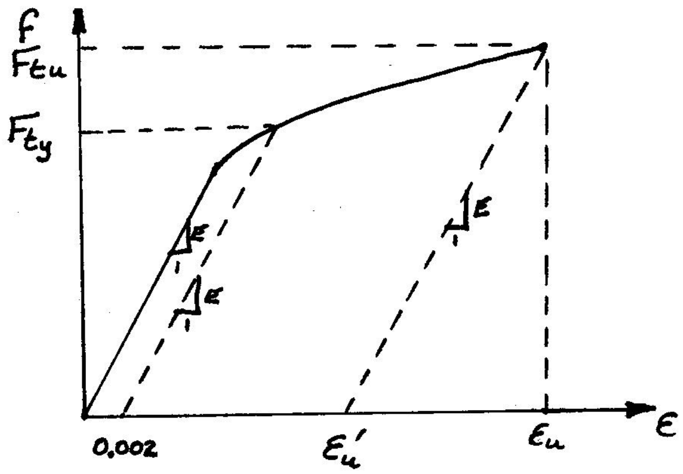
COZZONE STRESSES



Using the Ramberg-Osgood (Hill) stress-strain formulation

$$\varepsilon_u' = \varepsilon_u - F_{tu}/E \qquad \text{—①}$$

$$n = \log_e\left(\frac{\varepsilon_u'}{0.002}\right) \Big/ \log_e\left(\frac{F_{tu}}{F_{ty}}\right) \qquad \text{—②}$$

$$\varepsilon_m = \frac{f_m}{E} + \varepsilon_u'\left(\frac{f_m}{F_{tu}}\right)^n \qquad \text{—③}$$

For a rectangular section, $(b \times h)$, the Cozzone bending moment is:-

$$M_{Coz} = \frac{bh^2}{12}\left(2f_m + f_o\right) \qquad \text{—④}$$

By integration of ③ over the section, yields the plastic bending moment:-

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$$M_{max} = \frac{b h^2 f_m}{2 \, \varepsilon_m^2} \left\{ \frac{1}{3} \cdot \left(\frac{f_m}{E}\right)^2 + \varepsilon_u' \cdot \left(\frac{n+1}{n+2}\right)\left(\frac{f_m}{F_{tu}}\right)^n \left(\frac{f_m}{E}\right) + \left(\varepsilon_u'\right)^2 \cdot \left(\frac{n}{2n+1}\right) \cdot \left(\frac{f_m}{F_{tu}}\right)^{2n} \right\} \quad \text{—⑤}$$

Setting ④ = ⑤ and rearranging yields:-

$$\frac{f_o}{f_m} = \left(\frac{6}{\varepsilon_m^2}\right) \cdot \left\{ \frac{1}{3}\left(\frac{f_m}{E}\right)^2 + \varepsilon_u' \cdot \left(\frac{n+1}{n+2}\right)\left(\frac{f_m}{F_{tu}}\right)^n \cdot \left(\frac{f_m}{E}\right) + \left(\varepsilon_u'\right)^2 \cdot \left(\frac{n}{2n+1}\right) \cdot \left(\frac{f_m}{F_{tu}}\right)^{2n} \right\} - 2$$

Thus from the material parameters

- F_{tu}
- F_{ty}
- E
- ε_u

the Cozzone stresses f_o and f_m can be determined.

The key is the sentence from Peery:-

"Cozzone has shown that it is sufficiently accurate to calculate f_o for a rectangular cross section and to use this value for <u>all</u> cross sections"

Using the above stress-strain formulation, we get:-

Tangent Modulus:-

$$E_t = \frac{1}{\left[1/E + \left(\frac{n}{F_{tu}}\right) \cdot \varepsilon_u' \cdot \left(\frac{f}{F_{tu}}\right)^{n-1} \right]} \quad Z⑥$$

Secant Modulus:-

$$E_s = \frac{1}{\left[1/E + \left(\frac{\varepsilon_u'}{F_{tu}}\right) \cdot \left(\frac{f}{F_{tu}}\right)^{n-1} \right]} \quad Z⑦$$

For compression buckling and shear buckling in the plastic range, the buckling formula is written as follows:-

$$f_{cr} = k \cdot \eta_p \cdot E \cdot \left(t/b\right)^2$$

Where:

For compression buckling, $\eta_p = \eta_c = \left(\frac{E_t}{E}\right)^{1/2}$

and,

For shear buckling, $\eta_p = \eta_s = \left(\frac{G_t}{G}\right)^{1/2}$

where :-

$$G = \frac{E}{2(1+\nu)}$$

$$k = k_c \text{ or } k_s \cdot \frac{\pi^2}{12(1-\nu^2)}$$

Alternate stress-strain curve formulation.

