CORPS OF ENGINEERS, U. S. ARMY

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by

Massachusetts Institute of Technology
Department of Civil and Sanitary Engineering
Soil Engineering Division

Contract No. DA-19-016-eng-2743

TECHNICAL REPORT NO. 67

UNDER CONTRACT
WITH

ARCTIC CONSTRUCTION AND FROST EFFECTS LABORATORY
NEW ENGLAND DIVISION
BOSTON, MASSACHUSETTS

FOR

OFFICE OF THE CHIEF OF ENGINEERS
AIRFIELDS BRANCH
ENGINEERING DIVISION
MILITARY CONSTRUCTION

June 1957

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PREFACE

This report is one of three reports describing results of investigations for the "Design, Fabrication, Exploratory Operation and Operational Manual of a Hydraulic Analog Computer." The investigation has been sponsored by the Arctic Construction and Frost Effects Laboratory, New England Division, Corps of Engineers, U. S. Army, under contract No. DA-19-016-Eng-2743 with the Massachusetts Institute of Technology through its Division of Industrial Cooperation under D.I.C. Project No. 5-7155.

The entire investigation is directed toward improvement of techniques for predicting subsurface temperatures, particularly with reference to frost and thaw penetration below airfield pavements. This report presents the results of studies related to frost penetration in multilayer soil profiles. Two other reports are (a) Design and Operation of an Hydraulic Analog Computer for Studies of Freezing and Thawing of Soils, and (b) Heat Transfer at the Air-Ground Interface with Special Reference to Airfield Pavements.

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ERRATA

Page	
Title Page	Word PROFILE should read PROFILES.
vi	Page number for BIBLIOGRAPHY is 33; should be page 49.

TABLE OF NOTATION

Symbol	Name	Usual Units
A	Combined cross-sectional area of standpipe and manometer tube	sq in.
a	Amplitude of temperature curve	deg F.
C	Volumetric heat	Btu per (cu ft)(deg F.)
C_{wt}	Weighted value of volumetric heat	Btu per (cu ft)(deg F.)
d	Distance between centers of adjacent "lumps"	ft
d_n	Thickness of lump "n"	ft
F	Freezing index	deg days F.
G_s	Thermal conductance between centers of adjacent lumps	Btu per (deg F.)(hr)
h	Hydraulic analog head	in.
k	Thermal conductivity	Btu per (hr) (ft) (deg F.)
L	Latent heat	Btu per cu ft
L_{wt}	Weighted value of latent heat	Btu per cu ft
R_n	Thermal resistance of lump "n" = d_n/k_n	(hr) (deg F.) per Btu
T_s	Analog time scale	min per hr
t	Time, duration of freezing period	days
t_o	Time at start of freezing period	days
t_1	Time at end of freezing period	days
t_a	Analog time	min
t_p	Time in nature	hr
U_n	Thermal energy removed from lump "n"	Btu
U_s	Analog thermal energy scale	cu in. per Btu
V_s	Analog temperature scale	in. per deg F.

TABLE OF NOTATION (continued)

Symbol	Name	Usual Units
v	Temperature	deg F.
v_o	Deg F. by which mean annual temperature exceeds freezing point of soil moisture	deg F.
v_s	Deg F. by which effective surface temperature is less than freezing point of soil moisture during freezing period = F/t.	deg F.
w	Water content	percent
X	Depth of frost penetration	ft
X_{MB}	Depth of frost penetration computed from Modified Berggren formula	ft
X_S	Depth of frost penetration computed from Stefan equation	ft
α	Thermal ratio	dimensionless
γ_d	Dry unit weight	lb per cu ft
λ	Correction coefficient	dimensionless
μ	Fusion parameter	dimensionless
ω	$\frac{2\pi}{365}$	radians per day

SYNOPSIS

This report presents the results of an investigation relative to the prediction of the depth of frost penetration in multilayer soil profiles. An adaptation of the Modified Berggren formula is presented and computed results are compared with "exact" solutions for the depth of freezing as determined by the hydraulic analog computer developed under the contract.

Nine cases were investigated, representing typical soil profiles below bituminous concrete airfield pavements. In seven cases, the pavement surface temperature was assumed to drop suddenly to a given value below freezing where it remained constant. In the remaining two cases, the surface temperature was assumed to vary sinusoidally over an annual period.

Results of these tests indicate that the proposed adaptation of the Modified Berggren formula underpredicts the frost depth by approximately 3 percent. However, this error is believed to be insignificant compared to those errors in the predicted depth of frost penetration in any practical case arising from uncertainties in the thermal properties of the soil, particularly the thermal conductivity, and from unknown pavement surface temperatures. These factors present the most fruitful areas for future research on the depth of frost penetration prediction.

I. INTRODUCTION

1-01. CONTRACT

On 2 October, 1953, the Massachusetts Institute of Technology through its Division of Industrial Cooperation entered on a cost-type research contract with the Arctic Construction and Frost Effects Laboratory, New England Division, Corps of Engineers, U. S. Army. This contract, No. DA-19-016-Eng-2743, is entitled "Design, Fabrication, Exploratory Operation and Operational Manual of a Hydraulic Analog Computer". This is one of three reports covering investigations at M.I.T. during the first two years of this contract.

Specifically, this report describes results of investigations included within the following paragraphs of Appendix "A" of the contract:

> "3. After completing the work enumerated in Paragraph 1 and 2 above, the contractor shall perform comprehensive studies of the operational use of the apparatus for several or all of the following investigations: (a) Make a detailed study of the effect of variables, such as stratification, frost heave, and surface temperature fluctuations on the depth of freezing and thawing; compare the results of these studies with various formulas and averaging techniques which have been composed to handle these complicated situations."

1-02. SCOPE

The investigation described herein is directed primarily at evaluating the effectiveness of an adaptation of the Modified Berggren formula for predicting the depth of frost penetration in multilayer soil profiles. It is assumed, a priori, that the thermal properties of the soil and the boundary temperature conditions are known. The "exact" hydraulic analog solution for a given soil profile is compared with the predicted solution for these assumptions.

Comparisons are made for both a step change in surface temperature and for a surface temperature varying sinusoidally to give a total of nine solutions for six different soil profiles, all having a 3-inch bituminous concrete surface.

1-03. M.I.T. PERSONNEL

Investigations under this contract have been carried out under the supervision of Harl P. Aldrich, Jr., Assistant Professor of Soil Mechanics, Department of Civil and Sanitary Engineering.

Mr. Rasmus S. Nordal, formally half-time Research Assistant in Civil Engineering, carried out the majority of the investigation described herein.

Dr. Aldrich and Mr. Nordal are responsible for the technical writeup.

II. GENERAL

Freezing and thawing below highway and airfield pavements presents problems of utmost importance to their design and maintenance in Northern climates. These problems have assumed increased importance as wheel loadings, traffic frequency, and cost of pavement construction and maintenance have increased. Furthermore, thermal problems in soil are receiving additional attention as a result of construction activity in permafrost areas.

It is generally recognized that three requirements must be met simultaneously for significant ice-segregation and frost heave to occur in a foundation soil: (1) below freezing temperatures must penetrate into the ground, (2) the soil must meet certain grain-size requirements to be "frost-susceptible" and (3) a source of water must be available. This report deals with the first requirement only, namely the prediction of the rate and maximum depth of frost penetration through stratified (multilayered) soil profiles beneath bituminous concrete pavements.

The problem of predicting the depth of frost penetration has been studied by numerous investigators during the last 30 years especially. All practical analytic solutions are derived for uniform soil. The most simple of these is the well-known Stefan formula:

$$X = \sqrt{\frac{2k_f \int v_s dt}{L}} \qquad (2\text{-}1)$$

which may be written in terms of the freezing index F, expressed in degree days, as follows:

$$X = \sqrt{\frac{48\, k_f F}{L}} \qquad (2\text{-}2)$$

This expression neglects the volumetric heat of the soil and therefore generally predicts depths of frost penetration which are too large.

The Modified Berggren formula (1) (2)* has been developed to account

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*Numbers in parenthesis refer to references in the Bibliography.

for volumetric heat as well as latent heat and may be written:

$$X = \lambda \sqrt{\frac{2kv_s t}{L}} \tag{2-3}$$

where:

λ; (lambda); dimensionless correction coefficient which is given in Figure 14 as a function of two important dimensionless parameters, α (alpha) and μ (mu):

$$\text{Thermal ratio,} \quad \alpha = \frac{v_o}{v_s}$$

$$\text{Fusion parameter,} \quad \mu = \frac{C}{L} v_s$$

Introducing F as before to yield an equation for practical use, we obtain:

$$X = \lambda \sqrt{\frac{48\,kF}{L}} \tag{2-4}$$

Numerous other expressions (4) (5) (8) have been presented for frost penetration in uniform soil under one-dimensional conditions.

No exact analytic solutions to the depth of frost penetration in multilayer soil profiles exist. Therefore, for this nonuniform case which occurs typically below highway and airfield pavements, semi-empirical computation techniques must be used. Since no exact solution for the multilayer case exists, there is no possibility of verifying, in a simple way, the reliability of the approximate procedure itself.

It is true that a numerical approach based on a finite difference approximation to the fundamental differential equations involved, will lead after considerable effort to an "exact" solution. However, with the construction of the hydraulic analog computer (6) a relatively simple tool becomes available for checking semi-empirical computational procedures.

Thus, we seek to compare a series of solutions obtained on the hydraulic analog computer with results of approximate mathematical procedures for a variety of multilayer soil profiles which may occur below pavements. The first part of the report, however, presents a study of the accuracy of the analog computer solution itself. Results are compared with known analytic solutions for a step change in surface temperature and with results of numerical analyses carried out on the IBM-701 computer for a surface temperature varying sinusoidally.

III. HYDRAULIC ANALOG COMPUTER

3-01. GENERAL

The hydraulic analog computer developed under this contract is described in detail in Reference (6). Only a brief description of the computer will be presented here after which its operation relative to this investigation is given.

The computer consists of a series of glass tube standpipes with expansion reservoirs about mid-height. Each finite layer of soil in a given subsurface profile is represented by one of these standpipes. Water level in a standpipe corresponds to temperature for the soil layer it represents. Flow of heat within the soil is represented in the computing device by the flow of water. The water level versus stored water characteristic of each standpipe and expansion reservoir is analogous, then, to the thermal energy versus temperature characteristic of the soil.

To represent resistance to heat flow between adjacent soil layers, a resistance to fluid flow between standpipes must be provided. In the hydraulic analog computer, laminar flow in a hydraulic resistance is maintained in openings of capillary size, which can be continuously varied in length to yield the desired resistance.

Before an actual problem can be solved on the computer, the soil profile and its thermal properties must be known or assumed. The computer can then be set up with the appropriate standpipes, latent heat reservoir openings, and hydraulic resistances. Ground surface temperature, represented by a water reservoir, can be programmed automatically to follow actual or assumed variations in surface temperature with time by means of a moving paper template and servo-motor system.

The programming and scaling procedure as well as photographs and drawings for the hydraulic analog computer are given in Reference (6).

3-02. CHECK SOLUTIONS, UNIFORM SOIL

Because the hydraulic analog computer solves a thermal problem by "lumping" the continuous subsurface soil profile into a finite number of elements, the solution is not exact in the strictest sense. This "lumping" error depends, among other things, on the size of each lump and on the magnitude of thermal gradients. These lumping errors were investigated by Scott (9). Since other minor errors are inevitable in the analog system, it becomes important to check its performance with known analytic solutions in order to gain confidence in the computer results.

A series of four analog solutions were run which demonstrate that computer errors are negligible as compared to the precision of the physical data available for a practical case. A summary of these tests, numbered 1 through 4, is given in Table I and Figures 1 through 4.* All tests were run assuming a homogeneous soil profile. Tests 1 and 2 were run under a step change in surface temperature while tests 3 and 4 are corresponding tests under a surface temperature varying sinusoidally.

Excellent agreement is obtained, Figures 1 and 2, between analog results and analytic solutions based on the Modified Berggren formula, Equation (2-3), for the step change in surface temperature. In Table I, an actual correction coefficient λ, equal to the depth of frost penetration X given in the analog solution divided by the depth of frost penetration X_S according to the Stefan Equation (2-1), is compared with λ_{MB} given by the Modified Berggren formula from Figure 14. The agreement is perfect.

Analog results for tests 3 and 4 under a surface temperature varying sinusoidally are compared with results of studies on the IBM-701 computer, Table I, since the Modified Berggren formula is not strictly applicable to this case. The agreement is not as good as before. However, the errors due to "lumping" of the soil profile and especially those caused by the physical performance of the latent heat wells become larger when the freezing process is followed by thawing. Gradients below the frozen zone are very small when thawing starts (for example see Figure 3 at a time equal to 250 days) which makes the lower boundary of the frozen soil very difficult to determine.

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*The short horizontal lines in Figures 1 through 13 are related to the analog solution and may be explained as follows. The left end of the line represents the time when the soil "lump" begins to freeze, in other words, the time when the latent heat well starts to drain. The right end gives the time when the "lump" is entirely frozen. In the case of thawing, the reverse is true.

3-03. SOLUTIONS FOR STRATIFIED SOIL

A program of nine solutions on the hydraulic analog computer was carried out on six different soil profiles, all having a 3-inch bituminous concrete surface. Two of these tests were run under a surface temperature varying sinusoidally over an annual period. Samples of the scaling and programming procedures for analog solutions No. 9 and No. 12 are given in Appendix B.

Results of the analog solutions are plotted in Figures 5 through 13. A comparison of these results with formulas, summarized in Table II, will be made following a discussion of the computational procedures for predicting frost depth according to the Stefan and Modified Berggren formulas.

IV. FORMULAS FOR DEPTH OF FROST PENETRATION IN MULTILAYER SOIL PROFILES

4-01. GENERAL

No exact analytic solution for the depth of frost penetration in multilayer soil profiles exists. However, approximate solutions based on formulas for homogeneous soil have been presented. The use of the Stefan equation and Modified Berggren formula for this purpose will be presented here.

4-02. STEP CHANGE IN SURFACE TEMPERATURE

A step change in surface temperature is defined as that case where the temperature of the soil is initially uniform and the surface temperature is suddenly changed from this initial temperature above freezing to some temperature below the freezing point.

a. Stefan Equation. The depth of frost penetration from the Stefan equation, which ignores the volumetric heat of the soil, may be written:

$$X_S = \sqrt{\frac{48k_f v_s t}{L}} = \sqrt{\frac{48k_f F}{L}} \tag{4-1}$$

for uniform soil under a step change in surface temperature. For stratified soil two approaches yielding identical results may be used:

(1) "St. Paul" Approach. The so-called "St. Paul" method* makes use of the following relationship for the partial freezing index required to freeze any layer "n":

$$F_n = \frac{L_n d_n}{24}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2} + \cdots\cdots \frac{d_n}{2k_n}\right).$$

or:

$$F_n = \frac{U_n}{24}\left(R_1 + R_2 + \cdots\cdots \frac{R_n}{2}\right) \tag{4-2}$$

*This method is presented in Chapter 6, Part XV of the Engineering Manual for Military Construction, Corps of Engineers, U. S. Army, dated October, 1954. The derivation of Equation (4-2) is also given on Page 44 of Reference 2.

where:

$U_n = L_n d_n$; the latent heat removed when freezing the n^{th} layer of thickness d_n.

