ACCELEROMETER MOUNTING AND DATA INTEGRITY

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ABSTRACT

A test program has been conducted to evaluate common accelerometer mounting techniques. The first part covers the immediate attachment of the accelerometer to the structure. The resonance frequencies obtained using solid studs, helicoils, insulated studs, adhesives, aluminum blocks and the effects of proper torque are given. The resonance frequencies are not lowered much by the use of any of these methods, even when using very soft adhesives, so that the data obtained is very good. In some installations it is not possible to have the interface to the structure in tension, but it may be loaded in other ways. These types of mountings have been found to have lower natural frequencies. In some cases they may be too low for the intended measurements, with resonances around 10 kHz.

It was realized that the outputs could be affected if the transducer is attached to a thin-wall structure. Theoretical curves have been calculated from which the effect of adding a mass to the structure can be obtained. A few tests were made and they support the general behavior given. The local resonances caused by attaching the transducers are low, and of the same order as the basic structural resonances, but additional to these. The curves presented will help in evaluating these resonances when it is necessary to use mountings of this type. A number of lessons were learned in this study. One of the more important ones is the value of a simple and inexpensive high frequency calibrator in evaluating accelerometer performance. The manufacturers' stated flat performance regimes are valid, but care should be exercised if frequencies above these are considered in any way.

INTRODUCTION

Theory and tests are an essential complement to each other in our knowledge of shock and vibration. Instrumentation is basic to our understanding of what is happening during any given test. Power spectral density curves and shock spectra are based on the readings of data as recorded from tests. They are valid between a lower and an upper limit. The instruments selected, their mounting installations, and where they are located relative to the structure play an important role in the quality of the data obtained. Some of the factors found important are described in this paper. The discussion and curves show commonly used methods that are acceptable and others which can give very poor data.

OBJECTIVES

This investigation was conducted with particular objectives in mind: first, to establish the validity of the use of specific accelerometer mounting techniques in common use with respect to the frequency response expected; second, to find the local frequencies of a cantilever block, with an accelerometer, attached to a thin aluminum skin; and third, to gain some insight on systems of this type to measure shock. It is felt that the objectives have been met.

Manufacturers specify, for each transducer type, a mounted resonance frequency and a frequency to which the data is flat within a few percent, using the best mounting technique. These have been found to be valid. Reductions of these occur when other mounting techniques are used.

A local resonance is caused when a transducer is attached to a thin-wall structure. A preliminary evaluation of this effect is given in the form of theoretical curves, which are supported by a few test points.

An understanding of all of the preceding is necessary in order to know what kind of vibration and shock data an accelerometer is capable of measuring. The

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following tables and curves will help in understanding which types of mountings give good data and which types should be avoided.

INSTRUMENTATION OF TESTS

The instrumentation used to achieve the objectives of this investigation included a small, high frequency vibration calibrator, a pulse generator for electrical pulse excitation, a standard shaker system, and a small pneumatic shock machine.

High Frequency Calibrator

A B&K 4290 calibrator was used to determine the integrity of the various accelerometer mounting methods. This system was used to vibrate test accelerometers in the frequency range from 200 Hz to above 50,000 Hz at acceleration levels below 0.76 g. A block diagram is shown in Figure 1. The calibrator is similar to a back-to-back calibration fixture, where the fixture is actually the moving element of the shaker. This moving element has (1) a driver coil wound around its base, (2) an internally mounted reference accelerometer, and (3) an accelerometer mounting surface on top. The lowest resonant frequency of the system was that of the reference accelerometer, 57 kHz.

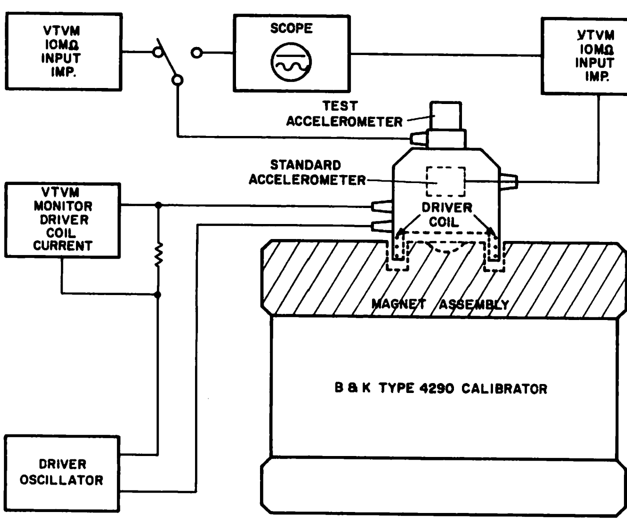

Fig. 1. Block Diagram for Comparison Calibration.

A Hewlett Packard audio signal generator, Model 205AG, was used to drive the calibrator at frequencies between 200 Hz and 23,000 Hz. This unit has an output of 5 watts up to 20,000 Hz. A 250 watt Optimation, Inc., digital oscillator-amplifier, Model RCD 1024/PA250, was used at frequencies from 23,000 Hz to above 50,000 Hz. The HP and the Optimation driver-oscillators provided the best impedance matching and the cleanest, distortion-free outputs of six different driver systems that were tried.

