

# SIMPLIFIED ANALYSIS OF Laterally Loaded Posts

Tom Bowling  
*Geotechnical Consultant<sup>1</sup>*

## ABSTRACT

Simplified equations for the strength and deflection of laterally loaded vertical posts embedded in soil foundations, using the modulus of subgrade reaction approach, are presented for simple post embedment and for posts with enlarged footings. Equations describing the elastic behaviour of similarly loaded embedded posts are also detailed. The aim of the paper is to provide a consolidated document containing information for the preliminary design of embedded posts prior to detailed geotechnical investigations and analysis or for the design of such posts where detailed investigations and analysis cannot be justified. The information provided in this paper is not generally available in basic soil mechanics textbooks.

## 1 INTRODUCTION

The design of posts subjected to lateral loading is a topic that can be encountered by young engineers or even more experienced engineers who are not geotechnical specialists. Examples of such laterally loaded posts include fence posts, light standards, electric transmission line poles and even some wind generator towers. Basic soil mechanics text books are often used by engineers as a guide to the clarification or solution of geotechnical problems that they may encounter in their professional work or as a background for their discussions with a geotechnical specialist. However a review of a number of such basic text books (see Bibliography) indicates that the analysis of laterally loaded posts is not dealt with in these books to any significant extent. Whilst information on this topic is covered in various technical papers and specialist publications it seems appropriate that at least some basic information on this topic should be consolidated in a reference document. This is the aim of this paper.

The analyses presented in the following provide simple, understandable and safe solutions to a number of problems associated with laterally loaded posts. Although the modulus of subgrade reaction used in the analyses is not an absolute soil property, depending as it does on the size of the loaded area amongst other things, the values used in the following (Table 1 for example) can give satisfactory results for typical post sizes that might be encountered in practical problems. As always, for critical problems in soil mechanics associated with major structures, more sophisticated design investigations should be carried out with the assistance of geotechnical specialists.

The loadings that post structures are subjected to are either effectively constant, such as the weight of cantilevered light fittings on a light standard, or intermittent, such as wind loading on most of these types of structures. The footings of these structures need to be designed so that the surrounding soil will resist the lateral loads, and in particular the resultant over-turning moments, with minimum deflection of the post.

Whilst the equations presented in this paper could also be applied to some aspects of the analysis of pile behaviour, the different intended structural action of piles, which principally support vertical loads with deep embedment, may make such an application inappropriate.

## 2 SIMPLE RIGID POST

We consider a square section vertical post projecting a distance  $h$  above a horizontal ground surface and embedded to a depth  $d$  into a uniform soil below the ground surface, Figure 1. The square section has a side dimension  $b$ . The top of the post is subjected to a horizontal force  $R$ . Due to this force the post will rotate slightly around a centre of rotation distant  $t$  below the ground surface. This rotation will mobilize resisting soil pressures which will develop on one side of the post above the point of rotation and on the opposite side of the post below the point of rotation. An origin for vertical co-ordinates  $z$  is taken at the point of rotation

We will first of all make the following assumptions:

- The resisting soil pressure is proportional to the soil deflection from the rest position, the constant of proportionality being a modulus of lateral subgrade reaction.

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<sup>1</sup> Author contact, tom.bwlng@gmail.com

- The modulus of lateral subgrade reaction increases linearly with depth below the ground surface so that at a depth  $(t - z)$  the modulus of lateral subgrade reaction is  $k(t - z)$ , see for example Terzaghi (1943) chapter 16.
- The post is rigid compared with the soil, ie the bending deflections of the post are small compared with soil deflections.
- Vertical stresses acting at the bottom of the post are negligible in resisting lateral forces, ie the square section dimension  $b$  is small compared with the embedment depth  $d$ .

The effects on the behaviour of the post when the last two assumptions are disregarded are discussed later in this paper.