Using: $\varepsilon_y' = \varepsilon_y - \dfrac{F_{ty}}{E} = 0.002$ (by def.)

n = same as Equ. 2

then: $$\varepsilon = \frac{f}{E} + \varepsilon_y' \left(\frac{f}{F_{ty}}\right)^n$$

Note: It is common in compression analysis to substitute F_{cy} for F_{ty}.

Thus we can write the η_c and η_s terms as follows:-

$$\eta_c = \left[\frac{1}{1 + \left(\frac{0.002\,E\,n}{F_{cy}}\right)\left(\frac{f_{cr}}{F_{cy}}\right)^{n-1}}\right]^{1/2} \quad \triangledown 1$$

$$\eta_s = \left[\frac{1}{1 + \left(\frac{0.002 \cdot G \cdot n}{F_{sy}}\right) \cdot \left(\frac{f_{scr}}{F_{sy}}\right)^{n-1}}\right]^{1/2}$$

where $F_{sy} \doteq 0.55\,F_{cy}$

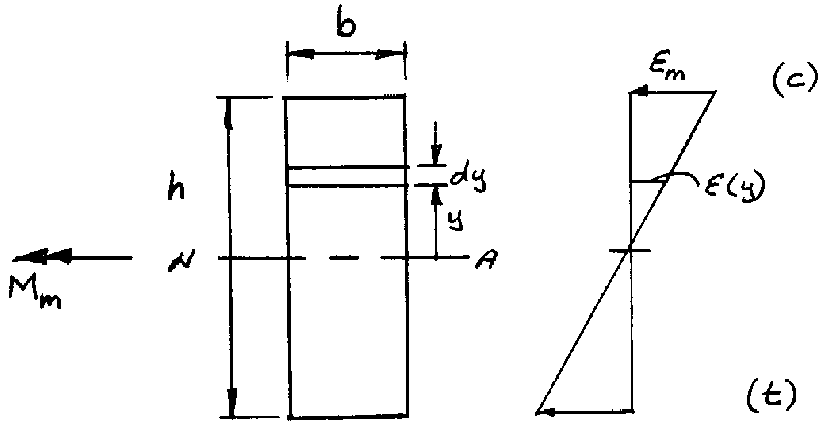
$\triangledown 1$ Ref. Michael CY Niu, "Airframe Stress Analysis and Sizing".

Much data exists for compression failure in terms of the $f_{0,7}$ stress where the 70% secant modulus cuts the stress-strain curve.

Using Equ. (7) and rearranging, we can write

$$f_{0,7} = F_{cy}\left[214.286 \cdot \frac{F_{cy}}{E_c}\right]^{\frac{1}{n-1}}$$

EQUIVALENT COZZONE STRESS FOR A RECTANGULAR BENDING SECTION



Beam Section Strain Distribution

From the Bernoulli-Euler theory of bending, also known as the Engineers' Theory of Bending (ETB) in which the assumption is made that plane sections remain plane, the strain distribution is linear. [1]

The maximum strain, at the outer fibres of the beam, is established by the maximum allowable compressive stress. The compressive stress is limited by the instability or collapse mode of the compression member located at the extreme fibre of the beam. This maximum compressive stress, and associated strain, is denoted as f_m and ε_m respectively.