The output of the built-in standard accelerometer was measured on a Ballantine true RMS volt meter, model 320, and the wave form was obtained from the scope output jack on the VTVM and monitored on a dual beam oscilloscope. The output of the accelerometer under test was measured on a Ballantine TRMS VTVM, model 320A, when the scope was not in the circuit. Both VTVM's have a 10 megohm input impedance.

The driver coil current was monitored on a VTVM by inserting an HP #11030A one ohm shunt in series with the driver coil. Figure 2 is a photo of the setup using the HP205AG oscillator, but without the oscilloscope.

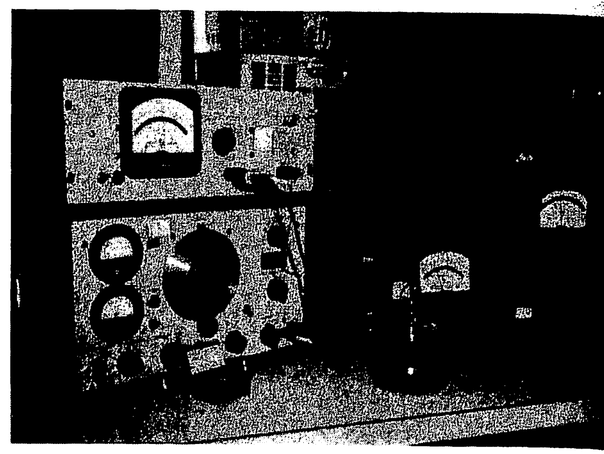

Fig. 2. B&K Shaker and Instrumentation.

The absolute system accuracy was approximately ± 2 db from 200 Hz to 12,000 Hz, but the repeatability from day to day was generally ± 0.5 db over the same frequency range. At resonance, or at frequencies above 12,000 Hz, the absolute accuracy might be off by ± 6 db. However, above 12,000 Hz, the repeatability was usually within ± 3 db, even at 20 db resonances. Even with errors as great as these, it was possible to determine the variations in resonant amplitudes and frequencies due to small changes in the thickness of bonding adhesives.

As part of this program several of each of eight different types of transducers were calibrated at frequencies at least as high as 21,000 Hz. From this data it was possible to separate calibrator system and accelerometer inaccuracies. As noted before, the standard accelerometer built into the calibrator resonated at 57,000 Hz. In many cases it was possible to excite this resonance by driving the calibrator at the 1/3 and 1/2 harmonics (19,000 Hz and 28,500 Hz). Because of this, some of the figures show discontinuous curves at these frequencies. Also, whenever the wave form became distorted, data points were omitted.

Electrical Pulse Calibrator

In an effort to minimize the possibility of any non-shock phenomena from being interpreted as shock data, single electrical pulses were introduced into the transducer-amplifier system. By this means, degradation of signals due to long lines, or the ringing of the electronics could be determined. The method used is similar to voltage insertion calibration methods except that single pulses of various shapes were used instead of continuous sinusoidal waves.

The pulses were generated by an arbitrary function generator and introduced into the accelerometer circuitry through a 100 ohm resistor, as shown in Figure 3. Both the input to the accelerometer circuitry and the output of the charge amplifier were captured on a storage oscilloscope for study.

The results of these tests indicated that single square pulses with durations as short as 40 microseconds caused no significant signal degradation or ringing. The worst case showed an overshoot of 5% and an undershoot of 10% when accelerometer cables both 3 feet and 30 feet in length were used

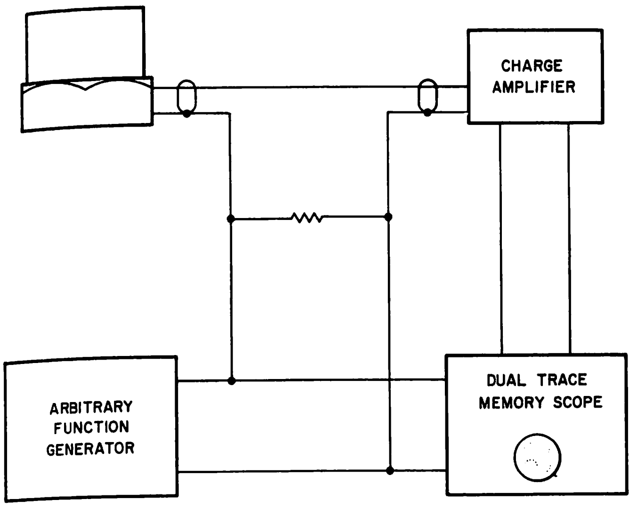

Fig. 3. Block Diagram for Electrical Shock Calibration.

Standard Vibration System

All of the vibration tests in which accelerometers were attached to curved panels were performed on an MB Model 2150 high frequency vibration test system. This system consists of an EA2150 vibration exciter and a Model 2750 amplifier. The system is operable from 5 Hz to 10,000 Hz, and has a force output of 150 pounds.

Standard Shock Machine

An AVCO shock machine, Model SM-005, was used to perform all of the mechanically induced shock tests.

INTERFACE EFFECTS

The immediate interfaces between the accelerometer and the structure are evaluated first. These include solid studs, adhesives, epoxies, etc., and the results are summarized in Table I. This table gives the resonant frequencies and data flatness limits for different types of accelerometers using several different mounting techniques. They are illustrated further in the associated figures, which also show the effect of other variables.