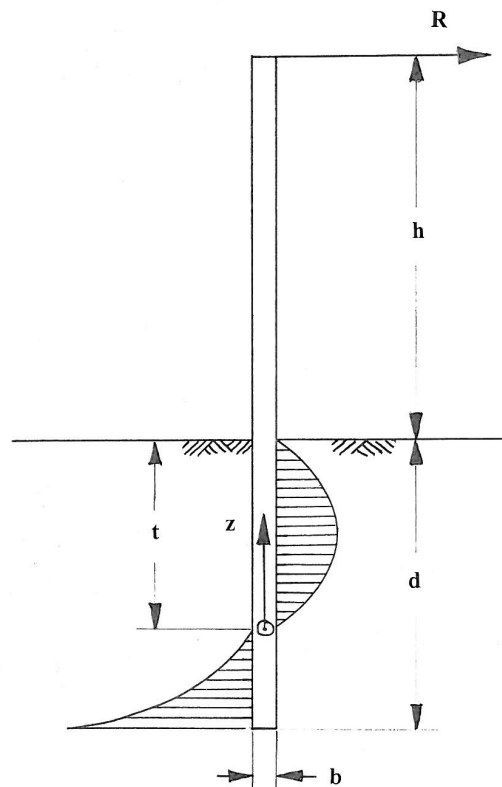


Figure 1: Post under lateral load.

Suggested values of the parameter  $k$ , the rate of increase of the modulus of lateral subgrade reaction with depth, are given in Table 1 for various soil types.

If the post rotates through an angle  $\alpha$  about the centre of rotation then the horizontal pressure developed in the soil will be given by

$$p = k \times (t - z) \times z \times \tan \alpha \quad (1)$$

The distribution of pressure is thus parabolic and is zero at the ground surface and at the centre of rotation. It can be noted that for small values of  $\alpha$ , which is usually a requirement for the design of laterally loaded vertical posts,  $\tan \alpha$  can be taken as equal to  $\alpha$  in radians.

The centre of rotation can be determined by equating the sum of horizontal forces on the post to zero. Thus we have:

$$\int_0^t k \times (t - z) \times z \times \tan \alpha \times b \cdot dz = R + \int_0^{(t-d)} k \times (t - z) \times z \times \tan \alpha \times b \cdot dz$$

This integration yields:

$$t = \frac{2}{3} \times d + \frac{2R}{k \times d^2 \times \tan \alpha \times b} \quad (2)$$

In most cases the second term in Equation (2) is small compared with the first term and  $t$  can be taken as equal to  $\frac{2}{3}d$ .

**Table 1**  
**Approximate values of parameter  $k$ , the increase in the modulus of lateral subgrade reaction with depth**  
Adapted from Sulzberger (1927)

Type of Soil	USC	$k$ in kPa/mm/m
Light peat and boggy soil	Pt	3 to 5
Heavy peat, fine loose sand	Pt & SP	5 to 8
Rubble, poorly compacted	GP	5 – 10
Clay wet	CL to CH	10 – 15
damp		20 – 25
dry		30 – 50
Sand clay with some gravel	SC to GC	40 – 50
Sand silt gravel mix	SW to GM	50 – 60
Fine gravel with sand	SM to GM	40 – 50
Medium gravel with sand	GM	50 – 60
Coarse gravel – sand	GM to GW	60 – 75
Coarse gravel with some sand	GW	75 – 100
Coarse gravel with some sand well compacted	GW	100 – 120

*The value of the modulus of lateral subgrade reaction at depth  $d$  is  $k \times d$  with  $d$  in metres. The modulus of vertical subgrade reaction for a foundation at depth  $d$  can be taken as  $1.2 \times k \times d$ . The parameter is applicable to depths up to about 8m in reasonably uniform soil.*

The resisting moment of the soil pressures around the centre of rotation is given by:

$$M = \int_0^t z \times k \times (t - z) \times z \times \tan \alpha \times b \cdot dz = k \times \tan \alpha \times b \left\{ \frac{t^2 d^2}{2} - \frac{2td^3}{3} + \frac{d^4}{4} \right\} \quad (3)$$

By substituting  $t = \frac{2}{3}d$ ,  $M$  reduces to:

$$M = k \times \tan \alpha \times b \times \frac{d^4}{36} \quad (4)$$

This is the basic equation for the moment of resistance, about the centre of rotation, of a vertical post embedded in soil.

The two maximum horizontal stresses developed in the soil are:

$$\begin{aligned} \text{At one third post depth, } p &= k \tan \alpha \frac{d^2}{9} \\ \text{At bottom of post, } p &= k \tan \alpha \frac{d^2}{3} \end{aligned} \quad (5)$$

Equation (4) can be used to derive an expression for the second term in Equation (2). Thus by equating the resisting moment derived from Equation (4) to  $R \times (h + \frac{2}{3}d)$  an expression for  $R$  is found. Substituting this value

of  $R$  into the second term of Equation (2) gives a value of  $\frac{d^2}{18(h + \frac{2}{3}d)}$  for this second term. As  $h$  for typical loaded posts is normally greater than  $d$  then the second term will be less than  $1/30$  of  $d$ , the depth of embedment.