$R_n = \frac{d_n}{k_n}$; thermal resistance.

If the freezing index for the locality in question is known, the frost depth can be determined by finding those layers whose sum of partial freezing indices equals the total for the period. The total freezing index for any period follows from Equation (4-2):

$$F = F_1 + F_2 + \ldots\ldots F_m$$

$$= \frac{U_1}{24}\left(\frac{R_1}{2}\right) + \frac{U_2}{24}\left(R_1 + \frac{R_2}{2}\right) + \ldots\ldots \frac{U_m}{24}\left(R_1 + R_2 + \ldots\ldots \frac{R_m}{2}\right)$$

$$= \sum_{n=1}^{m} \frac{U_n}{24}\left[\frac{R_n}{2} + \sum_{p=1}^{n-1} R_p\right] \qquad (4\text{-}3)$$

which is Equation (2-54) in Reference 2 when $U_n = L_n d_n$. It has been shown that the total freezing index may also be written:

$$F = \sum_{n=1}^{m} \frac{R_n}{24}\left[\frac{U_n}{2} + \sum_{p=n+1}^{m} U_p\right] \qquad (4\text{-}4)$$

(2) An Equivalent Alternate Approach. An entirely equivalent procedure to that given above is to determine an effective "L/k" for the frozen zone and compute the depth of frost penetration from Equation (4-1). The "St. Paul" method is preferred to this procedure because it is more direct. However, this alternate approach is fundamental to understanding the nomograph procedure (2, pg. 49) where the correction factor λ, is introduced.

Equation (4-1) for uniform soil may be re-written:

$$\frac{L}{k} = \frac{48}{X^2} F$$

We may substitute F from Equation (4-4) for the multilayer case to obtain:

$$\left(\frac{L}{k}\right)_{eff.} = \frac{2}{X^2}\left[\sum_{n=1}^{m} R_n\left(\frac{U_n}{2} + \sum_{p=n+1}^{m} U_p\right)\right]$$

or:

$$\left(\frac{L}{k}\right)_{eff.} = \frac{2}{X^2}\left[\frac{d_1}{k_1}\left(\frac{L_1d_1}{2} + L_2d_2 + \ldots\ldots + L_md_m\right)\right.$$

$$+ \frac{d_2}{k_2}\left(\frac{L_2d_2}{2} + L_3d_3 + \ldots\ldots + L_md_m\right)$$

$$\left. + \ldots\ldots\ldots + \frac{d_m}{k_m}\left(\frac{L_md_m}{2}\right)\right] \tag{4-5}$$

which is the same equation as that given in Step 5, page 49, of Reference 2. Thus, one may estimate the frost depth X, determine the $(L/k)_{eff.}$ from Equation (4-5), then compute X from Equation (4-1). If the computed value of X differs appreciably from that originally estimated, the computation may be repeated.

The procedure outlined above is identical to that suggested for use with the frost penetration nomograph (2) except that here, the correction coefficient λ, has been omitted which is consistent with an approach which ignores volumetric heat.

b. Latent Heat Plus Volumetric Heat of Frozen Soil. As a second somewhat less approximate approach, the heat removed from each frozen layer as it is cooled from the mean annual temperature to 32 F. may be added to the latent heat, L_nd_n. More specifically, we assume that the total heat removed from the n^{th} layer is now:

$$U_n = L_nd_n + C_nv_od_n \tag{4-6}$$

Thus, U_n from Equation (4-6) may be used in Equations (4-2) through (4-5) to give predicted depths of frost penetration somewhat closer to the actual. However, the heat given off while cooling the unfrozen soil and that released by the frozen soil as it cools from 32 F. is still neglected. Therefore, the predicted depths are still greater than the actual.

c. Adaptation of the Modified Berggren Equation. The depth of frost penetration from the modified Berggren equation may be written

$$X_{MB} = \lambda \sqrt{\frac{2kv_s t}{L}} = \lambda \sqrt{\frac{48kF}{L}} = \lambda X_S \qquad (4\text{-}7)$$

for uniform soil.

In the development of the Modified Berggren formula the surface temperature was assumed to change suddenly from v_o degrees above freezing to v_s degrees below freezing where it remained constant. This case is referred to as a step-change in surface temperature. The relationship among the dimensionless parameters α, μ, and λ, Figure 14, is strictly applicable only to this case and:

$$\alpha = \frac{v_o}{v_s} \quad \text{and} \quad \mu = \frac{C}{L} v_s$$

If the soil is stratified, however, and the surface temperature steps suddenly from v_o degrees above freezing to v_s degrees below freezing, the following procedure for determining a frost depth vs time curve is suggested:

Step 1. Compute weighted values of volumetric heat and latent heat within the depth of frost penetration X for which the time is desired:

$$C_{wt} = \frac{C_1 d_1 + C_2 d_2 + \ldots\ldots + C_m d_m}{X}$$

$$L_{wt} = \frac{L_1 d_1 + L_2 d_2 + \ldots\ldots + L_m d_m}{X} \qquad (4\text{-}8)$$

Step 2. Determine α and μ from:

$$\alpha = \frac{v_o}{v_s} \qquad \mu = \frac{C_{wt}}{L_{wt}} v_s$$

Step 3. Enter Figure 14, to obtain λ.

Step 4. The time t required to freeze to depth X is now computed from:

$$t = \frac{F}{v_s \lambda^2}$$

where F is given by Equation (4-3) or (4-4). Note that this time is equal to the time computed from the Stefan equation divided by λ^2.

Sample computations using this procedure are given in Appendix B.

4-03. SURFACE TEMPERATURE VARYING SINUSOIDALLY

In reality a pavement surface experiences daily as well as seasonal temperature fluctuations through local variations in air temperature, wind velocity, precipitation, solar radiation, etc. Indeed, a sinusoidal variation in surface temperature over an annual period more nearly represents the true case. Thus, we examine the effect of such a variation on the computational procedures for the depth of frost penetration.

a. Stefan Equation. The depth of frost penetration from the Stefan equation is actually independent of the surface temperature variation. Therefore, all procedures given in earlier sections are entirely applicable for determining the frost depth for any partial freezing index for a freezing season.

b. Adaptation of Modified Berggren Equation. In computing the maximum depth of frost penetration below a typical highway or airfield pavement the following semi-empirical adaptation of the Modified Berggren formula is suggested. It is assumed that the pavement freezing index F, duration of freezing period t, mean annual air temperature, and thermal properties of the soil are given. The layers making up the profile will be numbered consecutively downward starting with the Portland-cement concrete or bituminous concrete layer as No. 1.

Step 1. Compute an effective $\frac{L}{k}$ from Equation (4-5).

Step 2. Compute weighted values of C and L within the estimated depth of frost penetration from Equation (4-8).

Step 3. Compute the effective values of α and μ from:

$$\alpha = \frac{v_o t}{F} \qquad \text{and} \qquad \mu = \frac{C_{wt} F}{L_{wt} t}$$

Step 4. Determine the correction coefficient λ from Figure 14.

Step 5. Compute the depth of frost penetration from:

$$X = \lambda \sqrt{\frac{48\ F}{\left(\frac{L}{k}\right)_{eff.}}}$$

If the computed depth differs appreciably from the assumed depth, steps 1 through 5 may be repeated. Seldom are more than two cycles necessary.

A sample computation using this procedure is given in Appendix B.

V. RESULTS AND CONCLUSIONS

5-01. DISCUSSION OF RESULTS

A summary of results of this investigation for the multilayer soil profiles is given in Table II. It is believed that the most satisfactory method of comparing results of these tests is to determine the actual values of the correction coefficient λ defined by:

$$\lambda = \frac{X}{X_S}$$

where:

- X : "actual" depth of frost penetration at any time (for any freezing index) from the hydraulic analog computer
- X_S: depth of frost penetration at the same time (for the same freezing index) according to the Stefan formula.

and compare this value with the ratio:

$$\lambda_{MBD} = \frac{X_{MB}}{X_S}$$

where:

- X_{MB}: depth of frost penetration (for the same freezing index) according to the Modified Berggren formula.*

Thus, when λ_{MBD} equals λ, the Modified Berggren formula yields an "exact" result for the depth of frost penetration.

Several arbitrary values of X for each test have been selected in order to make this comparison, the results of which are given in the last column of Table II. It can be seen that λ_{MBD} is generally smaller than λ (negative percentage) which indicates that the Modified Berggren formula is predicting too shallow. More specifically, if the maximum recorded depth X for each of the nine tests is selected for comparison, the mean result is - 2.5% with a variation of from - 5% to + 1%.

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*The term λ_{MB} is reserved for the correction coefficient from the Modified Berggren formula as given in Figure 14. When the soil profile is uniform, $\lambda_{MB} = \lambda_{MBD}$.

5-02. CONCLUSIONS

Although the number of cases examined in this investigation is limited, the writers believe that these tests demonstrate the general validity of the proposed computational procedure for the depth of frost penetration based on the Modified Berggren formula. The adaptation for the multilayer soil profile proposed herein appears to yield results which are slightly too shallow. Nevertheless, this error is believed to be insignificant compared to those errors in the predicted depth of frost penetration arising from uncertainties in predicting the thermal properties of soil, in particular the thermal conductivity, and in determining the pavement surface temperature from air temperature and micrometeorological data.

Therefore, it is believed that future research related to the prediction of the depth of frost penetration should be directed toward these problems.

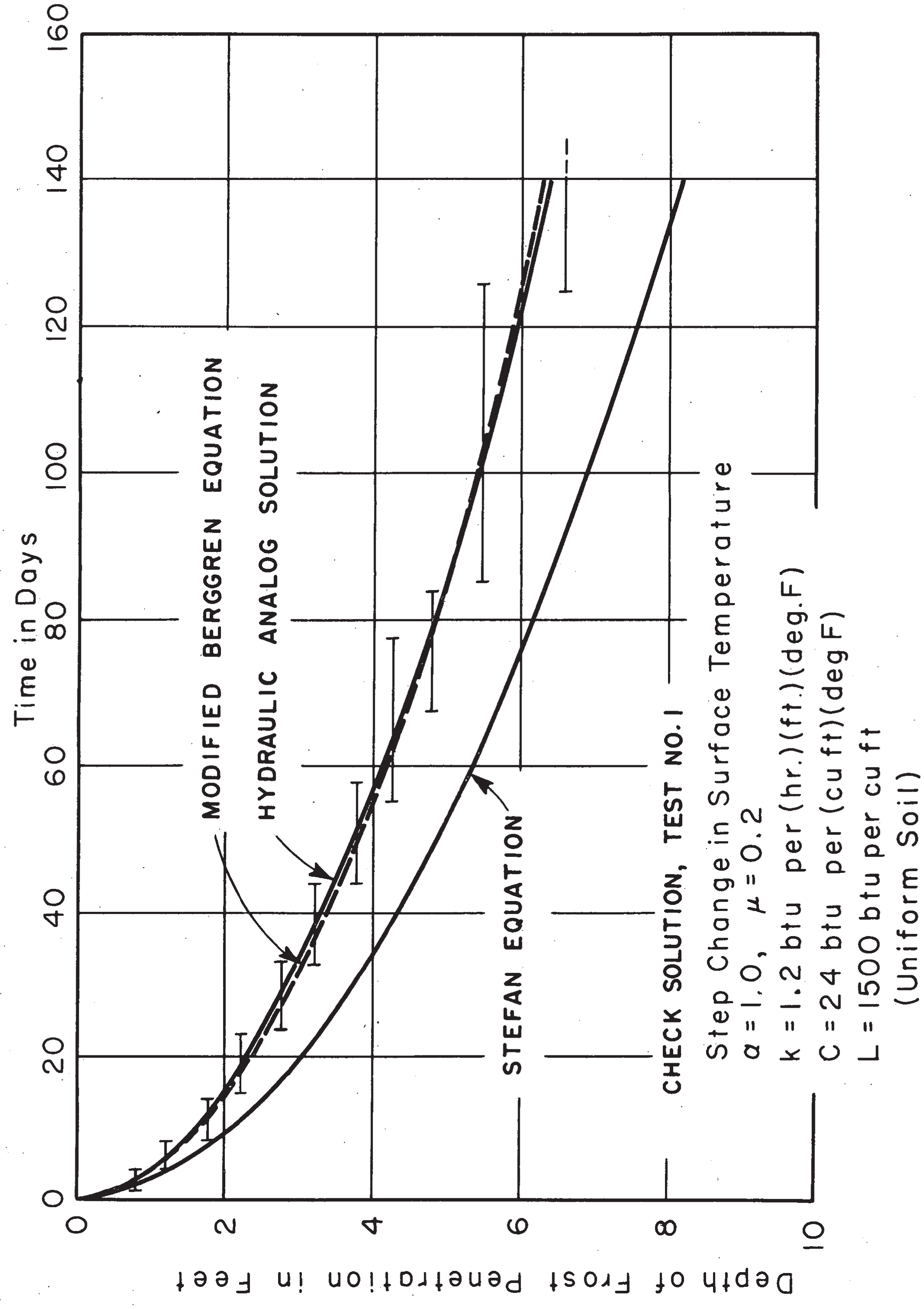
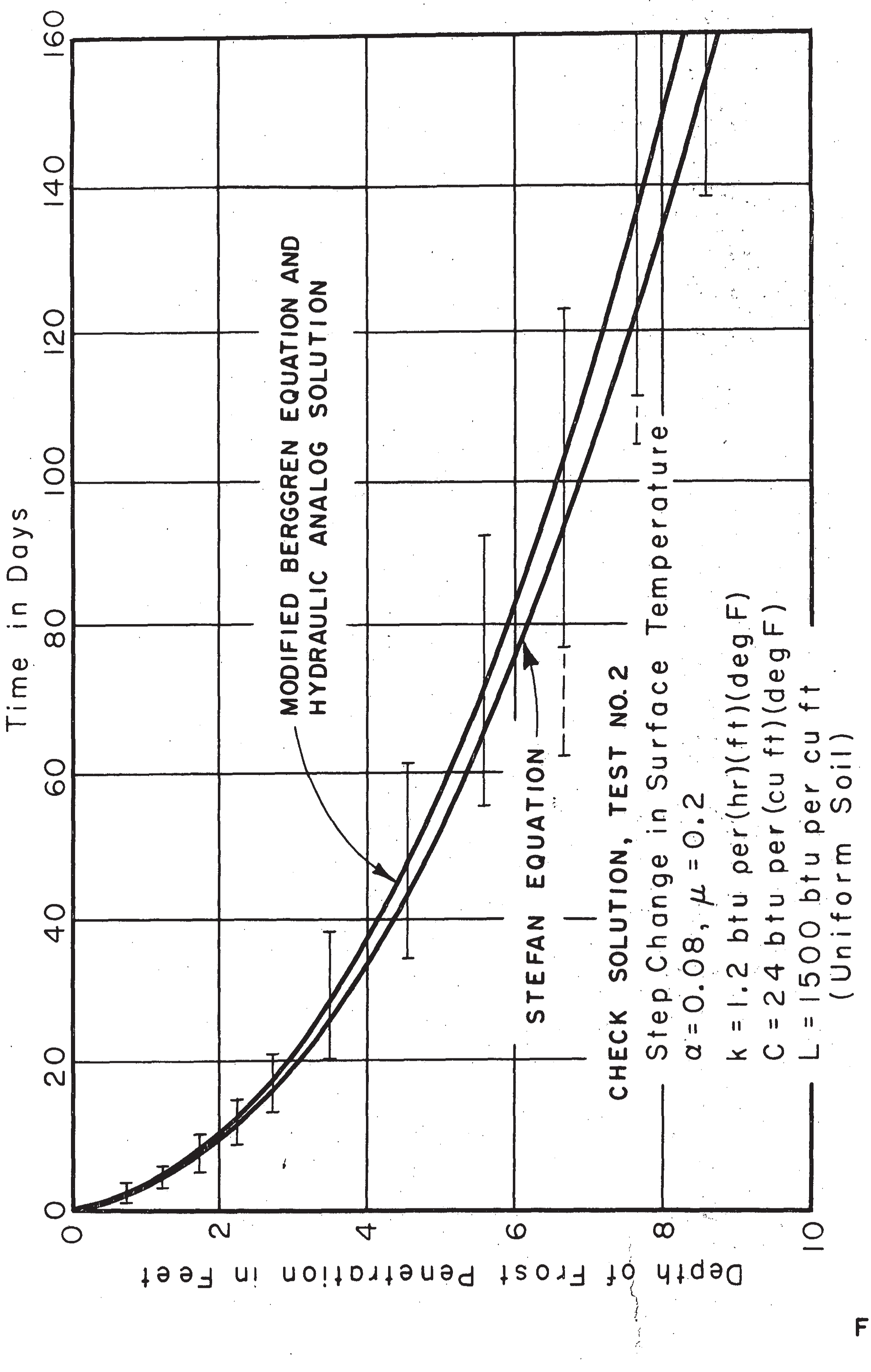
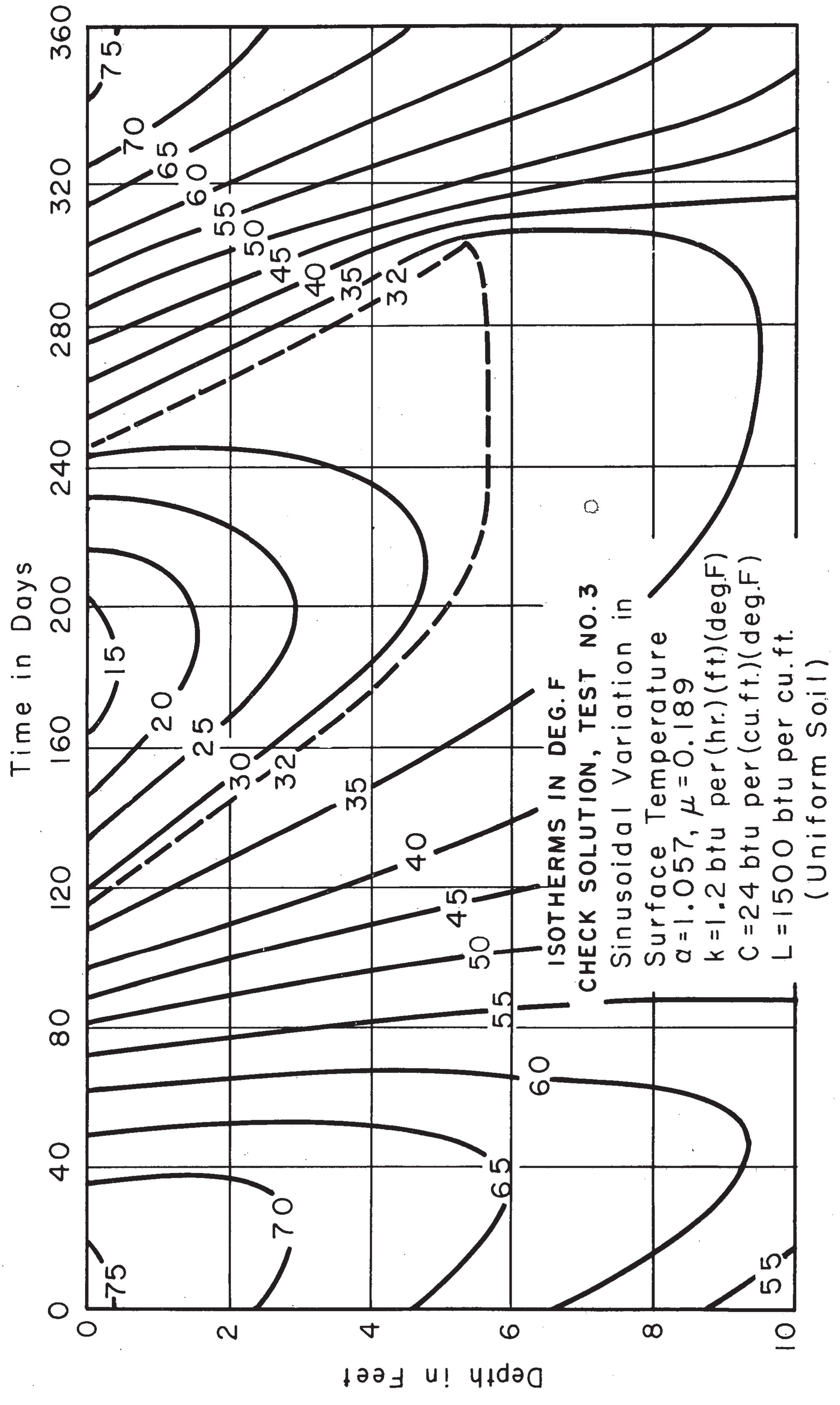

