5.2 Elastic buckling of an ideal column or struct with pinned end

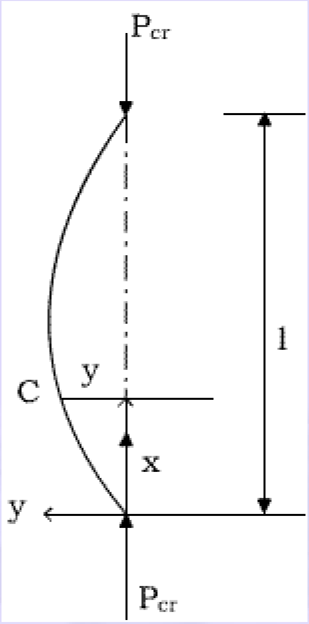

Fig 5.2 Buckling of pin-ended column

The classical Euler analysis of the elastic behaviour of an idealized, pin-ended, uniform strut makes the following assumptions.

- The material is homogeneous and linearly elastic (i.e. it obeys Hooke's Law).
- The strut is perfectly straight and there are no imperfections.
- The loading is applied at the centroid of the cross section at the ends.

We will assume that the member is able to bend about one of the principal axes. (See Fig. 5.2). Initially, the strut will remain straight for all values of P, but at a particular value $P = P_{cr}$, it buckles. Let the buckling deformation at a section distant x from the end B be y.

The bending moment at this section = $P_{cr}.y$

The differential equation governing the deformation can be obtained by considering moment equilibrium about point C as

$$P_{cr}\, y + EI \frac{d^2 y}{dx^2} = 0 \qquad \textbf{(5.1)}$$

The general solution for this differential equation is given by

$$y = A_1 \text{Cos}\, x \sqrt{\frac{P_{cr}}{E1}} + B_1 \text{Sin}\, x \sqrt{\frac{P_{cr}}{E1}} \qquad \textbf{(5.2)}$$

Where A_1 and B_1 are constants.

Since $y = 0$ when x = 0, A_1 = 0.

Also y = 0 when x = L gives

$$B_1 \text{Sin}\, L \sqrt{\frac{P_{cr}}{E1}} = 0$$

Thus, either B_1 = 0 or $B_1 \text{Sin}\, L \sqrt{\frac{P_{cr}}{E1}} = 0$

B_1 = 0 means $y = 0$ for all values of x (i.e. the column remains straight).

Alternatively

This equation is satisfied only when

$$L \sqrt{\frac{P_{cr}}{E1}} = 0, \pi, 2\pi, ...$$

This gives **(5.3)**

$$p_{cr} = \frac{\pi^2 EI}{L^2}, \frac{4\pi^2 EI}{L^2}, ... \frac{n^2 \pi^2 EI}{L^2}$$

Where n is any integer.

