9.3 Two-way Slabs (Part I)

This section covers the following topics.

- Introduction
- Analysis and Design
- Features in Modeling and Analysis
- Distribution of Moments to Strips

9.3.1 Introduction

The slabs are presented in two groups: **one-way slabs** and **two-way slabs**. The one-way slabs are presented in Section 9.2. When a rectangular slab is supported on all the sides and the length-to-breadth ratio is less than two, it is considered to be a two-way slab. The slab spans in both the orthogonal directions. A circular slab is a two-way slab. In general, a slab which is not falling in the category of one-way slab, is considered to be a two-way slab.

Rectangular two-way slabs can be divided into the following types.

1) Flat plates: These slabs do not have beams between the columns, drop panels or column capitals. Usually, there are spandrel beams at the edges.
2) Flat slabs: These slabs do not have beams but have drop panels or column capitals.
3) Two-way slabs with beams: There are beams between the columns. If the beams are wide and shallow, they are termed as band beams.

For long span construction, there are ribs in both the spanning directions of the slab. This type of slabs is called waffle slabs.

The slabs can be cast-in-situ (cast-in-place). Else, the slabs can be precast at ground level and lifted to the final height. The later type of slabs is called lift slabs. A slab in a framed building can be a two-way slab depending upon its length-to-breadth (L / B) ratio. Two-way slabs are also present as mat (raft) foundation.

The following sketches show the plan of various cases of two-way slabs. The spanning directions in each case are shown by the double headed arrows.

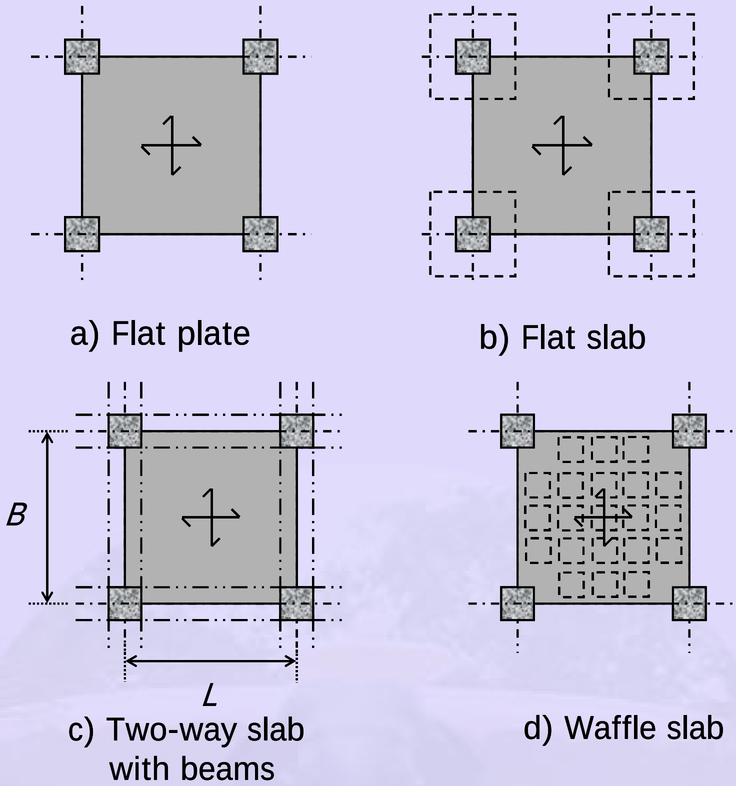


Figure 9-3.1 Plans of two-way slabs

The absence of beams in flat plates and flat slabs lead to the following advantages.

1) Formwork is simpler
2) Reduced obstruction to service conduits
3) More flexibility in interior layout and future refurbishment.

Two-way slabs can be post-tensioned. The main advantage of prestressing a slab is the increased span-to-depth ratio. As per **ACI 318-02** (*Building Code Requirements for Structural Concrete*, American Concrete Institute), the limits of span-to-depth ratios are as follows.

For floors	42
For roofs	48.

The values can be increased to 48 and 52, respectively, if the deflection, camber and vibration are not objectionable. The following photographs show post-tensioned flat plate and flat slab.

(a) Flat plate

(b) Flat slab

Figure 9-3.2 Post-tensioned two-way slabs

(Courtesy: VSL India Pvt. Ltd.)

9.3.2 Analysis and Design

Analysis

The analysis of two-way slabs is given in **Section 31, IS:456 - 2000**, under “Flat Slabs”. The analysis is applicable to flat plates, flat slabs and two-way slabs with deflecting beams. For two-way slabs with beams, if the beams are sufficiently stiff, then the method (based on moment coefficients) given in Annex D, **IS:456 – 2000**, is applicable.

