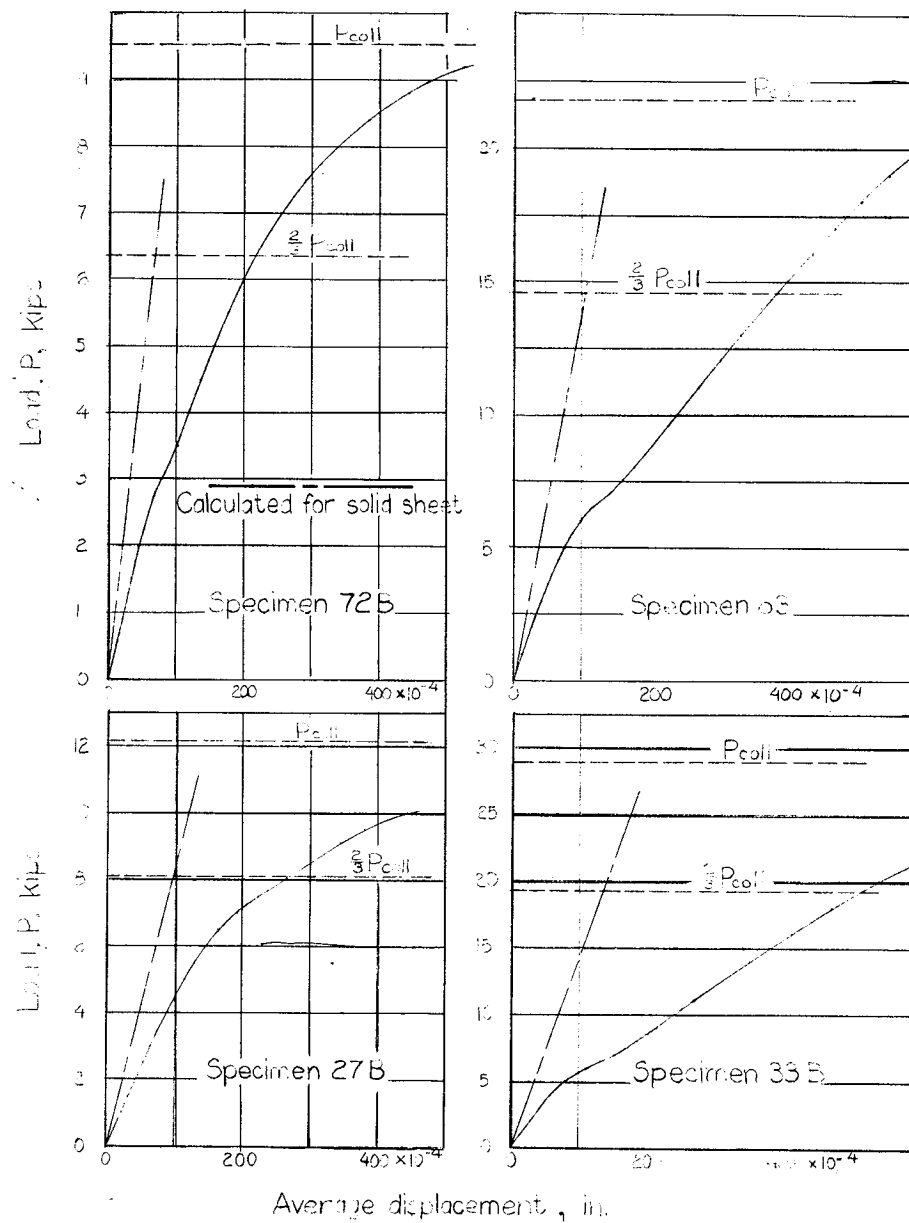


Figure 23.- Load-displacement curves for flanged-hole webs with rod supports and reamed bolt holes.