Thus at a distance y from the neutral axis (NA)

$$\varepsilon(y) = \frac{\varepsilon_m}{h/2} \cdot y$$

[1] This is apparently also referred to as the Bernoulli-Hypothesis, in some academic circles. (Ref. RM Rivello)

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or
$$y = \frac{h/2}{\mathcal{E}_m} \cdot \mathcal{E}(y)$$
$$= \left(\frac{h}{2\mathcal{E}_m}\right) \cdot \left[\frac{f}{E} + \mathcal{E}_y'\left(\frac{f}{F_{cy}}\right)^n\right] \quad 2 \; ⑧$$

Differentiating w.r.t. "f", yields:-
$$dy = \left(\frac{h}{2\mathcal{E}_m}\right)\left[\frac{1}{E} + \frac{n\,\mathcal{E}_y'}{F_{cy}^{\,n}} f^{n-1}\right] df \quad 2 \; ⑨$$

Determining the moment carried by 1/2 of the section, yields :-
$$\tfrac{1}{2} M_m = b \int_0^{h/2} f \cdot y \cdot dy$$

Substituting Equns ⑧ and ⑨ and changing the integration bounds to suit, yields:-
$$\tfrac{1}{2} M_m = b \left(\frac{h}{2\mathcal{E}_m}\right)^2 \int_0^{f_m} \left[\frac{f}{E} + \mathcal{E}_y'\left(\frac{f}{F_{cy}}\right)^n\right] \cdot \left[\frac{f}{E} + n\,\mathcal{E}_y'\left(\frac{f}{F_{cy}}\right)^n\right] df$$

Multiplying the brackets and gathering, yields:-
$$= b\left(\frac{h}{2\mathcal{E}_m}\right)^2 \int_0^{f_m} \left[\left(\frac{f}{E}\right)^2 + n\,\mathcal{E}_y'^{2}\left(\frac{f}{F_{cy}}\right)^{2n} + (n+1)\mathcal{E}_y'\left(\frac{f}{E}\right)\left(\frac{f}{F_{cy}}\right)^n\right] df$$

Integrating w.r.t. "f", yields:-

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$$\tfrac{1}{2} M_m = b\left(\frac{h}{2\varepsilon_m}\right)^2 \left[\frac{1}{3}\frac{f^3}{E^2} + \frac{n\,{\varepsilon_y'}^2}{(2n+1)F_{cy}^{2n}} f^{2n+1} + \varepsilon_y' \frac{n+1}{n+2}\frac{f^{n+2}}{E F_{cy}^{n}}\right]_0^{f_m}$$

Substituting the integration bounds into the function yields:-

$$M_m = \frac{bh^2}{2}\frac{f_m}{\varepsilon_m^2}\left\{\frac{1}{3}\left(\frac{f_m}{E}\right)^2 + (\varepsilon_y')^2\frac{n}{(2n+1)}\left(\frac{f_m}{F_{cy}}\right)^{2n} + \varepsilon_y'\left(\frac{n+1}{n+2}\right)\left(\frac{f_m}{F_{cy}}\right)^n\left(\frac{f_m}{E}\right)\right\}.$$

which is identical to Eqn. (5), but using the function terms ε_y' and F_{cy}.

In accordance with the derivation on page **1-COZ**, the $\sigma_{0.7}$ can also be written as follows in terms of the ultimate stress.

$$\sigma_{0.7} = [(0.4286\cdot\sigma_{tu}^{\ n})/(E_c\cdot\varepsilon_u')]^{(1/(n-1))}$$

Additional Observations

On inserted page 4-COZ above, a statement is made that the material shape power coefficient "n" can be calculated as per the original "full" definition of the stress-strain curve defined on page 1-COZ. From further reading of an aircraft manual from a firm that shall remain nameless, but for whom Prof Niu was a researcher, it has become evident that they pay particular attention to the "knee" of the $\sigma - \varepsilon$ curve and in fact define the inelastic portion of the curve by means of two zones. The first zone extends from their definition of the proportional limit, 0.0001 to 0.002 permanent set and the second zone extends from 0.002 to ε'_u. The usual definition of the proportional limit is 0.001 permanent set strain. It is over the first inelastic zone that the "n" for the formula given on page 4-COZ is intended. Using the overall or "full" definition of "n" does provide the correct inelastic correction outside the permanent set strain range of 0.0001 to 0.002.