The direct design method of analysing a two-way slab is not recommended for prestressed slabs. The equivalent frame method is recommended by **ACI 318-02**. It is given in **Subsection 31.5, IS:456 - 2000**. This method is briefly covered in this section for flat plates and flat slabs.

The slab system is represented by a series of two dimensional equivalent frames for each spanning direction. An equivalent frame along a column line is a slice of the building bound by the centre-lines of the bays adjacent to the column line.

The width of the equivalent frame is divided into a column strip and two middle strips. The column strip (CS) is the central half of the equivalent frame. Each middle strip (MS) consists of the remaining portions of two adjacent equivalent frames. The following figure shows the division in to strips along one direction. The direction under investigation is shown by the double headed arrow.

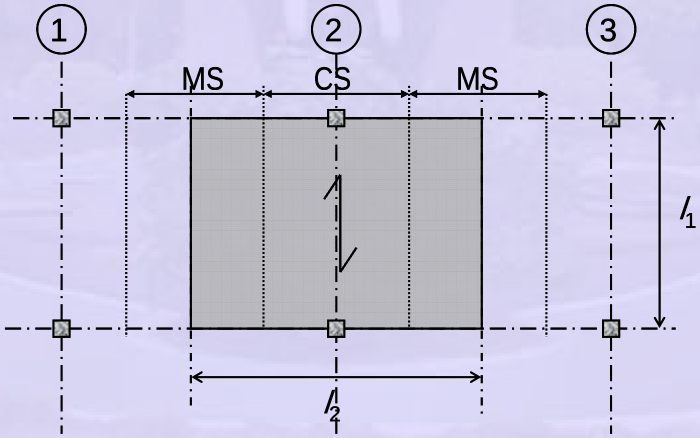


Figure 9-3.3 Equivalent frame along Column Line 2

In the above figure,

l_1 = span of the equivalent frame in a bay

l_2 = width of the equivalent frame. This is the tributary width for calculating the loads.

The following figure shows a typical elevation of an equivalent frame.

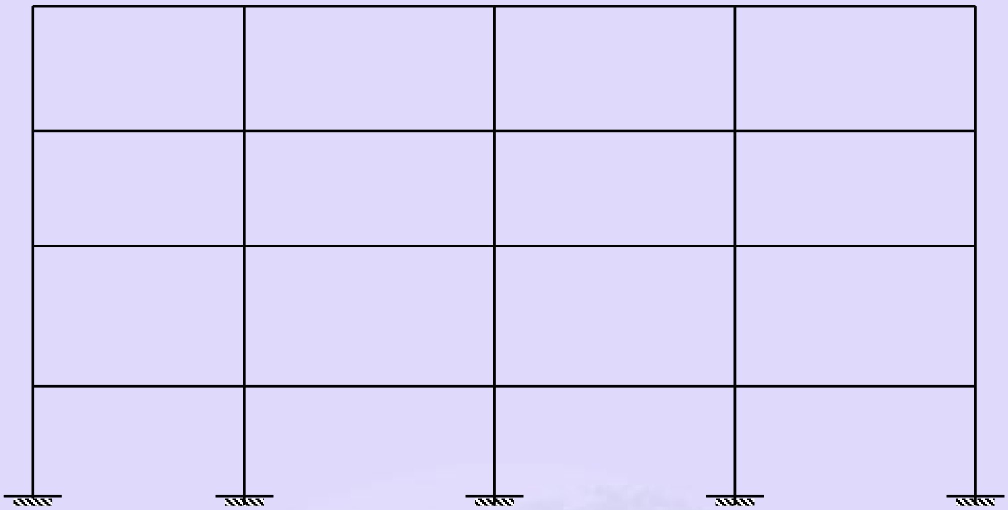


Figure 9-3.4 Elevation of an equivalent frame

The analysis is done for each typical equivalent frame. An equivalent frame is modelled by slab-beam members and equivalent columns. The equivalent frame is analysed for gravity load and lateral load (if required), by computer or simplified hand calculations. Next, the negative and positive moments at the critical sections of the slab-beam members are distributed along the transverse direction. This provides the design moments per unit width of a slab.

If the analysis is restricted to gravity loads, each floor of the equivalent frame can be analysed separately with the columns assumed to be fixed at their remote ends, as shown in the following figure. The pattern loading is applied to calculate the moments for the critical load cases. This is discussed later.

