9. LUG ANALYSIS

9.1 Introduction to Lug Analysis

Lugs are connector-type elements widely used as structural supports for pin connections. In the past, the lug strength was overdesigned since weight and size requirements were for the most part unrestricted. However, the refinement of these requirements have necessitated conservative methods of design.

This section presents static strength analysis procedures for uniformly loaded lugs and bushings, for double shear joints, and for single shear joints, subjected to axial, transverse, or oblique loading. Also listed is a section which applies to lugs made from materials having ultimate elongations of at least 5% in any direction in the plane of the lug. Modifications for lugs with less than 5% elongation are also presented. In addition, a short section on the stresses due to press fit bushings is presented.

9.2 Lug Analysis Nomenclature

F_{bru_L} = Lug ultimate bearing stress
F_{bry_L} = Lug yield bearing stress
F_{tux} = Cross grain tensile ultimate stress of lug material
F_{tyx} = Cross grain tensile yield stress of lug material
F_{bru} = Allowable ultimate bearing stress, MHB5
F_{bry} = Allowable yield bearing stress, MHB5
F_{tu} = Ultimate tensile stress
F_{nu_L} = Allowable lug net-section tensile ultimate stress
F_{ny_L} = Allowable lug net-section tensile yield stress
F_{bry_B} = Allowable bearing yield stress for bushings
F_{cy_B} = Bushing compressive yield stress
F_{bru_B} = Allowable bearing ultimate stress for bushings
F_{su_p} = Ultimate shear stress of the pin material
F_{tu_p} = Pin ultimate tensile stress
F_{tu_T} = Allowable ultimate tang stress
$F_{br_{max_L}}$ = Maximum lug bearing stress
$F_{br_{max_B}}$ = Maximum bushing bearing stress
$F_{s_{max_p}}$ = Maximum pin shear stress
$F_{b_{max_p}}$ = Maximum pin bending stress
P_{bru_L} = Allowable lug ultimate bearing load
P_{nu_L} = Allowable lug net-section ultimate load
P_{u_B} = Allowable bushing ultimate load
P_{u_L} = Allowable design ultimate load
$P_{u_{L_B}}$ = Allowable lug-bushing ultimate load
P_{us_p} = Pin ultimate shear load
P_{ub_p} = Pin ultimate bending load

P_{ub_p}	=	"Balanced design" pin ultimate bending load
P_{all}^{max}	=	Allowable joint ultimate load
P_T	=	Lug tang strength
P_{tru_L}	=	Allowable lug transverse ultimate load
P_{tru_B}	=	Allowable bushing transverse ultimate load
K_n	=	Net-tension stress coefficient
K_{b_p}	=	Plastic bending coefficient for pin
K_{b_T}	=	Plastic bending coefficient for tang
K_{br_L}	=	Plastic bearing coefficient for lug
K_{b_L}	=	Plastic bending coefficient for lug
K_{tru}	=	Transverse ultimate load coefficient
K_{try}	=	Transverse yield load coefficient
M_{max_p}	=	Maximum pin bending moment
M_{u_p}	=	Ultimate pin failing moment
A	=	Area, in.^{2}
a	=	Distance from edge of hole to edge of lug, inches
B	=	Ductility factor for lugs with less than 5% elongation
b	=	Effective bearing width, inches
D	=	Hole diameter of pin diameter, inches
E	=	Modulus of elasticity, psi
e	=	Edge distance, inches
f	=	Stress, psi
f_a	=	Cyclic stress amplitude on net section of given lug, lbs/in.^{2}
f_m	=	Mean cyclic stress on net section of given lug, lbs/in.^{2}
f_{max}	=	Maximum cyclic stress on net section of given lug, lbs/in.^{2}
f_{min}	=	Minimum cyclic stress on net section of given lug, lbs/in.^{2}
g	=	Gap between lugs, inches
$h_1 .. h_4$	=	Edge distances in transversely loaded lug, inches
h_{av}	=	Effective edge distance in transversely loaded lug
K	=	Allowable stress (or load) coefficient
k_1, k_2, k_3	=	Fatigue parameters
M	=	Bending moment, in.-lbs.
N	=	Fatigue life, number of cycles
P	=	Load, lbs.
R	=	Stress ratio, f_{min}/f_{max}
t_B	=	Bushing wall thickness, inches
t	=	Lug thickness, inches
w	=	Lug width, inches
α	=	Angle of load to axial direction, degrees
ϵ	=	Strain, inches/inch
ρ	=	Density, lbs/in.^{3}

Subscripts

all	= Allowable	opt	= Optimum
ax	= Axial	p	= Pin
B	= Bushing	s	= Shear
b	= Bending	T	= Tang
br	= Bearing	t	= Tensile
c	= Compression	tr	= Transverse
L	= Lug	u	= Ultimate
max	= Maximum	x	= Cross grain
n	= Net tensile	y	= Yield
o	= Oblique	1, 2	= Female and male lugs

9.3 Lug and Bushing Strength Under Uniform Axial Load

Axially loaded lugs in tension must be checked for bearing strength and for net-section strength. The bearing strength of a lug loaded in tension, as shown in Figure 9-1, depends largely on the interaction between bearing, shear-out, and hoop-tension stresses in the part of the lug ahead of the pin. The net-section of the lug through the pin must be checked against net-tension failure. In addition, the lug and bushing must be checked to ensure that the deformations at design yield load are not excessive.

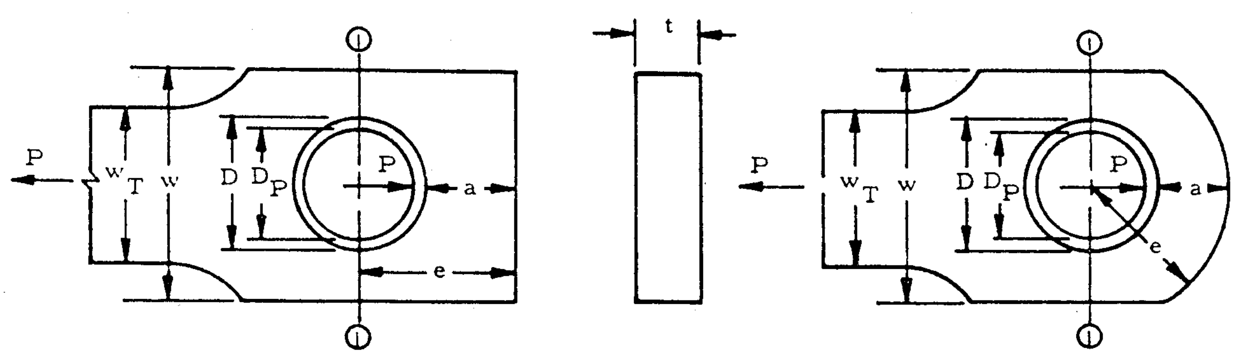


Figure 9-1. Schematics of Lugs Loaded in Tension

9.3.1 Lug Bearing Strength Under Uniform Axial Load

The bearing stresses and loads for lug failure involving bearing, shear-tearout, or hoop tension in the region forward of the net-section in Figure 9-1 are determined from the equations below, with an allowable load coefficient (K) determined from Figures 9-2 and 9-3. For values of e/D less than 1.5, lug failures are likely to involve shear-out or hoop-tension; and for values of e/D greater than 1.5, the bearing is likely to be critical. Actual lug failures may involve more than one failure mode, but such interaction effects are accounted for in the values of K. The lug ultimate bearing stress (F_{bru_L}) is

$$F_{bru_L} = K \frac{a}{D} F_{tux}, \quad (e/D < 1.5) \tag{9-1a}$$

$$F_{bru_L} = K F_{tux}, \quad (e/D \geqq 1.5) \tag{9-1b}$$

The graph in Figure 9-2 applies only to cases where D/t is 5 or less, which covers most of the cases. If D/t is greater than 5, there is a reduction in strength which can be approximated by the curves in Figure 9-3. The lug yield bearing stress (F_{bry_L}) is