As an example of the application of Equation (4) we can consider a post with square section of 0.2 m side embedded 2 m into a soil foundation and projecting 6m above the ground surface. If the soil foundation consists of sand, for which  $k = 40$  kPa per mm per m, we can determine the horizontal load at the top of the post which will produce a tilt of 0.01 radians (about half a degree) as follows

$$M = k.a.b. \frac{d^4}{36} = R.(6 + \frac{2}{3}.2)$$

$$M = 40,000 \times 0.01 \times 0.2 \times 16/36 = R \times 7.33$$

$$R = 4.85 \text{ kN and } M = 35.5 \text{ kN.m}$$

It can be noted that for consistent units  $k$  has been converted to kPa/m/m. Also in this example the second term in Equation (2) above is found to be 0.03 m which is about 2% of 1.33 m, the calculated simplified depth to the centre of rotation.

For design purposes the lateral resistance of soil near the ground surface cannot generally be relied upon because of likely disturbance during post installation. An inert zone of soil, perhaps up to half a metre deep, should be allowed below the ground surface to a level where the soil starts to become effective in providing lateral resistance.

If the value of  $k.tan\alpha$  from the expression for the maximum horizontal pressure at  $z = t/2$  is substituted in the expression for the moment of resistance (Equation 4), this moment is given by the expression:

$$M = pressure \times b \times \frac{d^2}{4} \quad (6)$$

In this expression the possibly less certain value of  $k.tan\alpha$  has been replaced by the horizontal soil pressure which can often be specified for a particular soil and site. For example if a simple Coulomb shear strength is assumed for the soil and the vertical soil pressure is taken as the soil weight density ( $\gamma$ ) times the total depth below ground surface, then the maximum horizontal pressure (horizontal stress for failure) can be obtained from a Mohr's circle analysis to give the following result:

$$\sigma_{h \max} = \gamma.depth \frac{(1 + \sin \phi)}{(1 - \sin \phi)} + 2c \frac{\cos \phi}{(1 - \sin \phi)} \quad (7)$$

More sophisticated analyses give larger values for  $\sigma_{h \max}$  (Brinch-Hansen, 1961) but Equation (7) is conservative.

If in the above example the soil material has a weight density of  $18 \text{ kN/m}^3$ , a  $\phi$  value of  $32^\circ$  and a small cohesion of 5 kPa and we assume an inert zone 0.5 m deep (inert with respect to lateral resistance but not with respect to weight) then the maximum horizontal (failure) stress at  $0.5 \text{ m} + 0.67 \text{ m}$  is given by

$$\sigma_{h \max} = 1.17 \times 18 \times \frac{(1 + \sin 32)}{(1 - \sin 32)} + 2 \times 5 \times \frac{\cos 32}{(1 - \sin 32)} = 87 \text{ kPa}$$

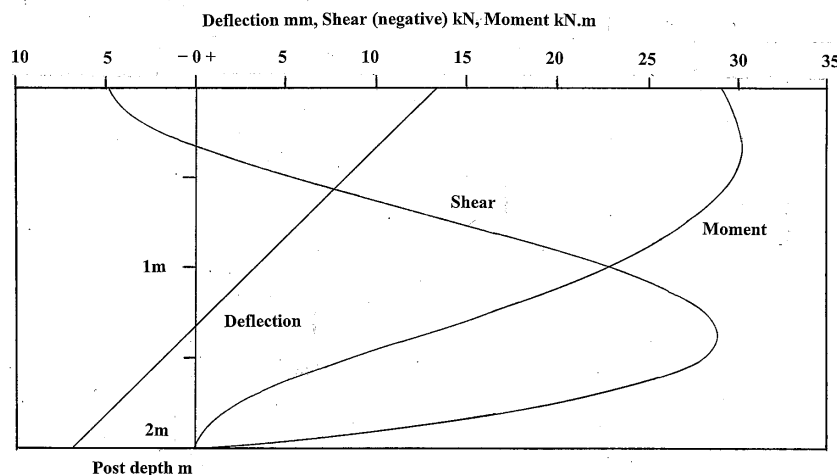


Figure 2: Distribution of shear and moment in the embedded section of the post with 4.85 kN horizontal load acting at 6m above the ground surface