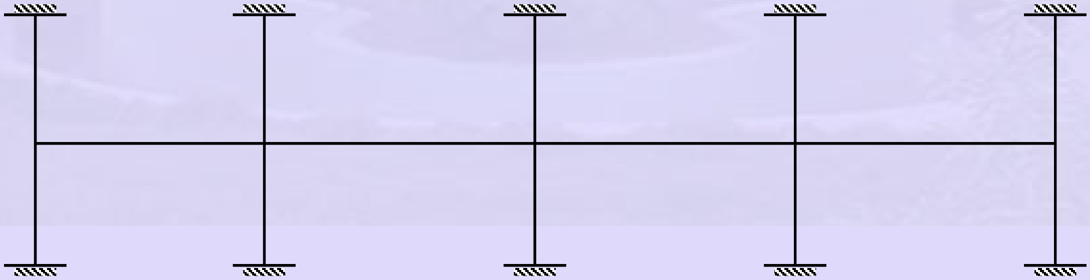


Figure 9-3.5 Simplified model of an equivalent frame

The steps of analysis of a two-way slab are as follows.

1) Determine the factored negative (M_u^-) and positive moment (M_u^+) demands at the critical sections in a slab-beam member from the analysis of an equivalent frame. The values of M_u^- are calculated at the faces of the columns. The values of M_u^+

are calculated at the spans. The following sketch shows a typical moment diagram in a level of an equivalent frame due to gravity loads.

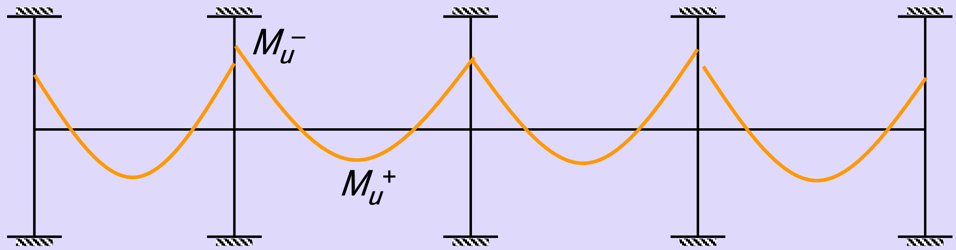


Figure 9-3.6 Typical moment diagram due to gravity loads

2) Distribute M_u^- to the CS and the MS. These components are represented as $M_{u,}{}^-{}_{CS}$ and $M_{u,}{}^-{}_{MS}$, respectively. Distribute M_u^+ to the CS and the MS. These components are represented as $M_{u,}{}^+{}_{CS}$ and $M_{u,}{}^+{}_{MS}$, respectively.

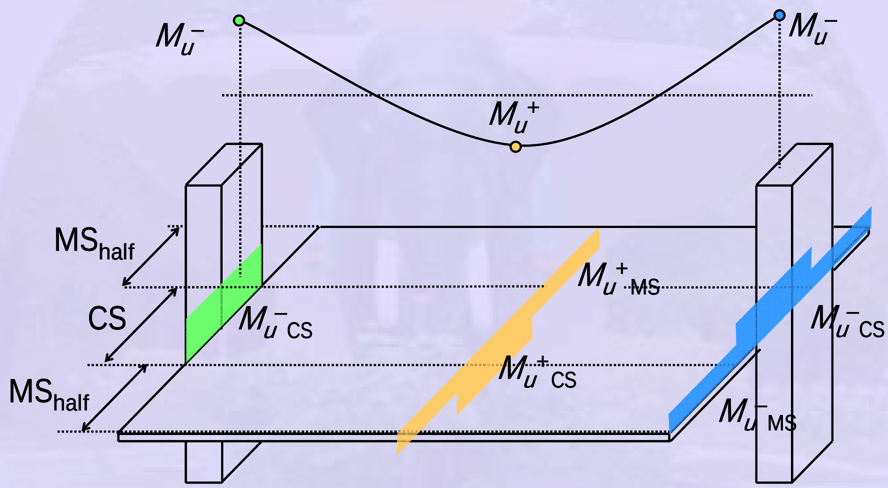


Figure 9-3.7 Distribution of moments to column strip and middle strips

3) If there is a beam in the column line in the spanning direction, distribute each of $M_{u,}{}^-{}_{CS}$ and $M_{u,}{}^+{}_{CS}$ between the beam and rest of the CS.