Time in Days
0
20
40
60
80
100
120
140
160
Depth of Frost Penetration in Feet
0
2
4
6
8
10
MODIFIED BERGGREN EQUATION
HYDRAULIC ANALOG SOLUTION
STEFAN EQUATION
CHECK SOLUTION, TEST NO. 1
Step Change in Surface Temperature
α = 1.0, μ = 0.2
k = 1.2 btu per (hr.)(ft.)(deg.F)
C = 24 btu per (cu ft)(deg F)
L = 1500 btu per cu ft
(Uniform Soil)

FIGURE 1



Time in Days
0
20
40
60
80
100
120
140
160
Depth of Frost Penetration in Feet
0
2
4
6
8
10
MODIFIED BERGGREN EQUATION AND
HYDRAULIC ANALOG SOLUTION
STEFAN EQUATION
CHECK SOLUTION, TEST NO. 2
Step Change in Surface Temperature
α = 0.08, μ = 0.2
k = 1.2 btu per (hr)(ft)(deg F)
C = 24 btu per (cu ft)(deg F)
L = 1500 btu per cu ft
(Uniform Soil)

FIGURE 2



Time in Days
Depth in Feet
ISOTHERMS IN DEG. F
CHECK SOLUTION, TEST NO. 3
Sinusoidal Variation in
Surface Temperature
a=1.057, μ=0.189
k=1.2 btu per (hr.)(ft.)(deg.F)
C=24 btu per (cu.ft.)(deg.F)
L=1500 btu per cu.ft.
(Uniform Soil)

FIGURE 3

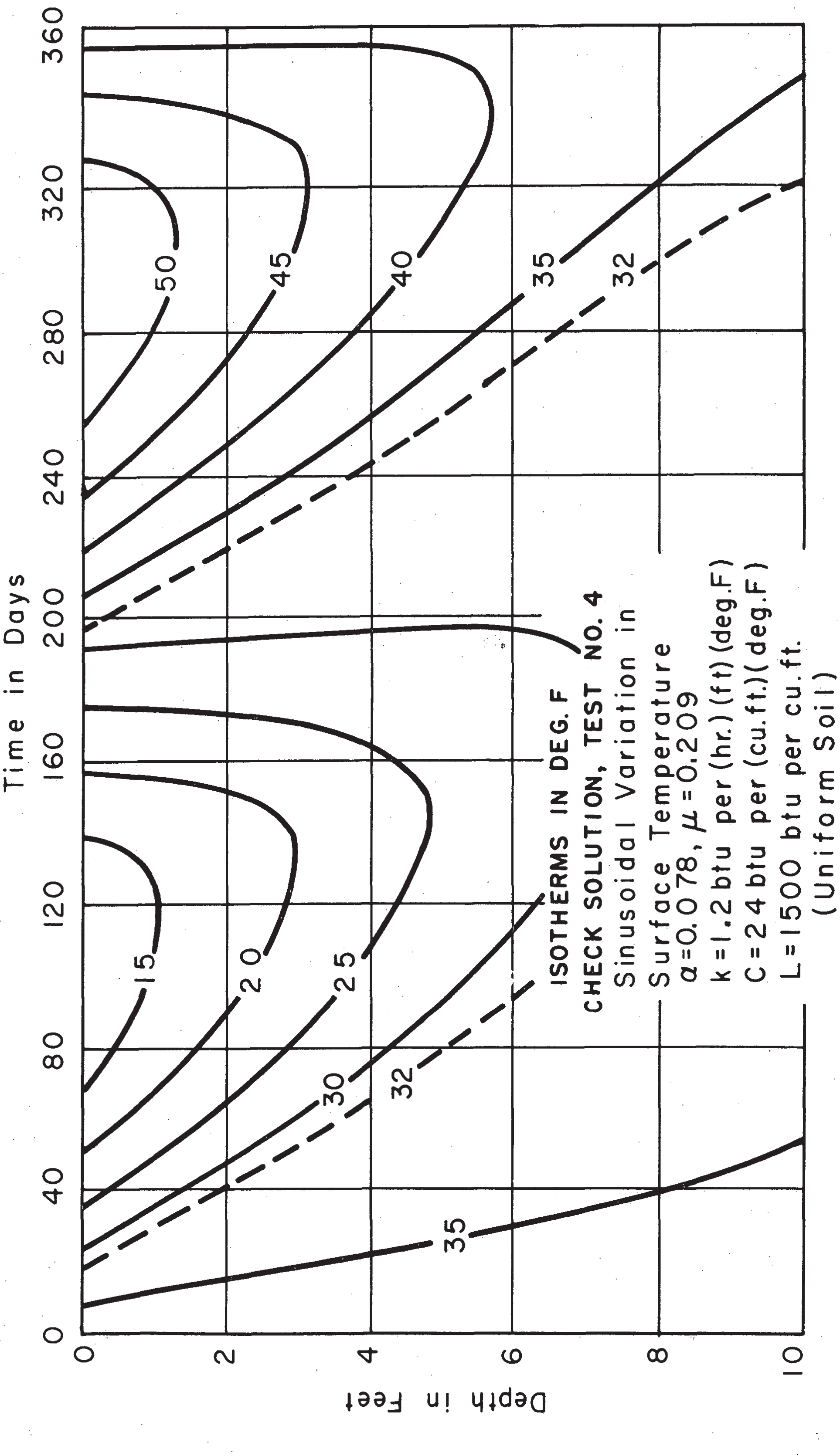

ISOTHERMS IN DEG. F
CHECK SOLUTION, TEST NO. 4
Sinusoidal Variation in
Surface Temperature
α=0.078, μ=0.209
k=1.2 btu per (hr.)(ft)(deg.F)
C=24 btu per (cu.ft.)(deg.F)
L=1500 btu per cu.ft.
(Uniform Soil)
Time in Days
Depth in Feet
0
40
80
120
160
200
240
280
320
360
0
2
4
6
8
10
15
20
25
30
32
35
50
45
40
35
32

FIGURE 4

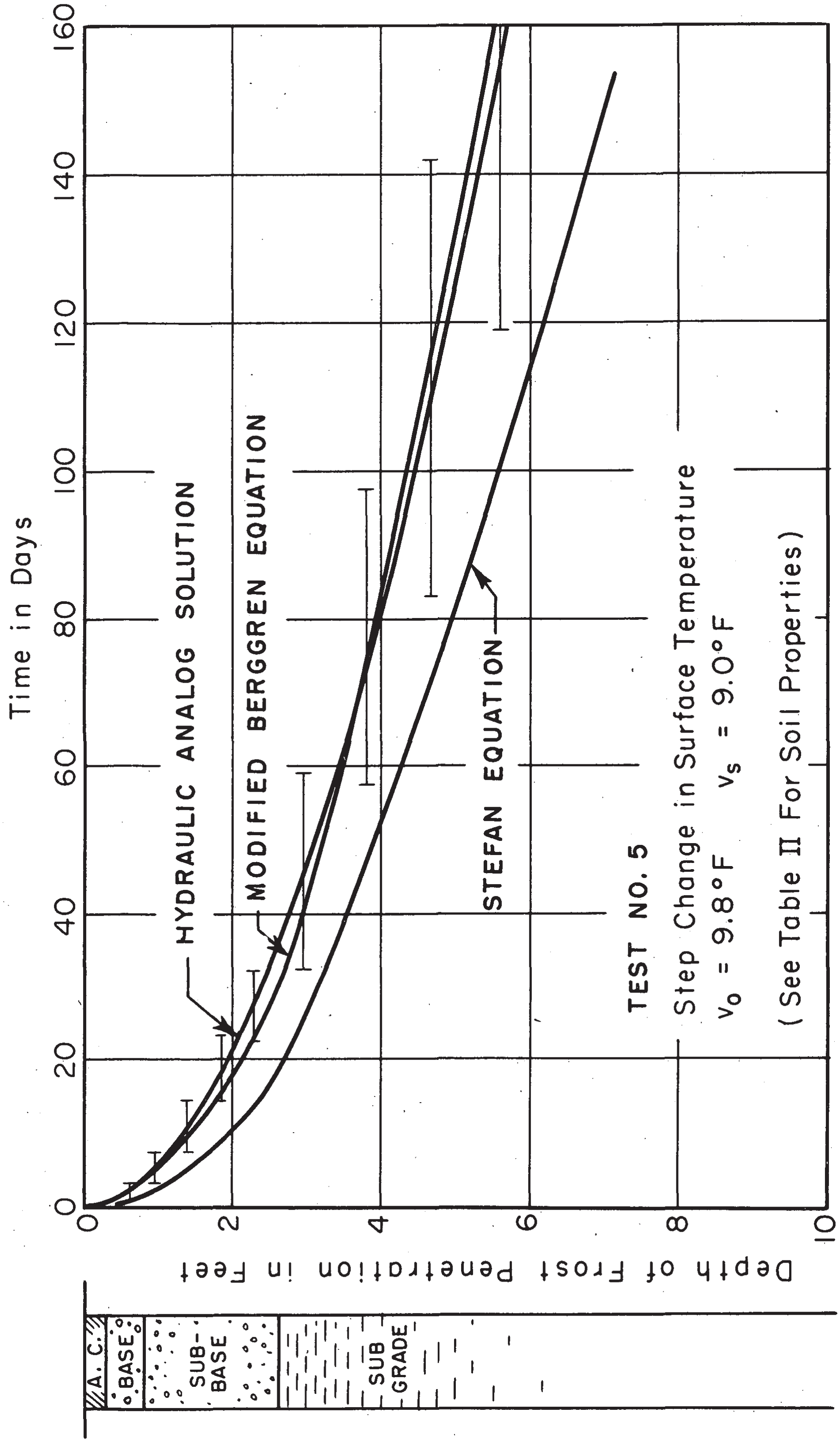

Time in Days
0
20
40
60
80
100
120
140
160
Depth of Frost Penetration in Feet
0
2
4
6
8
10
HYDRAULIC ANALOG SOLUTION
MODIFIED BERGGREN EQUATION
STEFAN EQUATION
TEST NO. 5
Step Change in Surface Temperature
v_o = 9.8°F v_s = 9.0°F
(See Table II For Soil Properties)
A. C.
BASE
SUB-BASE
SUB GRADE