The maximum moment of resistance is then obtained from Equation (5) as 17.3 kN.m or 2.21 kN at the top of the post, the lever arm now being  $(6 + 0.5 + \frac{2}{3}d)$  m. If the 4.85 kN load at the top of the post is to be achieved without exceeding the soil strength at say the  $\frac{1}{2}$ th depth then the depth of embedment has to be increased. This depth is found by substituting the value of  $\sigma_{hmax}$  from Equation (7), with *depth* equal to  $0.5\text{ m} + \frac{1}{3}d$ , as the *pressure* in Equation (6) and equating moment  $M$  to  $R \times (h + 0.5 + \frac{2}{3}d)$ . The solution of the resulting cubic equation gives  $d$  equal to 2.82 m and thus a total embedment of 3.32 m. In this case the angle of rotation for the post is reduced from 0.01 to 0.0029 radians. If the soil strength is not to be exceeded at the bottom of the post embedment for the 4.85 kN load, a similar analysis indicates that  $d$  would need to be 3.26 m giving a total embedment of 3.76 m.

The shear force and bending moment at a point in the post below the ground surface can be deduced by integrating the soil pressure distribution and the moment of the soil pressure distribution above the point and subtracting these integrated values from the shear force and bending moment at the same point in the post due to the external load  $R$ . Using the exact value for  $t$  (Equation 2) this gives the following equations:

$$\begin{aligned} \text{shear} &= R(1 - X^2) - \frac{k \tan \alpha b d^3}{3} X^2(1 - X) \\ \text{moment} &= R \left( h + Xd - \frac{X^3 d}{3} \right) - \frac{k \tan \alpha b d^4}{36} (4X^3 - 3X^4) \end{aligned} \quad (8 \text{ and } 9)$$

In these equations  $X$  is the ratio of the depth below the ground surface of the point under consideration to the total depth of the post embedment, equal to  $(t-z)/d$  using the symbols in Figure 1. The exact value for  $t$  is used in deriving the above equations to obtain zero moment and zero shear at the bottom of the post, i.e. at  $X = 1$ .

For the post described in the above example, with a horizontal load of 4.85 kN acting at its top, the distributions of shear force and moment in the post below the ground surface are shown in Figure 2. The linear deflection of the (rigid) post below the ground surface is also shown in this figure. The locations of the maximum moment and the maximum shear in the embedded section of post can be obtained by differentiating Equations (8) and (9) and equating to zero. Values of the maximum moment and the maximum shear can then be deduced. It can be seen in Figure 2 that, for this example, the maximum moment in the post is about 4% larger than the moment in the post at the ground surface and is at a depth of about  $1/6^{\text{th}}$  of the embedment depth. The maximum shear force is at about  $2/3^{\text{rd}}$  the embedment depth and is about 6 times the 4.85 kN shear force in the post at the ground surface.

If a vertical post is intended to resist mainly a horizontal force  $R$  close to or at ground level then in Equation (3)  $M = R \times t$ . If in Equation (3) in this form we substitute the expression for  $R$  that can be deduced from Equation (2), we obtain a value of  $\frac{2}{3}d$  for  $t$  and then

$$R = k \times \tan \alpha \times b \times \frac{d^3}{24} \quad (10)$$

In this case the two maximum horizontal stresses are  $(k \times \tan \alpha \times \frac{9d^2}{64})$  at  $\frac{3}{8}d$  and  $(k \times \tan \alpha \times \frac{d^2}{4})$  at  $d$ .

The above analyses have been for a square section post. If the post has a circular cross section with diameter  $D$  then the derived equations are applicable if  $b$  is replaced with  $\pi D/4$ .

In the analysis so far it has been tacitly assumed, for short term loading at least, that gaps will form between the post and the soil on the unloaded upper two thirds of the post and on the unloaded lower one third of the post. In practice this will often be the case as most soils have some cohesion enabling them to maintain such gaps in the short term. In the case of a post under a constant lateral load the gaps between the soil and the post will gradually close or will fill with loose material so that active pressures acting on the post may become a permanent feature of the foundations. In such cases, and in particular for larger structures, a reduction in foundation moment capacity due to the development of active pressures may need to be allowed for. The relative size of these active pressure moments will now be considered.