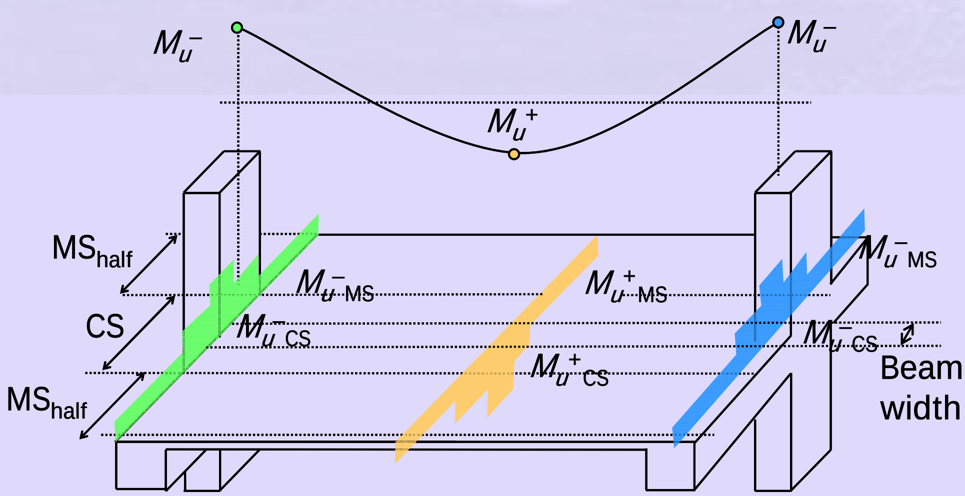


Figure 9-3.8 Distribution of moments to beam, column strip and middle strips

4) Add the moments $M_{u,}{}^{-}{}_{MS}$ and $M_{u,}{}^{+}{}_{MS}$ for the two portions of the MS (from adjacent equivalent frames).
5) Calculate the design moments per unit width of the CS and MS.

Design

Once the design moments per unit width of the CS and MS are known, the steps of design for prestressing steel are same as that for one-way slab. The profile of the tendons is selected similar to that for continuous beams. The flexural capacity of prestressed slab is controlled by total amount of prestressing steel and prestress rather than by tendon distribution. But the tendon distribution effects the load balancing. Some examples of tendon distribution are shown.