FIGURE 5

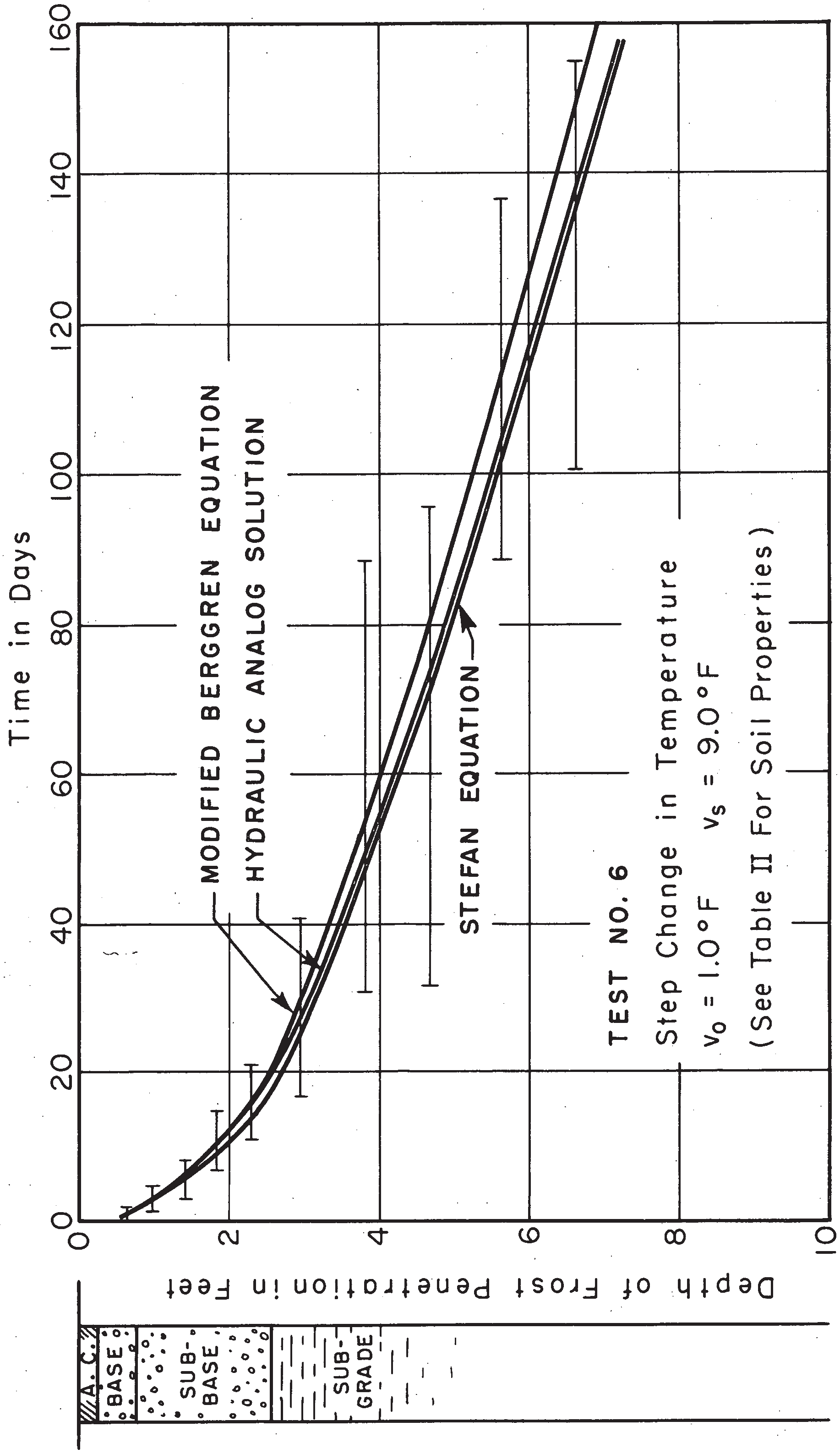
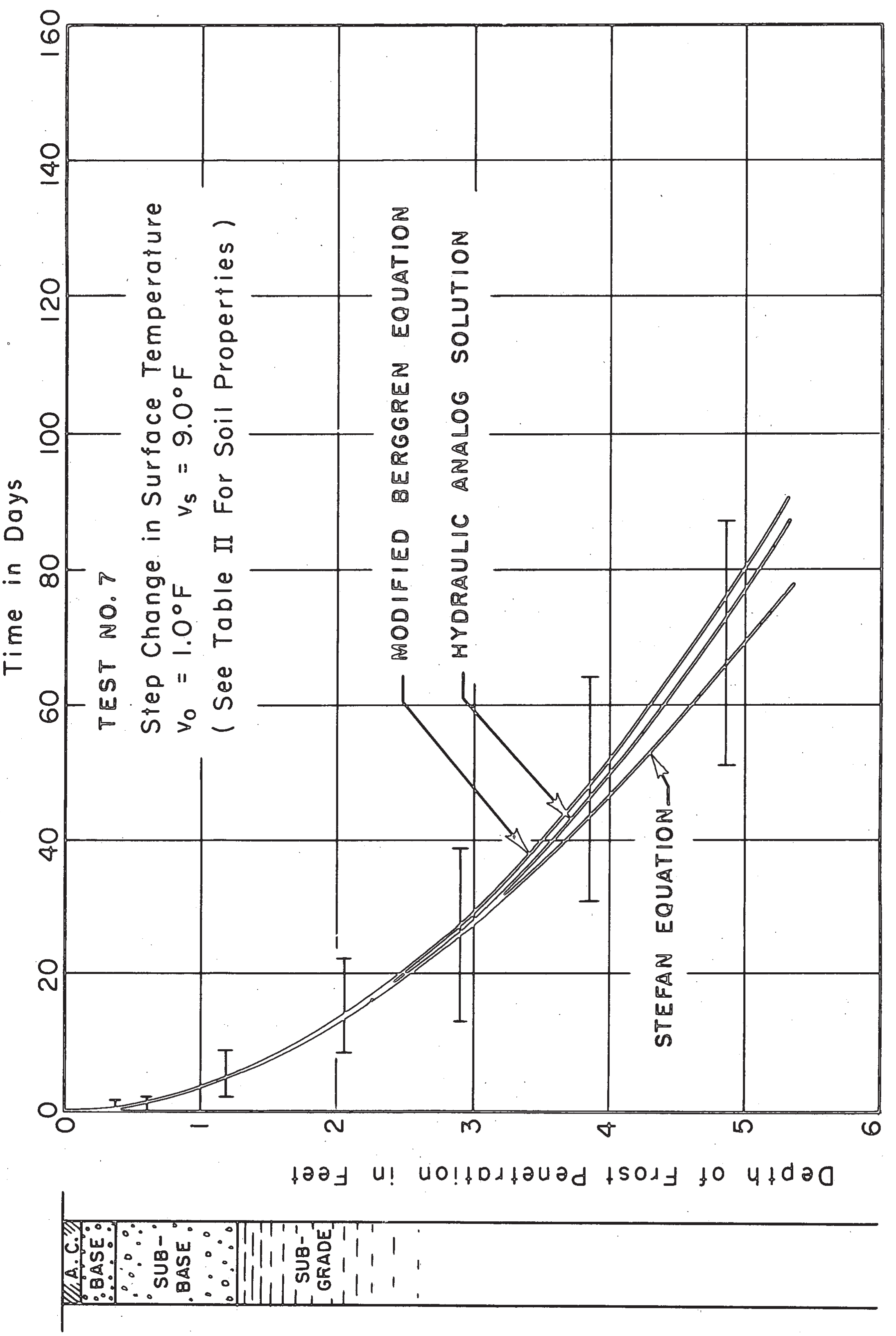
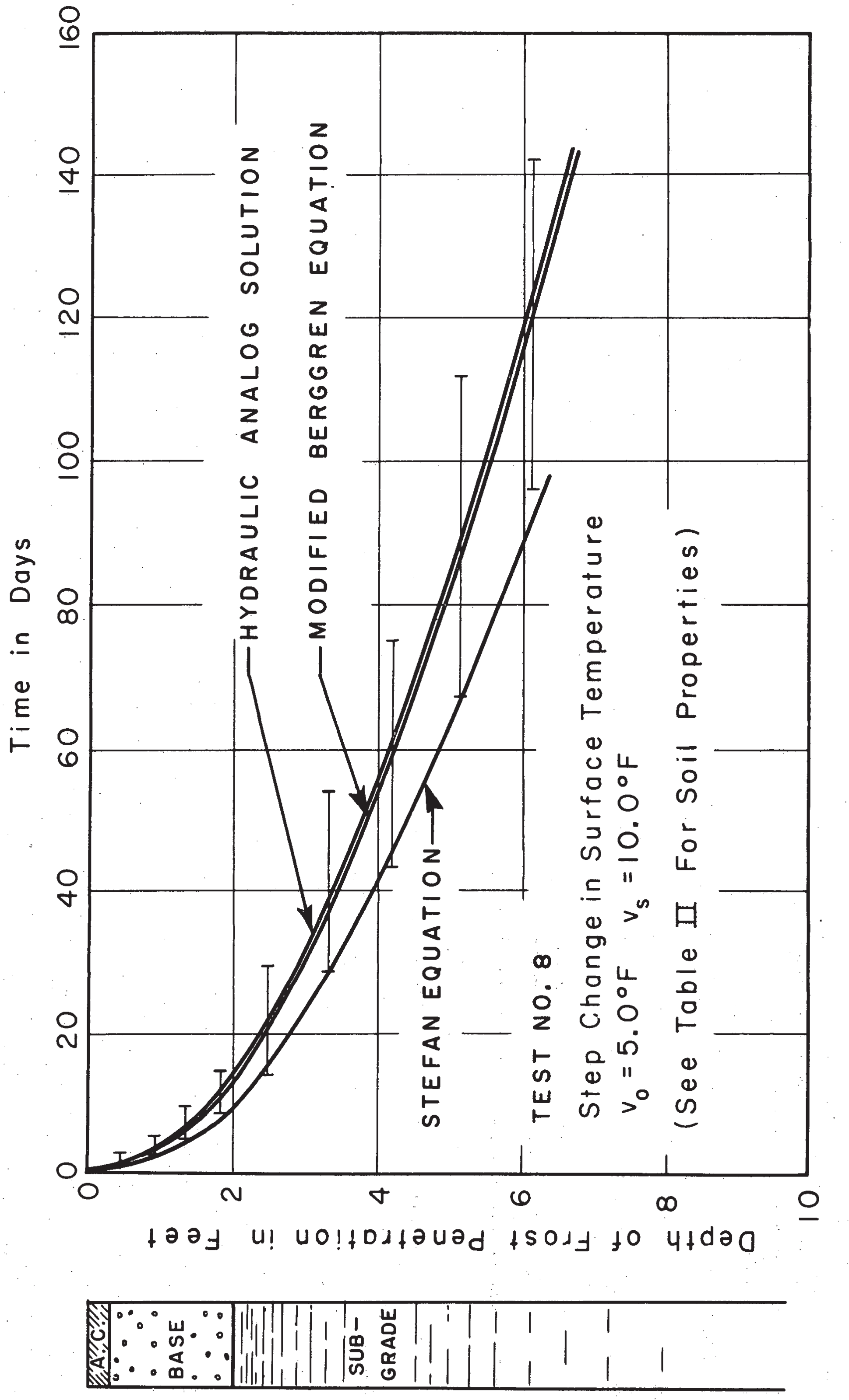

Time in Days
0
20
40
60
80
100
120
140
160
Depth of Frost Penetration in Feet
0
2
4
6
8
10
MODIFIED BERGGREN EQUATION
HYDRAULIC ANALOG SOLUTION
STEFAN EQUATION
TEST NO. 6
Step Change in Temperature
v_o = 1.0°F v_s = 9.0°F
(See Table II For Soil Properties)
A. C.
BASE
SUB-BASE
SUB-GRADE

FIGURE 6



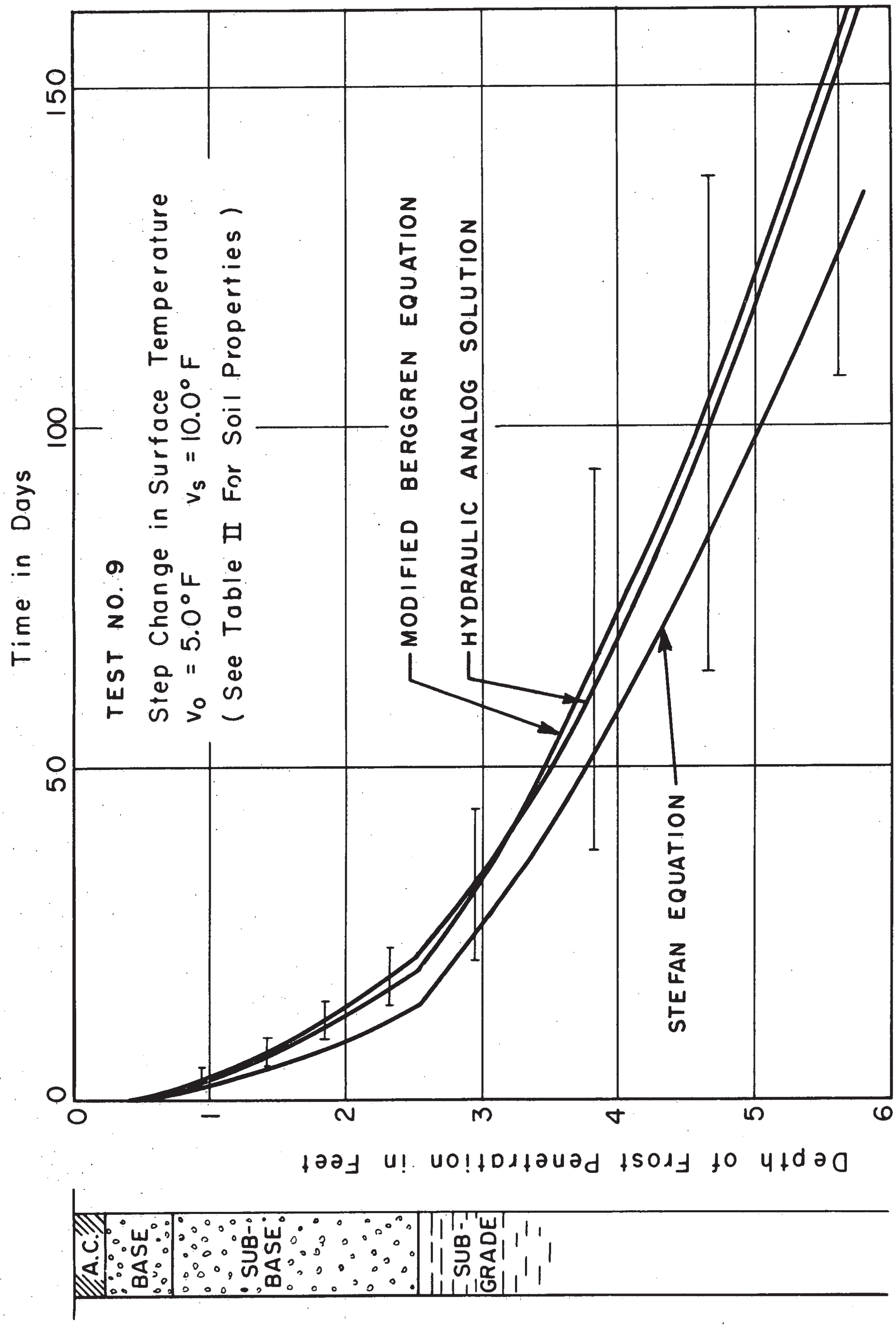
Time in Days
0
20
40
60
80
100
120
140
160
TEST NO. 7
Step Change in Surface Temperature
v_o = 1.0°F v_s = 9.0°F
(See Table II For Soil Properties)
MODIFIED BERGGREN EQUATION
HYDRAULIC ANALOG SOLUTION
STEFAN EQUATION
0
1
2
3
4
5
6
Depth of Frost Penetration in Feet
A.C.
BASE
SUB-BASE
SUB-GRADE