For a fully cohesionless soil, active pressures can be considered to develop at the locations of the above described gaps leading to a reduction in the moment capacity of the embedded post. While the additional horizontal loads acting on the embedded post will increase the value of  $t$  slightly it will be assumed in the following that  $t = \frac{2}{3}d$ . For the upper two thirds the triangularly distributed active pressure will have a resultant

of  $\frac{1}{2}K_a\gamma(\frac{2}{3}d)^2b$  acting at  $2/9d$  from the centre of rotation of the post in which  $K_a$  is the coefficient of active earth pressure. For the lower one third of the post we will assume that the soil bearing against the upper two thirds of the post is restrained by the deflected post and cannot apply a vertical load to any loose soil which might produce active pressure against the bottom one third of the post. Thus a triangularly distributed active pressure will only develop over the bottom third of the post. This pressure will have a resultant of  $\frac{1}{2}K_a\gamma(\frac{1}{3}d)^2b$  acting at  $2/9d$  from the centre of rotation. The combined moment of these two active forces is thus  $(5/81)K_a\gamma bd^3$ .

The proportional reduction in the moment capacity of the post will be  $(5/81)K_a\gamma bd^3$  divided by  $k \tan \alpha \cdot b \cdot d^4/36$  which reduces to:

$$\text{Proportional reduction in moment capacity} = \frac{20K_a\gamma}{9k \tan \alpha d} \quad (11)$$

This reduction is generally only of the order of a few percent and can be ignored. Thus in the above example, with  $\phi = 32^\circ$  and  $\gamma = 18 \text{ kN/m}^3$ , the reduction will be about 1.5% of the previously calculated 35.5 kN.m moment capacity of the post foundation.

### 3 POST WITH ENLARGED EMBEDMENT

In the above analyses the weight of the post and any vertical loads acting on it have been assumed to be supported principally by the soil at the base of the post and possibly also by some friction between the vertical embedded faces of the post and the surrounding soil.

We now consider the case where the embedded section of the post is enlarged to increase its lateral resistance and where the weight of this enlargement and of the post and any vertical loads on it need to be considered in the analysis of overturning resistance. The enlargement will result in a distribution of vertical pressures on the base of the enlargement that will contribute to the resistance of the post to an over-turning moment as well as resisting the vertical loads on the post.

We consider a post with an enlarged square footing, side **B**, through the depth of embedment **d** as shown in Figure 3. The enlarged footing is assumed to be symmetrical about the post axis. We again consider a slight rotation of the post due to a horizontal load acting on the post. The centre of rotation will now be to one side of the vertical axis of the post. The soil resistance on the vertical faces of the footing will be parabolically distributed in accordance with Equation (1) for a simple post embedment. The centre of rotation of the footing can thus be taken to be at a depth of  $2/3d$ . On the base of the footing there will develop a triangularly distributed pressure over an extent  $y_0$  from the centre of rotation to the edge of the enlarged footing. This pressure at a distance  $y$  from the centre of rotation will be  $k_b \cdot y \cdot \tan \alpha$  in which  $k_b$  is the (constant) vertical modulus of subgrade reaction for the foundation soil at the depth of the footing base. The value of  $k_b$  can be taken as 1.2 times the value of the lateral modulus of subgrade reaction at the level of the base (see note on Table 1). The total vertical reaction will thus be  $\frac{1}{2} \cdot k_b \cdot \tan \alpha \cdot B \cdot y_0^2$  which will be equal to **W**, the weight of the footing and post plus any vertical load acting on the post. The value of  $y_0$  will thus be given by

$$y_0 = \sqrt{\frac{2W}{k_b \tan \alpha B}} \quad (12)$$

The moment of this base pressure distribution about the centre of the footing, that is about the point where the line of action of **W** intercepts the base of the footing, will be  $W(B/2 - y_0/3)$ . The maximum base pressure will be  $k_b \cdot y_0 \cdot \tan \alpha$ .