$$F_{bry_L} = K \frac{a}{D} F_{tyx}, \quad (e/D < 1.5) \tag{9-2a}$$

$$F_{bry_L} = K F_{tyx}, \quad (e/D \geqq 1.5) \tag{9-2b}$$

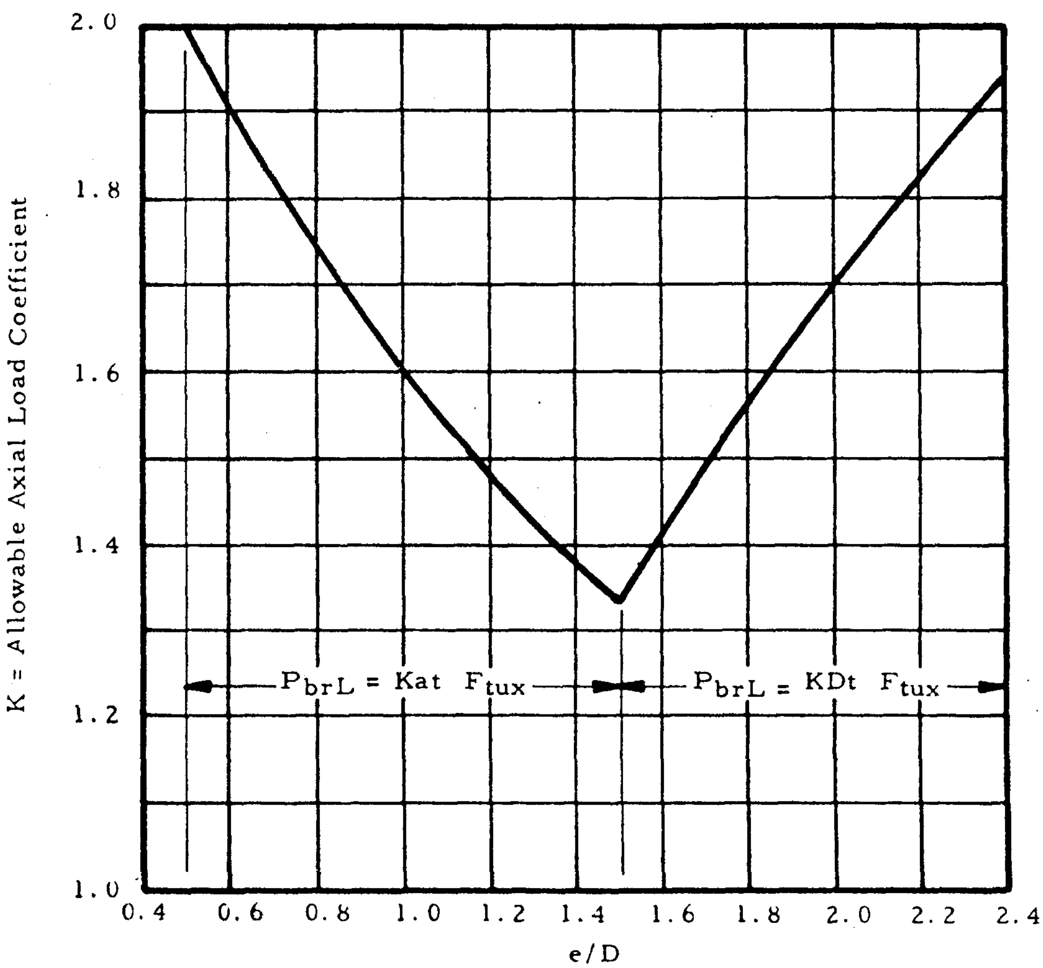


Figure 9-2. Allowable Uniform Axial Load Coefficient

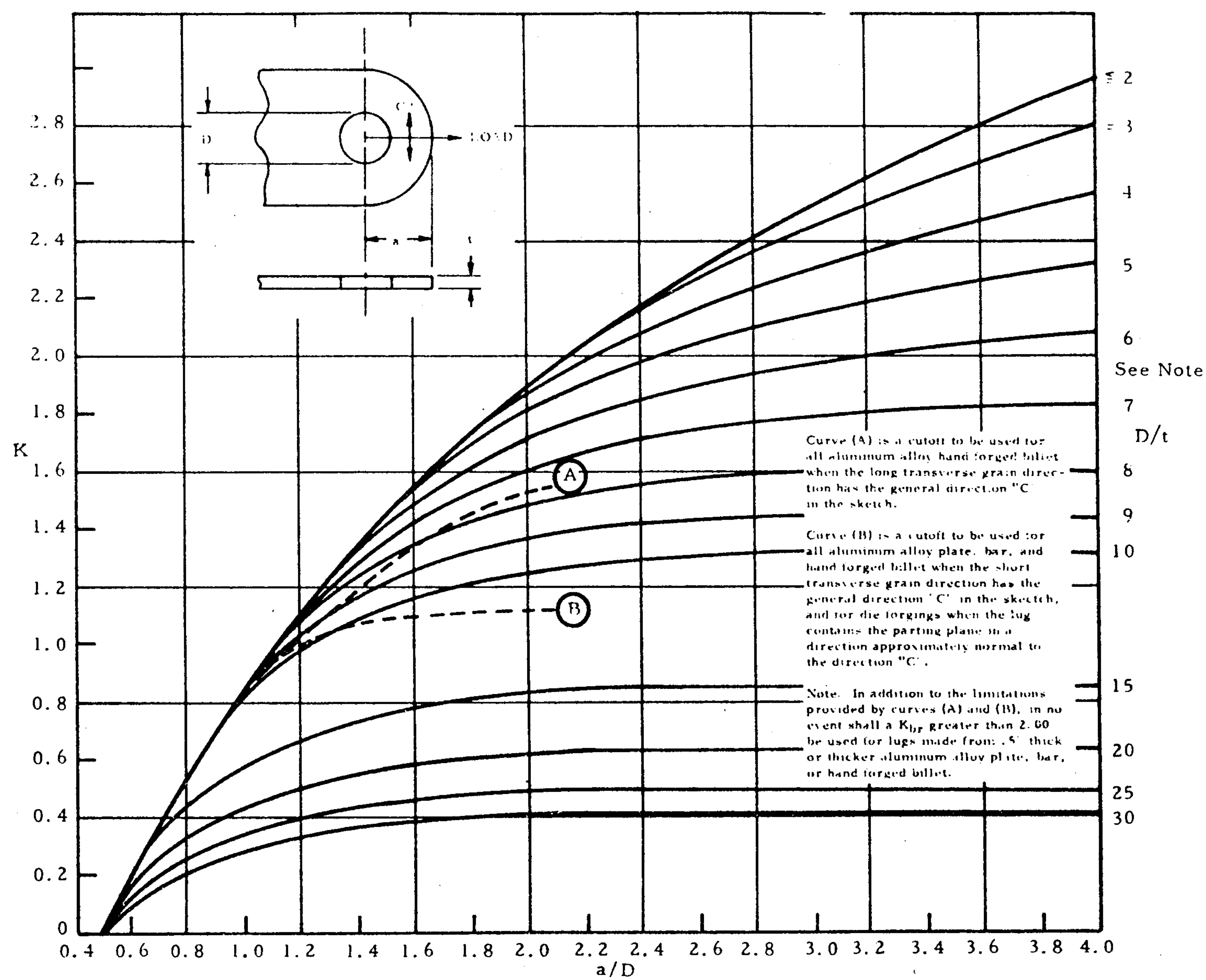


Figure 9-3. Bearing Efficiency Factors of Lugs, Aluminum Alloys, and Alloy Steel with F_{tu} < 160 KSI

The allowable lug ultimate bearing load (P_{bru_L}) for lug failure in bearing, shear-out, or hoop tension is

$$P_{bru_L} = F_{bru_L} Dt, \text{ (if } F_{tux} \leqq 1.304 F_{tyx}) \qquad (9\text{-}3a)$$

$$P_{bru_L} = 1.304 F_{bry_L} Dt, \text{ (if } F_{tux} > 1.304 F_{tyx}) \qquad (9\text{-}3b)$$

P_{bru_L}/Dt should not exceed either F_{bru} or $1.304 F_{bry}$, where F_{bru} and F_{bry} are the allowable ultimate and yield bearing stresses for the lug material for $e/D = 2.0$, as given in MIL-HDBK-5 or other applicable specification.

Equations (9-3a) and (9-3b) apply only if the load is uniformly distributed across the lug thickness. If the pin is too flexible and bends excessively, the load on the lug will tend to peak up near the shear faces and possibly cause premature failure of the lug.

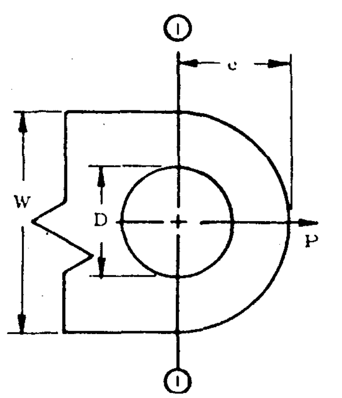
A procedure to check the pin bending strength in order to prevent premature lug failure is given in Section 9.4 entitled "Double Shear Joint Strength Under Uniform Axial Load."

9.3.2 <u>Lug Net-Section Strength Under Uniform Axial Load</u>

The allowable lug net-section tensile ultimate stress (F_{nu_L}) on Section 1-1 in Figure 9-4a is affected by the ability of the lug material to yield and thereby relieve the stress concentration at the edge of the hole.

$$F_{nu_L} = K_n F_{tu} \qquad (9\text{-}4)$$

K_n, the net-tension stress coefficient, is obtained from the graphs shown in Figure 9-4 as a function of the ultimate and yield stress and strains of the lug material in the direction of the applied load. The ultimate strain (ϵ_u) can be obtained from MIL-HDBK-5.



(a)

Figure 9-4. Net Tension Stress Coefficient

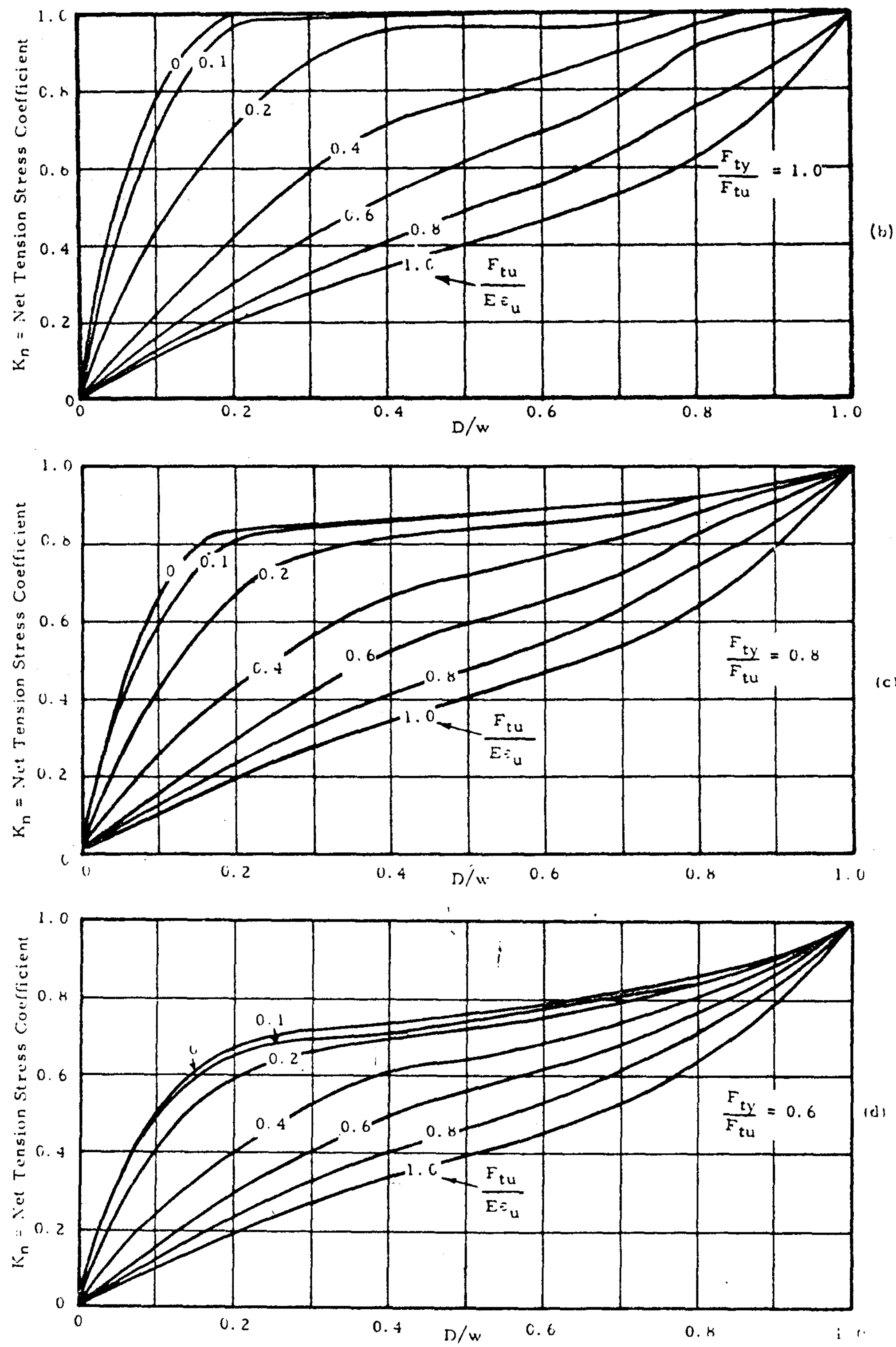


Figure 9-4. Net Tension Stress Coefficient (concluded)

The allowable lug net-section tensile yield stress (F_{ny_L}) is

$$F_{ny_L} = K_n F_{ty} \quad (9\text{-}5)$$

The allowable lug net-section ultimate load (P_{nu_L}) is

$$P_{nu_L} = F_{nu_L} \; (w\text{-}D)t, \;\; (\text{if } F_{tu} \leqq 1.304 \; F_{ty}) \quad (9\text{-}6a)$$

$$P_{nu_L} = 1.304 \; F_{ny_L} \; (w\text{-}D)t, \;\; (\text{if } F_{tu} > 1.304 \; F_{ty}) \quad (9\text{-}6b)$$

9.3.3 Lug Design Strength Under Uniform Axial Load

The allowable design ultimate load for the lug (P_{u_L}) is the lower of the values obtained from Equations (9-3) and (9-6).

$$P_{u_L} \leqq P_{bru_L} \text{ (Equations (9-3a) and (9-3b), or } P_{nu_L} \text{ (Equations (9-6a) and (9-6b))} \quad (9\text{-}7)$$

9.3.4 Bushing Bearing Strength Under Uniform Axial Load

The allowable bearing yield stress for bushings (F_{bry_B}) is restricted to the compressive yield stress (F_{cy_B}) of the bushing material, unless higher values are substantiated by tests.

The allowable bearing ultimate stress for bushings (F_{bru_B}) is

$$F_{bru_B} = 1.304 \; F_{cy_B} \quad (9\text{-}8)$$

The allowable bushing ultimate load (P_{u_B}) is

$$P_{u_B} = 1.304 \; F_{cy_B} \; D_p t \quad (9\text{-}9)$$

This assumes that the bushing extends through the full thickness of the lug.

9.3.5 Combined Lug-Bushing Design Strength Under Uniform Axial Load

The allowable lug-bushing ultimate load ($P_{u_{L_B}}$) is the lower of the loads obtained from Equations (9-7) and (9-9).

$$P_{u_{L_B}} \leqq P_{u_L} \text{ (Equation (9-7), or } P_{u_B} \text{ (Equation (9-9))} \quad (9\text{-}10)$$

9.4 Double Shear Joint Strength Under Uniform Axial Load

The strength of a joint such as the one shown in Figure 9-5 depends on the lug-bushing ultimate strength ($P_{u_{L_B}}$) and on the pin shear and pin bending strengths.

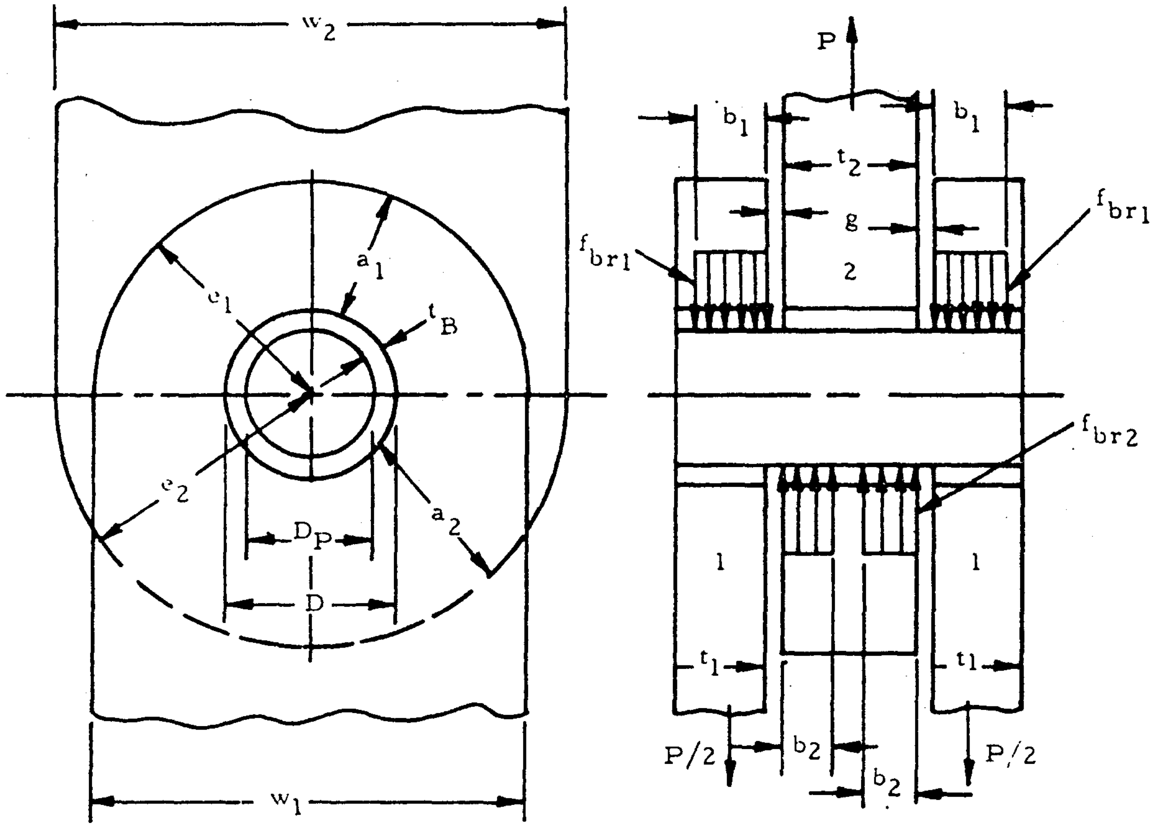


Figure 9-5. Double Shear Lug Joint

9.4.1 Lug-Bushing Design Strength for Double Shear Joints Under Uniform Axial Load

The allowable lug-bushing ultimate load ($P_{u_{L_B}}$) for the joint is computed, using Equation (9-10). For the symmetrical joint shown in the figure, Equation (9-10) is used to calculate the ultimate load for the outer lugs and bushings ($2P_{u_{L_{B1}}}$) and the ultimate load for the inner lug and bushing ($P_{u_{L_{B2}}}$). The allowable value of $P_{u_{L_B}}$ for the joint is the lower of these two values.

$$P_{u_{L_B}} \leqq 2P_{u_{L_{B1}}} \text{ (Equation (9-10)), or } P_{u_{L_{B2}}} \text{ (Equation (9-10))} \qquad (9\text{-}11)$$

9.4.2 Pin Shear Strength for Double Shear Joints Under Uniform Axial Load

The pin ultimate shear load (P_{us_p}) for the symmetrical joint shown in Figure 9-5 is the double shear strength of the pin:

$$P_{us_p} = 1.571 \, D_p^2 \, F_{su_p} \tag{9-12}$$

where F_{su_p} is the ultimate shear stress of the pin material.