FIGURE 7



Time in Days
0
20
40
60
80
100
120
140
160
Depth of Frost Penetration in Feet
0
2
4
6
8
10
HYDRAULIC ANALOG SOLUTION
MODIFIED BERGGREN EQUATION
STEFAN EQUATION
TEST NO. 8
Step Change in Surface Temperature
$v_o = 5.0°F \quad v_s = 10.0°F$
(See Table II For Soil Properties)
A.C.
BASE
SUB-GRADE

FIGURE 8



Time in Days
0
50
100
150
Depth of Frost Penetration in Feet
0
1
2
3
4
5
6
TEST NO. 9
Step Change in Surface Temperature
v_o = 5.0° F v_s = 10.0° F
(See Table II For Soil Properties)
MODIFIED BERGGREN EQUATION
HYDRAULIC ANALOG SOLUTION
STEFAN EQUATION
A.C.
BASE
SUB-BASE
SUB-GRADE

FIGURE 9

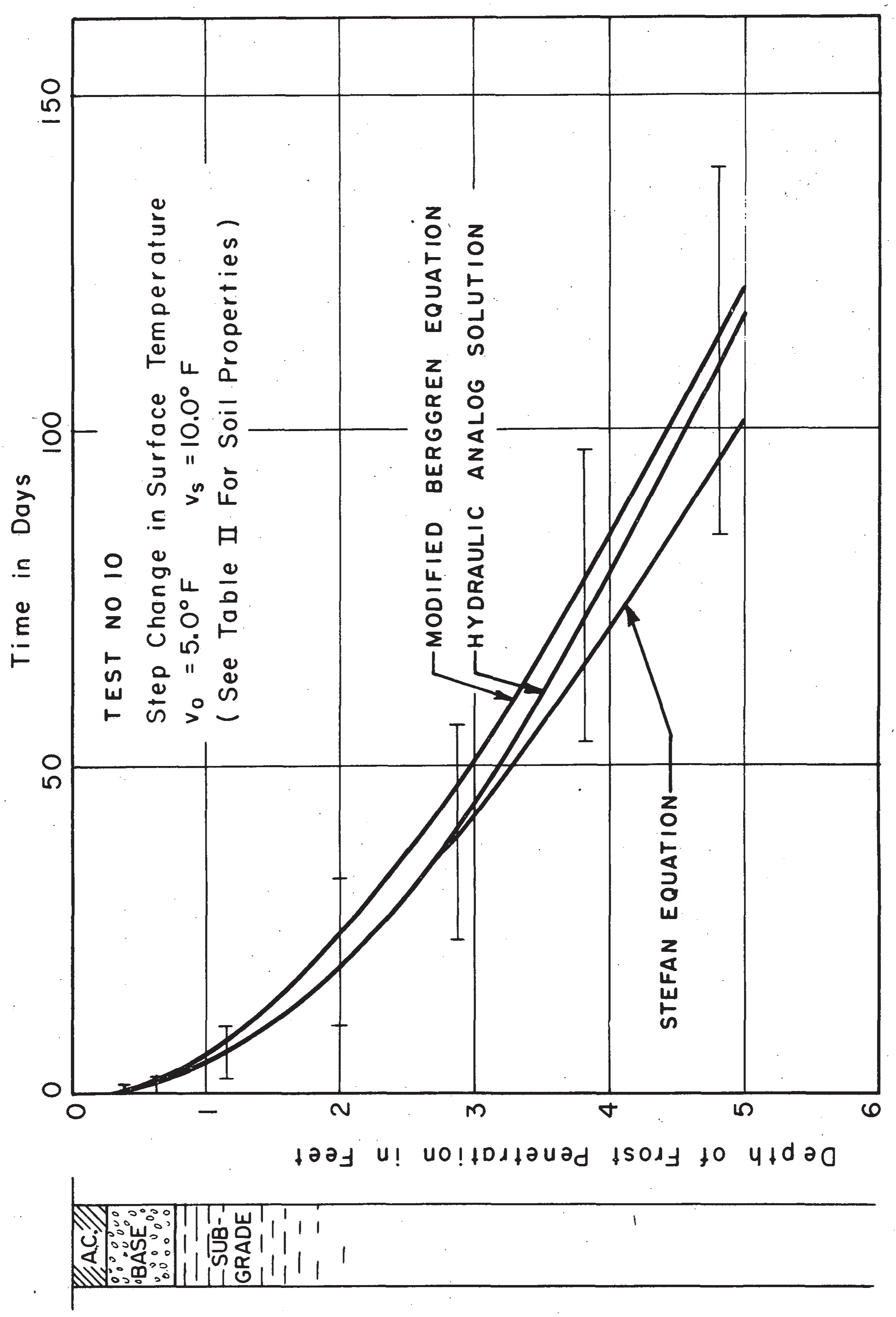
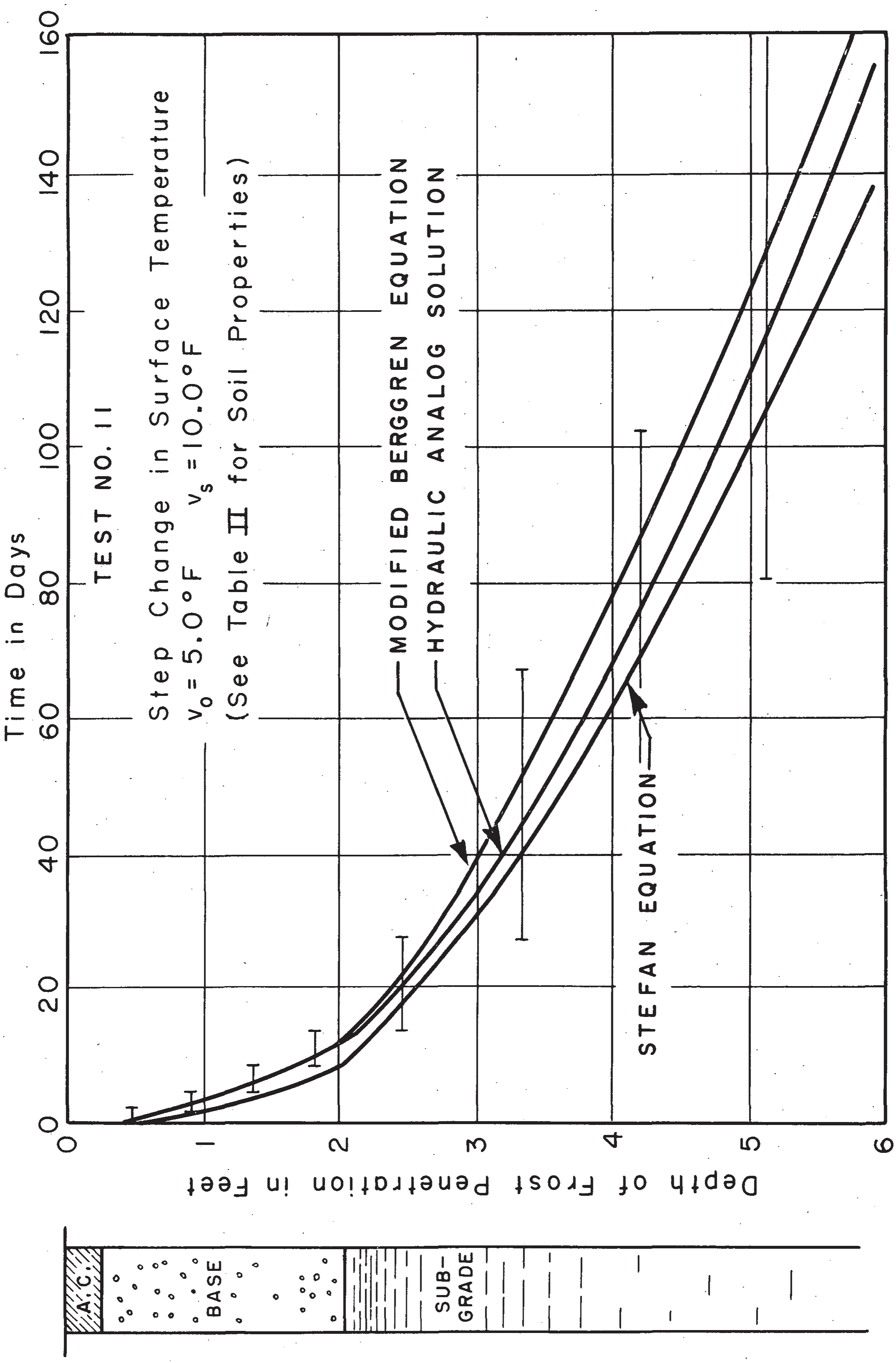

Time in Days
0
50
100
150
Depth of Frost Penetration in Feet
0
1
2
3
4
5
6
TEST NO 10
Step Change in Surface Temperature
v_o = 5.0° F v_s = 10.0° F
(See Table II For Soil Properties)
MODIFIED BERGGREN EQUATION
HYDRAULIC ANALOG SOLUTION
STEFAN EQUATION
A.C.
BASE
SUB-GRADE

FIGURE 10



Time in Days
0
20
40
60
80
100
120
140
160
TEST NO. 11
Step Change in Surface Temperature
$v_o = 5.0°F$ $v_s = 10.0°F$
(See Table II for Soil Properties)
MODIFIED BERGGREN EQUATION
HYDRAULIC ANALOG SOLUTION
STEFAN EQUATION
Depth of Frost Penetration in Feet
0
1
2
3
4
5
6
A.C.
BASE
SUB-GRADE

FIGURE 11

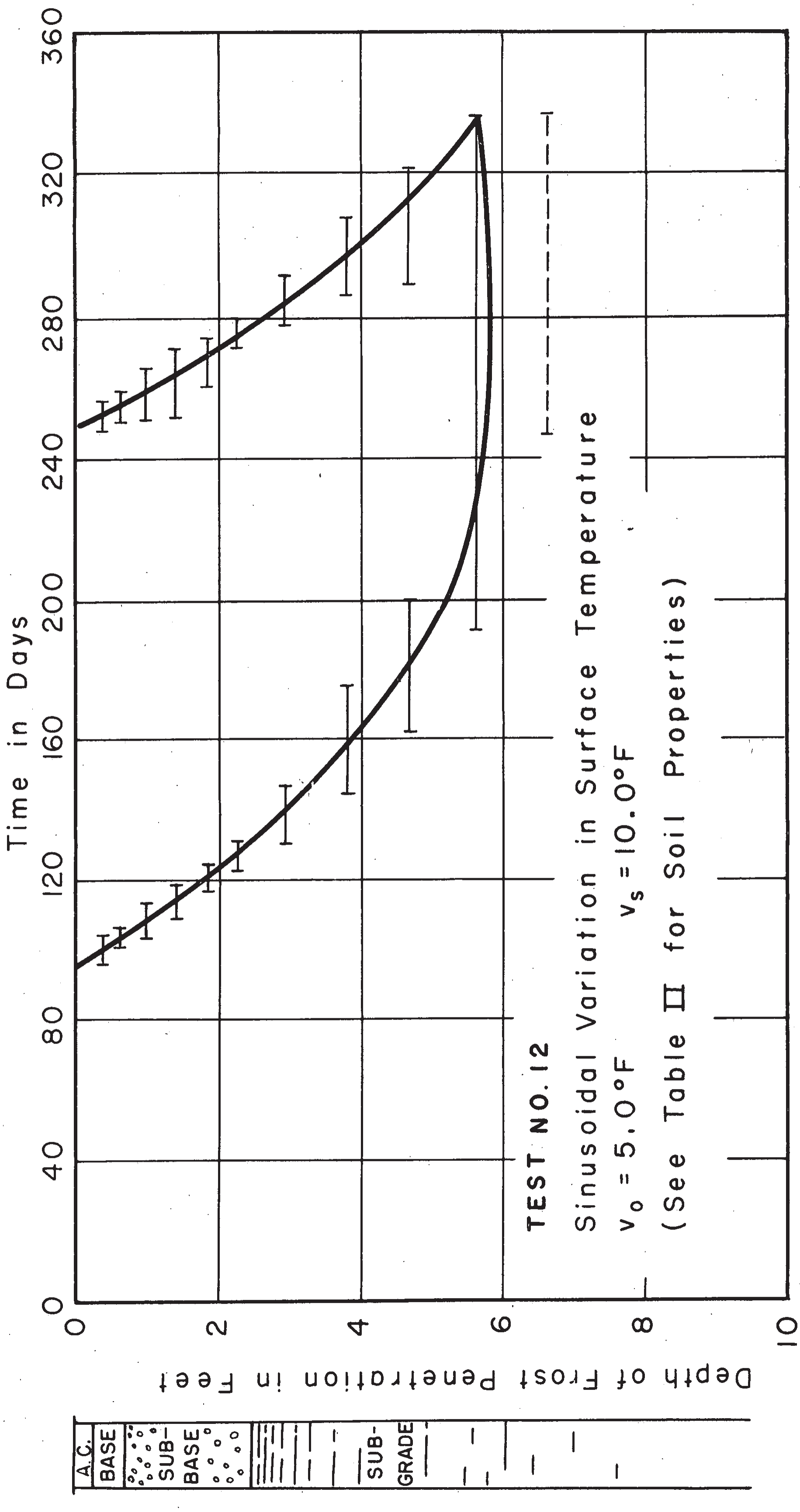

Time in Days
0
40
80
120
160
200
240
280
320
360
Depth of Frost Penetration in Feet
0
2
4
6
8
10
TEST NO. 12
Sinusoidal Variation in Surface Temperature
v₀ = 5.0°F
vₛ = 10.0°F
(See Table II for Soil Properties)
A.C.
BASE
SUB-BASE
SUB-GRADE