TENSION-COMPRESSION

Solid Studs

This is the standard threaded stud. It is very well known and has been extensively evaluated (1,3). The response characteristics are good and form the basis of comparison. Curves for different accelerometer types are given in Figure 4. The improvement of high frequency response above 5 kHz with the use of oil at the interface is well known (3). All of the tests above 5 kHz had a thin film of oil added to the contact areas.

TABLE I

MOUNTING VARIABLE EFFECTS

Loading Mode	Interface Material	Data Flat To 0.5 db (6%)	Data Flat To 1.0 db (12%)	Resonance Frequency kHz	Accelerometer Weight Grams	Ref. Fig.
Tension-Compression	Solid stud + oil film with or without Helicoil.	9.5	13	32	32	4
		12	15.5	32 (17)[1,2]	28	4
		17	20	50 (15)[3]	21	4
		(13)	(16)	80 (34)[4]	13	4
		21	26	50	2.2	4
	Insulated stud	7	10	22	32	
		(7)	(10)	27[2]	28	
		12	14	30[3]	21	5
		(11)	(17)	(32)[4]	13	
	Al. Block 0.75 x 0.75 x 0.625 inches high	(9.5)	(10.7)	See Note 3	21	5
		(15)	(17)	(27.5)	13	
		12	16.5	42	2.2	
	Rigid[5] adhesive	9	11.5	31.5	28	
		16.5-23	20-29	42	21	6
	Soft[6] adhesive	18	21	44	21	7
Tension-Compression + Bending	See Fig. 8	3	4	9	2.2	9
Shear + Bending	Four #4 screws	4	5.5	11.2	-	11
	Eastman 910	5	7	20	-	11
	Epon 934	5	7	19	-	11
	Epon 946	5	6.5	19	-	11

Notes:
1. Subresonances and their effects, are given in parentheses. When the accelerometer is rigidly mounted, the resonances are narrow and do not seriously affect the sensitivity.
2. This accelerometer has a local resonance of the case at 17 kHz, which appeared on all records.
3. This accelerometer has a local resonance of the case at 15 kHz, which appeared on all records.
4. This accelerometer has a subresonance which is at about 34 kHz when mounted with a solid stud. It decreases in frequency when a softer structure is used under the accelerometer.
5. See Figure 6 for types of rigid adhesives.
6. See Figure 7 for type of soft adhesive.

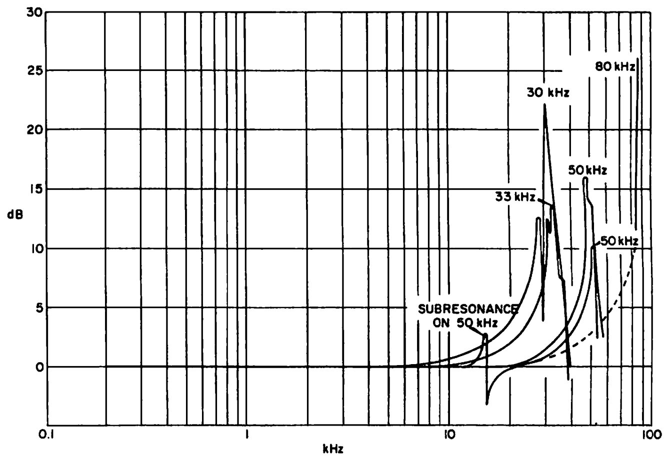

Fig. 4. Tension-Compression/Solid Studs & Oil/ Basic Accelerometer Resonances.

Helicoils

Aluminum structures are often assembled with helicoils for threads in repeated use or to develop greater strength in a threaded connection. They do not affect the mounted natural frequencies by any significant amounts. The g levels, weights, etc., can be used to calculate the need for such thread reinforcements when necessary.

Insulated Studs

These are used to electrically insulate an accelerometer from the parent structure. Mechanically they act as springs, and lower the mounted resonance. This effect is shown in Figure 5.

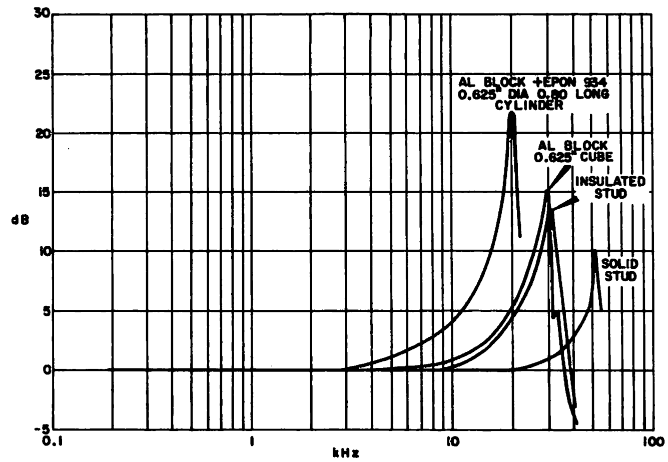

Fig. 5. Tension-Compression/Elastic Effect of Structure Under Accelerometer.

Adhesives

Adhesive behavior was evaluated. In general, adhesives were found to be much better than expected with respect to frequency response. They were evaluated using a design very similar to a cementing stud, which was made of 3/4 inch hex stainless steel stock, 0.187 inch thick. One side was flat, the other had an integral #10-32 stud with which it was fastened onto the shaker head. The accelerometer was cemented onto the flat side. The results are considered equivalent to the use of a regular cementing stud.