9.4.3 Pin Bending Strength for Double Shear Joints Under Uniform Axial Load

Although actual pin bending failures are infrequent, excessive pin deflections can cause the load in the lugs to peak up near the shear planes instead of being uniformly distributed across the lug thickness, thereby leading to premature lug or bushing failures at loads less than those predicted by Equation (9-11). At the same time, however, the concentration of load near the lug shear planes reduces the bending arm and, therefore, the bending moment in the pin, making the pin less critical in bending. The following procedure is used in determining the pin ultimate bending load.

Assume that the load in each lug is uniformly distributed across the lug thickness ($b_1 = t_1$, and $2b_2 = t_2$). For the symmetrical joint shown in Figure 9-5, the resulting maximum pin bending moment is

$$M_{max P} = \frac{P}{2} \left(\frac{t_1}{2} + \frac{t_2}{4} + g \right) \tag{9-13}$$

The ultimate failing moment for the pin is

$$M_{u_p} = 0.0982 \, k_{b_p} \, D_p^3 \, F_{tu_p} \tag{9-14}$$

where k_{b_p} is the plastic bending coefficient for the pin. The value of k_{b_p} varies from 1.0 for a perfectly elastic pin to 1.7 for a perfectly plastic pin, with a value of 1.56 for pins made from reasonably ductile materials (more than 5% elongation).

The pin ultimate bending load (P_{ub_p}) is, therefore,

$$P_{ub_p} = \frac{0.1963 \, k_{b_p} \, D_p^3 \, F_{tu_p}}{\left(\frac{t_1}{2} + \frac{t_2}{4} + g \right)} \tag{9-15}$$

If P_{ub_p} is equal to or greater than either $P_{u_{L_B}}$ (Equation (9-11) or P_{us_p} (Equation (9-12)), then the pin is a relatively strong pin that is not critical in bending, and no further pin bending calculations are required. The allowable load for the joint (P_{all}) can be determined by going directly to Equation (9-19a).

If P_{ub_p} (Equation (9-15)) is less than both $P_{u_{L_B}}$ (Equation (9-11)) and P_{us_p} (Equation (9-12)), the pin is considered a relatively weak pin, critical in bending. However, such a pin may deflect sufficiently under load to shift the c. g. of the bearing loads toward the shear faces of the lugs, resulting in a decreased pin bending moment and an increased value of P_{ub_p}. These shifted loads are assumed to be uniformly distributed over widths b_1 and $2b_2$, which are less than t_1 and t_2, respectively, as shown in Figure 9-5. The portions of the lugs and bushings not included in b_1 and $2b_2$ are considered ineffective. The new increased value of pin ultimate bending load is

$$P_{ub_p} = \frac{0.1963\, k_{b_p}\, D_p^3\, F_{tu_p}}{\left(\frac{b_1}{2} + \frac{b_2}{2} + g\right)} \qquad (9\text{-}15a)$$

The maximum allowable value of P_{ub_p} is reached when b_1 and b_2 are sufficiently reduced so that P_{ub_p} (Equation (9-15a)) is equal to $P_{u_{L_B}}$ (Equation (9-11)), provided that b_1 and $2b_2$ are substituted for t_1 and t_2, respectively, At this point we have a balanced design where the joint is equally critical in pin-bending failure or lug-bushing failure.

The following equations give the "balanced design" pin ultimate bending load ($P_{ub_{p\,max}}$) and effective bearing widths ($b_{1\,min}$ and $2b_{2\,min}$):

$$P_{ub_{p\,max}} = 2C \sqrt{\frac{P_{ub_p}}{C}\left(\frac{t_1}{2} + \frac{t_2}{4} + g\right) + g^2} - 2Cg \qquad (9\text{-}16)$$

where

$$C = \frac{P_{u_{L_{B1}}}\; P_{u_{L_{B2}}}}{P_{u_{L_{B1}}}\, t_2 + P_{u_{L_{B2}}}\, t_1}$$

The value of P_{ub_p} on the right hand side of Equation (9-16) and the values of $P_{u_{L_{B1}}}$ and $P_{u_{L_{B2}}}$ in the expression for C are based on the assumption that the full thicknesses of the lugs are effective and have already been calculated. (Equations (9-10) and (9-15)).

If the inner lug strength is equal to the total strength of the two outer lugs ($P_{u_{L_{B2}}} = 2P_{u_{L_{B1}}}$), and if g = 0, then

$$P_{ub_{p\,max}} = \sqrt{P_{ub_p} P_{u_{L_{B2}}}} \tag{9-17}$$

The "balanced design" effective bearing widths are

$$b_{1\,min} = \frac{P_{ub_{p\,max}} t_1}{2P_{u_{L_{B1}}}} \tag{9-18a}$$

$$2b_{2\,min} = \frac{P_{ub_{p\,max}} t_2}{P_{u_{L_{B2}}}} \tag{9-18b}$$

where $P_{ub_{p\,max}}$ is obtained from Equation (9-16) and $P_{u_{L_{B1}}}$ and $P_{u_{L_{B2}}}$ are the previously calculated values based on the full thicknesses of the lugs. Since any lug thicknesses greater than $b_{1\,min}$ or $b_{2\,min}$ are not considered effective, an efficient static strength design would have $t_1 = b_{1\,min}$ and $t_2 = 2b_{2\,min}$.

The allowable joint ultimate load (P_{all}) for the double-shear joint is obtained as follows:

If P_{ub_p} (Equation (9-15) is greater than either $P_{u_{L_B}}$ (Equation (9-11) or P_{us_p} (Equation (9-12)), then P_{all} is the lower of the values of $P_{u_{L_B}}$ or P_{us_p}.

$$P_{all} \leqq P_{u_{L_B}} \text{ (Equation (9-11) or } P_{us_p} \text{ (Equation (9-12))} \tag{9-19a}$$

If P_{ub_p} (Equation (9-15)) is less than both $P_{u_{L_B}}$ and P_{us_p}, then P_{all} is the lower of the values of P_{us_p} and $P_{ub_{p\,max}}$.

$$P_{all} \leqq P_{us_p} \text{ (Equation (9-12) or } P_{ub_{p\,max}} \text{ (Equation (9-16))} \tag{9-19b}$$

9.4.4 Lug Tang Strength for Double Shear Joints Under Uniform Axial Load

If Equation (9-19a) has been used to determine the joint allowable load, then we have a condition where the load in the lugs and tangs is assumed uniformly distributed. The allowable stress in the tangs is F_{tu_T}. The lug tang strength (P_T) is the lower of the following values.

$$P_T = 2F_{tu_T} w_{T1} t_1 \tag{9-20a}$$

$$P_T = F_{tu_{T2}} w_{T2} t_2 \tag{9-20b}$$

If Equation (9-19b) was used to determine the joint allowable load, the tangs of the outer lugs should be checked for the combined axial and bending stresses resulting from the eccentric application of the bearing loads. Assuming that the lug thickness remains constant beyond the pin, a load (P/2) applied over the width b_1 in each outer lug will produce the following bending moment in the tangs:

$$M_1 = \frac{P}{2} \left(\frac{t_1 - b_1}{2} \right)$$

A simple, but generally conservative, approximation to the maximum combined stress in the outer lug tangs is

$$F_{t_{T1}} = \frac{P}{2w_{T1} \; t_1} + \frac{6M_1}{k_{b_T} w_{T1} t_1^2} \tag{9-21}$$

where k_{b_T}, the plastic bending coefficient for a lug tang of rectangular cross-section, varies from 1.0 for a perfectly elastic tang to 1.5 for a perfectly plastic tang, with a value of 1.4 representative of rectangular cross sections with materials of reasonable ductility (more than 5% elongation). The allowable value of $F_{t_{T1}}$ is $F_{tu_{T1}}$. The lug tang strength is the lower of the following values:

$$P_T = \frac{2F_{tu_{T1}} \; w_{T1} \; t_1}{1 + \frac{3}{k_{b_T}} \left(1 - \frac{b_{1\,min}}{t_1} \right)} \tag{9-22a}$$

$$P_T = F_{tu_{T2}} w_{T2} t_2 \tag{9-22b}$$

where $b_{1\,min}$ is given by Equation (9-18a)

9.5 Single-Shear Joint Strength Under Uniform Axial Load

In single-shear joints, lug and pin bending are more critical than in double-shear joints. The amount of bending can be significantly affected by bolt clamping. In the cases considered in Figure 9-6, no bolt clamping is assumed, and the bending moment in the pin is resisted by socket action in the lugs.

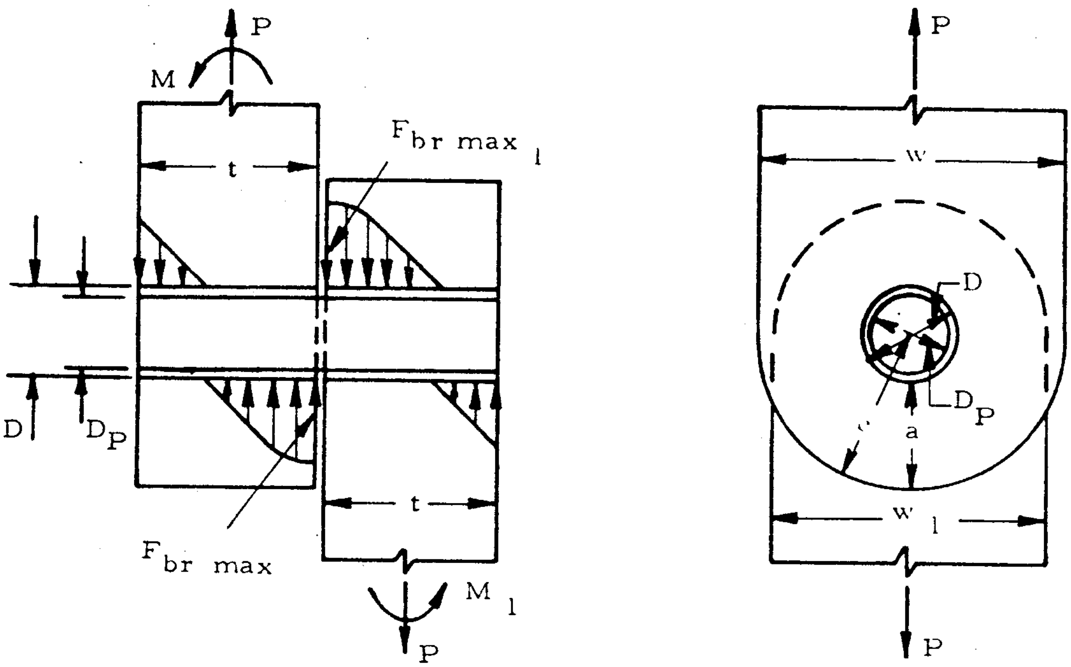


Figure 9-6. Single Shear Lug Joint

In Figure 9-6 a representative single-shear joint is shown, with centrally applied loads (P) in each lug, and bending moments (M and M_l) that keep the system in equilibrium. (Assuming that there is no gap between the lugs, $M + M_l = P(t + t_l)/2$). The individual values of M and M_l are determined from the loading of the lugs as modified by the deflection, if any, of the lugs, according to the principles of mechanics.

The strength analysis procedure outlined below applies to either lug. The joint strength is determined by the lowest of the margins of safety calculated for the different failure modes defined by Equations (9-23) through (9-27).

9.5.1 Lug Bearing Strength for Single Shear Joints Under Uniform Axial Loads

The bearing stress distribution between lug and bushing is assumed to be similar to the stress distribution that would be obtained in a rectangular cross section of width (D) and depth (t), subjected to a load (P) and moment (M). At ultimate load the maximum lug bearing stress ($F_{br\ max\ L}$) is approximated by

$$F_{br\ max\ L} = \frac{P}{Dt} + \frac{6M}{k_{br_L}\ Dt^2} \qquad (9\text{-}23)$$

where k_{br_L} is a plastic bearing coefficient for the lug material, and is assumed to be the same as the plastic bending coefficient (k_{b_L}) for a rectangular section.

