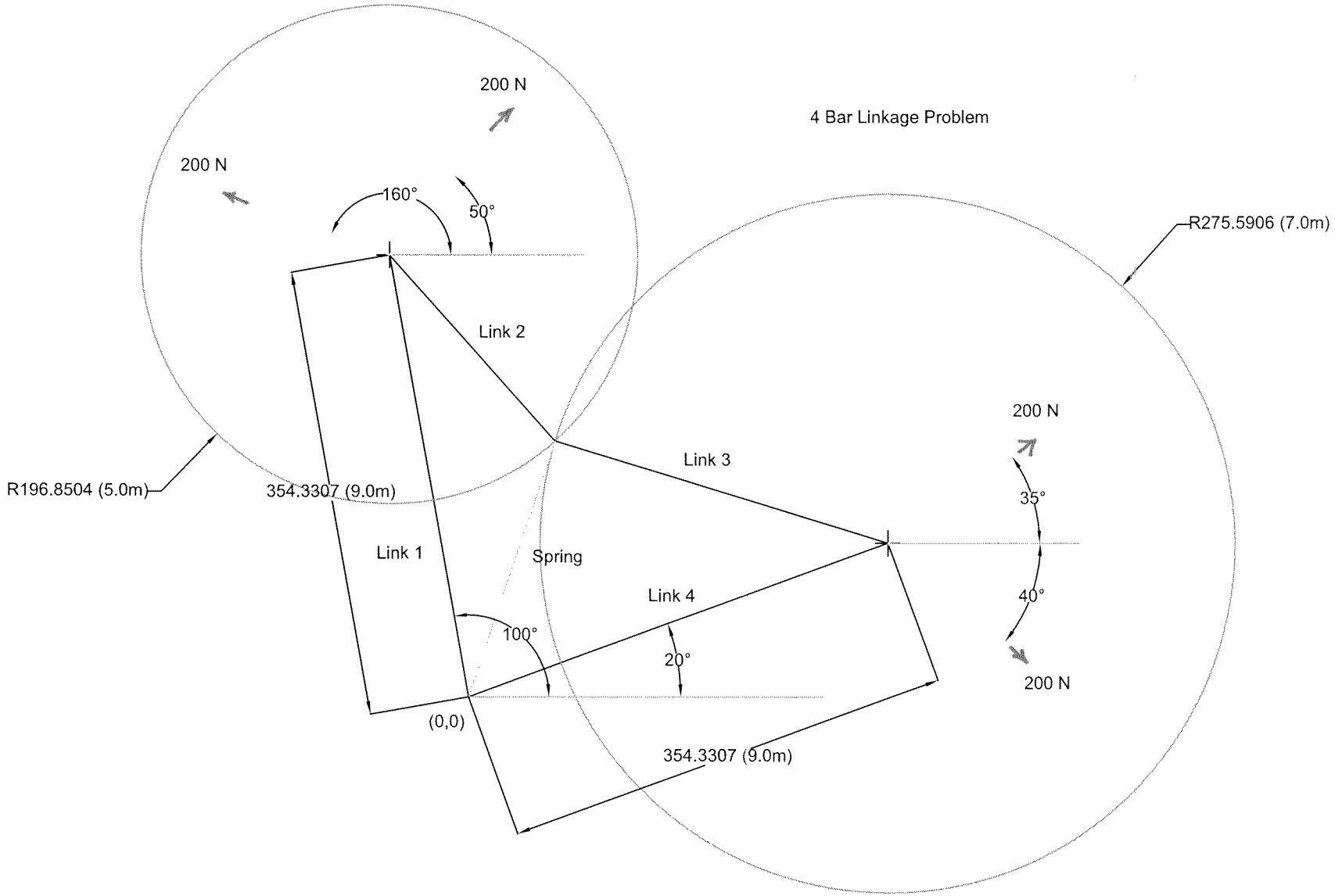

4 Bar Linkage Problem
200 N
200 N
160°
50°
R275.5906 (7.0m)
Link 2
200 N
Link 3
R196.8504 (5.0m)
354.3307 (9.0m)
35°
Link 1
Spring
Link 4
40°
100°
20°
200 N
(0,0)
354.3307 (9.0m)

21 October 2011

STEP 1: determine the geometry of 4 bar link system.

