However, the constant B must be taken negative $(-)$; thus, the buckling stress will be lower for the tee in this orientation.

$$M_{cr} = \frac{130{,}000}{L_b}\left[B + \sqrt{1 + B^2}\right]$$

$$B = -2.3\left(\frac{7.05}{L_b}\right)\sqrt{\frac{13.3}{0.398}} = -\frac{93.7}{L_b}$$

Setting M_{cr} equal to $M_r = 11.8$ ft-kips, then solving for L_b gives $L_r = 68$ ft.

The nominal strength relationship for the tee having its stem in compression is not presented in graphical form; however, M_n remains constant at $Q_s F_y S_{xc}$ (that is, 11.8 ft-kips) to L_r, then decreases according to M_{cr}. ■■

9.13* LATERAL BRACING DESIGN

The questions of what constitutes bracing and how to design bracing continue to be major concerns of practicing engineers. The subject is included in this chpater because a major item of concern in lateral bracing design is the restraint required to prevent lateral-torsional buckling in beams. The development of this section, however, is applicable to the bracing of columns as well as beams. In Sec. 6.8 the concept of *braced* and *unbraced* systems was briefly discussed in regard to the effective length factor K. In the following discussion, the emphasis in on *braced* systems; that is, the overall structural system is braced by cross bracing or attachment to an adjoining system that is braced. The bracing requirements for frames are treated in Chapter 14. Bracing for individual beams or columns may consist of cross bracing where the axial stiffness of the bracing elements is utilized; it may be provided at discrete locations by flexural members framing in transverse to the member being braced, wherein both axial and flexural stiffnesses of the bracing member are utilized; or such bracing may be provided continously by material such as light gage roof decking or wall panels.

Little is available in specifications but point bracing has been treated by Zuk [9.39], Winter [9.40], Massey [9.41], Pincus [9.42], Galambos [9.43], Urdal [9.44], Lay and Galambos [9.45], Taylor and Ojalvo [9.46], Hartmann [9.47], Mutton and Trahair [9.48], Medland and Segedin [9.49], and Plaut [9.73]. Recent practical treatment has been provided by Yura [9.50], Lutz and Fisher [9.51], Ales and Yura [9.52], Clarke and Bridge [9.53], and Yura [9.72]. What follows is largely a combination of the work of Winter [9.40], Galambos [9.43], and Yura [9.50, 9.72].

Point Bracing for Elastic Columns and Beams

Consider the axially loaded column of Fig. 9.13.1a where the top and bottom of the member are assumed to be supported in such a way that no side movement occurs at one end relative to the other. Such restraint would constitute a *braced* system. The bracing to create such restraint may be considered as a spring at the top that is capable of developing a horizontal reaction equal to the spring constant k times the deflection Δ. When the brace has a large spring constant (that is, the brace is very stiff) the deflection Δ could be close to zero and yet the spring may provide a large enough

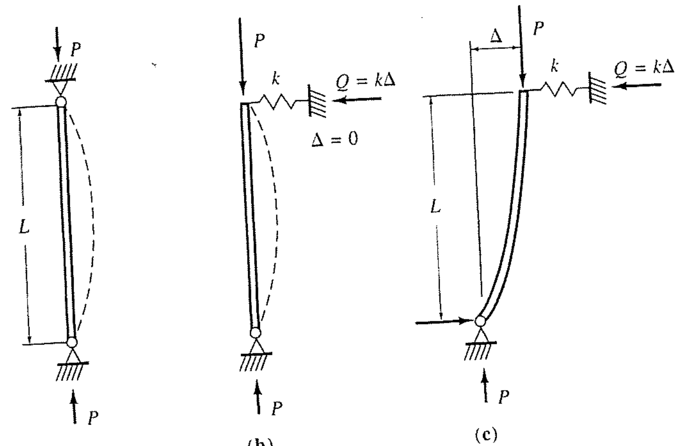


Figure 9.13.1 Bracing for a single-story column.

horizontal force to prevent any side motion (sideway) at the top. This would be the situation in Fig. 9.13.1b. The equilibrium requirement is shown in Fig. 9.13.1c, wherein a sidesway is shown. If one imagines this as a slightly deflected position, then in order to have equilibrium, it is required that

$$P\Delta = QL = (k\Delta)L \tag{9.13.1}$$

If $(k\Delta)L$ is less than $P\Delta$, sidesway occurs. If $(k\Delta)L$ is greater than $P\Delta$, no sidesway occurs, and the column would be considered braced. The ideal brace, then, would be one that has just enough stiffness k to prevent movement (at the top in this example); that is,

$$k = \frac{P}{L} \tag{9.13.2}$$

The maximum load for which bracing would be required is the elastic bucking load P_{cr}, or the load causing yielding or inelastic buckling if that is lower than the elastic P_{cr}. Thus the largest required stiffness k_{ideal} is

$$k_{ideal} = \frac{P_{cr}}{L} \tag{9.13.3}$$

The concept is shown by the plot of P vs kL in Fig. 9.13.2, wherein when k exceeds k_{ideal}, P_{cr} is reached and the column buckles without end translation (sidesway); in other words, it is a *braced* system. When k is less than k_{ideal}, a sidesway deflection will occur such that $P = kL$; in other words, a so-called *unbraced* system. The major treatment of unbraced systems is in Chapter 14, devoted to rigid frames.