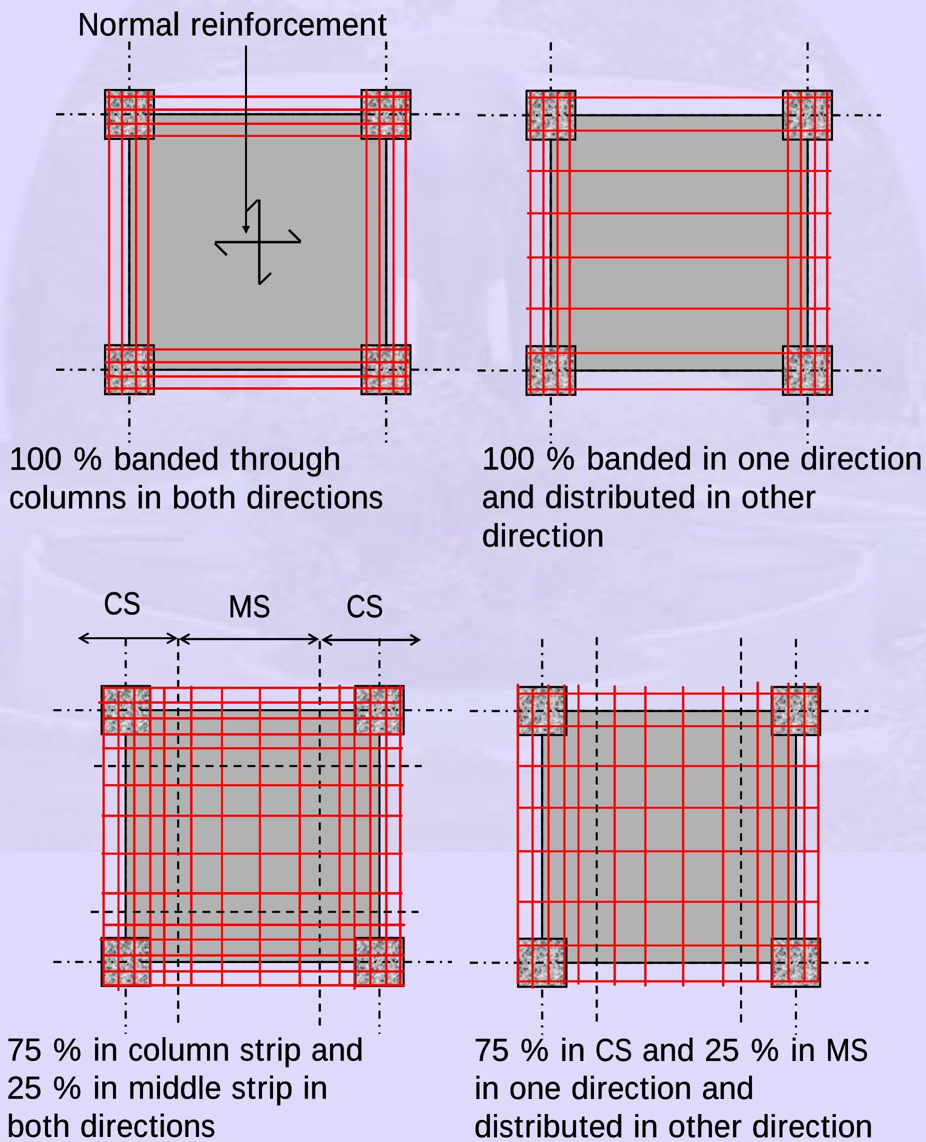


Figure 9-3.9 Typical tendon layouts

Maximum spacing of tendons or groups of tendons should be limited to 8*h* or 1.5 m, whichever is less. Here, *h* is the thickness of the slab. A minimum of two tendons shall be provided in each direction through the critical section for **punching shear** around a column. The critical section for punching shear is described in Section 9.4, Two-way Slabs (Part II). Grouping of tendons is permitted in band beams.

A minimum amount of non-prestressed reinforcement is provided in each direction based on temperature and shrinkage requirement. As per **IS:456 - 2000**, **Clause 26.5.2.1**, the minimum amount of reinforcement ($A_{st,min}$ in mm^2) for unit width of slab is given as follows.

$$A_{st,min} = 0.15\%\ 1000h \quad \text{for Fe 250 grade of steel}$$
$$= 0.12\%\ 1000h \quad \text{for Fe 415 grade of steel.}$$

The ducts for placing the individual strands are oval shaped to maintain the eccentricity, reduce frictional losses and convenient placement of crossing ducts. The ducts are not commonly grouted as the use of unbonded tendon is not detrimental in buildings.

The following photo shows the ducts for the prestressing tendons and the non-prestressed reinforcement in a two-way slab.

Figure 9-3.10 Reinforcement in a two-way slab
(Courtesy: VSL India Pvt. Ltd.)

9.3.3 Features in Modelling and Analysis

The features in modelling and analysing an equivalent frame are discussed.

Gross section versus cracked section

For determining the stiffness of a slab-beam member, the gross section can be considered in place of the cracked section. This is to simplify the calculation of moment of inertia of the section.

Equivalent column

The actual column needs to be replaced by an equivalent column to consider the flexibility of the transverse beam in the rotation of the slab. The portions of the slab in the MS rotate more than the portions in the CS because of the torsional deformation of the transverse beam. In the following figure, the size of the arrows qualitatively represents the rotation of the slab. Note that the rotation is higher away from the column.

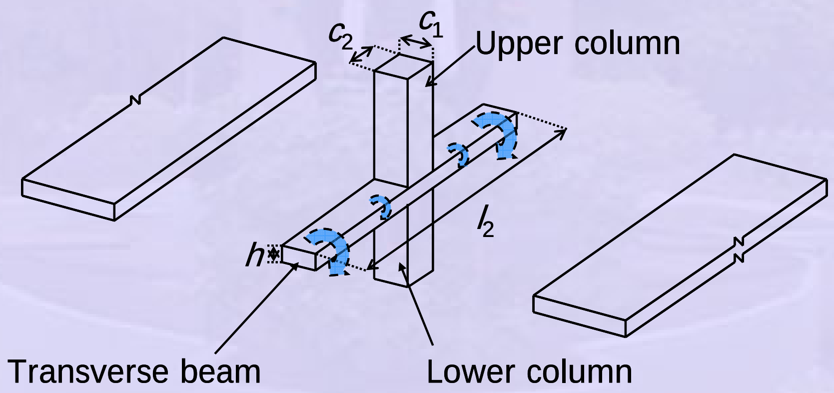
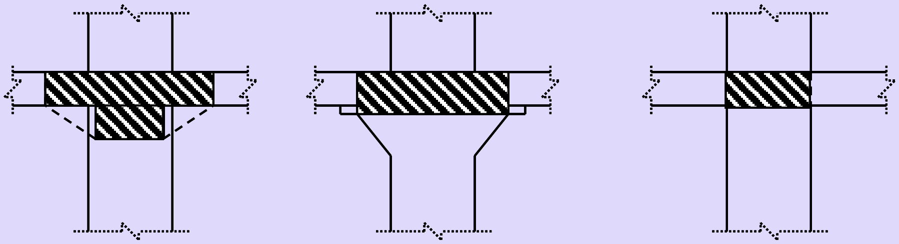