The allowable ultimate value of $F_{br\,max_L}$ is either F_{bru_L} (Equations (9-1a) (9-1b)) or 1.304 F_{bry_L} (Equations (9-2a) (9-2b)), whichever is lower.

9.5.2 Lug Net-Section Strength for Single Shear Joints Under Uniform Axial Load

At ultimate load the nominal value of the outer fiber tensile stress in the lug net-section is approximated by

$$F_{t\,max} = \frac{P}{(w-D)t} + \frac{6M}{k_{b_L}\,(w-D)t^2} \quad \left(= \frac{P}{A} + \frac{Mc}{I}\,\frac{1}{t} \right) \qquad (9\text{-}24)$$

where k_{b_L} is the plastic bending coefficient for the lug net-section.

The allowable ultimate value of $F_{t\,max}$ is F_{nu_L} (Equation (9-4)) or 1.304 F_{ny_L} (Equation (9-5)), whichever is lower.

9.5.3 Bushing Strength for Single Shear Joints Under Uniform Axial Load

The bearing stress distribution between bushing and pin is assumed to be similar to that between the lug and bushing. At ultimate bushing load the maximum bushing bearing stress is approximated by

$$F_{br\,max\,B} = \frac{P}{D_p t} + \frac{6M}{k_{br_L}\,D_p t^2} \qquad (9\text{-}25)$$

where k_{br_L}, the plastic bearing coefficient, is assumed the same as the plastic bending coefficient (k_{b_L}) for a rectangular section.

The allowable ultimate value of $F_{br\,max\,B}$ is 1.304 F_{cy_B}, where F_{cy_B} is the bushing material compressive yield strength.

9.5.4 Pin Shear Strength for Single Shear Joints Under Uniform Axial Load

The maximum value of pin shear can occur either within the lug or at the common shear face of the two lugs, depending upon the value of M/Pt. At the lug ultimate load the maximum pin shear stress ($F_{s\,max\,P}$) is approximated by

$$F_{s\,max\,P} = \frac{1.273\,P}{D_p^{\,2}}, \quad \left(\text{if } \frac{M}{Pt} \leq 2/3\right) \qquad (9\text{-}26a)$$

$$F_{s\,max\,P} = \frac{1.273\,P}{D_p^{\,2}} \; \frac{\left(\sqrt{\left(\frac{2M}{Pt}\right)^2 + 1} - 1\right)}{\left(\frac{2M}{Pt} + 1 - \sqrt{\left(\frac{2M}{Pt}\right)^2 + 1}\right)}, \quad \left(\text{if } \frac{M}{Pt} > 2/3\right) \qquad (9\text{-}26b)$$

Equation (9-26a) defines the case where the maximum pin shear is obtained at the common shear face of the lugs, and Equation (9-26b) defines the case where the maximum pin shear occurs away from the shear face.

The allowable ultimate value of $F_{b\,max\,p}$ is F_{su_p}, the ultimate shear stress of the pin material.

9.5.5 Pin Bending Strength for Single Shear Joints Under Uniform Axial Load

The maximum pin bending moment can occur within the lug or at the common shear faces of the two lugs, depending on the value of M/Pt. At the lug ultimate load the maximum pin bending stress ($F_{b\,max\,p}$) is approximated by

$$F_{b\,max\,p} = \frac{10.19M}{k_{bp}\,D_p^{\,3}} \left(\frac{Pt}{2M} - 1\right), \quad \left(\text{if } \frac{M}{Pt} \leq 3/8\right) \tag{9-27a}$$

$$F_{b\,max\,p} = \frac{10.19M}{k_{bp}\,D_p^{\,3}} \; \frac{\left(\sqrt{\left(\frac{2M}{Pt}\right)^2 + 1} - 1\right)}{\frac{2M}{Pt}}, \tag{9-27b}$$

$$\left(\text{if } \frac{M}{Pt} > 3/8\right)$$

where k_{bp} is the plastic bending coefficient for the pin.

Equation (9-27a) defines the case where the maximum pin bending moment is obtained at the common shear face of the lugs, and Equation (9-27b) defines the case where the maximum pin bending moment occurs away from the shear face, where the pin shear is zero.

The allowable ultimate value of $F_{b\,max\,p}$ is F_{tu_p}, the ultimate tensile stress of the pin material.

9.6 Example of Uniform Axially Loaded Lug Analysis

Determine the static strength of an axially loaded, double shear joint, such as shown in Section 9.4, with dimensions and material properties given in Table 9-1.

Table 9-1. Dimensions and Properties

	Female Lugs, 1	Male Lug, 2	Bushings, 1 and 2	Pin
Material	2024-T351 Plate	7075-T651 Plate	Al. Bronze	4130 Steel
F_{tu}	64000 psi (X-grain)	77000 psi (X-grain)	110,000 psi	125,000 psi
F_{ty}	40000 psi (X-grain)	66000 psi (X-grain)	60,000 psi	103,000 psi
F_{cy}			60,000 psi	
F_{su}				82,000 psi
E	10.5×10^6 psi	10.3×10^6		29×10^6 psi
ε_u	0.12	0.06		
D or D_p	D = 1.00 in.	D = 1.00 in.	D_p = 0.75 in., D = 1.00 in.	D_p = 0.75 in.
e	1.25 in.	1.50 in.		
a	0.75 in.	1.00 in.		
w = W_T	2.50 in.	3.00 in.		
t	0.50 in.	0.75 in.	0.50 and 0.75 in.	
g	0.10 in.			

(1) Female Lugs and Bushings

F_{tux} = 64,000 psi; 1.304 F_{tyx} = 1.304 x 40000 = 52160 psi.

a) Lug Bearing Strength (Equations (9-2a) and (9-3b))

$$\frac{e_1}{D} = \frac{1.25}{1.00} = 1.25; \text{ therefore } K_1 = 1.46 \text{ (from Figure 9-2)}$$

$$P_{bru_{L_1}} = 1.304 \times 1.46 \times 0.75 \times 40000 \times 1.00 \times 0.50 = 28600 \text{ lbs.}$$

b) Lug Net-Section Tension Strength (Equations (9-5) and (9-6b))

$$\frac{D}{w_1} = \frac{1.00}{2.50} = 0.40; \quad \frac{F_{ty}}{F_{tu}} = \frac{40000}{64000} = 0.625;$$

$$\frac{F_{tu}}{E\varepsilon_u} = \frac{64000}{10.5 \times 10^6 \times 0.12} = 0.051; \text{ therefore, } k_{n_1} = 0.74$$

(by interpolation from Figure 9-4)

$$P_{nu_L} = 2 \times 1.304 \times .74 \times 4000 \times (2.5 - 1.0) \times .5 = 57,898 \text{ lbs.}$$

c) Lug Design Strength (Equation (9-7))

$$P_{u_{L1}} = P_{bru_{L1}} = 28600 \text{ lbs.}$$

d) Bushing Bearing Strength (Equation (9-9))

$$P_{u_{B1}} = 1.304 \times 60000 \times 0.75 \times 0.50 = 29300 \text{ lbs.}$$

e) Combined Lug-Bushing Design Strength (Equation (9-10))

$$P_{u_{L_{B1}}} = P_{u_{L1}} = 28600 \text{ lbs.}$$

(2) Male Lug and Bushing

$$F_{tux} = 77000 \text{ psi}; \; 1.304\ F_{tyx} = 1.304 \times 66000 = 86100 \text{ psi.}$$

a) Lug Bearing Strength (Equations (9-1b) and (9-3a))

$$\frac{e_2}{D} = \frac{1.50}{1.00} = 1.50; \text{ therefore, } K_2 = 1.33 \text{ (from Figure 9-7)}$$

$$P_{bru_{L2}} = 1.33 \times 77000 \times 1.00 \times 0.75 = 77000 \text{ lbs.}$$

b) Lug Net-Section Tension Strength (Equations (9-4) and (9-6a))

$$\frac{D}{w_2} = \frac{1.00}{3.00} = 0.333; \; \frac{F_{ty}}{F_{tu}} = \frac{66000}{77000} = 0.857;$$

$$\frac{F_{tu}}{E\varepsilon_u} = \frac{77000}{10.3 \times 10^6 \times 0.06} = 0.125; \text{ therefore } K_{n_2} = 0.87$$

(by interpolation from Figure 9-4)

$$P_{nu_L} = 1.304 \times 87 \times 66000 \times (3.0 - 1\ 0) \times .75 = 112,313 \text{ lbs.}$$

c) Lug Design Strength (Equation (9-7))

$$P_{u_{L2}} = P_{bru_{L2}} = 77000 \text{ lbs.}$$

d) Bushing Bearing Strength (Equation (9-9))

$$P_{u_{B2}} = 1.304 \times 60000 \times 0.75 \times 0.75 = 44000 \text{ lbs.}$$

e) Combined Lug-Bushing Design Strength (Equation (9-10))

$$P_{u_{L_{B2}}} = P_{u_{B2}} = 44000 \text{ lbs.}$$

(3) Joint Analysis

a) Lug-Bushing Strength (Equation (9-11))

$$P_{u_{L_B}} = P_{u_{L_{B2}}} = 44000 \text{ lbs.}$$

b) Pin Shear Strength (Equation (9-12))

$$P_{us_p} = 1.571 \times (0.75)^2 \times 82000 = 72400 \text{ lbs.}$$

c) Pin Bending Strength (Equation (9-15))

The pin ultimate bending load, assuming uniform bearing across the lugs, is

$$P_{ub_p} = \frac{0.1963 \times 1.56 \times (0.75)^3 \times 125000}{0.25 + 0.1875 + 0.10} = 30100 \text{ lbs.}$$

Since P_{ub_p} is less than both $P_{u_{L_B}}$ and P_{us_p}, the pin is a relatively weak pin which deflects sufficiently under load to shift the bearing loads toward the shear faces of the lugs. The new value of pin bending strength is, then,

$$P_{ub\ max} = 2C \times \left(\sqrt{\frac{30100}{C} \times (0.25 + 0.1875 + 0.10) + (0.10)^2} - 0.10\right),$$

(from Equation (9-16)) where $C = \dfrac{28600 \times 44000}{286 \times 0.75 + 44000 \times 0.050}$

$= 29000$ lbs/in.

Therefore, $P_{ub_p\ max} = 2 \times 29000 \times (0.754 - 0.10) = 37900$ lbs.

The "balanced design" effective bearing widths are

$$b_{1_{min}} = \frac{37900 \times 0.50}{2 \times 28600} = 0.331 \text{ in. (from Equation (9-18a))}$$

$$2b_{2\ min} = \frac{37900 \times 0.75}{44000} = 0.646 \text{ in. (from Equation (9-18b))}$$

Therefore, the same value of $P_{ub_{p\ max}}$ would be obtained if the thickness of each female lug was reduced to 0.331 inches and the thickness of the male lug reduced to 0.646 inches.

d) Joint Strength (Equation (9-19b))

The final allowable load for the joint, exclusive of the lug tangs, is

$$P_{all} = P_{ub_{p\,max}} = 37900 \text{ lbs.}$$

(4) Lug Tang Analysis

$$P_T = \frac{2 \times 64000 \times 2.50 \times 0.50}{1 + \frac{3}{1.4} \times \left(1 - \frac{0.331}{0.500}\right)} = 92700 \text{ lbs. (from Equation (9-22a))}$$

or

$$P_T = 77000 \text{ z } 3.00 \times 0.75 = 173300 \text{ lbs. (from Equation (9-22b))}$$

Therefore, the lug tangs are not critical and the allowable joint load remains at 37900 pounds.

9.7 Lug and Bushing Strength Under Transverse Load

Transversely loaded lugs and bushings are checked in the same general manner as axially loaded lugs. The transversely loaded lug, however, is a more redundant structure than an axially loaded lug, and it has a more complicated failure mode. Figure 9-7 illustrates the different lug dimensions that are critical in determining the lug strength.