Next, extend the concept to a two-story column within a braced system, as shown in Fig. 9.13.3. When no displacement occurs at mid-height, i.e., full bracing is provided, the column will buckle at a load nearly equal to

$$P_{cr} = \frac{\pi^2 EA}{(L/r)^2} \tag{9.13.4}$$

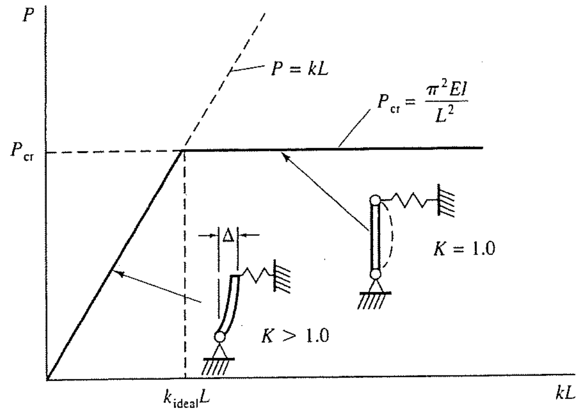


Figure 9.13.2 Brace stiffness relative to concept of "braced" ($K = 1.0$) and "unbraced" ($K > 1.0$) systems for column hinged at top and bottom.

In other words, one may imagine that a hinge exists at mid-height. Buckling occurs when the column snaps into the two half-wave mode of Fig. 9.13.3c.

Taking moments about the imaginary hinge location with the column deflected by an amount Δ, as in Fig. 9.13.3b, gives

$$P_{cr}\Delta = \frac{Q}{2}L \tag{9.13.5}$$

Since $Q = k\Delta$,

$$P_{cr}\Delta = \frac{(k\Delta)L}{2} \tag{9.13.6}$$

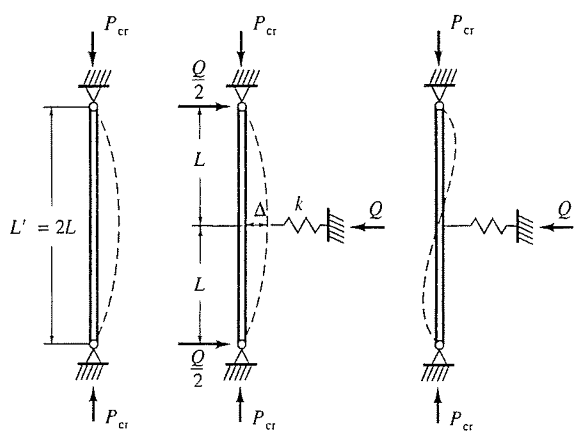


Figure 9.13.3 Mid-height brace for a two-story column.

As in the one-story column, if k_{ideal} is the necessary stiffness to create a nodal point (zero deflection) at mid-height of the two-story column, then

$$k_{\text{ideal}} = \frac{2P_{\text{cr}}}{L} \tag{9.13.7}$$

For situations with more than two equal spans, the same procedure may be used to obtain k_{ideal}. Examination of Fig. 9.13.4 for three equal spans will show that the spring forces Q can act either in the same or in opposite directions. Assuming they act in the *same* direction (Fig. 9.13.4a), using imaginary hinges at one-third span points, and taking moments when slightly deflected at the brace points, gives

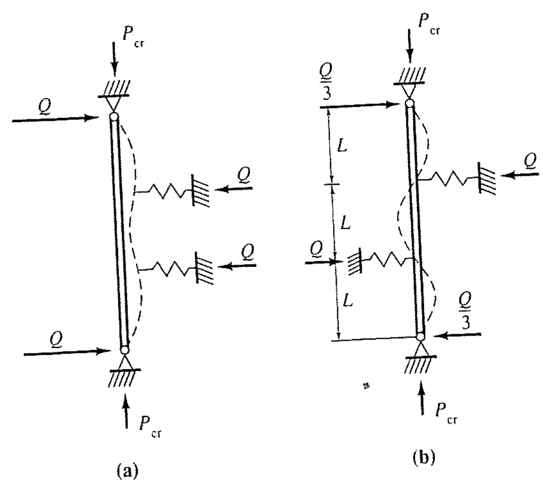


Figure 9.13.4 Column braced to make three equal spans.

$$QL = (k\Delta)L = P_{\text{cr}}\Delta; \quad k_{\text{ideal}} = \frac{P_{\text{cr}}}{L} \tag{9.13.8}$$

Assuming the forces Q acting in *opposite* directions (Fig. 9.13.4b) gives

$$QL/3 = (k\Delta)L = P_{\text{cr}}\Delta; \quad k_{\text{ideal}} = \frac{3P_{\text{cr}}}{L} \tag{9.13.9}$$

The configuration requiring the highest spring constant is the correct one, that which will permit the highest critical load. If a lesser stiffness is used, an alternate buckling mode will occur at a lower load, accompanied by displacement at the springs.