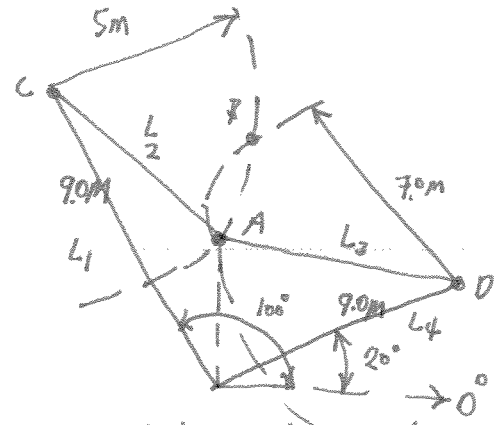


figure 1: linkage system geometry

- pts A & B are circle intersections centred at the ends of L_1, L_4:
- I know the orientation of L_1 & L_2 and the lengths!

pts C: centre of circle at the end of L_1.

$x_C = 9.0 \cos(100) = -1.56283\ m$

$y_C = 9.0 \sin(100) = 8.86327\ m$

pts D: centre of circle at the end of L_4.

$x_D = 9.0 \cos(20) = 8.45723\ m$

$y_D = 9.0 \sin(20) = 3.07818\ m$

equation of circle thru L_1:

$(x - -1.56283)^2 + (y - 8.86327)^2 = 5.0^2$

$(x + 1.56283)^2 + (y - 8.86327)^2 = 25.0$ ← circle L_1

equation of circle thru L_4:

$(x - 8.45723)^2 + (y - 3.07818)^2 = 7.0^2$

$(x - 8.45723)^2 + (y - 3.07813)^2 = 49.0$ ← circle L_4

STEP 2: compute the intersection points of the geometry.

$(x + 1.56283)^2 + (y - 8.86327)^2 =$ circle thru L_1 (i.e. 9.0m @ 100°)

$(x - 8.45723)^2 + (y - 3.07813)^2 =$ circle thru L_4 (i.e. 9.0m @ 20°)

pts A: (1.76525, 5.13180) m lower intersection pts

pts B: (3.33275, 7.84688) m upper intersection pts

verify: $(1.76525 + 1.56283)^2 + (5.13180 - 8.86327)^2 = 25.0$

$11.07612 + 13.92387 = 25.0$

$24.99999 = 25.0$ correct!

$$(3.33275 - 8.45723)^2 + (7.8468 + {}^{-}3.07813)^2 = 49.0$$

$$26.26050 + 22.74021 = 49.0$$

$$49.00051 \approx 49.0 \quad \text{correct!}$$

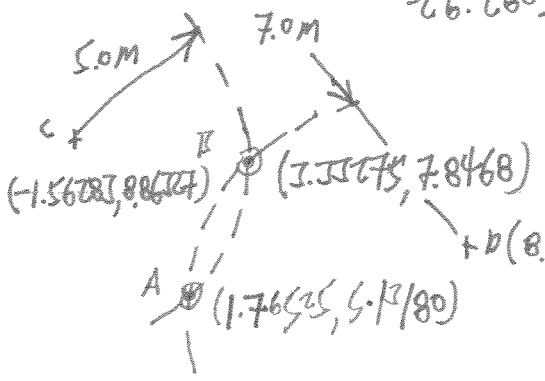


figure 1: circular intersection pts

- this tied down the pts A & pts II intersections, which defines the motion of the spring, A → II.
- so the input rope force will stretch the spring from A to II, regardless, we have no dynamic information to govern the linkage, i.e. mass, accelerations from rest, etc so this is a static analysis with motion impending.