As an example we consider a footing block of concrete ( $\gamma = 24 \text{ kN/m}^3$ ) 1.5 m deep below the ground surface and 1.7 m square. We can calculate the over-turning moment that it will resist for 0.01 radians rotation if  $k$  is 30kPa/mm/m and  $k_b$  is  $1.2 \times 30 \times 1.5 = 54 \text{ kPa/mm}$  as follows:

$$W = 24 \times 1.5 \times 1.7^2 = 104 \text{ kN}$$

$$\text{Side moment of resistance using Equation (4)} = 30,000 \times 1.7 \times 0.01 \times 1.5^4/36 = 71.7 \text{ kN.m}$$

$$y_0 = \sqrt{(2 \times 104 / (54000 \times 0.01 \times 1.7))} = 0.476 \text{ m}$$

$$\text{Base moment of resistance} = 104 \times (1.7/2 - 0.476/3) = 71.9 \text{ kN.m}$$

$$\text{Total moment of resistance of footing} = 143.6 \text{ kN.m}$$

$$\text{Maximum base pressure} = 54,000 \times 0.476 \times 0.01 = 257 \text{ kPa}$$

A similar analysis can be applied to a cylindrical footing, with axis vertical and coinciding with the axis of the pole, with a diameter **D** and depth **d**. As before the side moment of resistance about the centre of rotation,  $M_s$ , is given by Equation (4) for a circular post:

$$M_s = \frac{\pi}{4} D \times k \times \tan \alpha \times \frac{d^4}{36} \quad (13)$$

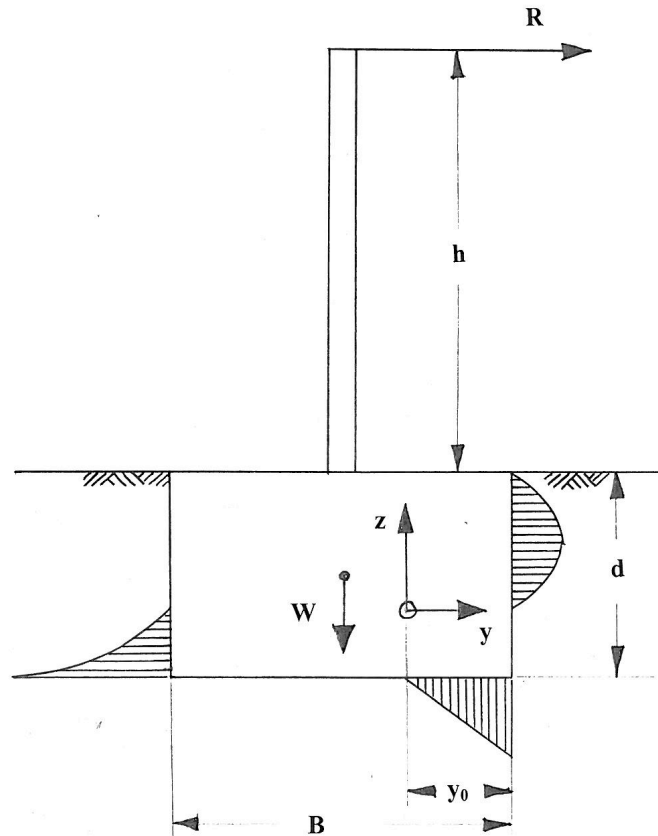


Figure 3: Post with enlarged embedment.

On the circular base the vertical foundation pressure is distributed over a segment with a maximum width in the radial direction of  $y_0$  and for which the segment chord subtends an angle  $2 \times \theta_0$  at the circle centre. Then it can be shown that:

$$W = k_b \times \tan \alpha \times \frac{D^3}{8} \times C(\theta_0) \quad \text{in which } C(\theta_0) \text{ is given by}$$

$$C(\theta_0) = (\sin \theta_0 - \frac{1}{3} \sin^3 \theta_0 - \theta_0 \cos \theta_0) \quad (14)$$

$$y_0 = \frac{D}{2} \times (1 - \cos \theta_0)$$

Equations (14) can be solved for  $\theta_0$  and  $y_0$  and it can then be shown that the base moment of resistance of the base pressure about the centre of the footing is given by:

$$M_B = k_b \times \tan \alpha \times \frac{D^4}{32} \left( \frac{1}{4} (2\theta_0 - \sin 2\theta_0) - \frac{\sin^3 \theta_0 \cos \theta_0}{3} \right) \quad (15)$$