By the same process, k_{ideal} may be determined for any number of equal spans. In general,

$$k_{\text{ideal}} = \frac{\beta P_{\text{cr}}}{L} \tag{9.13.10}$$

where β varies from 1 for one span to 4 for infinite equal spans. The variation is given in Fig. 9.13.5.

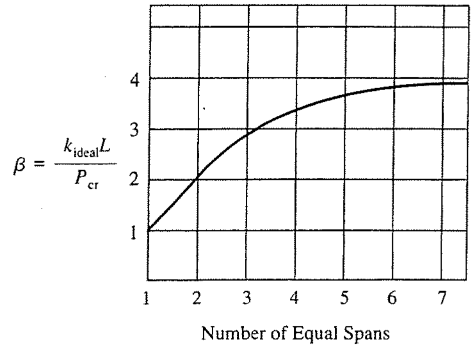


Figure 9.13.5 Variation of required spring constant for column with number of equal unbraced spans.

Thus Eqs. 9.13.3, 9.13.7, 9.13.9, and 9.13.10 give the ideal brace *stiffness* to prevent translation at the points where the braces act.

In addition to stiffness, a brace must provide adequate *strength*. The strength Q required of an ideal brace is

$$Q = k_{\text{ideal}}\Delta \tag{9.13.11}$$

but until buckling occurs, Δ is zero (see Fig. 9.13.6); therefore there will be no brace force in the ideal system until buckling occurs.

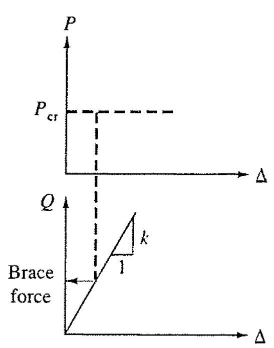


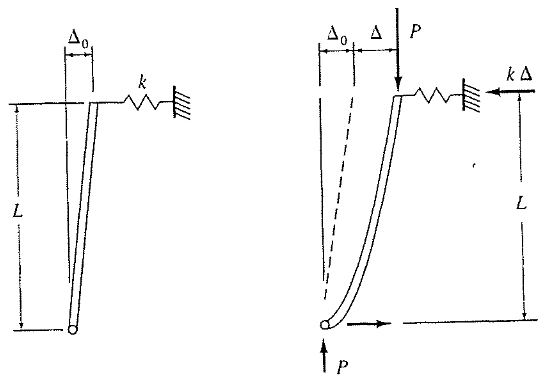
Figure 9.13.6 Brace force relative to column load for ideal system.

Compression members in real structures are not perfectly straight, perfectly aligned vertically, nor perfectly loaded as assumed in calculations; there is always an initial crookednesss. In other words, Δ is not zero even when there is no compressive load P acting. Reexamine the single-story column of Fig. 9.13.1 assuming there is an initial deflection Δ_0 that exists even when P is zero. Then, as shown in Fig. 9.13.7, equilibrium requires

$$(k\Delta)L = P(\Delta + \Delta_0) \tag{9.13.12}$$

for $P = P_{cr}$,

$$k_{\text{reqd}} = \frac{P_{cr}}{L}\left(1 + \frac{\Delta_0}{\Delta}\right) \tag{9.13.13}$$



(a) No load applied (b) When load is applied

Figure 9.13.7 Column with initial crookedness Δ_0.

Since $k_{ideal} = P_{cr}/L$, Eq. 9.13.13 then becomes

$$k_{reqd} = k_{ideal}\left(1 + \frac{\Delta_0}{\Delta}\right) \tag{9.13.14}$$

which is the *stiffness* requirement for compression members having initial crookedness Δ_0.

The *strength* requirement is then

$$Q = k_{reqd}\Delta = k_{ideal}\left(1 + \frac{\Delta_0}{\Delta}\right)\Delta$$

$$Q = k_{ideal}(\Delta + \Delta_0) \tag{9.13.15}$$

Normal tolerances on crookedness of compression members would vary from 1/500 to 1/1000 of the length [9.41]. The AISC *Code of Standard Practice** indicates acceptable out-of-plumbness to be $L/500$. Considering accidental eccentricity of loading, Winter [9.41] suggests taking Δ_0 from 1/250 to 1/500 of the length.

Load and Resistance Factor Design—Point Bracing

To obtain design equations useful in LRFD, Winter [9.40] has suggests $\Delta = \Delta_0 = L/500$. Substitution of this in Eqs. (9.13.14) and (9.13.15) gives design equations.

1. For *stiffness* k_{reqd},

$$k_{reqd} = 2k_{ideal} \tag{9.13.16}$$

* *Code of Standard Practice for Steel Buildings and Bridges,* American Institute of Steel Construction, June 10, 1992 (Section 7.11.3.1).