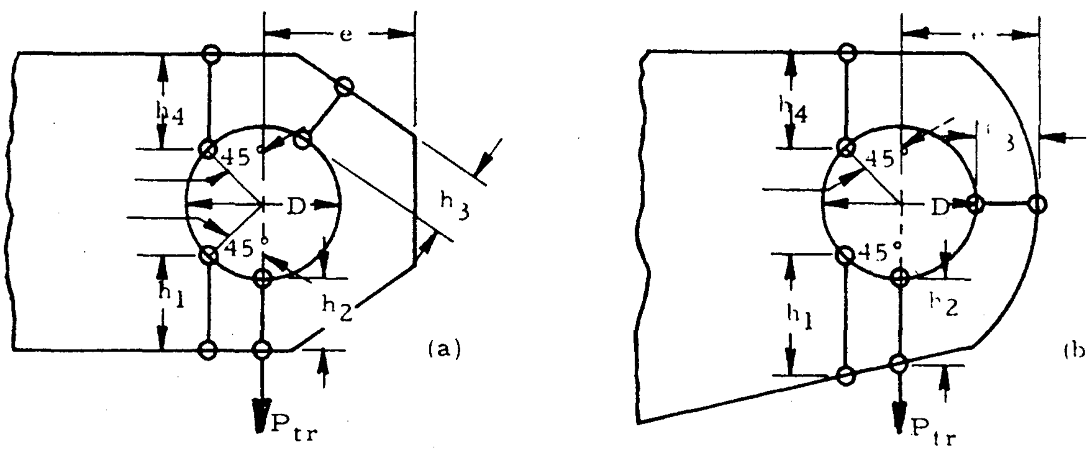


Figure 9-7. Schematic of Lugs Under Transverse Loads

9.7.1 Lug Strength Under Transverse Load

The lug ultimate bearing stress (F_{bru_L}) is

$$F_{bru_L} = K_{tru} F_{tux} \qquad (9\text{-}28)$$

where K_{tru}, the transverse ultimate load coefficient, is obtained from Figure 9-8 as a function of the "effective" edge distance (h_{av}):

$$h_{av} = \frac{6}{3/h_1 + 1/h_2 + 1/h_3 + 1\ h/4}$$

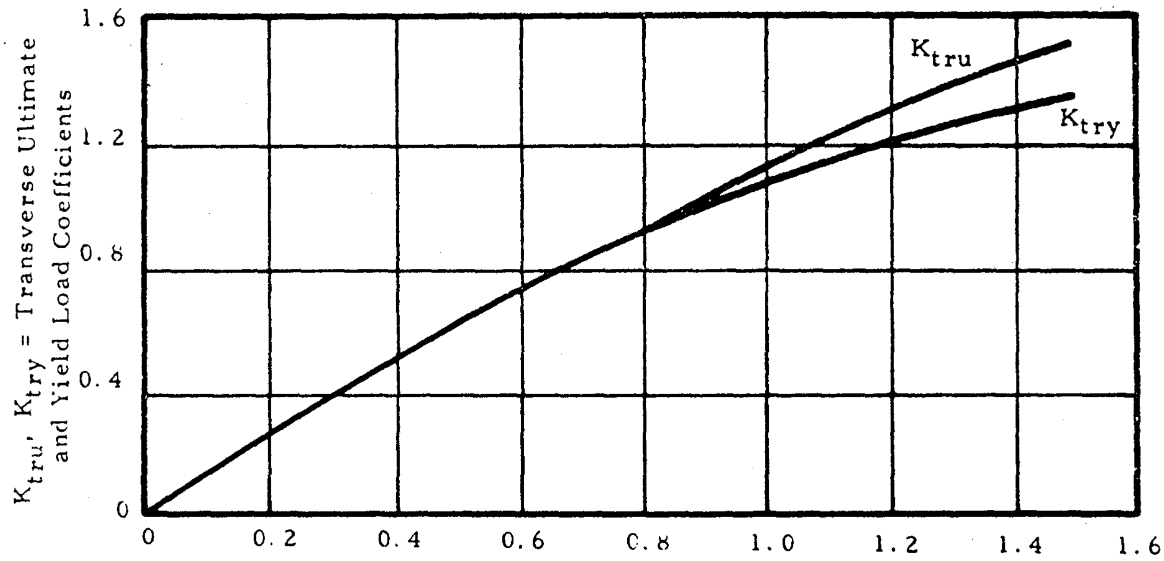


Figure 9-8

The effective edge distance can be found by using the nomograph in Figure 9-9. The nomograph is used by first connecting the h_1 and h_2 lines at the appropriate value of h_1 and h_2. The intersection with line A is noted. Next connect the h_3 and h_4 lines similarly, and note the B line intersection. Connecting the A and B line intersection gives the value of h_{av} to be read at the intersection with the h_{av} line. The different edge distances (h_1, h_2, h_3, h_4) indicate different critical regions in the lug, h_1 being the most critical. The distance h_3 is the smallest distance from the hole to the edge of the lug. If the lug is a concentric lug with parallel sides, h_{av}/D can be obtained directly from Figure 9-10 for any value of e/D. In concentric lugs, $h_1 = h_4$ and $h_2 = h_3$.

The lug yield bearing stress (F_{bry_L}) is

$$F_{bry_L} = K_{try} F_{tyx} \qquad (9\text{-}29)$$

where K_{try}, the transverse yield load coefficient, is obtained from Figure 9-9.

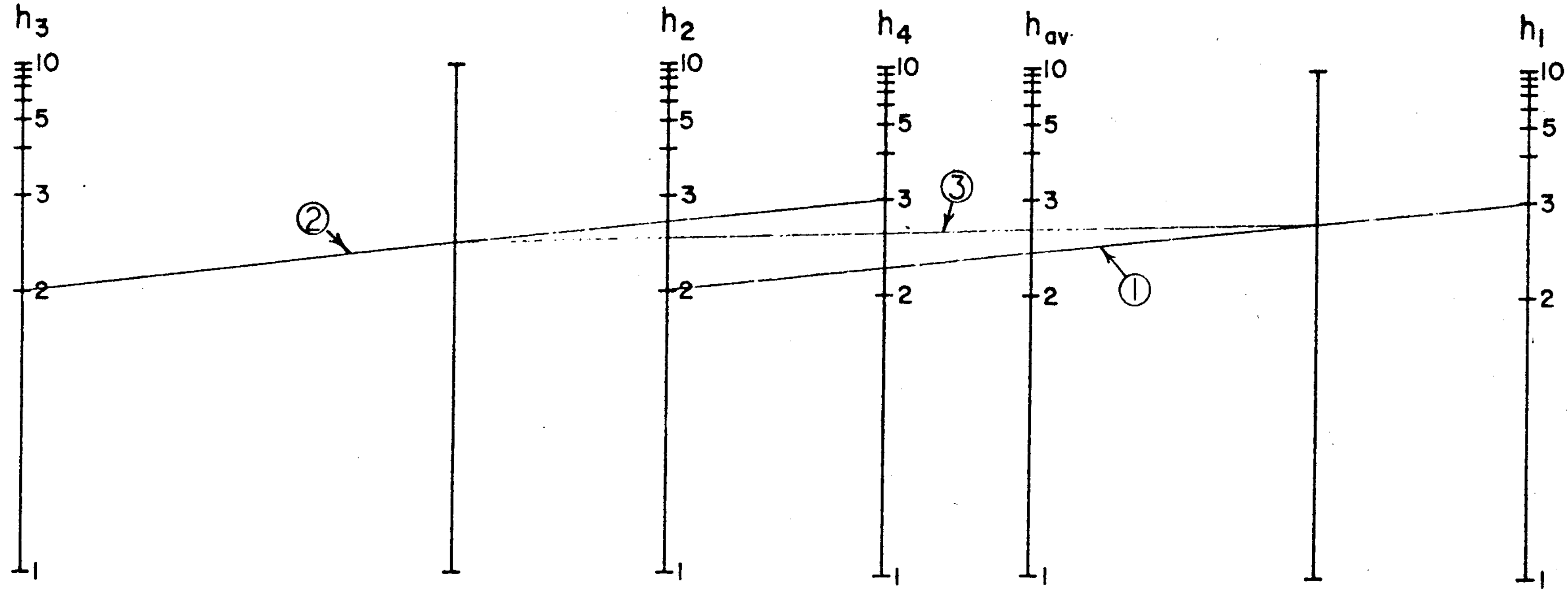
The allowable lug transverse ultimate load (P_{tru_L}) is

$$P_{tru_L} = F_{br_L} Dt \text{ (if } F_{tux} \leqq 1.304 F_{tyx}) \qquad (9\text{-}30a)$$

$$P_{tru_L} = 1.304 F_{bry_L} Dt \text{ (if } F_{tux} > 1.304 F_{tyx}) \qquad (9\text{-}30b)$$

where F_{bru_L} and F_{bry_L} are obtained from Equations (9-28) and (9-29)).

EFFECTIVE EDGE DISTANCE



$$h_{av} = \frac{6}{3/h_1 + 1/h_2 + 1/h_3 + 1/h_4}$$

Figure 9-9 Nomograph for Effective Edge Distance

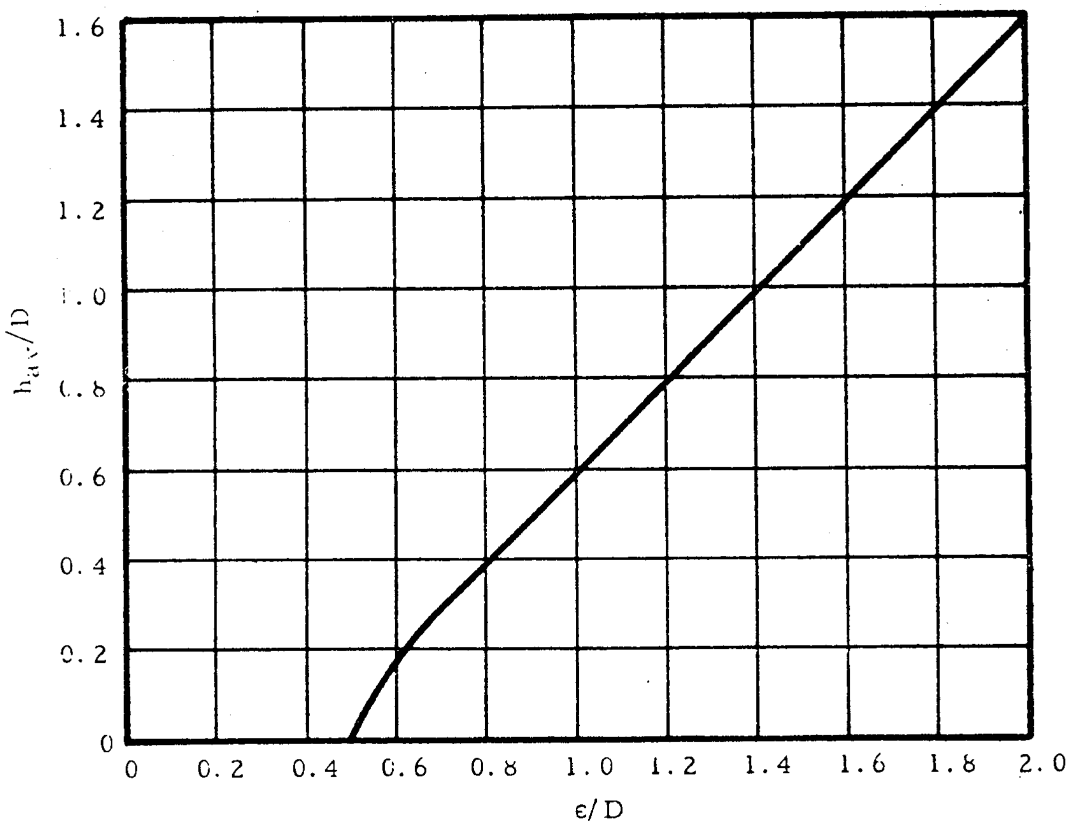


Figure 9-10. Effective Edge Distance

If the lug is not of constant thickness, then A_{av}/A_{br} is substituted for h_{av}/D on the horizontal scale of the graph in Figure 9-8, where A_{br} is the lug bearing area, and

$$A_{av} = \frac{6}{3/A_1 + 1/A_2 + 1/A_3 + 1/A_4}$$

A_1, A_2, A_3, and A_4 are the areas of the sections defined by h_1, h_2, h_3, and h_4, respectively.

The values of K_{tru} and K_{try} corresponding to A_{av}/A_{br} are then obtained from the graph in Figure 9-8 and the allowable bearing stresses are obtained as before from Equations (9-28) and (9-29)).

9.7.2 Bushing Strength Under Transverse Load

The allowable bearing stress on the bushing is the same as that for the bushing in an axially loaded lug and is given by Equation (9-8). The allowable bushing ultimate load (P_{tru_B}) is equal to P_{u_B} (Equation (9-9)).

9.8 Double Shear Joints Under Transverse Load

The strength calculations needed for double shear joint strength analysis are basically the same as those needed for axially loaded. Equations (9-11) through (9-19) can be used; however, the maximum lug bearing stresses at ultimate and yield loads must not exceed those given by Equations (9-28) and (9-29).

9.9 Single Shear Joints Under Transverse Load

The previous discussion on double shear joint applies to single shear joint strength analysis except the equations to be used are now Equations (9-23) through (9-27).

9.10 Lug and Bushing Strength Under Oblique Load

The analysis procedures used to check the strength of axially loaded lugs and of transversely loaded lugs are combined to analyze obliquely loaded lugs such as the one shown in Figure 9-11. These procedures apply only if α does not exceed 90°.

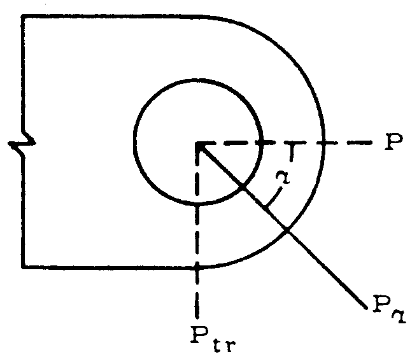


Figure 9-11. Obliquely Loaded Lug

9.10.1 Lug Strength Under Oblique Load

The obliquely applied load (P_α) is resolved into an axial component ($P = P_\alpha \cos \alpha$) and a transverse component ($P_{tr} = P_\alpha \sin \alpha$). The allowable ultimate value of P_α is P_{α_L} and its axial and transverse components satisfy the following equation:

$$\left(\frac{P}{P_{u_L}}\right)^{1.6} + \left(\frac{P_{tr}}{P_{tru_L}}\right)^{1.6} = 1 \qquad (9\text{-}31)$$

where P_{u_L} is the strength of an axially loaded lug (Equation (9-7)) and P_{tru_L} is the strength of a transversely loaded lug (Equations (9-30a), (9-30b)). The allowable load curve defined by Equation (9-31) is plotted on the graph in Figure 9-12.