FIGURE 12

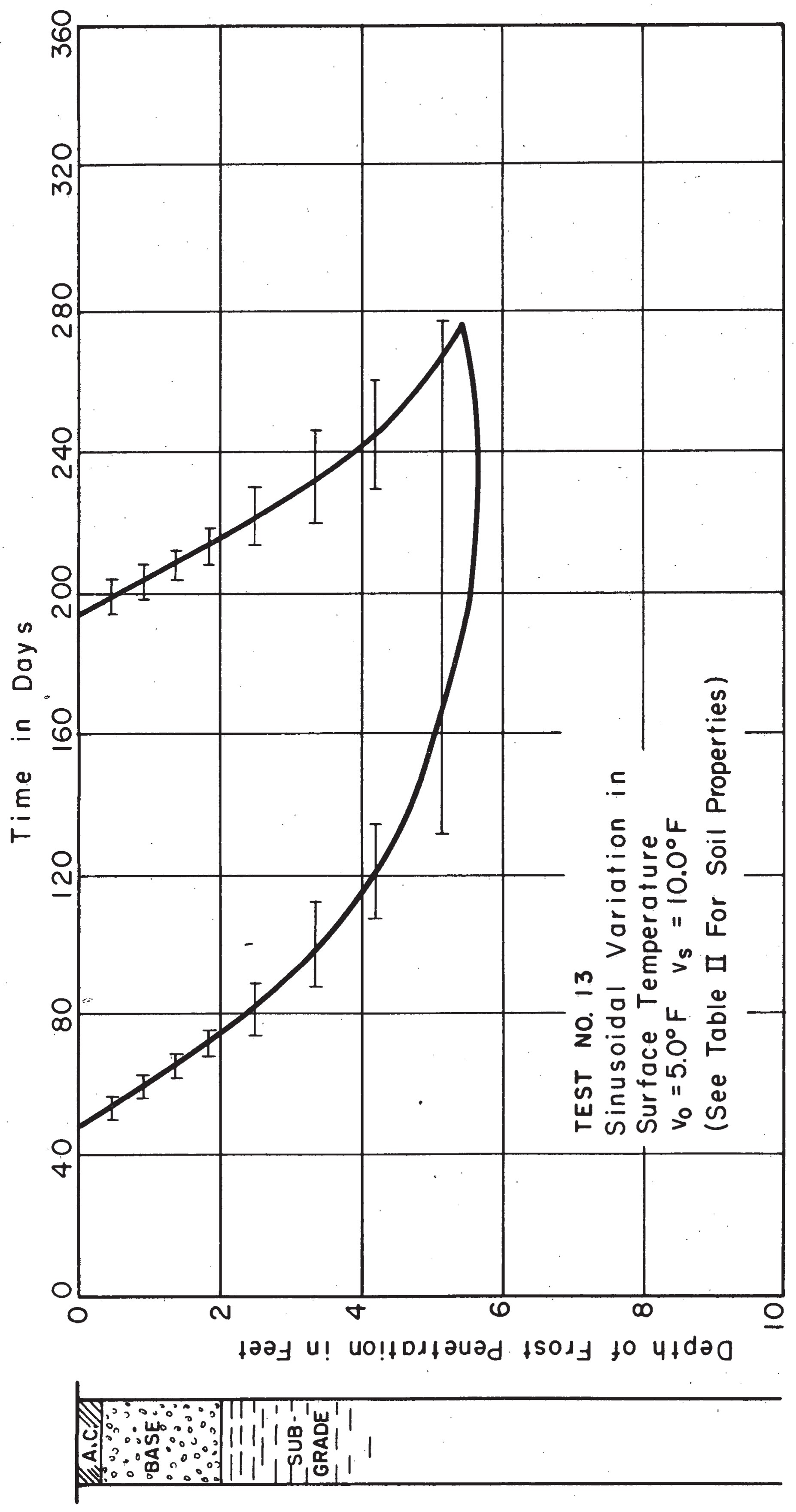
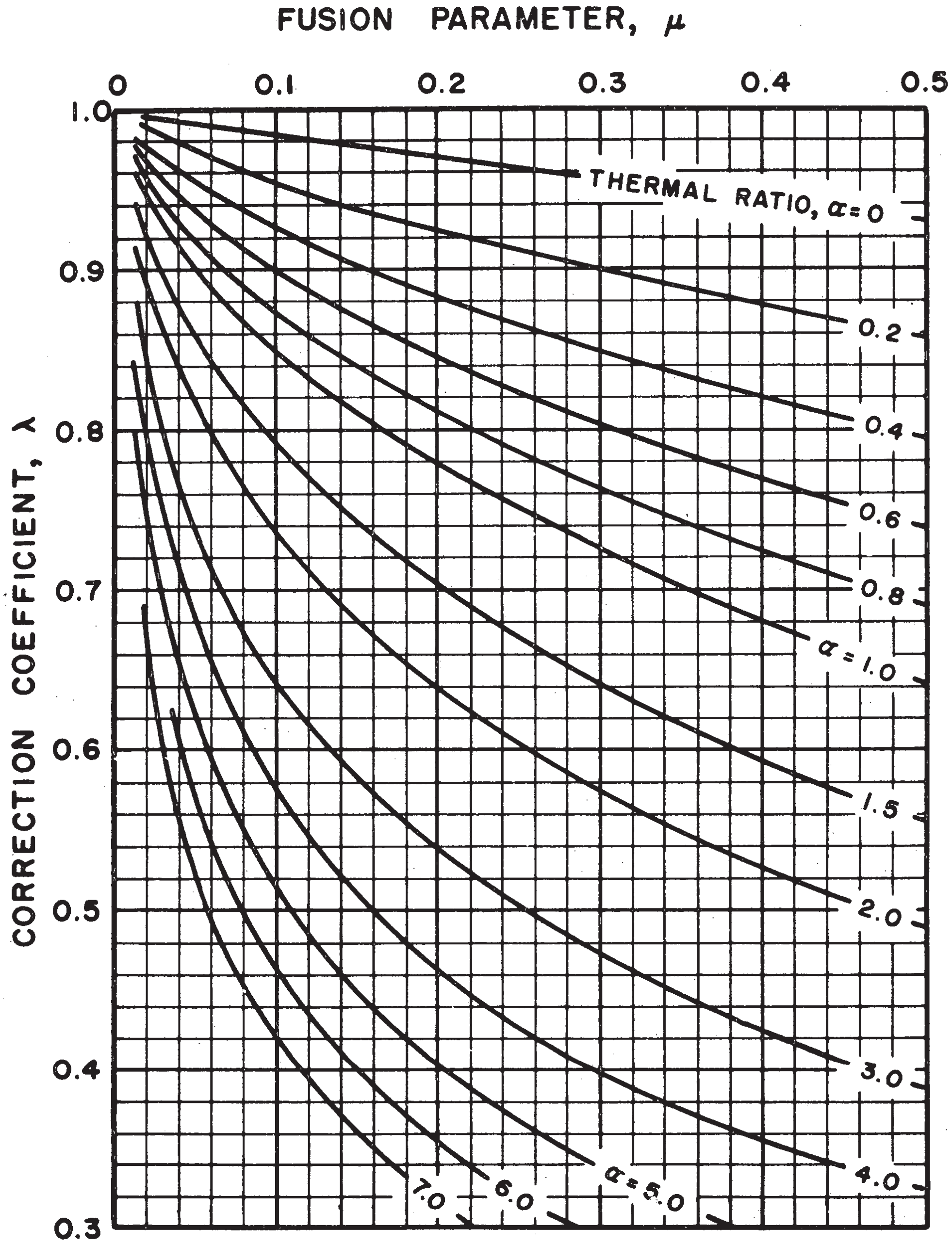

Time in Days
0
40
80
120
160
200
240
280
320
360
Depth of Frost Penetration in Feet
0
2
4
6
8
10
TEST NO. 13
Sinusoidal Variation in
Surface Temperature
v_0 = 5.0°F v_s = 10.0°F
(See Table II For Soil Properties)
A.C.
BASE
SUB-GRADE

FIGURE 13



CORRECTION COEFFICIENT IN THE MODIFIED BERGGREN FORMULA

FIGURE 14

Table I

SUMMARY OF CHECK SOLUTIONS

A. Step Change in Surface Temperature

Test No.	Test Results	v_o (deg F.)	v_s (deg F.)	α	μ	$\lambda = \frac{X}{X_S}$	λ_{MB}
1	Figure 1	12.5	12.5	1.0	0.2	0.78 $\pm$	0.78
2	Figure 2	1.0	12.5	0.08	0.2	0.95	0 95

B. Sinusoidal Variation of Surface Temperature

Test No.	Test Results	v_o (deg F.)	v_s (deg F.)	α	μ	$\lambda = \frac{X}{X_S}$	λ_{IBM}
3	Figure 3	12.5	12.5	1.057	0.189	0.757	0.73 $\pm$
4	Figure 4	1.0	12.5	0.078	0.209	0.95	0.96 +

Soil Profile:

Homogeneous: γ_d = 110 lb per cu ft

ω = 9.6%

Thermal Properties:

Thermal Conductivity, k = 1.2 Btu per (hr) (ft) (deg F.)

Volumetric Heat, C = 24 Btu per (cu ft) (deg F.)

Latent Heat, L = 1500 Btu per cu ft

Table II

SUMMARY OF RESULTS OF TESTS ON STRATIFIED SOIL(1)

A. Step Change in Surface Temperature

Test No.	Soil Profile(2)	Assumed Soil Properties: w	γ_d	L	C	k	v_o	v_s	α	μ	X	$\lambda_{MB}^{(3)}$	$\lambda_{MBD} = \frac{X_{MB}}{X_S}$	$\lambda = \frac{X}{X_S}$	$\frac{\lambda_{MBD} - \lambda}{\lambda_{MBD}}$
5	6" Base 21.5" Subbase	5% 7	110 120	850 1200	23 25	1.00 1.30	9.8	9.0	1.09	0.221 0.172	2.55 5.07	0.755 0.785	0.84 0.77	0.77 0.78	+ 8% - 1%
6	"Dry" Subgrade	10	120	1700	27	1.70	1.0	9.0	0.11	0.221 0.172	2.55 5.07	0.94 0.95	0.93 0.94	0.95 0.99	- 2% - 5%
7	6" Base "Dry" Subgrade	5 10	110 120	850 1700	23 27	1.00 1.70	1.0	9.0	0.11	0.166 0.160	3.27 4.33	0.955 0.956	0.96 0.93	0.98 0.96	- 2% - 3%
8	21.5" Base "Dry" Subgrade	7 10	120 120	1200 1700	25 27	1.30 1.70	5.0	10.0	0.5	0.242 0.187 0.181	2.05 4.57 5.63	0.85 0.87 0.875	0.87 0.87 0.86	0.86 0.86 0.85	+ 1% + 1% + 1%
9	6" Base 21.5" Subbase "Wet" Subgrade	5 7 20	110 120 102	850 1200 2900	23 25 27	1.00 1.30 1.70	5.0	10.0	0.5	0.221 0.128	2.55 5.55	0.855 0.90	0.92 0.90	0.895 0.91	+ 3% - 1%
10	6" Base "Wet" Subgrade	5 20	110 102	850 2900	23 27	1.00 1.70	5.0	10.0	0.5	0.124 0.109	2.25 3.75	0.90 0.91	0.89 0.90	0.98 0.945	- 10% - 5%
11	21.5" Base "Wet" Subgrade	7 20	120 102	1200 2900	25 27	1.30 1.70	5.0	10.0	0.5	0.242 0.142 0.119	2.05 3.55 5.60	0.85 0.892 0.903	0.92 0.90 0.90	0.93 0.96 0.94	- 1% - 6% - 4%

B. Sinusoidal Variation in Surface Temperature

Test No.	Soil Profile(1)	Assumed Soil Properties: w	γ_d	L	C	k	v_o	v_s	α	μ	X	λ_{MB}	$\lambda_{MBD} = \frac{X_{MB}}{X_S}$	$\lambda = \frac{X}{X_S}$	$\frac{\lambda_{MBD} - \lambda}{\lambda_{MBD}}$
12	6" Base 21.5" Subbase "Wet" Subgrade	5% 7 20	110 120 102	850 1200 2900	23 25 27	1.00 1.30 1.70	5.0	10.0	0.5	0.126	5.80	0.90	0.90	0.94	- 4%
13	21.5" Base "Wet" Subgrade	7 20	120 102	1200 2900	25 27	1.30 1.70	5.0	10.0	0.5	0.119	5.60	0.90	0.90	0.91	- 1%

(1) Test Results are plotted in Figures bearing the same number as the Test No.

(2) All profiles have 3 inches of bituminous concrete; assume C = 28, k = 0.80.

(3) λ_{MB} is obtained from Figure 14.

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APPENDIX A
SINUSOIDAL VARIATION IN SURFACE TEMPERATURE

When the surface freezing index F and the thermal ratio α are known, the equivalent sine curve representing the surface temperature variation may be computed, although the direct computation involves the solution of a trigonometric equation by trial and error. However, the main part of the computations may be carried out once and for all so that solutions to the final equation may be obtained directly from a simple chart.

The sinusoidal surface temperature function (Figure A-1) may be written:

$$v = a \sin \omega t \qquad \text{(A-1)}$$

where:

v = temperature variation from the mean annual temperature, deg F.

a = amplitude of sine curve, deg F.

$\omega = \frac{2\pi}{365}$ radians per day

t = time in days

The deg F. by which the mean temperature is below the freezing point during the freezing period is designated by v_s while the deg F. by which the mean annual temperature exceeds the freezing point is v_o. The thermal ratio α is defined as:

$$\alpha = \frac{v_o}{v_s} \qquad \text{(A-2)}$$

The freezing index F is defined by the following equations:

$$F = \int_{t_o}^{t_1} (v - v_o)\, dt$$

where $(t_1 - t_o)$ is the duration of the freezing period. Substituting Equation (A-1) and noting that $v_o = a \sin \omega t_o$, we have:

$$F = \int_{t_o}^{t_1} (a \sin \omega t - a \sin \omega t_o)\, dt$$

which, on integration, becomes:

$$F = \frac{a}{\omega} (\cos \omega t_o - \cos \omega t_1) - a \sin \omega t_o (t_1 - t_o) \qquad \text{(A-3)}$$

Furthermore, the freezing index may be written:

$$F = v_s(t_1 - t_o) = \frac{v_o}{\alpha} (t_1 - t_o) = \frac{a \sin \omega t_o}{\alpha} (t_1 - t_o) \qquad \text{(A-4)}$$

and the following expressions are apparent from Figure A-1:

$$t_1 = \frac{\pi}{\omega} - t_o \qquad \text{(A-5)}$$

$$\cos \omega t_1 = - \cos \omega t_o \qquad \text{(A-6)}$$

Finally, by combining equations (A-3) through (A-6) and solving for the thermal ratio α in terms of ωt_o:

$$\alpha = \frac{(\pi - 2\omega t_o) \tan \omega t_o}{2 - (\pi - 2\omega t_o) \tan \omega t_o} \qquad \text{(A-7)}$$

For a given value of α the value of t_o must be found by trial and error. However, Figure A-2 has been prepared such that the value of t_o may be read directly for a given α.

