
Title: **Coulomb Friction in User Subroutine *FRIC* Using the Penalty Method**

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Introduction

This technical note illustrates how to implement Coulomb friction in user subroutine **FRIC** using the penalty method. The two-dimensional problem is presented first. Once the equations are understood, it is straightforward to extend the two-dimensional formulation to a three-dimensional formulation, i.e., sliding is permitted in two directions.

The input files and example user subroutines have been updated and checked for ABAQUS Version 6.2.

Analysis of the Model

The ideal frictional behavior between two contacting bodies is shown in Figure 1.

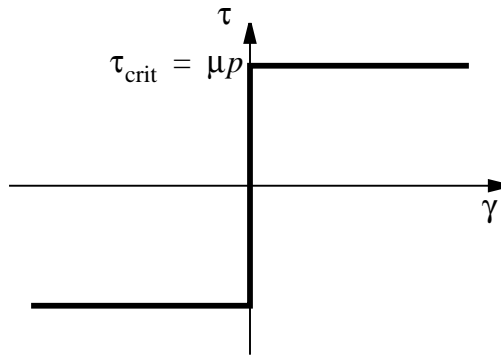


Figure 1. Ideal Frictional Behavior Between Two Contacting Bodies

Figure 1 shows that if the frictional shear stress $|\tau| < \tau_{\text{crit}}$, no relative motion occurs. In the penalty formulation of Coulomb friction, we approximate the condition of no relative sliding motion with a “stiff elastic behavior” as shown in Figure 2. Therefore, a small amount of elastic slip $|\gamma^{el}| \leq \gamma_{\text{crit}}$ is permitted although there should be no slipping at all. The elastic slip is therefore related to the frictional shear stress through the relation

$$\tau = k_s \gamma^{el}, \quad (\text{Eq. 1})$$

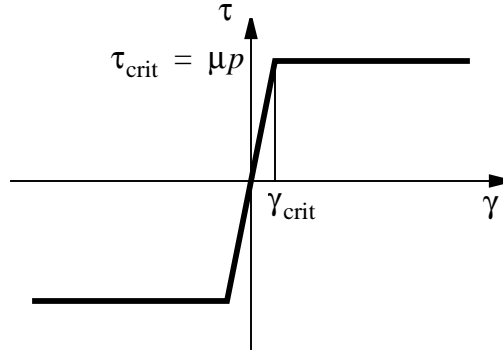


Figure 2. “Stiff Elastic Behavior”

where

$$k_s(p) = \frac{\tau_{\text{crit}}}{\gamma_{\text{crit}}} = \frac{\mu p}{\gamma_{\text{crit}}} . \quad (\text{Eq. 2})$$

The behavior remains elastic as long as $|\tau| < \tau_{\text{crit}}$. Linearization of Equation 1 yields

$$d\tau = k_s d\gamma^{el} + \frac{\gamma^{el}}{\gamma_{\text{crit}}} \mu dp . \quad (\text{Eq. 3})$$

The above expressions hold as long as $|\tau| \leq \tau_{\text{crit}}$. If $|\tau| > \tau_{\text{crit}}$, then the frictional shear stress is given by

$$\tau = \frac{\gamma}{|\gamma|} \tau_{\text{crit}} = \frac{\gamma}{|\gamma|} \mu p , \quad (\text{Eq. 4})$$

where γ is the total slip (the coefficient $\gamma/|\gamma|$ in front of τ_{crit} is used to determine the direction of the frictional stress). The elastic slip (see Figure 3) is then given by

$$\gamma^{el} = \frac{\tau}{k_s} , \quad (\text{Eq. 5})$$

where τ is given by Equation 4. The incremental “plastic” slip which we will denote by $\Delta\gamma^{sl}$ is then calculated as follows. From Figure 3 it can be seen that the incremental “plastic” slip is given by

$$\Delta\gamma^{sl} = \bar{\gamma}^{el} + \Delta\gamma - \frac{\tau}{k_s} , \quad (\text{Eq. 6})$$