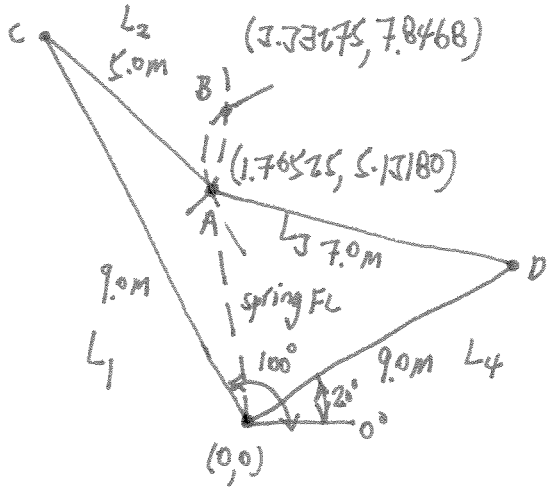


figure 2: 4 Bar Linkage System geometry

STEP 3: compute spring length and stretch

$$\text{Tol}_{FL} = \sqrt{1.76525^2 + 5.13180^2} = 5.42692 \text{ m}$$

$$\text{Tol}_{ST} = \sqrt{(3.33275)^2 + (7.8468)^2} = 8.52523 \text{ m}$$

$$\delta = 8.52523 - 5.42692 \text{ m} = 3.09831 \text{ m}$$

$FL_{spring} = 5.42692$ m — FL_{sp}

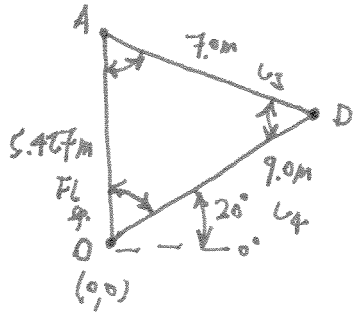
$\delta = 3.07831$ m — δ_{sp}

STEP 4: determine the orientation on links 2 & 3.

law of cosines: $L_4^2 = FL_{spr}^2 + L_3^2 - 2\, FL_{spr} \cdot L_3^2 \cos A$

$$9.0^2 = 5.427^2 + 7.0^2 - 2(5.427)(7.0)\cos A$$

$$\therefore \cos A = {}^{-}0.01151 \rightleftharpoons A = 91.92168 \approx 92°$$



law of cosines: $5.427^2 = 9.0^2 + 7.0^2 - 2(9.0)(7.0)\cos D$

$$\therefore \cos D = 0.79800 \quad \vee \quad D = 37.06071 \approx 37°$$

$m\angle O = 180 - 92 - 37 = \underline{51^\circ}$

- this fully defines ΔOAD; the left hand quantities for ΔOCA are done in similar fashion;
- note we are given L_4 @ 20° and L_1 @ 100° inclination,
- then it follow that: $O' = 100 - 20 - 51 = 29^\circ$;

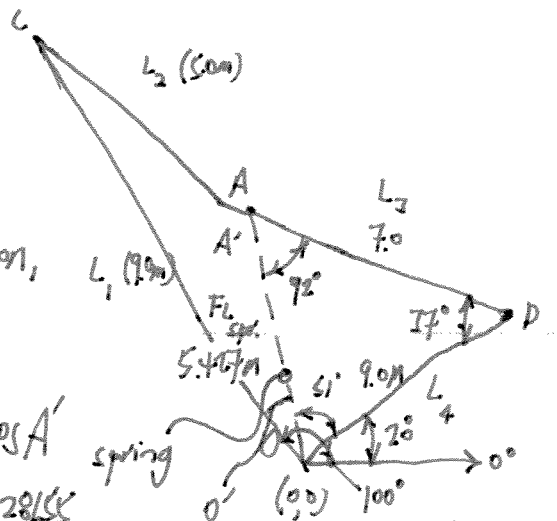


figure 4: geometry LH & RH of spring

Law of cosines: $9.0^2 = 5.0^2 + 5.427^2 - 2(5.0)(5.427)\cos A'$

$\therefore \cos A = -0.48918 \Rightarrow A' = 119.2855^\circ$

$m\angle C = 180 - 119.2855 - 29 = 31.7145^\circ$.

- this fully defines all vertices and element lengths!
- need to be careful in getting a proper orientation to the datum,
- all elements are measured relative to the RHS positive;