Figure 9-3.11 Isometric view of a slab-column junction

The transverse beam need not be a visible beam, but a part of the slab in the transverse direction, bounded by the edges of the column or column capital. In presence of beam or column capital or in absence of beam, the cross-section of the modelled transverse beam is taken as shown in the following sketches.



a) In presence of transverse beam

b) In presence of column capital

c) In absence of transverse beam

Figure 9-3.12 Cross-section of modelled transverse beam

The flexibility of the equivalent column is equal to the sum of the flexibilities of the actual column and the transverse beam.

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t} \qquad \textbf{(9-3.1)}$$

Here,

K_{ec} = flexural stiffness of the equivalent column

ΣK_c = $K_{c,upper}$ + $K_{c,lower}$

$K_{c,upper}$ = flexural stiffness of the upper column

$K_{c,lower}$ = flexural stiffness of the lower column

K_t = torsional stiffness of the transverse beam.

An approximate expression for the flexural stiffness of a column (K_c) is given below.

$$K_c = \frac{4E_cI_c}{L-2h} \qquad \textbf{(9-3.2)}$$

Here,

E_c = modulus of concrete

L = length of the column

h = thickness of the slab

I_c = moment of inertia of the column.

An approximate expression for torsional stiffness of the transverse beam (K_t) is given below.

$$K_t = \frac{9E_cC}{l_2\left(1-\dfrac{c_2}{l_2}\right)^3} \qquad \textbf{(9-3.3)}$$

Here,

C = equivalent polar moment of inertia of transverse beam

c_2 = dimension of column in the transverse direction

l_2 = width of equivalent frame.

For a rectangular section, the expression of C is given below.

$$C = \Sigma\left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} \quad \textbf{(9-3.4)}$$

Here, x and y are the smaller and larger dimensions of the transverse beam. The expression of C is a lower bound estimate, that is, the calculated value is always lower than the actual moment of inertia of the transverse beam. For a transverse beam of compound section, the value of C is the summation of the individual values of the component rectangles. The splitting of the compound section into component rectangles should be such, so as to maximise the value of C. For the following two cases of splitting, select the larger value of C.

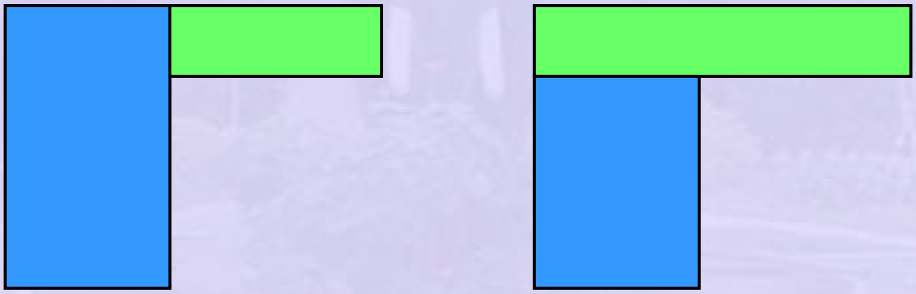


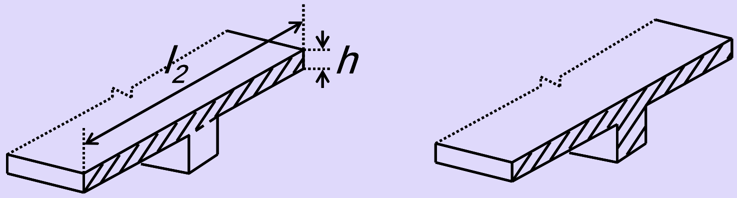
Figure 9-3.13 Component rectangles of a compound section

If there is a beam in the column strip in the spanning direction, then K_t is replaced by K_t (I_{sb} / I_s).

Here,

I_s = moment of inertia of slab without the projecting portion of the beam (shaded area in Sketch (a) of the following figure)

I_{sb} = moment of inertia of slab considering it as a T-section (shaded area in Sketch (b) of the following figure).