The adhesives were divided roughly into two classes; rigid and soft. The rigids have a modulus of elasticity from 30 to 50 ksi; the softs, around 5 ksi. Another characteristic of rigid adhesives is their low elongation, which generally is less than 5%. Eastman 910 is classed by behavior as a rigid adhesive. The rubbery adhesives have an elongation over 10% when tested in bulk at slow strain rates.

The rigid adhesives, as a class, are flat to 10 kHz. Thickness of the bond line has a very slight effect on the natural frequency and consequently on the region of flat response. This is shown in Figure 6. As long as there are no shock loads applied to the bond lines or they are very small, these adhesives can be relied upon to provide good vibration data.

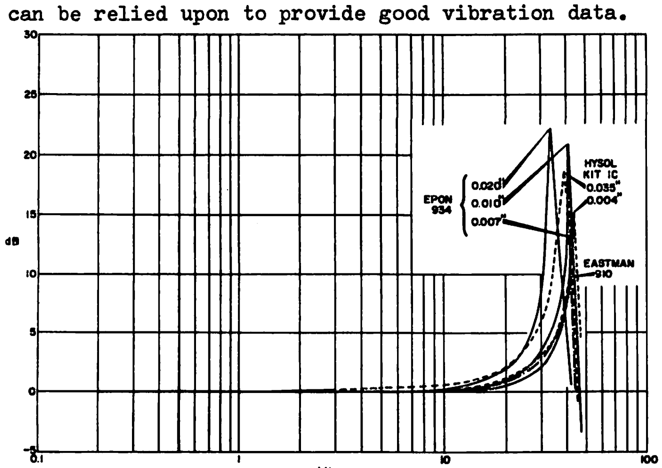

Fig. 6. Tension-Compression/Rigid Adhesives/ Accelerometer Resonance 50 kHz.

The rubbery adhesives are more sensitive to the bond thickness. In the beginning it was not believed that they would be of any value because their resonance frequency was expected to be very low. Only one, which has been used as a shock attenuator, was included. When about 0.005 inch thick or less, the Epon 946 behaved as a rigid adhesive. At 0.020 inch it is flat to only 7 kHz. The frequency response is shown in Figure 7.

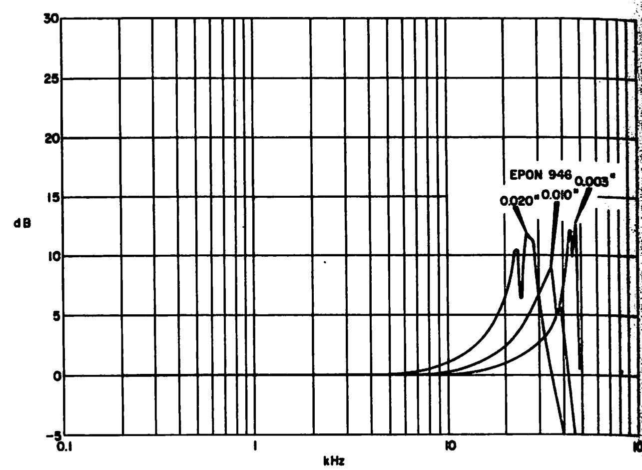

Fig. 7. Tension-Compression/Soft Adhesives/ Accelerometer Resonance 50 kHz.

Blocks

The structural spring effect is most easily seen when accelerometers are mounted on different sizes of aluminum blocks. Some of this can be seen in Figure 5. The thicker the block, the softer the spring, and the lower the limit of output flatness. Normally the reason for using a block is that it is not possible to stud mount the accelerometer or it is desired to make readings in more than one direction at the same point. This latter practice should be discouraged or used only when the backup is solid. Only aluminum blocks were evaluated. It is felt that the small advantage gained in the larger modulus of steel is more than offset by the lowering of the natural frequency from the additional weight. As can be seen from the curves, a good frequency response can be obtained using an aluminum block when its height is kept as low as possible. Blocks of fiber materials, etc., may find use in special applications where a lower frequency response is adequate [1, 2A].

Torque

The maximum possible g level and the specified torque are related to each other. If the vibration level is below 1 g, "finger tight" gives the same frequency response as the manufacturers' recommended torque. To get this same response at a higher g level, it is necessary to tighten it down further to keep the force applied by the accelerometer to the stud within the preload range.

TENSION - COMPRESSION AND BENDING

This design represents an accelerometer mounted to the side of a block, where the base of the block is in the above noted complicated stress condition.

A few designs along these lines, one shown in Figure 8, were attached to the B&K shaker head and vibrated. The results of this one, where the centroid of the test weights was located on the centerline of the shaker head, are given in Figure 9.

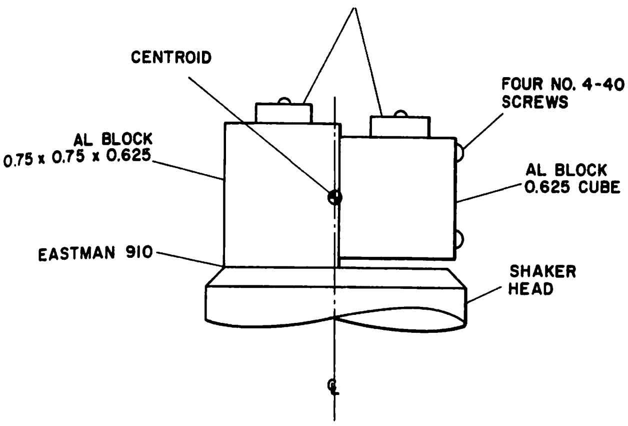

Fig. 8. Tension-Compression & Bending/Test Specimen.