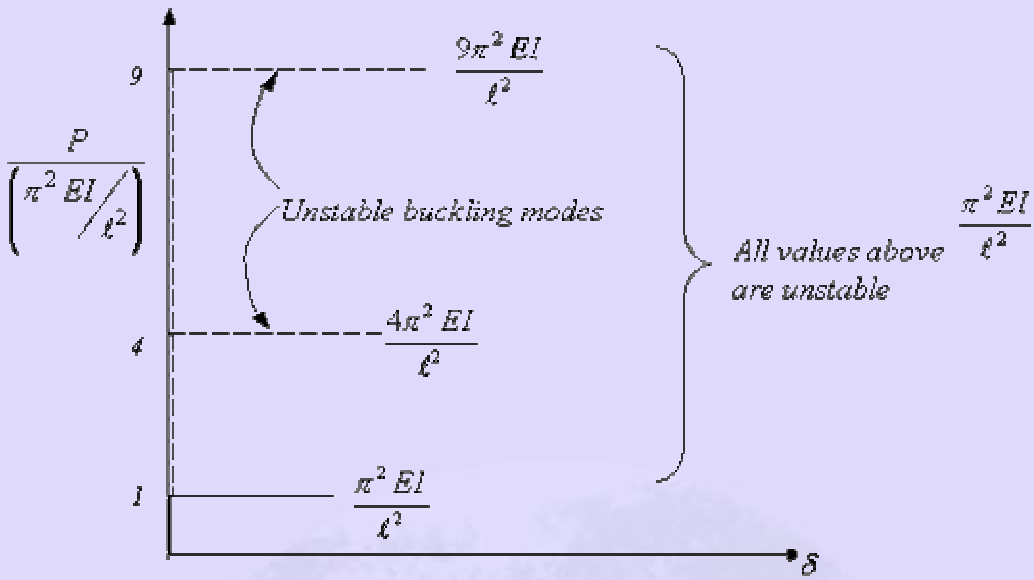

Fig 5.3 Buckling load Vs centre lateral deflection relationship

While there are several buckling modes each corresponding to n = 1, 2, 3… (See Fig. 5.3) the lowest stable buckling mode corresponds to n = 1.

The lowest value of the critical load (i.e. the load causing buckling) is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \textbf{\textit{(5.4)}}$$

Thus the Euler buckling analysis for a **"straight"** strut; will lead to the following conclusions:

1. The strut can remain straight for any value of P.
2. Under incremental loading, when P reaches a value of $P_{cr} = \pi^2 EI / L^2$ the strut can buckle in the shape of a half-sine wave; the amplitude of this buckling deflection is indeterminate.

3. At higher values of the loads given by $n^2\pi^2 EI / L^2$ other sinusoidal buckled shapes (n half waves) are possible. However, it is possible to show that the column will be in unstable equilibrium for all values of $P > \pi^2 EI / L^2$ whether it be straight or buckled. *This means that the slightest disturbance will cause the column to deflect away from its*

original position. Elastic Instability may be defined in general terms as a condition in which the structure has no tendency to return to its initial position when slightly disturbed, even when the material is assumed to have an infinitely large yield stress. Thus $P_{cr} = \pi^2 EI / L^2$ represents the maximum load that the strut can usefully support.

It is often convenient to study the onset of elastic buckling in terms of the mean applied compressive stress (rather than the force). The mean compressive stress at buckling, f_{cr}, is given by

$$f_{cr} = \frac{p_{cr}}{A} = \frac{\pi^2}{AL^2} = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 E}{\lambda^2} \qquad \textbf{\textit{(5.5)}}$$

Where *A* = area of cross section of the strut, *r* = radius of gyration of the cross section, ($I = Ar^2$) and $\lambda = L / r$

λ = the slenderness ratio of the column defined by

The equation $f_{cr} = (\pi^2 E) / \lambda^2$, implies that the critical stress of a column is inversely proportional to the square of the slenderness ratio of the column (see Fig. 5.4).

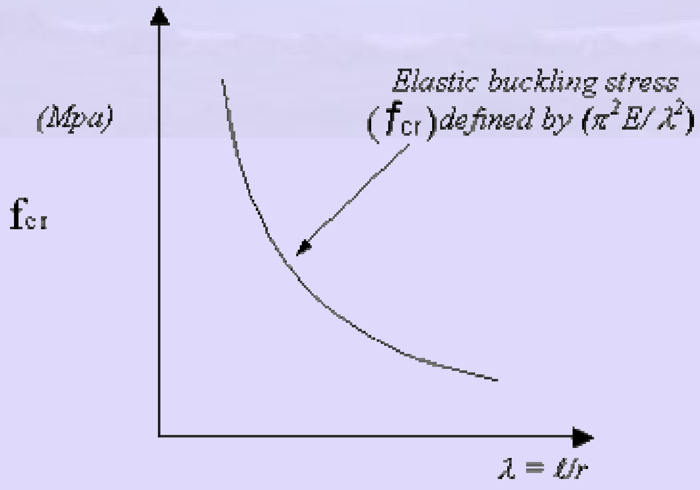

Fig 5.4 Euler buckling relation between f_{cr} and λ