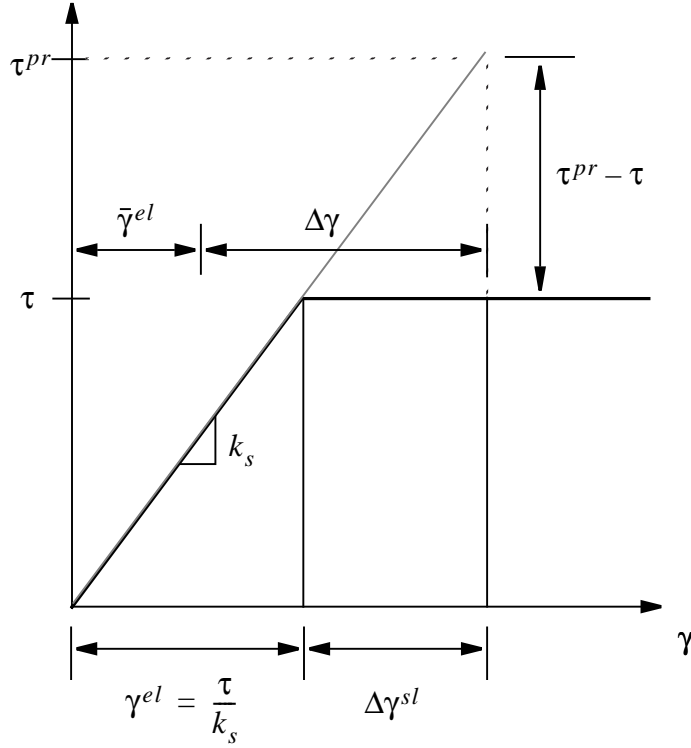


Figure 3. Computation of Incremental “Plastic” Slip

where τ is given by Equation 4, $\bar{\gamma}^{el}$ is the elastic slip at the start of the iteration, and $\Delta\gamma$ is the current slip increment passed to user subroutine **FRIC**. By defining the elastic predictor to be

$$\tau^{pr} = k_s(\bar{\gamma}^{el} + \Delta\gamma), \quad (\text{Eq. 7})$$

Equation 6 can be written as

$$\Delta\gamma^{sl} = \frac{\tau^{pr} - \tau}{k_s}. \quad (\text{Eq. 8})$$

The above equations are depicted graphically in Figure 3.

Linearization of Equation 4 yields

$$d\tau = \frac{\gamma}{|\gamma|} \mu dp. \quad (\text{Eq. 9})$$

The above equations for the two-dimensional problem are implemented in the user subroutine **FRIC** shown in Listing 1.

For the three-dimensional problem, the equations developed so far should be modified due to the existence of another slip direction. We will assume for simplicity that the frictional response is the same in both directions, i.e. isotropic friction. For three-dimensional, isotropic, Coulomb friction the equation corresponding to Equation 1 is

$$\tau_i = k_s \gamma_i^{el}, \quad \text{for } i = 1, 2, \quad (\text{Eq. 10})$$

where k_s is given by Equation 2. Linearization of the above equation yields

$$d\tau_i = k_s d\gamma_i^{el} + \frac{\gamma_i^{el}}{\gamma_{crit}} \mu dp, \quad \text{for } i = 1, 2 \quad (\text{Eq. 11})$$

Again, the above expressions holds if $\tau_{eq} \leq \tau_{crit}$, where

$$\tau_{eq} = \sqrt{\tau_1^2 + \tau_2^2}. \quad (\text{Eq. 12})$$

If $\tau_{eq} > \tau_{crit}$, then the frictional shear stresses are given by

$$\tau_i = \frac{\gamma_i}{\gamma_{eq}} \tau_{crit} = \frac{\gamma_i}{\gamma_{eq}} \mu p \quad (\text{Eq. 13})$$

where

$$\gamma_{eq} = \sqrt{\gamma_1^2 + \gamma_2^2} \quad (\text{Eq. 14})$$

and γ_i are the components of the total slip. The equivalent incremental “plastic” slip, $\Delta\gamma_{eq}^{sl}$, can then be obtained as shown in Figure 4. From Figure 4, the equivalent incremental “plastic” slip is obtained by subtracting γ_{crit} from the equivalent total slip, i.e.,

$$\Delta\gamma_{eq}^{sl} = \gamma_{eq} - \gamma_{crit}. \quad (\text{Eq. 15})$$

By defining the elastic predictor to be

$$\tau_{eq}^{pr} = k_s \gamma_{eq}, \quad (\text{Eq. 16})$$

Equation 15 can be written as

$$\Delta\gamma_{eq}^{sl} = \frac{\tau_{eq}^{pr} - \tau}{k_s}. \quad (\text{Eq. 17})$$