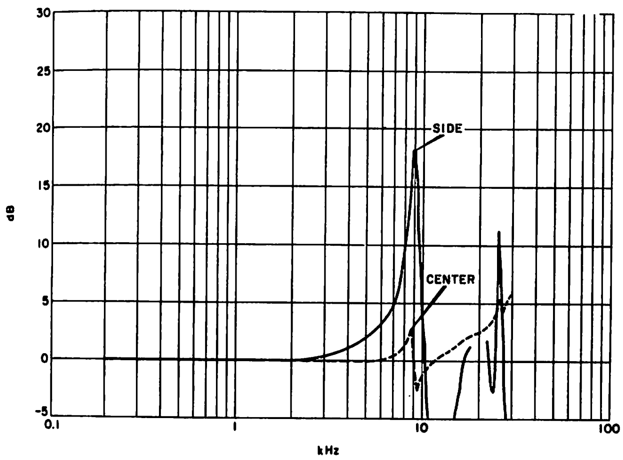

Fig. 9. Tension-Compression and Bending/Eastman 910 Base. Side Block Attached with Screws/Centroid Aligned with Shaker Head Centroid.

The additional bending in the base lowers the resonance frequency from the pure tension-compression case. Because of the normally low cross axis sensitivities of accelerometers, the motion recorded by the "side" accelerometer may not be seen by the accelerometer represented by the side block. If there is an accelerometer on "top", it will see this motion as recorded by the "center" trace. The exact frequency and magnitude of this motion will depend on the geometry and flexibility of the base.

SHEAR AND BENDING

This test configuration is shown in Figure 10. It consists of two blocks mounted symmetrically with respect to a center one. The center one is attached to the shaker. As far as the side ones are concerned, they are backed by a solid aluminum structure which is infinitely rigid at the plane of symmetry. A larger block at the center may have given a higher resonance frequency, but this was not verified.

This configuration represents the basis of comparison for those cases when it is necessary to measure vibration or shock in the plane of the surface. The backup in these tests was effectively rigid.

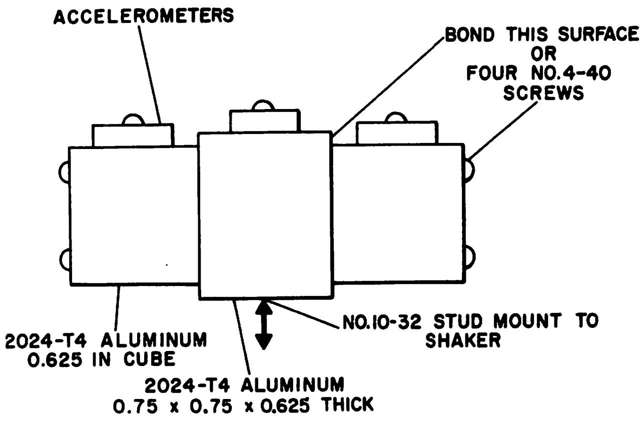

Fig. 10. Shear and Bending/Solid Backup Structure Test Specimen.

Screwed Down Block

This method appears to be somewhat inferior to the bonded attachments. Apparently some motion occurs at the contact surface despite the preload. Figure 11 shows a comparison with a typical bonded attachment mentioned below.

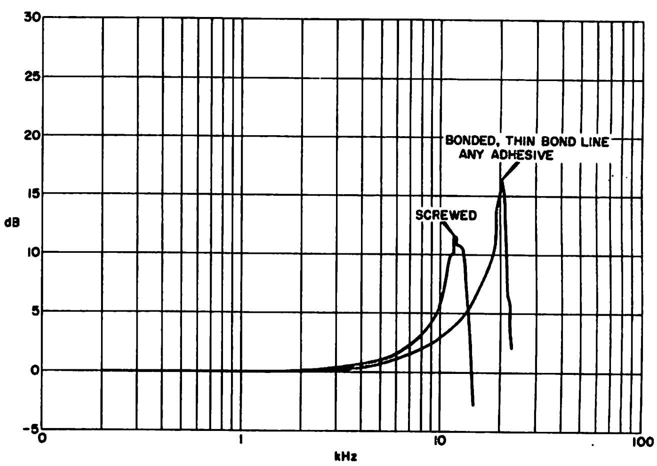

Fig. 11. Shear and Bending/Solid Backup Structure/Bonded and Screwed Mounting Comparison.

Bonded Block

Only thin bond line thicknesses were investigated. There appeared to be very little difference in the behavior of different adhesives. This may be because of the thin bond line and not because of material differences as seen in the tension-compression case.