$m\angle D = 37^\circ$

$m\angle A = 92^\circ$

$m\angle A' = 119.29^\circ$

$m\angle C = 31.71$

$m\angle O' = 29^\circ$

$180^\circ - (100^\circ + 31.71) = 48.29^\circ \approx 48^\circ$

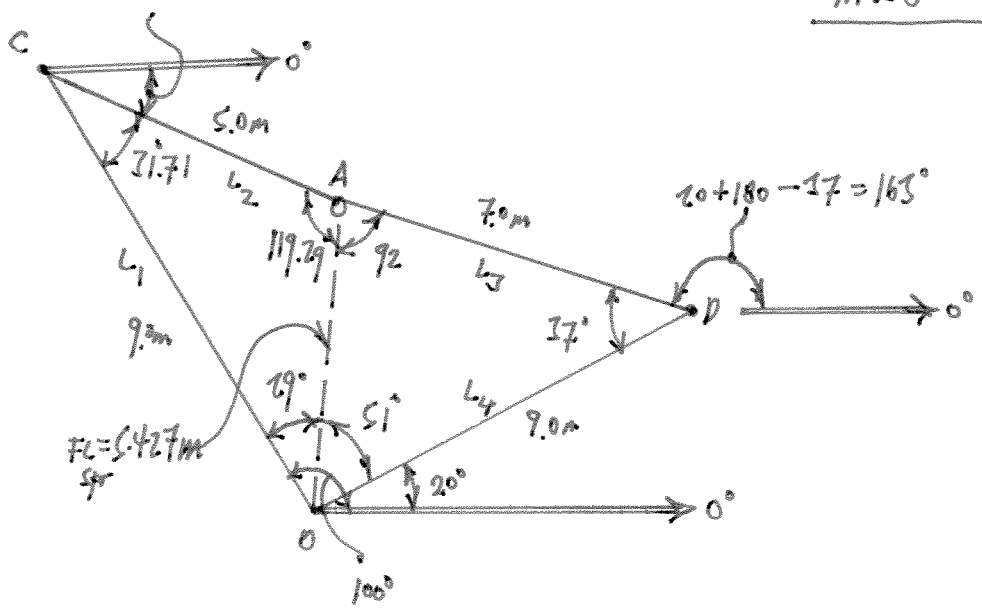


figure 5: 4 Bar Linkage System – Fully Defined

NOTE: verify quantities using AutoCAD dwg of layout – 24 October 2011.

Computation of Member Loads

vertex C:

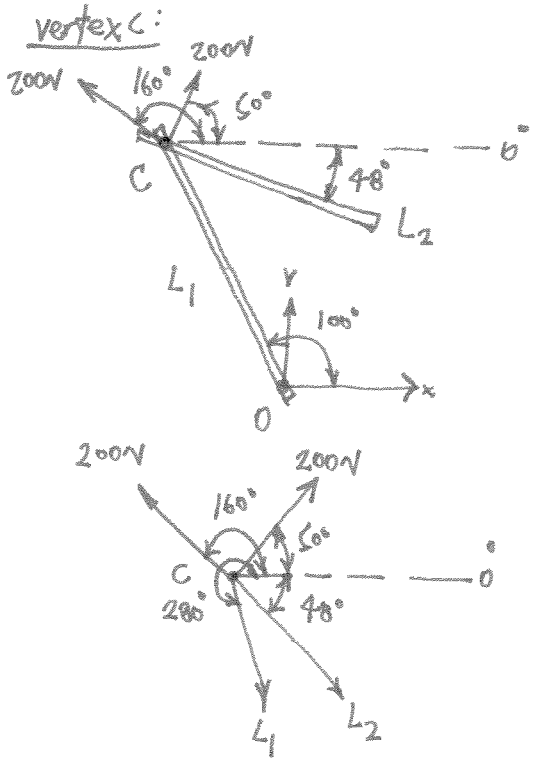


figure 6: Joint C and Free Body Diagram

$(\rightarrow) F_x$	$(\uparrow) F_y$
$200\cos(50)$	$200\sin(50)$
$200\cos(160)$	$200\sin(160)$
$L_1\cos(280)$	$L_1\sin(280)$
$L_2\cos(312)$	$L_2\sin(312)$
$\rightarrow \sum F_x$	$(\uparrow) \sum F_y$

$$\rightarrow \sum F_x = 128.55752 + -187.93852 + 0.17365 L_1 + 0.66913 L_2 = 0$$

$$\uparrow \sum F_y = 153.20889 + 68.40401 - 0.98481 L_1 - 0.74314 L_2 = 0$$

$$0.17365 L_1 + 0.66913 L_2 = 59.38100$$
$$-0.98481 L_1 - 0.74314 L_2 = -221.61292$$

$$\begin{pmatrix} 0.17365 & 0.66913 \\ -0.98481 & -0.74314 \end{pmatrix} \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} = \begin{pmatrix} 59.38100 \\ -221.61292 \end{pmatrix}$$

HP41CX → XEQ MATRIX, XEQ SIMEQ

$\therefore L_1 = 196.56\,N$ & $L_2 = 37.73\,N$

$L_1 = 196.56\,N$ (tensile) L_1

$L_2 = 37.73\,N$ (tensile) L_2

vertex D:

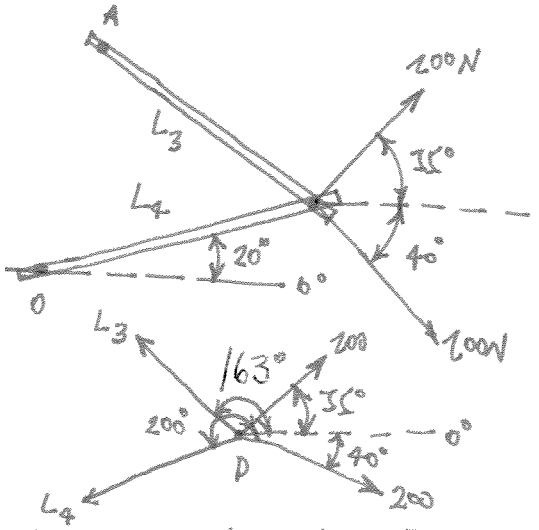


figure 7: Joint D and Free Body Diagram

$(\rightarrow) F_x$	$(\uparrow) F_y$
$100\cos(35)$	$100\sin(35)$
$L_3\cos(163)$	$L_3\sin(163)$
$L_4\cos(200)$	$L_4\sin(200)$
$100\cos(310)$	$100\sin(310)$
$\rightarrow \sum F_x$	$(\uparrow) \sum F_y$

$$\rightarrow \sum F_x = 163.87041 - 0.95630 L_3 - 0.93969 L_4 + 153.20889 = 0$$
$$\uparrow \sum F_y = 114.71529 + 0.29237 L_3 - 0.34202 L_4 - 128.55752 = 0$$

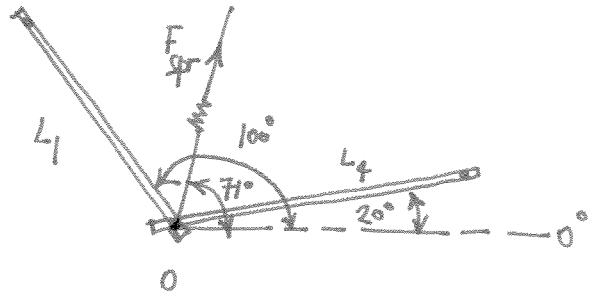
$$-0.95630 L_3 - 0.93969 L_4 = -317.07930$$
$$0.29237 L_3 - 0.34202 L_4 = 13.84191$$

$$\begin{pmatrix} -0.95630 & -0.93969 \\ 0.29237 & -0.34202 \end{pmatrix} \begin{pmatrix} L_3 \\ L_4 \end{pmatrix} = \begin{pmatrix} -317.07930 \\ 13.84191 \end{pmatrix}$$

HP41Cx → XEQ MATRIX, XEQ SIMEQ

$$\therefore L_3 = 201.792\,N \;\&\; L_4 = 132.028\,N$$

Vertex O:



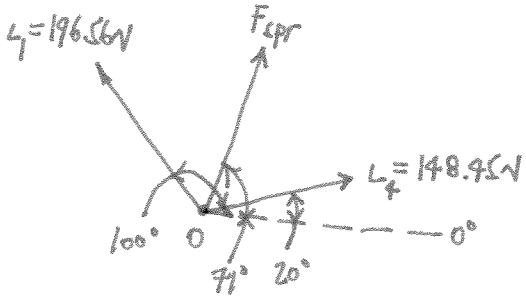


figure 8: Joint O (origin) and Free Body Diagram

$L_3 = 201.8N$ (tensile) L_3

$L_4 = 132.0N$ (tensile) L_4

$\rightarrow F_x$	$\uparrow F_y$
$132.07\cos(20)$	$132.07\sin(20)$
$F_{spr}\cos(71)$	$F_{spr}\sin(71)$
$196.56\cos(100)$	$196.56\sin(100)$
$\rightarrow \sum F_x$	$\uparrow \sum F_y$

$$\sum F_x = 124.06762 + 0.32557 F_{spr} - 34.13229 = 0$$

$$F_{spr} = \frac{-124.06762 + 34.13229}{0.32557}\,N$$

$$F_{spr} = -276.24N \quad \text{spring load}$$

∴ the spring load in the 4 Bar Linkage is 276.2N (compressive)

$$F_{spr} = -276.2N \angle 71°$$