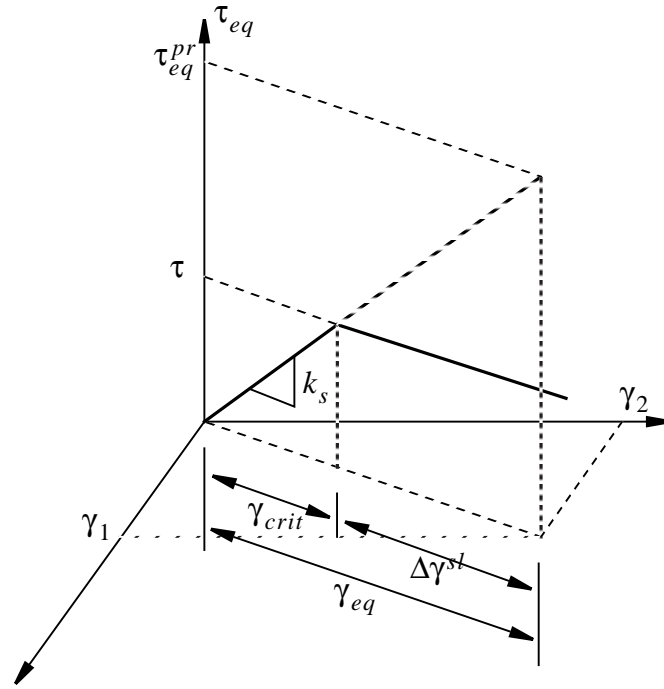


Figure 4. Computation of Equivalent Incremental “Plastic” Slip

The components of the incremental “plastic” slip are computed by assuming that the direction of the incremental “plastic” slip is equal to the direction of the total slip, i.e.,

$$\Delta\gamma_i^{sl} = \frac{\gamma_i}{\gamma_{eq}} \Delta\gamma_{eq}^{sl}, \quad \text{for } i = 1, 2. \quad (\text{Eq. 18})$$

Linearization of the above equation yields

$$d\tau_i = \frac{\mu p}{\gamma_{eq}} \left(\delta_{ij} - \frac{\gamma_i}{\gamma_{eq}} \frac{\gamma_j}{\gamma_{eq}} \right) d\gamma_j + \frac{\gamma_i}{\gamma_{eq}} \mu dp. \quad (\text{Eq. 19})$$

The above equations are implemented for three-dimensional Coulomb friction using the Penalty method as shown in Listing 2.