The maximum stress on the base =  $k_b y_0 \tan \alpha$

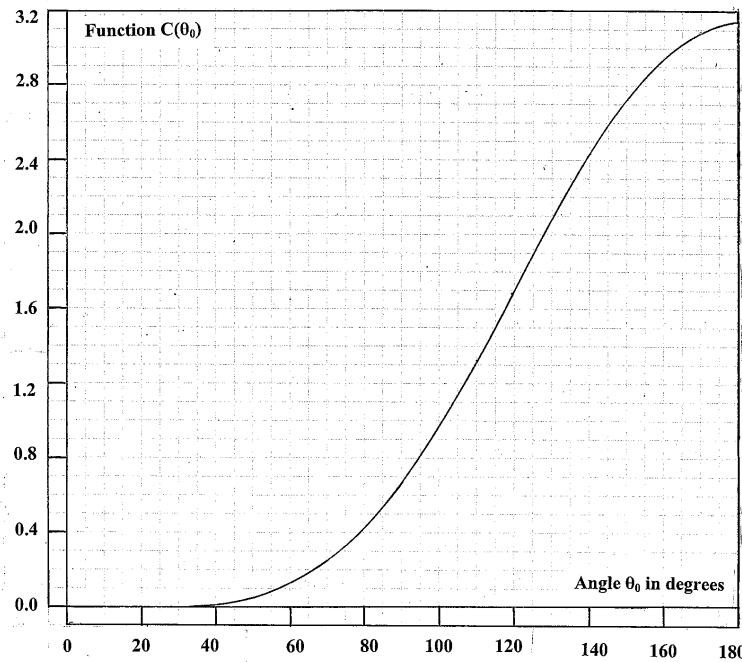


Figure 4: Plot of function  $C(\theta_0)$  against angle  $\theta_0$

A plot of function  $C(\theta_0)$  against angle  $\theta_0$  is shown in Figure 4 which can assist in solving the first of Equations (14).

#### 4 ANALYSIS OF POST BENDING

In the discussions so far it has been assumed that bending deflections of the post can be neglected when compared with deflections of the soil. If the embedded vertical post bends, to the extent that its deflections are comparable with the deflections of the foundation soil, then bending and shear conditions in the embedded part of the post will be different to those that would be derived from the equations so far developed. For this case of post bending the following analytical approach can be used.

We assume that the vertical post extends indefinitely into the foundation soil. From basic beam analysis the following equation is used (see for example Gere & Timoshenko, 1991)

$$EI \frac{d^4 y}{dz^4} = q \quad (16)$$

in which  $q$  is the intensity of the lateral load acting on the post,  $E$  is the elastic modulus of the post material and  $I$  is the second moment of area of the post cross-section. With the coordinate system shown in Figure 5,  $q$  will equal  $-kzyb$ , where  $b$  is the dimension of the side of the square section post, and the equation becomes:

$$EI \frac{d^4 y}{dz^4} + kzyb = 0$$

A closed form solution of this equation is not available and it is simplified by assuming that the modulus of lateral subgrade reaction is constant through the depth of the post embedment and equal to  $k_1$ . The differential equation then becomes:

$$EI \frac{d^4 y}{dz^4} + k_1 yb = 0$$

This equation can be rewritten as

$$\frac{d^4 y}{dz^4} + 4\beta^4 y = 0 \quad (17)$$



in which  $\beta = \sqrt[4]{\frac{k_l b}{4EI}}$  and  $1/\beta$  is a characteristic depth for the post.

If the post has a circular cross section of diameter  $D$  then as before  $\pi D/4$  is substituted for  $b$ .

The applicable solution ( $y$  decreasing with depth) for the above equation is:

$$y = e^{-\beta z} (A \cos \beta z + B \sin \beta z) \quad (18)$$

$A$  and  $B$  are determined by loading conditions for the post. Thus if the bending moment in the post at the ground surface level is  $M_0$  and the horizontal shear force at the same location is  $V_0$  then the bending moment at any location  $z$  below the ground surface is given by:

$$M_z = -e^{-\beta z} \left( \frac{V_0}{\beta} \sin \beta z - M_0 (\sin \beta z + \cos \beta z) \right) \quad (19)$$