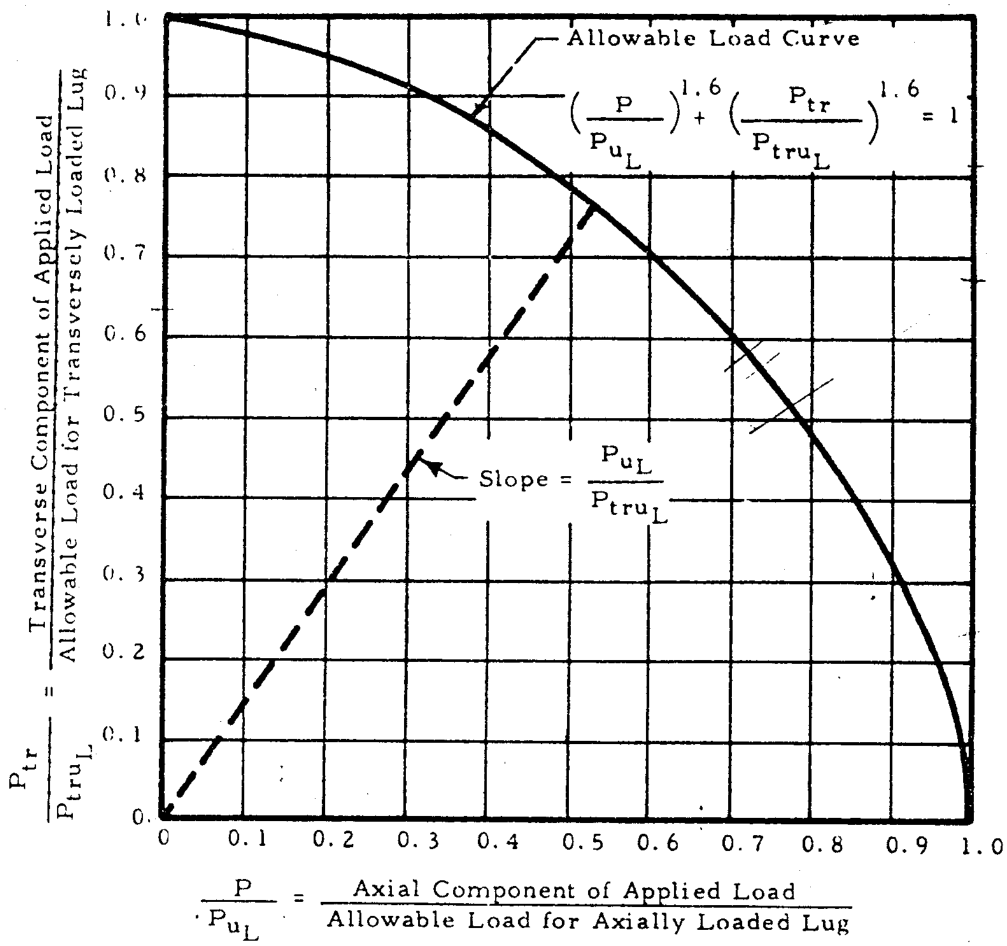


Figure 9-12. Allowable Load Curve

For any given value of α the allowable load (P_{α_t}) for a lug can be determined from the graph shown in Figure 9-12 by drawing a line from the origin with a slope equal to (P_{u_L}/P_{tru_L}). The intersection of this line with the allowable load curve (point 1 on the graph) indicates the allowable values of P/P_{u_L} and P_{tr}/P_{tru_L}, from which the axial and transverse components, P and P_{tr}, of the allowable load can be readily obtained.

9.10.2 Bushing Strength Under Oblique Load

The bushing strength calculations are identical to those for axial loading (Equations (9-8) and (9-9)).

9.11 Double Shear Joints Under Oblique Load

The strength calculations are basically the same as those for an axially loaded joint except that the maximum lug bearing stress at ultimate load must not exceed P_{α_L}/Dt, where P_{α_L} is defined by Equation (9-31). Use Equations (9-11) through (9-19)).

9.12 Single Shear Joints Under Oblique Load

The previous discussion on double shear joints applies to single shear joint strength analysis except the equations to be used are now Equations (9-23) through (9-27).

9.13 Multiple Shear and Single Shear Connections

Lug-pin combinations having the geometry indicated in Figure 9-13 should be analyzed according to the following criteria:

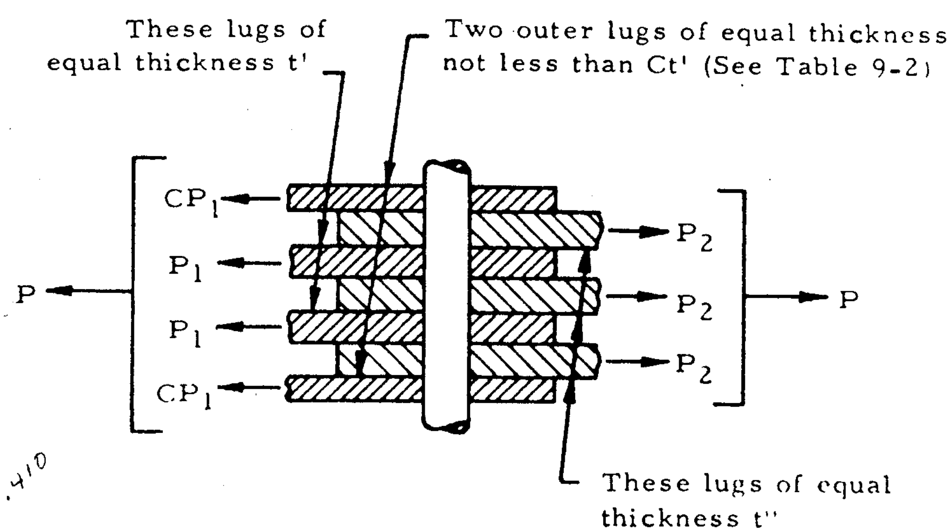
(1) The load carried by each lug should be determined by distributing the total applied load P among the lugs as indicated in Figure 9-13, b being obtained in Table 9-2. This distribution is based on the assumption of plastic behavior (at ultimate load) of the lugs and elastic bending of the pin, and gives approximately zero bending deflection of the pin.

(2) The maximum shear load on the pin is given in Table 9-2.

(3) The maximum bending moment in the pin is given by the formulae

$$M = \frac{P_1 b}{2}$$ where b is given in Table 9-2.

t_1 t_2



$t_1/t_2 = 1.13$ $t_3/t_2 = .410$

Figure 9-13. Schematic of Multiple Shear Joint in Tension

Total number of lugs including both sides	C	Pin Shear	b
5	.35	$.50 P_1$	$.28 \frac{t'+t''}{2}$
7	.40	$.53 P_1$	$.33 \frac{t'+t''}{2}$
9	.43	$.54 P_1$	$.37 \frac{t'+t''}{2}$
11	.44	$.54 P_1$	$.39 \frac{t'+t''}{2}$
∞	.50	$.50 P_1$	$.50 \frac{t'+t''}{2}$

9.14 Axially Loaded Lug Design

This section presents procedures for the optimized design of lugs, bushings and pin in a symmetrical, double-shear joint, such as shown in Figure 9-5, subjected to a static axial load (P). One design procedure applies to the case where the pin is critical in shear, the other to the case where the pin is critical in bending. A method is given to help determine which mode of pin failure is more likely, so that the appropriate design procedure will be used.

Portions of the design procedures may be useful in obtaining efficient designs for joints other than symmetrical, double-shear joints.

9.14.1 Axial Lug Design for Pin Failure

An indication of whether the pin in an optimized joint design is more likely to fail in shear or in bending can be obtained from the value of R (Equation (9-32)). If R is less than 1.0, the pin is likely to fail in shear and the design procedure for joints with pins critical in shear should be used to get an optimized design. If R is greater than 1.0, the pin is likely to be critical in bending and the design procedures for joints with pins critical in bending should be used.

$$R = \frac{\pi F_{su_p}}{k_{b_p} F_{tu_p}} \left(\frac{F_{su_p}}{F_{br\,all\,1}} + \frac{F_{su_p}}{F_{br\,all\,2}} \right) \qquad (9\text{-}32)$$

where F_{su_p} and F_{tu_p} are the ultimate shear and ultimate tension stresses for the pin material, k_{b_p} is the plastic bending coefficient for the pin, and $F_{br\,all\,1}$

and $F_{br\,all_2}$ are allowable bearing stresses in the female and male lugs. The value of $F_{br\,all_1}$ can be approximated by the lowest of the following three values:

$$K\,F_{tux_1}\frac{D}{D_p}\ ;\ 1.304\,K\,F_{tyx_1}\frac{D}{D_p}\ ;\ 1.304\,F_{cy_{B_1}}$$

where F_{tux_1} and F_{tyx_1} are the cross-grain tensile ultimate and tensile yield stress for female lugs, $F_{cy_{B_1}}$ is the compressive yield stress of the bushings in the female lugs, and K is obtained from Figure 9-14. Assume $D = D_p$ if a better estimate cannot be made. $F_{br\,all_2}$ is approximated in a similar manner.

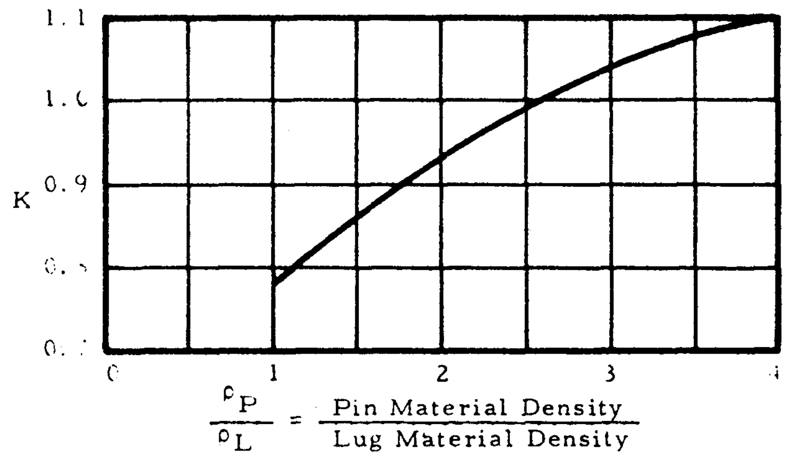


Figure 9-14. Allowable Bearing Coefficient

9.14.1.1 Axial Lug Design for Pin Failure in the Shearing Mode

Pin and Bushing Diameter

The minimum allowable diameter for a pin in double shear is

$$D_p = 0.798\sqrt{\frac{P}{F_{su_p}}} \tag{9-33}$$

The outside diameter of the bushing is $D = D_p + 2t_B$, where t_B is the bushing wall thickness.

Edge Distance Ratio (e/D)

The value of e/D that will minimize the combined lug and pin weight is obtained from Figure (9-15)(a) for the case where lug bearing failure and pin shear failure occur simultaneously. The lug is assumed not critical in net tension, and the bushing is assumed not critical in bearing.