Listing 1. Implementation of Two-Dimensional Coulomb Friction Using Penalty Method.

```

*HEADING
TECHNOTE_FRICTION_3: IMPLEMENTATION OF 2-D COULOMB FRICTION
USING PENALTY METHOD
*RESTART,WRITE
*NODE
1,0.,0.
2,0.,1.0
100,0.,0.
*ELEMENT,TYPE=B21,ELSET=BEAM
1,1,2
*BEAM SECTION,SECT=CIRC,ELSET=BEAM,MATERIAL=ELAS
0.1,
*MATERIAL,NAME=ELAS
*ELASTIC
30.E6,0.3
*surface,type=node, NAME=CNS
1,
*rigid body,analytical surface=RIGS,REF NODE=100
*surface,TYPE=SEGMENTS,NAME=RIGS
START,-1.0,0.
LINE,200.,0.
*CONTACT PAIR, INTERACTION=INT1
CNS,RIGS
*SURFACE INTERACTION, NAME=INT1
1.0,
*FRICTION,USER,DEPVAR=1,PROPERTIES=1
0.5,
*****
*STEP,NLGEOM,UNSYMM=YES
ESTABLISH CONTACT
*STATIC
1.,1.
*EL PRINT
S,E
*EL FILE,F=100
S,E
*CONTACT PRINT
CSTRESS,CDISP
*CONTACT FILE,F=100
CSTRESS,CDISP
*BOUNDARY
100,1,2,0.0
100,6,6,0.0
2,1,1,0.0
2,2,2,-1.5E-4
2,6,6,0.0
*PRINT,CONTACT=YES
*END STEP
*****
*STEP,NLGEOM
X SLIDE
*STATIC
.1,1.

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```

*BOUNDARY,OP=NEW
100,1,2,0.0
100,6,6,0.0
2,1,1,4.0
2,2,2,-1.5E-4
2,6,6,0.0
*PRINT,CONTACT=YES
*END STEP
*****
*STEP,NLGEOM
X SLIDE (2)
*STATIC
.1,1.
*BOUNDARY,OP=NEW
100,1,2,0.0
100,6,6,0.0
2,1,1,0.0
2,2,2,-1.5E-4
2,6,6,0.0
*PRINT,CONTACT=YES
*END STEP
C
C      PUT SUBROUTINE FRIC INTO SEPARATE FORTRAN FILE
C
      SUBROUTINE FRIC(LM,TAU,DDTDDG,DDTDDP,DSLIP,SED,SFD,
1      DDTDDT,PNEWDT,STATEV,DGAM,TAULM,PRESS,DPRESS,DDPDDH,
2      SLIP,KSTEP,KINC,TIME,DTIME,NOEL,CINAME,SLNAME,
3      MSNAME,NPT,NODE,NPATCH,COORDS,RCOORD,DROT,TEMP,
4      PREDEF,NFDIR,MCRD,NPRED,NSTATV,CHRLNGTH,PROPS,NPROPS)
C
      INCLUDE 'ABA_PARAM.INC'
C
      CHARACTER*80 CINAME,SLNAME,MSNAME
      DIMENSION TAU(NFDIR),DDTDDG(NFDIR,NFDIR),DDTDDP(NFDIR),
1      DSLIP(NFDIR),DDTDDT(NFDIR,2),STATEV(*),
2      DGAM(NFDIR),TAULM(NFDIR),SLIP(NFDIR),
3      COORDS(MCRD),RCOORD(MCRD),DROT(2,2),TEMP(2),
4      PREDEF(2,*),TIME(2),PROPS(NPROPS)
C
      PARAMETER(ZERO=0.0D0,PRECIS=1.D-14,XKS=1.D6)
C
C      IMPLEMENTATION OF COULOMB
C      FRICTION USING PENALTY METHOD
C
C      VARIABLES USED:
C          XMU      = COEFFICIENT OF FRICTION
C          GCRIT    = CRITICAL ELASTIC SLIP
C          STIFF     = ARTIFICIAL STIFFNESS
C          GAMMA     = TOTAL SLIP
C          STATEV(1) = ELASTIC SLIP
C          TAUCRIT   = CRITICAL FRICTIONAL STRESS
C
C
C          XMU      = PROPS(1)
          IF (LM .EQ. 2) THEN

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```

C
C          GAP IS OPENED AT START
C          OF THE CURRENT INCREMENT
C
C      IF (XMU .LE. PRECIS) RETURN
C      END IF
C
C      LM      = 0
C      GCRIT = 0.005*CHRLNGTH
C
C          CHECK IF PRESSURE IS NON-POSITIVE
C
C      IF (PRESS .LE. ZERO) THEN
C          STATEV(1) = ZERO
C          GAMMA      = DGAM(1)
C          IF (XMU .LE. PRECIS) THEN
C              DDTDDG(1,1) = ZERO
C              DDTDDP(1)   = XMU*GAMMA/GCRIT
C          ELSE
C              DDTDDG(1,1) = XKS
C              DDTDDP(1)   = ZERO
C          END IF
C          TAU(1)          = ZERO
C          DSLIP(1)        = ZERO
C          RETURN
C      ELSE
C
C          COMPUTE FOR CRITICAL STRESS