This refined usage of the $\sigma-\varepsilon$ curve is extended into the manner in which they calculate the material Cozzone stresses. They also correct for sections that are not rectangular, as asserted by the original Cozzone assumptions.

As a caveat to what the unnamed Big Aircraft Company is doing, it must be noted that many, many other very successful aircraft companies use the method as originally postulated by Cozzone (and presented herein), and is considered to be valid and accurate. This author has also used the method to successfully design and analyse bending components that have been tested to destruction. The method has also been used by the author to "calibrate" aluminium beam section work published on the Cornell University (USA) website. This work was done by the Civil Engineering (Built Environment) Faculty. Their theoretical analysis was performed using inelastic FE Analysis and calibrated against collapse failure laboratory tests. There are additional notes elsewhere in these notes on the analysis of I-, C-, and Z-section flanged beams

Another fact that comes to mind here is the question of material strength properties and allowables. The material strength data used by companies, and in the Mil-Handbooks and more recently the MMPDS volumes, are statistically guaranteed values for the alloy and/or from the approved company supplier. When **laboratory tests** are being done, specific **specimen material properties are required** if **accurate** correlation between analysis and test is required.

When only the f_o Cozzone stress is required at rupture, then a slightly reduced version of the formula on p 2-COZ can be written as follows: -

$$f_{ou}/\sigma_{tu} = (6/\varepsilon_u^2)\cdot\{1/3\cdot(\sigma_{tu}/E)^2 + \varepsilon_u'\cdot((n+1)/(n+2))\cdot(\sigma_{tu}/E) + (n/(2\cdot n+1))\cdot(\varepsilon_u')^2\} - 2$$

10.1.1 Inelastic Buckling Correction

Up to now inelastic buckling calculations were determined in most American textbooks with the use of dimensionless $F_{cr}/F_{0,7}$ curves. The introduction of the spreadsheet as an engineering tool has made it possible to directly use the 4 key strength parameters of a material to solve for inelastic buckling stress.

The Hill stress-strain formulation from the 4 parameters σ_{tu}, σ_{ty}, ε_u, E_c yields the material exponent "**n**" and the ultimate material "set" value of ε'_u .

In general, inelastic buckling formulas of all types have the following form: -

$$\sigma_{cr}/\eta_c \quad = \quad k_c \cdot E_c \cdot (t/b)^2 \text{ ------------------(A)}$$

For the given geometry of the panel the RHS of (A) is a constant, which we will call D_2. So: -

$$D_2 \quad = \quad k_c \cdot E_c \cdot (t/b)^2$$

The η_c term in turn is a function of the material inelastic parameters and σ_{cr} itself, so an iterative process is required to calculate σ_{cr} that matches the buckling equation (A).

In many of the classical textbooks on airframe analysis the value of η_c is given as: -

$$\eta_c \quad = \quad E_t/E_c \text{ ------------------------------(B)}$$

or,

$$\eta_c \quad = \quad E_s/E_c \text{ ------------------------------(B')}$$

While in others it is given as: -

$$\eta_c \quad = \quad [E_t/E_c]^{1/2} \text{ ---------------------------(C)}$$

And more confusingly, some company manuals have fine-tuned these correction functions to even more complex functions based on the stress-strain curve.

For the purposes of these notes and for illustration only, the inelastic correction (B) will be used: -

Thus: -

$$E_t/E_c \quad = \quad \{1/[1+(\varepsilon'_u\cdot E_c\cdot n/\sigma_{cu})\cdot(\sigma_{cr}/\sigma_{cu})^{n-1}]\}$$

Substituting into Equ. (A) yields: -

$$\sigma_{cr} \cdot [1 + (\varepsilon'_u\cdot E_c\cdot n/\sigma_{cu})\cdot(\sigma_{cr}/\sigma_{cu})^{n-1}] \quad = \quad D_2$$

Thus: -

$$\sigma_{cr} + [(\varepsilon'_u\cdot E_c\cdot n/\sigma_{cu})\cdot(1/\sigma_{cu})^{n-1}]\cdot\sigma_{cr}^{n} \quad = \quad D_2$$

The terms in the [ ] brackets make up a constant that will be called D_1, as follows: -