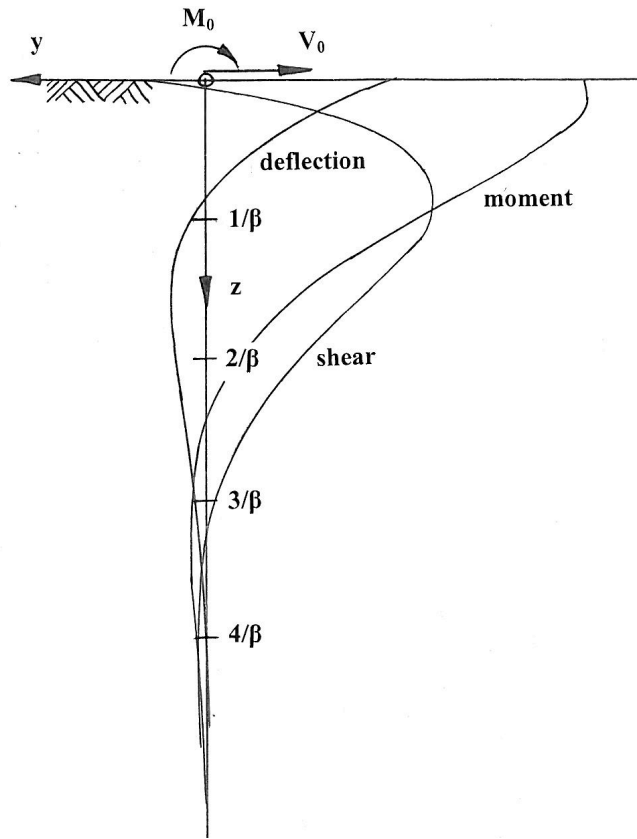


Figure 5: Bending of post subjected to ground surface shear and moment.

The shear force at the same location is given by:

$$V_z = e^{-\beta z} (2M_0 \beta \sin \beta z - V_0 (\cos \beta z - \sin \beta z)) \quad (20)$$

and the deflection at this location is given by:

$$y = \frac{2\beta}{k_l b} e^{-\beta z} (M_0 \beta (\sin \beta z - \cos \beta z) - V_0 \cos \beta z) \quad (21)$$

As an example we consider a square section post having a side of 0.2 m, and made of material having an elastic modulus of 20 GPa, embedded in a soil having an average modulus of lateral subgrade reaction of 40 kPa/mm. At its point of entry into the soil the post is acted on by a horizontal shear force of 4.85 kN and a moment of 29.1 kN.m. We will determine the deflection, shear and bending moment distributions for the embedded section of the post.

The value of  $\beta$  is found to be  $0.9306 \text{ m}^{-1}$ . The deduced distributions of deflection, shear and bending moment are illustrated in Figure 5. These distributions can be compared with the corresponding distributions for a rigid post shown in Figure 2. Particular values for this example in the distributions shown in Figure 5 are described in the following.

The maximum deflection of the post at the ground surface is 7.4 mm, the deflection is zero at a depth of 0.93 m and a maximum deflection of 1.3 mm in the opposite direction is obtained at a depth of 1.8 m. The deflection is again zero at a depth of 4.3 m.

The shear force decreases from the ground surface value of 4.85 kN to  $-17.5 \text{ kN}$  at a depth of 1 m then increases to zero at a depth of 3.5 m becoming slightly positive but of diminishing magnitude at greater depths.

The bending moment in the post increases slightly to  $29.3 \text{ kN.m}$  at a depth of 0.1 m then decreases to zero at a depth of 2.6 m becoming slightly negative before returning to zero at a depth of 6 m.

It can be seen that the deflection, the shear force, and the bending moment in the post become negligible at a depth of about 4 m in this example. In general depths can be considered in non-dimensional form as multiples of the characteristic depth  $1/\beta$ , as shown in Figure 4, so that actions in the post become negligible at a depth of about  $4/\beta$ .

If only the shear force  $V_0$  acts at the ground surface, i.e. the post is very short above the ground surface and is acting as an anchorage for horizontal forces, then the bending moment and shear force at any location  $z$  below the ground surface are given by:

$$M_z = -\frac{V_0}{\beta} e^{-\beta z} \sin \beta z \quad (22)$$

$$V_z = V_0 e^{-\beta z} (\cos \beta z - \sin \beta z)$$

In this case the maximum bending moment occurs at a depth of  $\pi/4\beta$  and is given by:

$$M_{\max} = -\frac{V_0}{\beta} e^{-\pi/4} \sin(\pi/4) = -\frac{V_0}{\beta} 0.3224 \quad (23)$$

The maximum deflection of the post at the ground surface is given by:

$$y_{\max} = -\frac{2V_0\beta}{k_1 b} \quad (24)$$

## 5 REFERENCES

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