C          AND ARTIFICIAL STIFFNESS
C
C          TAUCRIT = XMU*PRESS
C          STIFF    = TAUCRIT/GCRIT
C      ENDIF
C
C          COMPUTE FOR THE TOTAL SLIP
C          AND FRICTIONAL SHEAR STRESS
C
C      GAMMA = STATEV(1) + DGAM(1)
C      TAU(1) = STIFF*GAMMA
C
C          CHECK IF THE FRICTIONAL STRESS
C          EXCEEDS THE CRITICAL STRESS
C
C      IF (ABS(TAU(1)) .LT. TAUCRIT) THEN
C
C          BEHAVIOR REMAINS ELASTIC
C
C          STATEV(1) = GAMMA
C          DDTDDG(1,1) = STIFF
C          DDTDDP(1)   = XMU*GAMMA/GCRIT
C          DSLIP(1)    = ZERO
C      ELSE
C
C          BEHAVIOR IS PLASTIC
C

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      TAU(1) = SIGN(TAUCRIT,GAMMA)
      STATEV(1) = TAU(1)/STIFF
      DDTDDG(1,1) = ZERO
      DDTDDP(1) = SIGN(XMU,GAMMA)
      DSLIP(1) = GAMMA - STATEV(1)
ENDIF
RETURN
END

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Listing 2. Implementation of Three-Dimensional Coulomb Friction Using Penalty Method.

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*HEADING
TECHNOTE_FRICTION_4: IMPLEMENTATION OF 3-D COULOMB FRICTION
USING PENALTY METHOD
*RESTART,WRITE
*NODE
1,0.,0.,0.
2,0.,0.,1.0
100,0.,0.,0.
*ELEMENT,TYPE=B31,ELSET=BEAM
1,1,2
*BEAM SECTION,SECT=CIRC,ELSET=BEAM,MATERIAL=ELAS
0.1,
0.,1.,0.
*MATERIAL,NAME=ELAS
*ELASTIC
30.E6,0.3
*surface,type=node, NAME=CNS
1,
*rigid body,analytical surface=RIGS,REF NODE=100
*surface,TYPE=CYLINDER,NAME=RIGS
-100.,-100.,0.,0.,-100.,0.
-100.,0.,0.
START,0.,0.
LINE,200.,0.
*CONTACT PAIR, INTERACTION=INT1
CNS,RIGS
*SURFACE INTERACTION, NAME=INT1
1.0,
*FRICTION,USER,DEPVAR=2,PROPERTIES=1
0.5,
*****
*STEP,NLGEOM,UNSYMM=YES
ESTABLISH CONTACT
*STATIC
1.,1.
*EL PRINT
S,E
*EL FILE,F=100
S,E
*CONTACT PRINT
CSTRESS,CDISP
*CONTACT FILE,F=100
CSTRESS,CDISP
*BOUNDARY
100,1,6,0.0
2,1,1,0.0
2,2,2,0.0
2,4,6,0.0
2,3,3,-1.5E-4
*PRINT,CONTACT=YES
*END STEP
*****
*STEP,NLGEOM

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SLIDE IN X DIRECTION
*STATIC
0.1,1.
*BOUNDARY,OP=NEW
100,1,6,0.0
2,1,1,3.0
2,2,2,0.0
2,4,6,0.0
2,3,3,-1.5E-4
*PRINT,CONTACT=YES
*END STEP
*****
*STEP,NLGEOM
SLIDE IN Y DIRECTION
*STATIC
.1,1.
*BOUNDARY,OP=NEW
100,1,6,0.0
2,1,1,3.0
2,2,2,3.0
2,4,6,0.0
2,3,3,-1.5E-4
*PRINT,CONTACT=YES
*END STEP
*****
*STEP,NLGEOM
SLIDE BACK TO THE ORIGIN
*STATIC
.1,1.
*BOUNDARY,OP=NEW
100,1,6,0.0
2,1,1,0.0
2,2,2,0.0
2,4,6,0.0
2,3,3,-1.5E-4
*PRINT,CONTACT=YES
*END STEP
C
C      PUT SUBROUTINE FRIC INTO SEPARATE FORTRAN FILE
C
      SUBROUTINE FRIC(LM,TAU,DDTDDG,DDTDDP,DSLIP,SED,SFD,
1      DDTDDT,PNEWDT,STATEV,DGAM,TAULM,PRESS,DPRESS,DDPDDH,
2      SLIP,KSTEP,KINC,TIME,DTIME,NOEL,CINAME,SLNAME,
3      MSNAME,NPT,NODE,NPATCH,COORDS,RCOORD,DROT,TEMP,
4      PREDEF,NFDIR,MCRD,NPRED,NSTATV,CHRLNGTH,PROPS,NPROPS)
C
      INCLUDE 'ABA_PARAM.INC'
C
      CHARACTER*80 CINAME,SLNAME,MSNAME
      DIMENSION TAU(NFDIR),DDTDDG(NFDIR,NFDIR),DDTDDP(NFDIR),
1      DSLIP(NFDIR),DDTDDT(NFDIR,2),STATEV(*),
2      DGAM(NFDIR),TAULM(NFDIR),SLIP(NFDIR),
3      COORDS(MCRD),RCOORD(MCRD),DROT(2,2),TEMP(2),
4      PREDEF(2,*),TIME(2),PROPS(NPROPS)
C