PROBLEM AREAS

Local Resonances

An unexpected result of this investigation was the discovery of accelerometer subresonances. These occur below the natural frequency of the accelerometer but above the region of advertised measurement flatness. They are typical for a given model but may vary slightly in frequency and amplitude from instrument to instrument. In some they may be quite insignificant. Others may have amplitudes as large as 20 db. No case has been found so far where they would be of concern for the measurement, however no data should be obtained above the flat frequency range specified by the manufacturer unless calibrations of the type described herein are performed. A case resonance is shown in Figure 4 and Figure 12.

Burr Resonances

A wide resonance of a few db amplitude around 4 to 9 kHz was always found to be caused by some metal burr at the edge of a block or insulated stud, or from some other undesirable foreign interference. Often their presence could not be seen nor felt. The hump on the frequency response curve was always removed after a meticulous cleaning had been done. See Figure 12.

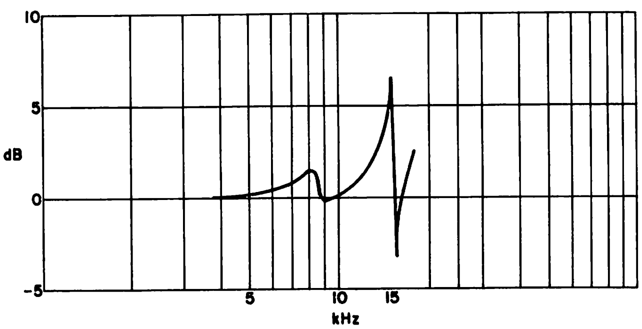

Fig. 12. Tension-Compression/Burr Resonance on Edge of Insulated Stud and 15 kHz Resonance in System.

Pseudo Cross-Axis Sensitivity

Another general phenomenon that affects both shock and vibration measurements is the danger of having inputs from what could be called pseudo cross axis sensitivity. Outputs of this nature are quite common.

The design of accelerometers is such that they inherently give very low outputs from motion in the plane perpendicular to the axis of measurement. This assumes ideal mounting conditions. Many times the practical attachment will allow identical motion to be set up by either a longitudinal or lateral input. Consider the installation shown in Figure 13. Assume that due to forces in the structure, point A moves suddenly downward. Because of the inertia of the accelerometer, it will follow the dynamics of the mass-spring system of the mounting. It can easily be seen that an identical motion can be caused by point A moving suddenly to the right. In this case, we are getting a large output from an instrument set to measure longitudinal motion from a lateral input.

Shock Sensitivity of Adhesives

A problem area needing work is the sensitivity of adhesives to shock-type loadings. It has been known

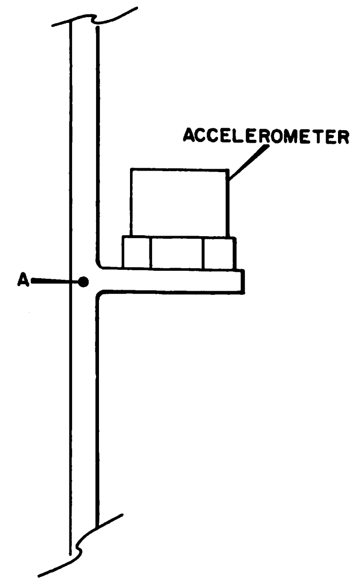

Fig. 13. Pseudo Cross-Axis Sensitivity.

for some time that certain adhesives will stand shock loading where others will fail. The limits and parameters affecting this behavior in bond line thicknesses have not previously been clarified. It is important to know this so that proper measurements can be made in the areas of interest.

Finite Thickness Backup

The preceding discussion has taken for granted that the inputs to the accelerometer from the structure are not affected by the presence of the instrument. When there is sufficient mass in the immediate vicinity of the accelerometer mounting area, the outputs are as described. At the other extreme, everybody would be suspicious of the validity of the outputs from a one ounce pickup on an 0.016 inch flat aluminum skin. Obviously adding such a heavy mass onto a thin flexible skin is going to distort any motion present in the original structure. The effect on intermediate cases is more of direct interest. Stathopoulos has treated the problem of total response from the low stiffnesses of structures (4).

Kempner, Sheng and Pohle have calculated the deflections caused by line loadings on cylindrical shells (5). It was possible to calculate the stiffnesses of the local shell structure using the curves and thus obtain the resonance frequencies of a given mass-spring system. The curves are given in Figures 14 and 15 with the aluminum block size and structure radius as parameters.

Measurement Normal to Surface

The calculated curves of Figure 14 were obtained as follows. A resonance frequency is given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{cycles/sec}$$

where

k = stiffness lb/in
m = mass $lb\text{-}sec^2/in$

The stiffness is obtained as follows (4). The curves are plotted as

$$\frac{w\,10^{-3}}{P/Er^2} = C$$

where

$$w = \frac{\Delta}{r}$$

Δ = deflection, inches
r = radius, inches
E = modulus of elasticity, lb/in^2
P = applied load, lb
C = nondimensional number