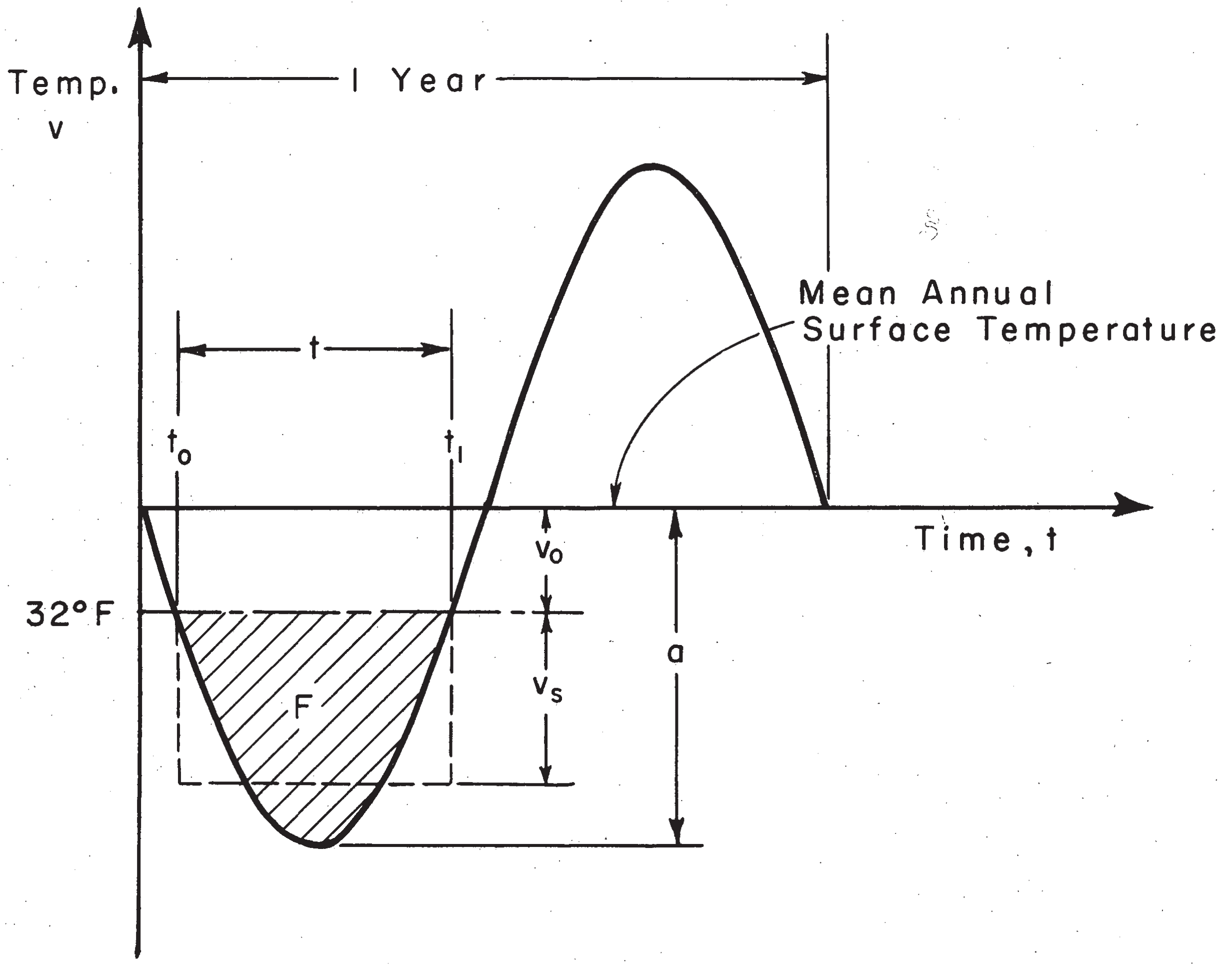
Once t_o is known, the temperature amplitude may be determined from the following equation (A-1):

$$a = \frac{v_o}{\sin \omega t_o}$$

or

$$a = \frac{\alpha v_s}{\sin \omega t_o} = \left(\frac{\alpha}{\sin \omega t_o}\right) \left(\frac{F}{182.5 - 2\, t_o}\right) \qquad \text{(A-8)}$$

By choosing the time scale for a computer solution, the speed of the kymograph, and the temperature scale, the actual dimensions of the sine curve may be determined for programming the solution.



SINE CURVE FOR ASSUMED ANNUAL VARIATION IN SURFACE TEMPERATURE.

FIGURE A-1

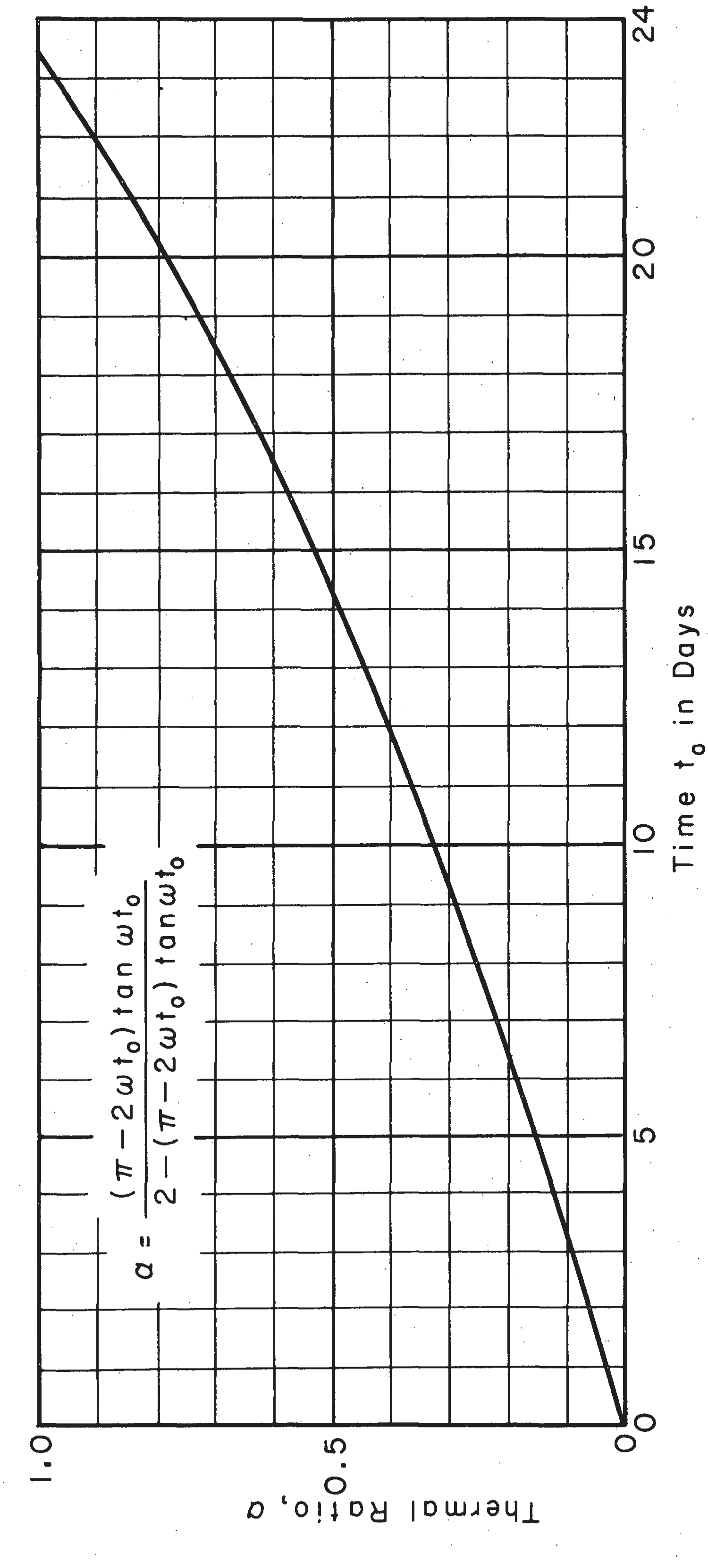

$a = \frac{(\pi - 2\omega t_o)\tan \omega t_o}{2-(\pi - 2\omega t_o)\tan \omega t_o}$
Time t_o in Days
Thermal Ratio, a
0
5
10
15
20
24
0
0.5
1.0

FIGURE A-2

APPENDIX B
SAMPLE COMPUTATIONS

STEP CHANGE IN SURFACE TEMPERATURE

TEST NO. 9

Soil Profile and Assumed Thermal Properties:

		w	γ_d	L	C	k
3"	Bituminous Concrete	--	--	--	28	0.8
6"	Base	5	110	850	23	1.0
21.5"	Subbase	7	120	1200	25	1.3
	'Wet' Subgrade	20	102	2900	27	1.7

Temperature Conditions:

$v_o = 5$ F

$v_s = 10$ F

Step change in surface temperature from 37 F. to 22 F.

Hydraulic Analog Solution:

The scaling for this problem is given on the accompanying 'Scaling Sheet' for 'Hydraulic Analog Computer.'

Temperature Scale $V_s = 0.2$ inches per degree F.

Thermal Energy Scale $U_s = 0.00195$ cu in. per Btu.

Time Scale $T_s = 0.5$ min per day

Observations made during the solution are shown on the accompanying 'Observation Sheets.'

The water temperature during the test remained about 0.6 C below the desired 28 C. This temperature deviation causes an error of about 0.5 percent which is too small to justify a correction.

Results of the analog solution are plotted in Figure 9.

Stefan Equation:

d	$X = \Sigma d$	k	L	$R = \frac{d}{k}$	$\Sigma R + \frac{R}{2}$	F^*	ΣF	$t = \frac{\Sigma F}{v_s}$
0.25	0.25	0.8	0	0.31	0.16	0	0	0
0.50	0.75	1.0	850	0.50	0.56	10	10	1
1.80	2.55	1.3	1200	1.38	1.50	135	145	14.5
1.50	4.05	1.7	2900	0.88	2.63	447	592	59.2
1.50	5.55	1.7	2900	0.88	3.51	636	1228	122.8
0.25	5.80	1.7	2900	0.15	4.03	127	1355	135.5
0.42	6.20	1.7	2900	0.25	4.23	215	1570	157.0

* $F_n = \frac{L_n d_n}{24}\left(\Sigma R + \frac{R}{2}\right)$

See Figure 9 for plotted results.

Adaptation of Modified Berggren Equation:

$$\alpha = \frac{v_o}{v_s} = 0.5 \qquad \mu = \frac{C_{wt}}{L_{wt}} v_s = 10\,\frac{C_{wt}}{L_{wt}}$$

d	$X = \Sigma d$	C	C_{wt}	L	L_{wt}	μ	λ_{MB}	ΣF	$t = \frac{\Sigma F}{v_s \lambda_{MB}^2}$
0.25	0.25	28	28	0	0	--	--	0	0
0.50	0.75	23	24.7	850	567	0.392	0.795	10	1.6
1.80	2.55	25	24.9	1200	1015	0.221	0.855	145	20
1.50	4.05	27	25.7	2900	1712	0.150	0.890	592	75
1.50	5.55	27	26.0	2900	2030	0.128	0.897	1228	153
0.25	5.80	27	26.1	2900	2070	0.126	0.90	1355	167

See Figure 9 for plotted results.

SINUSOIDAL VARIATION IN SURFACE TEMPERATURE

TEST NO. 12

Soil Profile and Assumed Thermal Properties:

(Same as Test No. 9)

Temperature Conditions:

$v_o = 5$ F

$v_s = 10$ F

Sinusoidal variation in surface temperature

Surface Temperature Curve:

From Figure A-2:

$$\alpha = \frac{v_o}{v_s} = 0.5 \qquad \text{therefore} \qquad t_o = 14.35 \text{ days}$$

$$F = v_s\,(182.5 - 2t_o) = 1538 \text{ degree days}$$

$$a = \frac{v_o}{\sin \omega t_o} = \frac{5}{\sin \frac{2\pi}{365}\, 14.35} = 20.45 \text{ F}$$

Therefore, the following equation is obtained for the surface temperature variation:

$$v = 20.45 \sin \frac{2\pi}{365}\, t$$

Hydraulic Analog Solution:

The scaling for this problem is the same as that for Test No. 9 and is therefore given on the accompanying 'Scaling Sheet' for Test 9.

Observations made during the analog solution are shown on the accompanying 'Observation Sheets'. Data were taken during one surface temperature cycle. The initial temperature distribution in the ground was assumed on the basis of results of Test No. 3 which represents uniform soil. This apparently crude approximation was later found to yield errors which are very small.

It was difficult to get a good determination of the maximum depth of frost penetration in this test. Shortly after the thawing started at ground surface, gradients were very small at the freezing interface. Latent heat well No. 11 was 90 per cent drained and well No. 12 had just started to drain. As thawing continued downward from the surface, well No. 12 continued to drain into well No. 11 which complicated the usual procedure of determining the maximum depth of frost penetration. However, it is felt that the most reliable determination of the maximum depth of freezing can be obtained from observations at the start of the thawing period when 70 per cent of well No. 11 was frozen. Thus:

$$X = (5.61 - 0.51) + (0.7)(1.02) = \underline{5.81 \text{ ft}}.$$

The actual surface temperature curve as observed during the test deviated slightly from that intended yielding the following results:

$$F = 1568 \text{ degree days}$$

$$t = 157.5 \text{ days}$$

Therefore, actually:

$$v_s = 9.95\,F$$

$$\alpha = 0.503$$

$$\mu = 0.126 \quad \text{(see computations for Test No. 9)}$$

Results of the analog solution are plotted in Figure 12.

<u>Stefan Equation:</u>

From sample computations for Test No. 9, the depth of frost penetration corresponding to F = 1568 degree days is:

$$X_S = \underline{6.20 \text{ ft}}.$$

Therefore:

$$\lambda = \frac{X}{X_S} = \frac{5.81}{6.20} = \underline{0.94}$$

Adaptation of Modified Berggren Equation:

We start with Step 1 of the procedure suggested in Section 4-03-b:

Step 1.

Assume that the depth of frost penetration is 5.7 feet. Thus:

$$\left(\frac{L}{k}\right)_{eff.} = \frac{2}{(5.7)^2}\left[\frac{0.25}{0.8}\left(0 + \overset{425}{(850)(0.5)} + \overset{2150}{(1200)(1.79)} + \overset{9160}{(2900)(3.16)}\right)\right.$$

$$+\frac{0.5}{1.00}\left(212 + 2150 + 9160\right)$$

$$\left.+\frac{1.79}{1.30}\left(1075 + 9160\right) + \frac{3.16}{1.70}\left(4580\right)\right]$$

$$\left(\frac{L}{k}\right)_{eff.} = 1970$$

Step 2.

$$C_{wt} = \frac{(28)(0.25) + (23)(0.5) + (25)(1.79) + (27)(3.16)}{5.7} = 26.0$$

$$L_{wt} = \frac{0 + (850)(0.5) + (1200)(1.79) + (2900) + (3.16)}{5.7} = 2060$$

Step 3.

$$\alpha = \frac{v_o t}{F} = \frac{(5)(157.5)}{1568} = 0.503$$

$$\mu = \frac{C_{wt}F}{L_{wt}t} \; \frac{(26.0)(1568)}{(2060)(157.5)} = 0.126$$

Step 4.

From Figure 14, $\lambda_{MB} = 0.90$

Step 5.

$$X_{MB} = \lambda_{MB} \sqrt{\frac{48F}{\left(\frac{L}{k}\right)_{eff.}}} = 0.90 \sqrt{\frac{(48)(1568)}{1970}}$$

$$X_{MB} = \underline{5.6 \text{ ft}} .$$

This result is recorded in Table II.