```

```

C      DIMENSION GAMMA(2)
C
C      PARAMETER(ZERO=0.0D0,ASMALL=1.0D-27,PRECIS=1.D-14,XKS=1.D6)
C
C      IMPLEMENTATION OF 3-D COULOMB FRICTION USING PENALTY METHOD
C
C      VARIABLES USED:
C      XMU = COEFFICIENT OF FRICTION
C      GCRIT = CRITICAL ELASTIC SLIP
C      STIFF = ARTIFICIAL STIFFNESS
C      GAMMA(1:2) = TOTAL SLIP
C      STATEV(1:2) = ELASTIC SLIP
C      TAUCRIT = CRITICAL FRICTIONAL STRESS
C
C
C      XMU      = PROPS(1)
C      IF (LM .EQ. 2) THEN
C
C          GAP IS OPENED AT START
C          OF THE CURRENT INCREMENT
C
C      IF (XMU .LE. PRECIS) RETURN
C      END IF
C      LM      = 0
C      GCRIT = 0.005*CHRLNGTH
C
C          CHECK IF PRESSURE IS NON-POSITIVE
C
C      IF (PRESS .LE. ZERO) THEN
C          STATEV(1) = ZERO
C          STATEV(2) = ZERO
C          GAMMA(1)  = DGAM(1)
C          GAMMA(2)  = DGAM(2)
C          IF (XMU .LE. PRECIS) THEN
C              DDTDDG(1,1) = ZERO
C              DDTDDG(1,2) = ZERO
C              DDTDDG(2,1) = ZERO
C              DDTDDG(2,2) = ZERO
C              DDTDDP(1)   = XMU*GAMMA(1)/GCRIT
C              DDTDDP(2)   = XMU*GAMMA(2)/GCRIT
C          ELSE
C              DDTDDG(1,1) = XKS
C              DDTDDG(1,2) = ZERO
C              DDTDDG(2,1) = ZERO
C              DDTDDG(2,2) = XKS
C              DDTDDP(1)   = ZERO
C              DDTDDP(2)   = ZERO
C          END IF
C          TAU(1)          = ZERO
C          TAU(2)          = ZERO
C          DSLIP(1)        = ZERO
C          DSLIP(2)        = ZERO
C          RETURN
C      ELSE
C

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```

C          COMPUTE FOR CRITICAL STRESS
C          AND ARTIFICIAL STIFFNESS
C
      TAUCRIT = XMU*PRESS
      STIFF   = TAUCRIT/GCRIT
ENDIF

C          COMPUTE FOR THE TOTAL SLIP,
C          FRICTIONAL SHEAR STRESS,
C          AND THE EQUIVALENT SHEAR STRESS
C
      GAMMA(1) = STATEV(1) + DGAM(1)
      GAMMA(2) = STATEV(2) + DGAM(2)
      TAU(1)   = STIFF*GAMMA(1)
      TAU(2)   = STIFF*GAMMA(2)
      TAUEQV   = SQRT(TAU(1)**2 + TAU(2)**2)

C          CHECK IF THE FRICTIONAL STRESS
C          EXCEEDS THE CRITICAL STRESS
C
      IF (TAUEQV .LT. TAUCRIT) THEN

C          BEHAVIOR REMAINS ELASTIC
C
          STATEV(1) = GAMMA(1)
          STATEV(2) = GAMMA(2)
          DDTDDG(1,1) = STIFF
          DDTDDG(1,2) = ZERO
          DDTDDG(2,1) = ZERO
          DDTDDG(2,2) = STIFF
          DDTDDP(1)   = XMU*GAMMA(1)/GCRIT
          DDTDDP(2)   = XMU*GAMMA(2)/GCRIT
          DSLIP(1)    = ZERO
          DSLIP(2)    = ZERO
      ELSE

C          BEHAVIOR IS PLASTIC
C
          GAMEQV = SQRT(GAMMA(1)**2 + GAMMA(2)**2)
          TAU(1) = GAMMA(1)*TAUCRIT/GAMEQV
          TAU(2) = GAMMA(2)*TAUCRIT/GAMEQV
          DDTDDG(1,1) = TAUCRIT/GAMEQV*(1 - (GAMMA(1)/GAMEQV)**2)
          DDTDDG(1,2) = -TAUCRIT/GAMEQV*(GAMMA(1)/GAMEQV)*
&          (GAMMA(2)/GAMEQV)
          DDTDDG(2,1) = DDTDDG(1,2)
          DDTDDG(2,2) = TAUCRIT/GAMEQV*(1 - (GAMMA(2)/GAMEQV)**2)
          DDTDDP(1)   = XMU*GAMMA(1)/GAMEQV
          DDTDDP(2)   = XMU*GAMMA(2)/GAMEQV

C          COMPUTATION OF THE ELASTIC
C          AND PLASTIC SLIP
C
          STATEV(1) = GAMMA(1)*GCRIT/GAMEQV
          STATEV(2) = GAMMA(2)*GCRIT/GAMEQV
          DGSLEQ    = GAMEQV - GCRIT

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      DSLIP (1) = GAMMA (1) *DGSLEQ/GAMEQV  
      DSLIP (2) = GAMMA (2) *DGSLEQ/GAMEQV  
ENDIF  
RETURN  
END
```