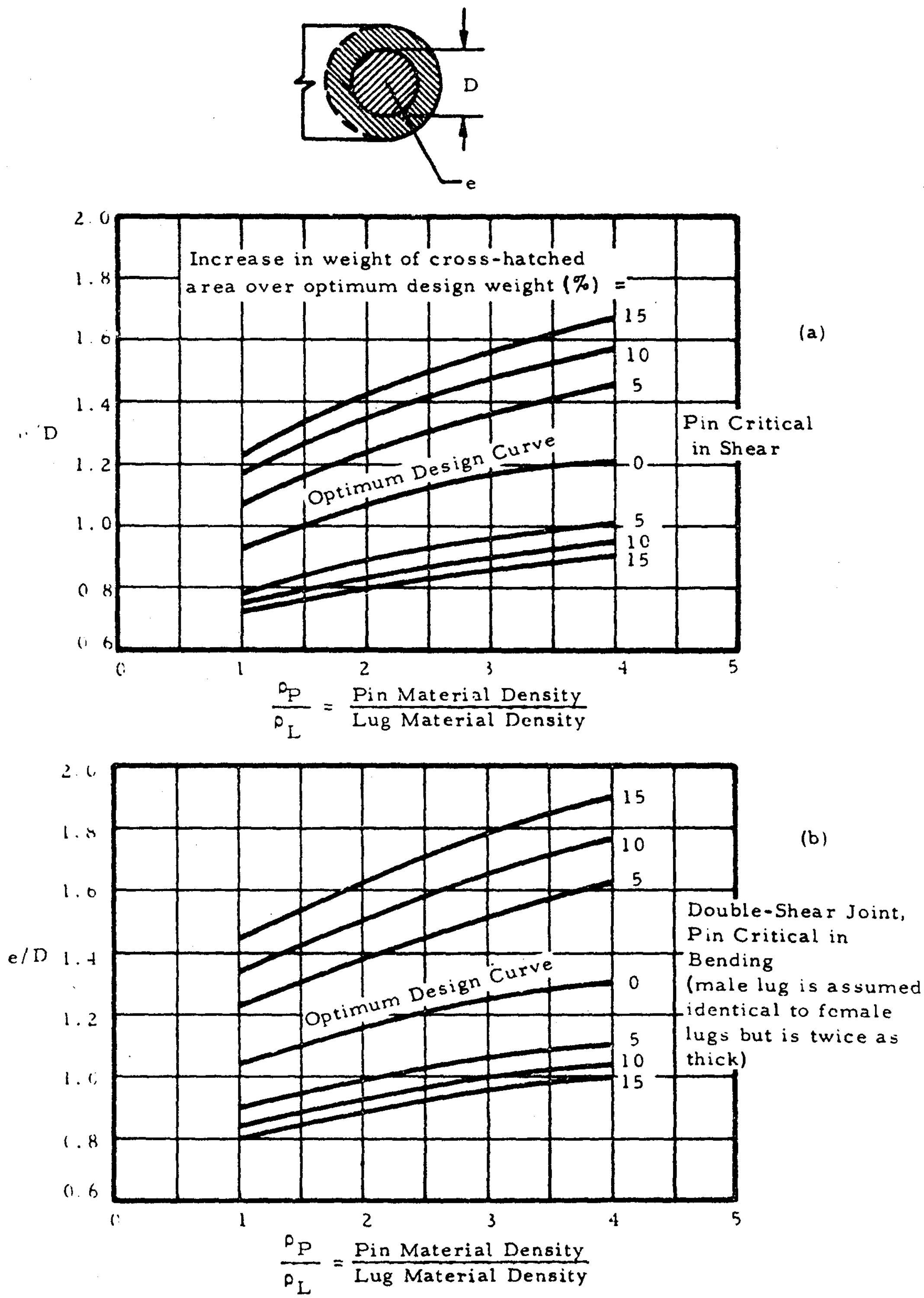


Figure 9-15. Edge Distance Ratio

The curves in Figure 9-15 apply specifically to concentric lugs ($a = e - D/2$, and $w = 2e$), but they can be used for reasonably similar lugs.

Allowable Loads

The allowable loads for the different failure modes (lug bearing failure, lug net-tension failure, and bushing failure) are determined from Equations (9-3), (9-6), and (9-9) in terms of the (unknown) lug thickness. The lowest of these loads is critical.

Lug Thicknesses

The required male and female lug thicknesses are determined by equating the applied load in each lug to the critical failure load for the lug.

Pin Bending

To prevent bending failure of the pin before lug or bushing failure occurs in a uniformly loaded symmetrical double-shear joint, the required pin diameter is

$$D_p = \sqrt[3]{\frac{2.55\ P}{k_{b_p}\ F_{tu_p}} \left(t_1 + \frac{t_2}{2} + 2g\right)} \qquad (9\text{-}34)$$

where k_{b_p} is the plastic bending coefficient for the pin. If the value of D_p from Equation (9-34) is greater than that from Equation (9-33), the joint must be redesigned because the pin is critical in bending.

Reduced Edge Distance

If the allowable bushing load (Equation (9-9)) is less than the allowable lug load (Equation (9-3)), a reduced value of e, obtained by using the curve shown in Figure 9-16 for optimum e/D, will give a lighter joint in which lug bearing failure and bushing bearing failure will occur simultaneously. The previously calculated pin diameter and lug thicknesses are unchanged.

Reduced Lug Width

If the lug net-tension strength (Equation (9-6)) exceeds the bearing strength (Equation (9-3)), the net-section width can be reduced by the ratio of the bearing strength to the net-tension strength.

9.14.1.2 Axial Lug Design for Pin Failure in the Bending Mode

Pin and Bushing Diameters (First Approximation)

A first approximation to the optimum pin diameter is shown in Equation (9-35).

$$D_p = \sqrt[4]{\frac{1.273}{k_{bp}} \left(\frac{P}{F_{tu_p}}\right)^2 \left(\frac{F_{tu_p}}{F_{t\,all_1}} + \frac{F_{tu_p}}{F_{t\,all_2}}\right)} \qquad (9\text{-}35)$$

where $F_{t\,all_1}$ is either F_{tux_1} or 1.304 F_{tyx_1}, whichever is smaller; and $F_{t\,all_2}$ is either F_{tux2} or 1.304 F_{tyx2}, whichever is smaller. This approximation becomes more accurate when there are no bushings and when there is no gap between lugs.

The first approximation to the outside diameter of the bushing is $D = D_p + 2t_B$.

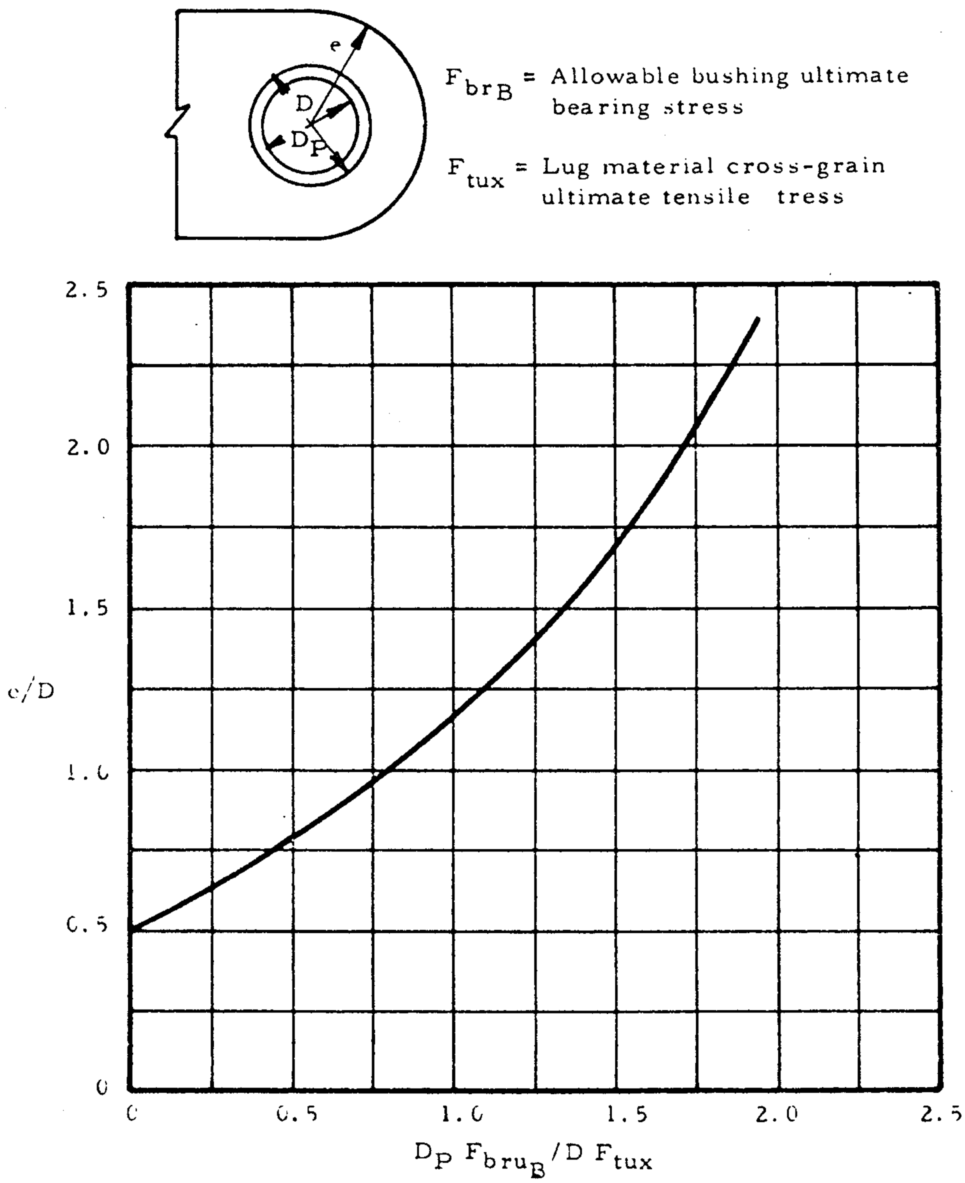


Figure 9-16. Edge Distance Ratio

Edge Distance Ratio (e/D)

The value of e/D that will minimize the combined lug and pin weight is obtained from Figure (9-15)(b) for the case of symmetrical double-shear joints in which lug bearing failure and pin bending failure occur simultaneously. The lug is assumed not critical in tension and the bushing is assumed not critical in bearing.

The curves apply specifically to concentric lugs ($a = e - D/2$, and $w = 2e$), but can be used for reasonably similar lugs.

Allowable Loads (First Approximation)

The allowable loads for the different failure modes (lug bearing failure, lug net-tension failure, and bushing failure) are determined from Equations (9-3), (9-6), and (9-9), in terms of the (unknown) lug thickness. The lowest of these loads is critical.

Lug Thicknesses (First Approximation)

The first approximation to the required male and female lug thicknesses are determined by equating the applied load in each lug to the lowest allowable load for the lug.

Pin Diameter (Second Approximation)

The second approximation to the pin diameter is obtained by substituting the first approximation lug thicknesses into Equation (9-34).

Final Pin and Bushing Diameters and Lug Thicknesses

The final optimum pin diameter is very closely approximated by

$$D_{p\ opt} = 1/3\ D_p\ (\text{Equation (9-35)}) + 2/3\ D_p\ (\text{Equation (9-34)}) \qquad (9\text{-}36)$$

An average value, however, is generally sufficient. If the final optimum value is not a standard pin diameter, choose the next larger standard pin and bushing.

The final lug thicknesses corresponding to the standard pin and bushing are then determined.

Pin Shear

The pin is checked for shear strength (Equation (9-33)).

Reduced Edge Distance

If the bushing bearing strength (Equation (9-9)) is less than the lug bearing strength (Equation (9-3)), a reduced value of e/D, obtained from the curve in Figure 9-16, will give a lighter joint. The pin diameter and lug thicknesses are unchanged.

Reduced Lug Width

If the lug net-tension strength (Equation (9-6)) exceeds the lug bearing strength (Equation (9-3)), the net-section width can be reduced by the ratio of the bearing strength to the net-tension strength.

9.14.1.3 Example of Axially Loaded Lug Design

Using the same materials for the lug, bushing and pin as mentioned in Section 9.6, and assuming the same allowable static load of 37900 pounds, a symmetrical double-shear joint will be designed to carry this load. A 0.10-inch gap is again assumed between the lugs. The bushing wall thickness is assumed to be 1/8 inch.

The lug will first be assumed to be concentric ($a = e - D/2$, and $w = 2e$) but the final minimum weight design will not necessarily be concentric.

Pin Failure Mode (Equation (9-32))

The pin is first checked to determine whether it will be critical in shear or bending, using Equation (9-32). Assuming $D = D_p$ as a first approximation, determine $F_{br\,all_1}$ and $F_{br\,all_2}$, using the graph in Figure 9-14 to determine K.

$$KF_{tux_1} = 1.02 \times 64000 = 65300 \text{ psi}; \; 1.304\, KF_{tyx_1}$$

$$= 1.304 \times 1.02 \times 40000 = 53100 \text{ psi};$$

$$1.304\, F_{cy_{B_1}} = 1.304 \times 60000 = 78200 \text{ psi}; \text{ therefore, } F_{br\,all_1} = 53100 \text{ psi}$$

$$KF_{tux_2} = 1.02 \times 77000 = 78500 \text{ psi}; \; 1.304\, KF_{tyx_2}$$

$$= 1.304 \times 1.02 \times 66000 = 87900 \text{ psi}$$

$$1.304\, F_{cy_{B_2}} = 1.304 \times 60000 = 78200 \text{ psi}; \text{ therefore } F_{br\,all_2} = 78200 \text{ psi}$$

Therefore,

$$R = \frac{\pi \times 82000}{1.56 \times 125000} \times \left(\frac{82000}{53100} + \frac{82000}{78200} \right) = 3.4 \text{ (Equation (9-32))}$$

Therefore, the design procedure for pins critical in bending applies.

Pin and Bushing Diameters - First Approximation (Equation (9-35))

$$D_p = \sqrt[4]{\frac{1.273}{1.56} \times \left(\frac{37900}{125000}\right)^2 \times \left(\frac{125000}{52160} + \frac{125000}{77000}\right)} = 0.741 \text{ in.}$$

$D = 0.741 + 2 \times 0.125 = 0.991$ in.

Edge Distance Ratiön (e/D)

The optimum value of e/D for both male and female lugs is 1.24 (Figure 9-15 (b)). Therefore a/D is 0.74 and w/D is 2.48 for a concentric lug (therefore, w = 2.46 in.).

Allowable Loads - Female Lugs and Bushings (First Approximation)

(a) Lug Bearing Strength (Equations (9-2a) and (9-36))

$P_{bru_{L_1}} = 1.304 \times 1.46 \times 0.74 \times 40000 \times 0.991\ t_1 = 55900\ t_1$ lbs.

where K = 1.46 is obtained from Figure 9-2 for e/D - 1.24

(b) Lug Net-Section Tension Strength (Equations (9-5) and (9-6b))

$K_{n_1} = 0.74$ (obtained by interpolation from the graphs shown in Figure 9-9-4) for

$$\frac{D}{w} = 0.403; \quad \frac{F_{ty}}{F_{tu}} = 0.625; \quad \frac{F_{tu}}{E\epsilon_u} = 0.051$$

$P_{nu_{L_1}} = 1.304 \times 40000 \times (2.46 - 0.991)\ t_1 = 56600\ t_1$ lbs.