$$k = \frac{P}{\Delta} = \frac{Er}{C\,10^3}$$

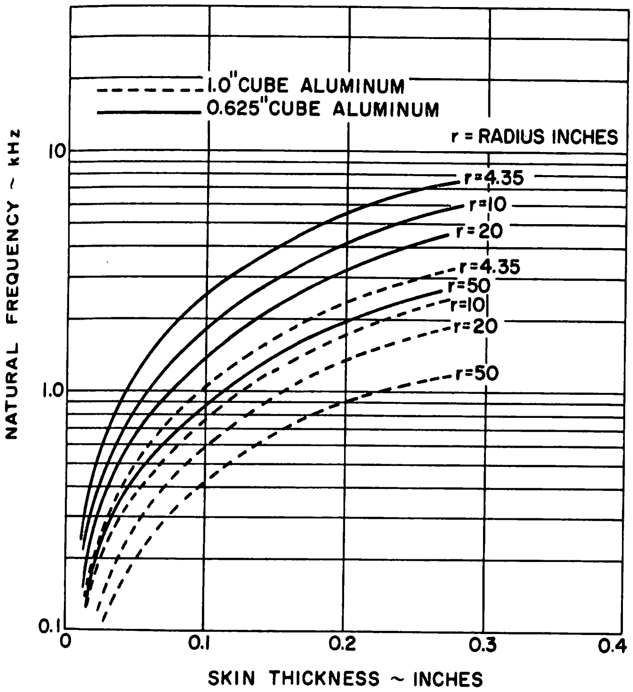

Fig. 14. Local Effects - Normal to Surface.

The mass has been taken as that of the aluminum block only.

The curves are based on the load being a line load. It distributes the force in the direction of the cylinder axis for the length of the block but does not take into account the width. The width has the effect of increasing the frequency. A comparison of this was made in the reference. As it was shown to be less than 15% for a representative case, no effort was made to correct for it here.

Addition of an accelerometer will increase the mass but not the contact area of the block. This effect can be estimated from the formula

$$f_{new} = f_{ref}\sqrt{\frac{m_{ref}}{m_{new}}}$$

These curves will help to evaluate the effect of the accelerometer attachment and will improve the mounting design to avoid the local effects.

A few calculations have been made comparing these curves with the empirical values obtainable from Levy and Jewell (6). It was stated there, that the dynamic stiffness was larger than the static stiffness. The curves of Figure 14 are based on the latter. The differences are fairly small considering the effects of all the parameters involved. More knowledge of these is necessary before greater accuracy can be obtained.

No direct tests were made to verify the curves. A few step force loadings were made on the test panels of Figure 16 and observed on an oscilloscope. These verified qualitatively the low values given by the curves.

Measurement Parallel to Surface

Figure 15 gives the results of the calculations made of the natural frequencies of a cantilever block as a function of skin thickness, radius, and block size for a number of representative values. The major influence is seen to be the block size, with the radius playing a secondary role.

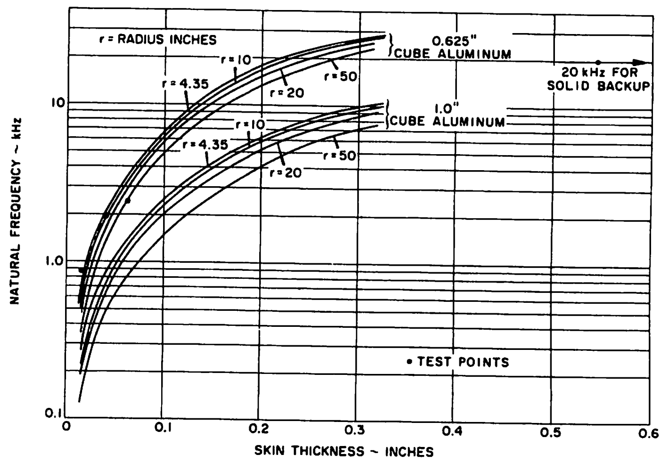

Fig. 15. Local Effects - Parallel to Surface (Cantilever).

The results are from Kempner, et al (5). Only the mass of the block has been calculated and its moment of inertia was taken around its own centroid. The frequency is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$$

where

k = stiffness of shell for moment loading, in-lb/rad

I = mass moment of inertia of added block, $lb\text{-}in\text{-}sec^2$

See Ref. 2B and 2C.

The shell stiffness was obtained as follows. The curves were given as a function of the parameters r/h and

$$\frac{\theta x 10^{-5}}{M/Er^3}$$

where

θ = slope, radians
M = applied axial moment, in-lb
E = modulus of elasticity, lb/in^2

The parameter for a given r/h can then be written as

$$\frac{\theta \; x10^{-5}}{M/Er^3} = C$$

The stiffness k is

$$k = \frac{M}{\theta} = \frac{Er^3}{Cx10^5}$$

With the block side designated "d", we then have

$$f_n = \frac{1}{2\pi}\sqrt{\frac{6\ Er^3}{10^5 C\ \rho d^5}} \quad \text{cycles/sec.}$$

where

ρ = mass density of block lb-sec^2/in^4

The curves are not very easy to read from the reference and it is felt that there is a fairly wide band of values around the curve which qualify as well. The block width effects have not been corrected, for the same reason as in the normal load case. This correction will increase the frequency.

A few tests were run with aluminum blocks mounted on three different thickensses of skin panels of the same radius. The specimen geometry is given in Figure 16 and a picture of the specimen on the shaker is shown in Figure 17.