(a) (b)

Figure 9-3.14 Sections for determining moments of inertia

Slab–beam members

The variation of the flexural moment of inertia of a slab-beam member is considered as follows.

The value of the moment of inertia (I) is constant (say equal to I_1) in the prismatic portion, that is, in between the faces of the columns or column capitals or drop panels. It is also constant, with a different value (say equal to I_2) in the region of a drop panel. The value varies in the region from the face of the column or column capital to the center line of the column. But it is approximated to a constant value equal to the following.

$$\frac{I_2}{\left(1-\frac{c_2}{l_2}\right)^2} \qquad \textbf{(9-3.5)}$$

Here,

I_2 = moment of inertia at the face of the column or column capital

c_2 = dimension of column in the transverse direction

l_2 = width of equivalent frame.

The following figure shows the variation of the moment of inertia of the slab beam member.

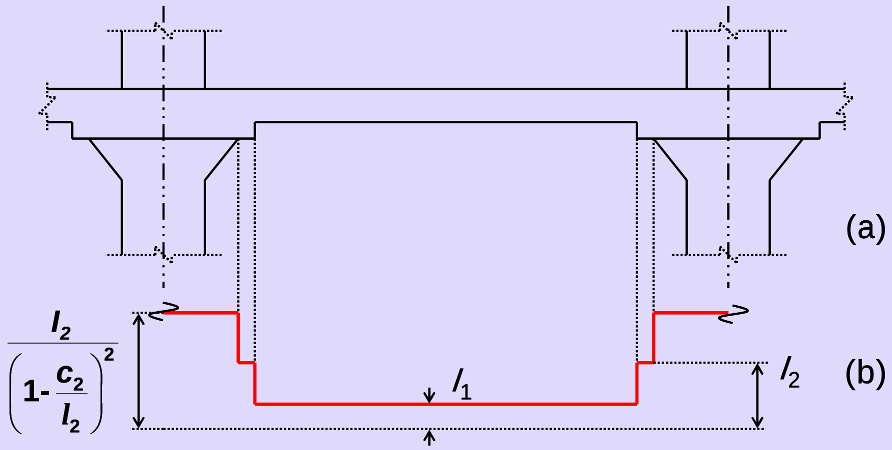


Figure 9-3.15 a) Elevation of equivalent frame, b) Variation of the moment of inertia of the slab-beam member

Arrangement of live load

Since the factored live load ($w_{u,\,LL}$) may not occur uniformly in all the spans in a floor, a distribution is considered to generate the maximum values of the negative (M_u^-) and positive moments (M_u^+) at the critical sections. If the distribution of $w_{u,\,LL}$ is known, then the load is applied accordingly. If the distribution is not known, then a **pattern loading** is considered based on the value of $w_{u,\,LL}$ with respect to that of the factored dead load ($w_{u,\,DL}$). Of course, the load case with $w_{u,\,LL}$ on all the spans should be also analysed.

1) For $w_{u,\,LL} \leq ¾\, w_{u,\,DL}$

The possible variation in $w_{u,LL}$ in the different spans is neglected. $w_{u,LL}$ is applied uniformly on all the spans.

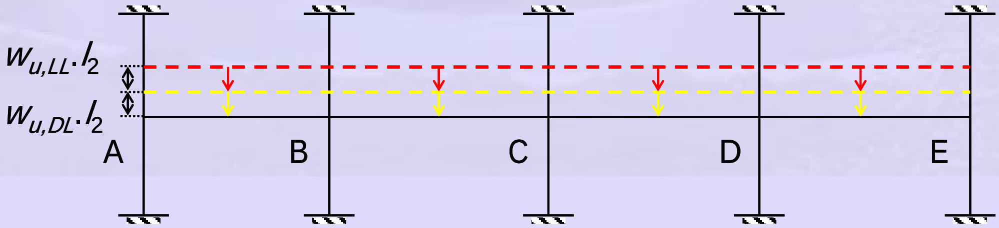
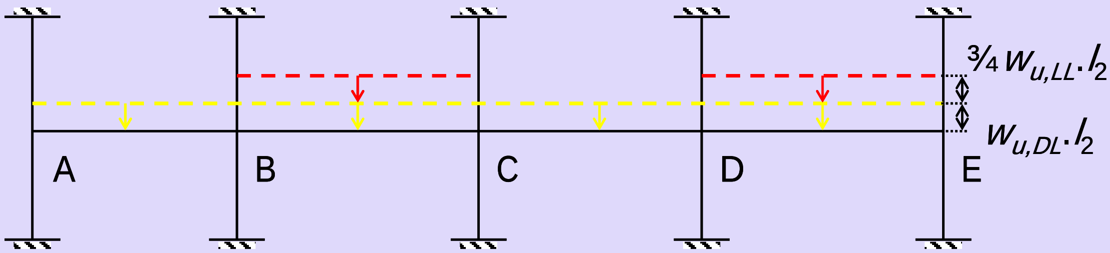