HYDRAULIC ANALOGUE COMPUTER
SCALING SHEET

TEST NO.: 9
DATE: July 20, 1955
COMPUTED BY: RN

PROBLEM: 4-layer Case, Wet Subgrade

TIME SCALE 1 MIN = 2 DAYS; 1°F = 0.2 INCH; 1 BTU = .00195 INCH³; 1 BTU/°F HR = 2.14 MIN/IN

SOIL PROFILE	L BTU/FT³	C BTU/°F FT³	k BTU/°F HR FT	SLIDE OPENING INCH	LATENT HEAT INCH³	LATENT HEAT BTU	STAND-PIPE AREA (INCH²)		ANALOGUE PROFILE
3" AC	0	28	0.8	0	0	0	0.068	3	1
6" Base γ = 110 pcf w = 5%	850	23	1.0	0.44	0.415	213	0.056	3	2
				0.44	0.415	213	0.056	3	3
21.5" Subgrade γ = 120 pcf w = 7%	1200	25	1.30	1.33	1.045	537	0.109	5.37	4
				1.33	"	"	0.109	5.37	5
				1.33	"	"	0.109	5.37	6
				1.33	"	"	0.109	5.37	7
Subgrade Wet γ = 102 pcf w = 20%	2900	27	1.70	7.42	4.83	2480	0.225	10.26	8
				7.42	"	"	0.225	10.26	9
				7.42	"	"	0.225	10.26	10
				8.85	5.75	2950	0.267	12.2	11
				8.85	"	"	0.267	12.2	12
							0.415	18.90	13
							0.415	18.90	14
							1.10	50.2	15
							1.26	57.4	16
							2.26	103	17
							4.32	197	18
							8.10	369	19
							13.8	630	20
				Water temperature = 28°C					

	G_s BTU/°F HR	R_w MIN/IN²	Setting IN.
1.5	6.4	0.34	16.18
3		0.61	15.69
3	4.0	0.54	15.78
3.69		0.64	15.80
5.37	2.91	0.74	15.26
5.37	2.91	0.74	15.41
5.31	2.91	0.74	15.46
7.82		0.91	15.91
10.26	1.99	1.08	15.40
10.26	1.99	1.08	15.92
11.23	1.815	1.18	Turns 0.35
12.2	1.67	1.28	0.37
15.55	1.312	1.63	0.60
18.90	1.08	1.98	0.50
34.55	0.59	3.63	1.50
53.8	0.38	5.60	4.55
80.2	0.254	8.45	2.50
150	0.136	15.70	6.00
28.3	0.072	29.70	13.00
500	0.041	52.2	22.00

Resistor Type B

Resistor Type A

HYDRAULIC ANALOGUE COMPUTER
OBSERVATION SHEET

OBSERVED BY: RN TEST NO. 9
DATE: July 20, 1955 SHEET 1 OF 2

PROBLEM: 4-layer case - Wet Subgrade
Step Change in Surface Temperature

DATE AND HOUR	ELAPSED TIME 1 MIN = 2 DAYS ANALOGUE (MIN)	PROTOTYPE (DAYS)	STANDPIPES: TEMPERATURE SCALE 0.2 INCH = 1° F 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	REMARKS
			Surf.	0.13	0.38	0.63	0.97	1.42	1.86	2.31	2.96	3.82	4.67	5.61	6.63	7.92	9.50	12.46	16.94	23.62	36.12	59.22	101.42	← Depth in Ft.
13:20	0	0	22	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	
					8"	30"	1:00	2:35	4:40	7:25	10:35	18:55	32:05	53:45	80:35	←	(Time when layer starts to freeze)							
					25"	50"	2:30	4:40	7:35	11:10	21:50	47:10	68:30	10:2-	122:40	←	(Time when layer completely frozen)							
	5	10	22	22.8	24.5	26.-	27.9	29.9	32	32.5	33	33.7	34.3	34.7	35.2	35.8	36.2	36.7	37.-	36.9	37	37	37	
	10	20	22	22.9	24.1	25.2	26.7	28.2	29.7	31.2	32	32.5	33	33.4	33.9	34.5	35.2	36.2	36.7	36.9	37	37	37	
	15	30	22	22.8	23.9	24.9	26.2	27.6	28.9	30.3	31.8	32.2	32.7	33.1	33.5	34.0	34.6	35.7	36.6	36.8	37	37	37	27.3°C (Test temp)
	20	40	22	22.4	23.5	24.4	25.7	27	28.3	29.7	31.1	32	32.4	32.7	33.1	33.7	34.1	35.3	36.2	36.7	37	37	37	
	25	50	22	22.4	23.4	24.3	25.5	26.6	27.8	29	30.2	32	32.3	32.6	32.9	33.4	33.9	35	36	36.5	37	37	37	27.3°C
	30	60	22	22.5	23.4	24.2	25.4	26.6	27.7	28.9	30.1	31.8	32.2	32.5	32.7	33.2	33.7	34.8	35.7	36.4	36.9	37	37	
	35	70	22	22.5	23.4	24.1	25.2	26.3	27.4	28.5	29.7	31.4	32.2	32.5	32.6	33.1	33.6	34.5	35.6	36.2	36.9	37	37	27.4°C
	40	80	22	22.4	23.4	24.2	25.2	26.3	27.4	28.5	29.6	31.2	32.2	32.5	32.7	33.1	33.5	34.9	35.3	36.-	36.8	36.9	37	
	45	90	22	22.8	23.5	24.3	25.2	26.2	27.2	28.3	29.4	31	32.2	32.3	32.7	33.0	33.5	34.2	35.1	35.8	36.8	36.9	37	
	50	100	22	22.5	23.2	24	24.9	25.8	26.9	27.9	28.9	30.4	31.8	32	32.3	32.7	33.1	34.1	35.1	35.7	36.7	36.9	37	27.4°C
	55	110	22	22.3	23	23.8	24.7	25.6	26.7	27.6	28.6	30	31.5	32	32.2	32.7	33	34	34.9	35.6	36.7	36.9	37	
	60	120	22	22.9	23.5	24.2	24.8	25.7	26.7	27.5	28.5	29.8	31.2	32.2	32.4	32.7	33	33.8	34.8	35.4	36.7	36.9	37	
	65	130	22	22.5	23.2	23.8	24.6	25.5	26.4	27.4	28.3	29.7	31.2	32.2	32.4	32.7	33	33.8	34.7	35.3	36.7	36.9	37	
	70	140	22	23	23.5	24.1	24.7	25.6	26.5	27.3	28.2	29.4	30.7	32	32.2	32.6	33	33.7	34.6	35.2	36.5	36.9	37	
	75	150	22	22.4	23.1	23.5	24.5	25.3	26.2	27	27.9	29.2	30.5	31.7	32.1	32.5	32.8	33.7	34.5	35.1	36.4	36.9	37	
	80	160	22	22.3	23	23.5	24.5	25.3	26.2	27	27.9	29	30.3	31.6	32	32.3	32.7	33.6	34.4	35.1	36.4	36.9	37	27.5°C
	85	170	22	22.5	23.1	23.5	24.4	25.2	26	26.8	27.7	28.8	30.1	31.3	32	32.3	32.6	33.5	34.3	34.9	36.3	36.9	37	
	90	180	22	22.4	23	23.5	24.3	25	25.8	26.7	27.5	28.7	29.9	31.2	32	32.3	32.6	33.4	34.2	34.8	36.2	36.8	37	
	95	190	22	22.3	22.9	23.5	24.3	25	25.9	26.7	27.5	28.6	29.9	31.1	32.2	32.4	32.6	33.3	34.2	34.7	36.2	36.8	37	
	100	200	22	22.5	23.1	23.5	24.3	25.1	25.8	26.7	27.4	28.5	29.6	30.8	32	32.3	32.6	33.3	34.1	34.7	36.2	36.8	37	
	105	210	22	22.3	23	23.5	24.4	25.1	25.8	26.7	27.4	28.5	29.6	30.7	31.8	32.2	32.5	33.2	34.-	34.6	36.1	36.8	37	

HYDRAULIC ANALOGUE COMPUTER
OBSERVATION SHEET

OBSERVED BY: RN TEST NO. 9

DATE: July 20, 1955 SHEET 2 OF 2

PROBLEM: 4-layer case, Wet Subgrade

DATE AND HOUR	ELAPSED TIME 1 MIN = 2 DAYS ANALOGUE (MIN)	PROTOTYPE (DAYS)	STANDPIPES: TEMPERATURE SCALE 0.2 INCH = 1° F 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	REMARKS
	110	220	22	22.5	23.-	23.5	24.3	25	25.7	26.5	27.2	28.2	29.4	30.5	31.6	32.-	32.4	33.1	34	34.6	36	36.8	36.9	27.5°C
	115	230	22	22.3	23	23.4	24.3	25	25.7	26.4	27.1	28.1	29.2	30.2	31.4	31.8	32.1	33	33.8	34.5	36	36.7	36.9	
	120	240	22	22.4	22.9	23.4	24.1	24.7	25.5	26.1	26.8	27.9	28.9	30.1	31.2	31.6	32	32.8	33.7	34.4	35.9	36.7	36.9	
	125	250	22	22.3	22.8	23.3	24	24.6	25.3	26	26.6	27.5	28.2	29	29.5	30.5	31.2	32.6	33.6	34.3	35.8	36.7	36.9	

HYDRAULIC ANALOGUE COMPUTER

OBSERVATION SHEET

OBSERVED BY: RN

TEST NO. 12

DATE: July 21, 1955

SHEET 1 OF 2

PROBLEM: 4-layer case, Wet Subgrade
Sinusoidal Variation in Surface Temperature

DATE AND HOUR	ELAPSED TIME 1 MIN = 2 DAYS: ANALOGUE (MIN)	PROTOTYPE (DAYS)	STANDPIPES: TEMPERATURE SCALE 0.2 INCH = 1° F: 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	REMARKS
			Surf.	0.13	0.38	0.63	0.97	1.42	1.86	2.31	2.96	3.82	4.67	5.61	6.63	7.92	9.50		16.94	23.62	36.12	59.72	101.42	← Depth in Ft.
12.49	0		43.5	43.5	43.5	43.3	43.2	43.1	43	43	42.7	42.5	42.3	42.3	42.1	41.8	41.4	40.3	39.1	37.9	37.2	37	37	27.7°C (Test Temp)
	10	0	37	37.2	37.6	38	38.4	38.9	39.3	39.5	39.7	40.2	40.4	40.6	40.8	40.8	40.7	40.2	39.3	38.3	37.2	37	37	
					18:55	20:25	22:20	25:00	28:40	31:50	35:20	42:25	51:40											
					21:00	23:00	26:20	29:20	32:30	35:50	43:20	57:00												
	20	20	29.6	29.7	31.2	32	32.6	33.3	33.9	34.5	34.9	35.7	36.4	37	37.5	38.2	38.6	39.2	39	38.5	37.3	37	37	
	30	40	24	24.5	26	27.1	28.7	30.4	32	32.5	32.8	33.5	34.1	34.6	35.1	35.9	36.6	37.8	38.5	38.4	37.5	37	37	
	35	50	21.9																					
	40	60	19.5	20.1	21.8	22.8	24.5	26.1	28	29.7	31.7	32.2	32.7	33.2	33.7	34.5	35.1	36.6	37.7	38.1	37.5	37	37	
	45	70	18.2																					
	50	80	17.2	18.-	19.4	20.5	22.3	24	25.6	27.3	29.2	31.7	32.2	32.6	33.1	33.7	34.3	35.6	36.9	37.5	37.5	37	37	28°C
	55	90	16.6																					
	60	100	16.7	17.4	18.7	19.6	21.4	22.9	24.4	25.9	27.7	29.9	32.2	32.5	32.8	33.4	33.9	35.1	36.3	37.1	37.5	37.1	37	
														65:50	93:00*									*Slightly starting
													70:35											
	65	110	17.4																					28.1°C
	70	120	18.5	19.3	20.3	21.1	22.2	23.4	24.7	25.9	27.3	28.9	30.6	32.1	32.5	32.9	33.5	34.5	35.8	36.6	37.4	37	37	
	75	130	20.6																					
	80	140	23.1	22.7	24.2	24.5	25.1	26	26.7	27.4	28.3	29.4	30.7	32.1	32.4	32.8	33.2	34.3	35.4	36.2	37.2	37	37	
	85	150	26.2																					
	90	160	29	29.2	29.3	29.4	29.4	29.7	29.8	30	30.2	30.6	31.1	31.6	32.1	32.5	32.9	34	35.1	35.9	37.1	37	37	
					94:00	95:10	95:25	95:40	100:-	105:40	108:35	113:15	114:00											
					97:00	99:20	102:10	104:20	106:35	109:10	115:30													
	95	170	32																					

HYDRAULIC ANALOGUE COMPUTER
OBSERVATION SHEET

OBSERVED BY: *RN* TEST NO. *12*

DATE: *July 21, 1955* SHEET *2* OF *2*

PROBLEM: *4-layer case, Wet Subgrade*

STANDPIPES: TEMPERATURE SCALE *0.2* INCH = 1° F

DATE AND HOUR	ELAPSED TIME (1 MIN = 2 DAYS) ANALOGUE (MIN)	PROTOTYPE (DAYS)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	REMARKS
	100	180	35.3	35	34	33	31.8	31.4	31.2	31.2	31.1	31.1	31.3	31.7	32.1	32.5	32.8	33.8	34.8	35.6	37	37	37	
	105	190	39.5																					
	110	200	42.7	42.3	40.8	39.5	38.1	36.6	35	33.5	31.5	31.3	31.4	31.7	32.1	32.4	32.8	33.6	34.6	35.3	36.8	37	37	28.3 °C
	115	210	46																					
	120	220	49	48.5	46.6	45.6	43.5	41.5	39.5	37.5	35.1	32.2	31.6	31.8	32.2	32.5	32.8	33.6	34.4	35.1	36.7	37	37	
												123:35	130:30	138:00	138:00*									* well No. 12 drained
	125	230	51.7																					out partly into well
	130	240	54	53.1	51.2	49.7	47.8	45.7	43.6	41.5	39	36.3	34.2	32.1	32.3	32.5	32.8	33.5	34.3	35	36.5	37	37	No. 11.
	135	250	55.5																					
	140	260	56.5	55.8	54.2	52.8	51.2	49.5	47.8	46.3	44.4	42.2	40.5	38.7	37.2	35.6	34.5	33.7	34.2	34.9	36.4	37	37	
	145	270	57																					
	150	280	57.2	56.7	55.5	54.5	53.5	52.4	51.2	50	48.6	47.6	45.8	44.5	43.3	41.7	40.2	37	35.4	35	36.2	37	37	
	155	290	56.5																					
	160	300	55	54.6	54	53.6	53.2	52.5	51.8	51.2	50.3	49.2	48.3	47.3	46.4	44.8	43.3	40	37.2	35.9	36.2	37	37	
	165	310	54.2																					
	170	320	52.1	52	51.4	51.2	51.1	50.7	50.3	50	49.5	48.8	48.2	47.6	46.8	45.7	44.5	41.5	38.8	36.9	36.2	36.9	37	
	175	330	48.7																					
	180	340	45.1	45.4	45	45.4	45.6	45.8	45.9	45.9	45.9	45.9	45.8	46.6	45.4	44.8	44.2	42.2	39.8	38	36.5	37	37	

Maximum: 70% of well No. 11 drained
well No. 12 just slightly started.