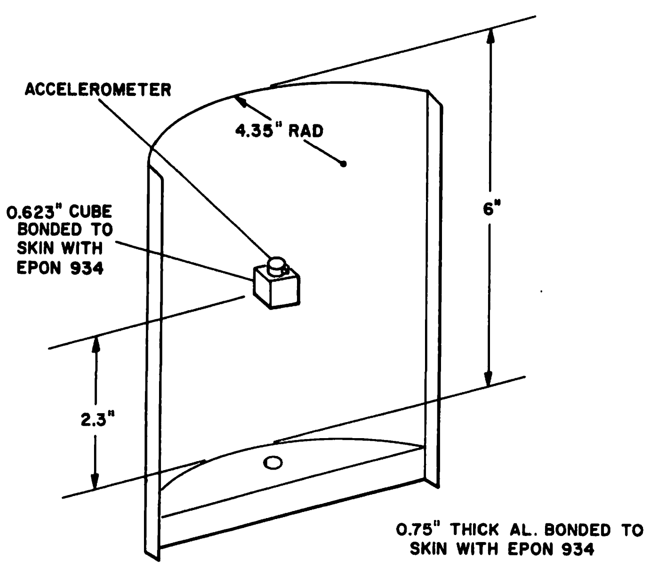

Fig. 16. Test Specimen - Cantilever on Skin Panel.

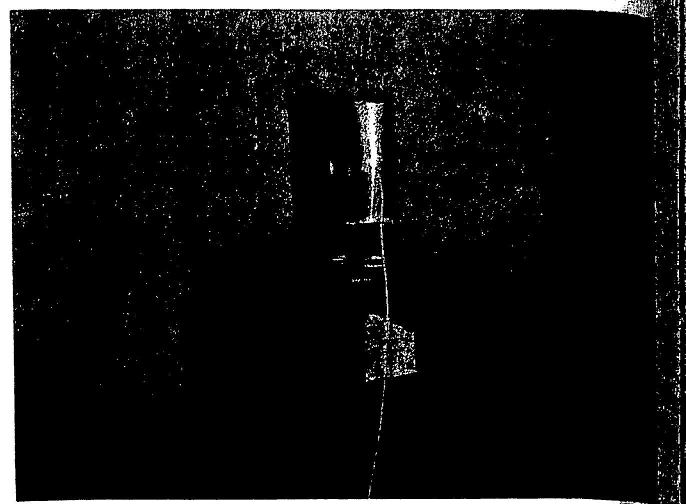

Fig. 17. Shear and Bending Vibration Test.

It is felt that the thinnest specimen gave a good resonance in the shaker test. The node lines have been sketched on the panel in Figure 17 and the increase in output can be seen in Figure 18 leading to the 873 Hz resonance. The other two specimens obviously had other modes in addition to the cantilever rocking expected. It is felt that this could still be seen in the 0.040 inch thick panel, but probably not in the 0.063 inch panel.

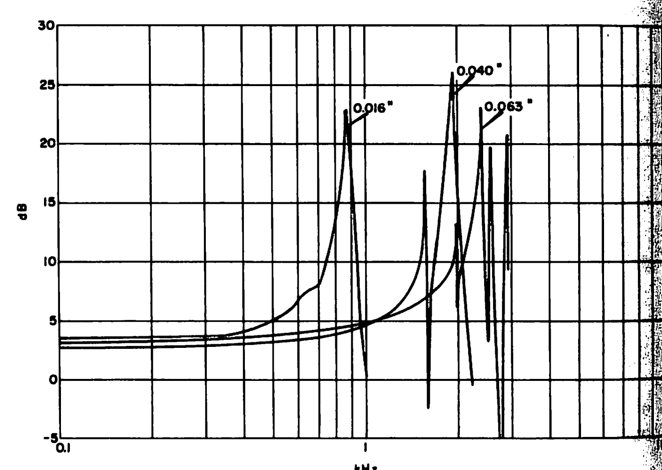

Fig. 18. Cantilever Effect Tests Vibrated at 5G PK.

Step forces were applied to the blocks to try to excite the cantilever modes and observe the behavior on an oscilloscope. Based on these the two thinner resonances appear correct, but the thicker is apparently too low.

A heavier 13 gram accelerometer was attached to the block on the thickest skin specimen and vibrated. The result is plotted in Figure 19 and compared with the result from the lighter 2.2 gram accelerometer. The difference is not as great as could be expected from the weight effect formula. This may again be because the resonance with the lighter accelerometer was precipitated too early.

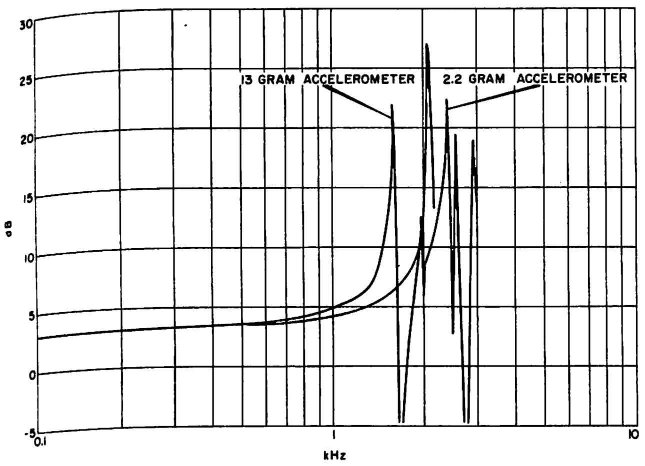

Fig. 19. Cantilever Weight Effects/.063 Inch Skin Vibrated at 5G PK.

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