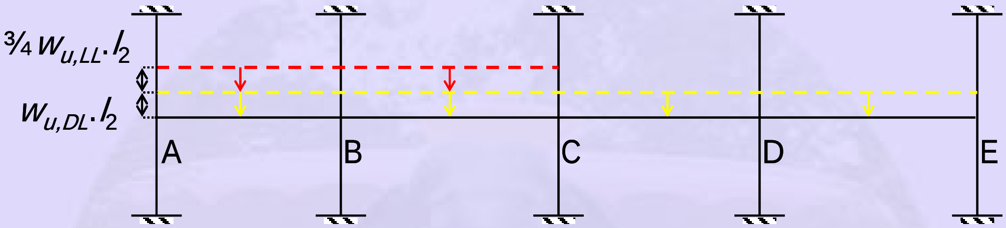
Figure 9-3.16 Distribution of live load for $w_{u,\,LL} \leq ¾\, w_{u,\,DL}$

2) For $w_{u\,LL} > ¾\, w_{u,DL}$

For maximum value of M_u^+ in a span, ¾ $w_{u,LL}$ is applied on the span and the alternate spans. For example, if the maximum value of M_u^+ in Span BC of the frame below is to be determined, then ¾ $w_{u,LL}$ is placed in Spans BC and DE. This distribution will also give the maximum value of M_u^+ in Span DE. For maximum value of M_u^- near the support, ¾ $w_{u\ LL}$ is applied on the adjacent spans only. For example, if the maximum value of M_u^- near Support B is to be determined, then ¾ $w_{u,LL}$ is placed in Spans AB and BC.



Distribution of live load for maximum M_u^+ in Spans BC and DE



Distribution of live load for maximum M_u^- near Support B

Figure 9-3.17 Distribution of live load for $w_{u\ LL}$ > ¾ $w_{u,DL}$

Critical section near a support

The critical section is determined as follows.

1) At interior support

At the face of support (column or column capital, if any), but not further than $0.175l_1$ from the center line of the column.

2) At exterior support

At a distance from the face of column not greater than half the projection of the column capital (if any).

9.3.4 Distribution of Moments to Strips

In absence of a rigorous analysis (say finite element analysis), the procedure for reinforced concrete slabs may be used to distribute the moments M_u^+ and M_u^- to the column strip (CS) and middle strip (MS). Of course, **ACI 318-02** does not recommend this procedure to be used for prestressed slabs.

Distribution of M_u^- at interior support

$$M_{u,}{}^{-}{}_{CS} = 0.75\ M_u^- \quad \textbf{(9-3.6)}$$

$$M_{u,}{}^{-}{}_{MS} = 0.25\ M_u^- \quad \textbf{(9-3.7)}$$

Here,

$M_{u,}{}^{-}{}_{CS}$ = negative moment in the CS

$M_{u,}{}^{-}{}_{MS}$ = total negative moment in the two MS at the sides.

Distribution of M_u^- at exterior support

If the width of the column or wall support is less than ¾ $l_{2,}$

$$M_{u,}{}^{-}{}_{CS} = M_u^- \quad \textbf{(9-3.8)}$$

$$M_{u,}{}^{-}{}_{MS} = 0. \quad \textbf{(9-3.9)}$$

If the width of the column or wall support is greater than ¾ $l_{2,}$ then M_u^- is uniformly distributed along the width l_2.

Distribution of M_u^+ at mid span

$$M_{u,}{}^{+}{}_{CS} = 0.60\ M_u^+ \quad \textbf{(9-3.10)}$$

$$M_{u,}{}^{+}{}_{MS} = 0.40\ M_u^+ \quad \textbf{(9-3.11)}$$

Here,

$M_{u,}{}^{+}{}_{CS}$ = positive moment in the CS

$M_{u,}{}^{+}{}_{MS}$ = total positive moment in the two MS at the sides.

The total moments in MS ($M_{u,}{}^{-}{}_{MS}$ and $M_{u,}{}^{+}{}_{MS}$) are distributed to the two middle strips at the sides of the equivalent frame, proportional to their widths. The combined MS from two adjacent equivalent frames is designed for the sum of the moments assigned to its parts.