(c) Bushing Bearing Strength (Equation (9-9))

$P_{u_{B_1}} = 1.304 \times 60000 \times 0.741\ t_1 = 58000\ t_1$ lbs.

Allowable Loads - Male Lug and Bushing (First Approximation)

(a) Lug Bearing Strength (Equations (9-1a) and (9-3a))

$$P_{bru_{L2}} = 1.46 \times 0.74 \times 77000 \times 0.991\ t_2 = 82500\ t_2 \text{ lbs.}$$

(b) Lug Net-Section Tension Strength (Equations (9-4) and (9-6a))

K_{n2} = 0.88 (obtained by interpolation from graphs shown in Figure 9-4) for

$$\frac{D}{w_2} = 0.403;\ \frac{F_{ty}}{F_{tu}} = 0.857;\ \frac{F_{tu}}{E\epsilon_u} = 0.125$$

$$P_{nu_{L2}} = 0.88 \times 77000 \times (2.46 - 0.991)\ t_2 = 99500\ t_2 \text{ lbs.}$$

(c) Bushing Bearing Strength (Equation (9-9))

$$P_{u_{B2}} = 1.304 \times 60000 \times 0.741\ t_2 = 58000\ t_2 \text{ lbs.}$$

Lug Thicknesses (First Approximation)

$$t_1 = \frac{37900}{2 \times 55900} = 0.339 \text{ in.};\ t_2 = \frac{37900}{58000} = 0.654 \text{ in.}$$

Pin Diameter - Second Approximation (Equation (9-34))

$$D_p = \sqrt[3]{\frac{2.55 \times 37900}{1.56 \times 125000}(0.339 + 0.327 + 0.200)} = 0.755 \text{ in.}$$

$$D = 0.7555 + 2 \times 0.125 = 1.0005 \text{ in.}$$

Final Pin and Bushing Diameter (Equation (9-36))

$$D_{popt} = \frac{0.741}{2} + \frac{0.755}{2} = 0.748 \text{ in. (Use 0.750 inch pin)}$$

$$D = 0.750 + 2 \times 0.125 = 1.000 \text{ in.}$$

Pin Shear (Equation (9-33))

$$D_p = 0.798 \sqrt{\frac{37900}{82000}} = 0.541 \text{ in.}$$

Therefore, the pin is not critical in shear.

Final Lug Thicknesses

$$t_1 = 0.339 \times \frac{0.991}{1.000} = 0.336 \text{ in.}$$

$$t_2 = 0.654 \times \frac{0.741}{0.750} = 0.646 \text{ in.}$$

Reduced Edge Distance

The lug tension strength (Equation (9-3)) exceeds the bushing strength (Equation (9-9)) for the male lug. Therefore, a reduced e/D can be obtained for the male lug shown in Figure 9-16.

$$\frac{D_p}{D} \frac{F_{bru_B}}{F_{tux}} = \frac{0.750}{1.000} \times \frac{1.304 \times 60000}{77000} = 0.762$$

Therefore, e/D = 0.97 (male lug)

Reduced Lug Width

The lug net-section tension strength (Equation (9-6)) exceeds the bearing strength (Equation (9-3)) for both the male and female lugs. Therefore, the widths can be reduced as follows:

$$w_1 = 1.00 + (2.48 - 1.00) \left(\frac{55900\ t_1}{56600\ t_1} \right) = 2.46 \text{ in.}$$

$$w_2 = 1.00 + (2.48 - 1.00) \left(\frac{82500\ t_2}{99500\ t_2} \right) = 2.23 \text{ in.}$$

Final Dimensions

D_p = 0.750 in.; D = 1.000 in.

t_1 = 0.336 in.; e_1 = 1.24 in.; w_1 = 2.46 in.

t_2 = 0.646 in.; e_2 = 0.97 in.; w_2 = 2.23 in.

Since w_2 is larger than $2e_2$, the final male lug is not concentric.

9.15 Analysis of Lugs with Less Than 5 PCT Elongation

The procedures given through Section 9-14 for determining the static strength of lugs apply to lugs made from materials which have ultimate elongations, ϵ_u, of at least 5% in all directions in the plane of the lug. This section describes procedures for calculating reductions in strength for lugs made from materials which do not meet the elongation requirement. In addition to using these procedures, special consideration must be given to possible further loss in strength resulting from material defects when the short transverse gain direction of the lug material is in the plane of the lug.

The analysis procedures for lugs made from materials without defects but with less than 5% elongation are as follows:

9.15.1 Bearing Strength of Axially Loaded Lugs with Less Than 5 PCT Elongation

(1) Determine F_{ty}/F_{tu} and ϵ_y/ϵ_u, using values of F_{ty}, F_{tu}, ϵ_y, and ϵ_u that correspond to the minimum value of ϵ_u in the plane of the lug.

(2) Determine the value of B, the ductility factor, from the graph shown in Figure 9-17.

(3) Determine a second value of B (denoted by $B_{.05}$) for the same values of F_{ty}, F_{tu}, and ϵ_y as before, but with $\epsilon_u = 0.05$.

(4) Multiply the bearing stress and bearing load allowables given by Equations (9-1a) through (9-3b) by $B/B_{.05}$ to obtain the corrected allowables.

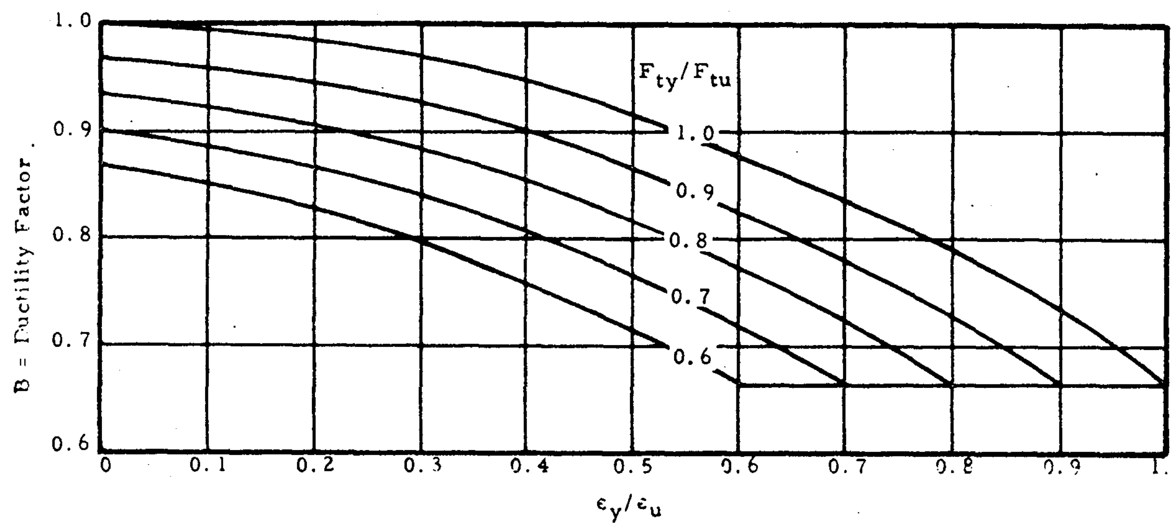


Figure 9-17. Ductility Factor

9.15.2 Net-Section Strength of Axially Loaded Lugs with Less Than 5 PCT Elongation

The procedure for determining net-section allowables is the same for all values of ϵ_u. The graphs in Figure 9-4 are used to obtain a value of K_n which is susbstituted in Equations (9-4) and (9-5). If the grain direction of the material is known, the values of F_{ty}, F_{tu}, and ϵ_u used in entering the graphs should correspond to the grain direction parallel to the load. Otherwise, use values corresponding to the minimum value of ϵ_u in the plane of the lug.

9.15.3 Strength of Lug Tangs in Axially Loaded Lugs with Less Than 5 PCT Elongation

The plastic bending coefficient for a rectangular cross section can be approximated by $k_{b_L} = 1.5B$, where B is obtained from Figure 9-17, in which y and u are the yield and ultimate strains of the lug tang material in the direction of loading. The maximum allowable value of k_{b_L} for a rectangle is 1.4.

9.15.4 Lug-Bushing Strength in Axially-Loaded Single-Shear Joint with Less Than 5 PCT Elongation

The values of k_{br_L} and k_{b_L} for rectangular cross sections are approximated by 1.5B, where B is determined from the graph as described in Figure 9-17. The maximum allowable values of k_{br_L} and k_{b_L} are 1.4.

9.15.5 Bearing Strength of Transversely Loaded Lugs with Less Than 5% Elongation (Equations (9-28) through (9-30b) in Section 9.7.1

The same procedure as that for the bearing strength of axially loaded lugs is used.

(1) Determine B and $B_{.05}$ as described for axially loaded lugs, where B corresponds to the minimum value of ϵ_u in the plane of the lug.

(2) Multiply the bearing stress and bearing load allowables given by Equations (9-28) through (9-30b) by $B/B_{0.5}$ to obtain the corrected allowables.

9.16 Stresses Due to Press Fit Bushings

Pressure between a lug and bushing assembly having negative clearance can be determined from consideration of the radial displacements. After assembly, the increase in inner radius of the ring (lug) plus the decrease in outer radius of the bushing equals the difference between the radii of the bushing and ring before assembly:

$$\delta = u_{ring} - u_{bushing} \qquad (9\text{-}36)$$

where

δ = Difference between outer radius of bushing and inner radius of the ring.

u = Radial displacement, positive away from the axis of ring or bushing.

Radial displacement at the inner surface of a ring subjected to internal pressure p is

$$u = \frac{D_p}{E_{ring}} \left[\frac{C^2 + D^2}{C^2 - D^2} + \mu_{ring} \right] \qquad (9\text{-}37)$$

Radial displacement at the outer surface of a bushing subjected to external pressure p is

$$u = -\frac{B_p}{E_{bush.}} \left[\frac{B^2 + A^2}{B^2 - A^2} - \mu_{bush.} \right] \qquad (9\text{-}38)$$

where

A = Inner radius of bushing	D = Inner radius of ring (lug)
B = Outer radius of bushing	E = Modulus of elasticity
C = Outer radius of ring (lug)	μ = Poisson's ratio

Substitute Equations (9-37) and (9-38) into Equation (9-36) and solve for p;

$$p = \frac{\delta}{\frac{D}{E_{ring}}\left(\frac{C^2 + D^2}{C^2 - D^2} + \mu_{ring}\right) + \frac{B}{E_{bush.}}\left(\frac{B^2 + A^2}{B^2 - A^2} - \mu_{bush.}\right)}$$

Maximum radial and tangential stresses for a ring subjected to internal pressure occur at the inner surface of the ring (lug).

$$F_r = -p \qquad F_t = p\left[\frac{C^2 + D^2}{C^2 - D^2}\right]$$

Positive sign indicates tension. The maximum shear stress at this point is

$$F_s = \frac{F_t - F_r}{2}$$

The maximum radial stress for a bushing subjected to external pressure occurs at the outer surface of the bushing is

$$F_r = -p$$

The maximum tangential stress for a bushing subjected to external pressure occurs at the inner surface of the bushing is

$$F_t = -\frac{2pB^2}{B^2 = A^2}$$

The allowable press fit stress may be based on:

(1) Stress Corrosion. The maximum allowable press fit stress in magnesium alloys should not exceed 8000 psi. For all aluminum alloys the maximum press fit stress should not exceed 0.50 F_{ty}.

(2) Static Fatigue. Static fatigue is the brittle fracture of metals under sustained loading, and in steel may result from several different phenomena, the most familiar of which is hydrogen embrittlement. Steel parts heat treated above 200 ksi, which by nature of their function or other considerations are exposed to hydrogen embrittlement, should be designed to an allowable press fit stress of 25% F_{tu}.

(3) Ultimate Strength. Ultimate strength cannot be exceeded, but is not usually critical in a press fit application.

(4) Fatigue Life. The hoop tension stresses resulting from the press fit of a bushing in a lug will reduce the stress range for oscillating loads, thereby improving fatigue life.

The presence of hard brittle coatings in holes that contain a press fit bushing or bearing can cause premature failure by cracking of the coating or by high press fit stresses caused by build-up of coating. Therefore, Hardcoat or HAE coatings should not be used in holes that will subsequently contain a press fit bushing or bearing.

Figures 9-18 and 9-19 permit determining the tangential stress, F_T, for bushings pressed into aluminum rings. Figure 9-18 presents data for general steel bushings, and Figure 9-19 presents data for the NAS 75 class bushings. Figure 9-20 gives limits for maximum interference fits for steel bushings in magnesium alloy rings.

9.17 Lug Fatigue Analysis

A method for determining the fatigue strength of 2024-T3 and 7075-T6 aluminum alloy lugs under axial loading is presented.

Figures 9-21 and 9-22 show the lug and the range of lug geometries covered by the fatigue strength prediction method. Fatigue lives for lugs having dimensional ratios falling outside the region shown should be corroborated by tests.

In this method the important fatigue parameters are k_1, k_2, and k_3 (see Figure 9-23).

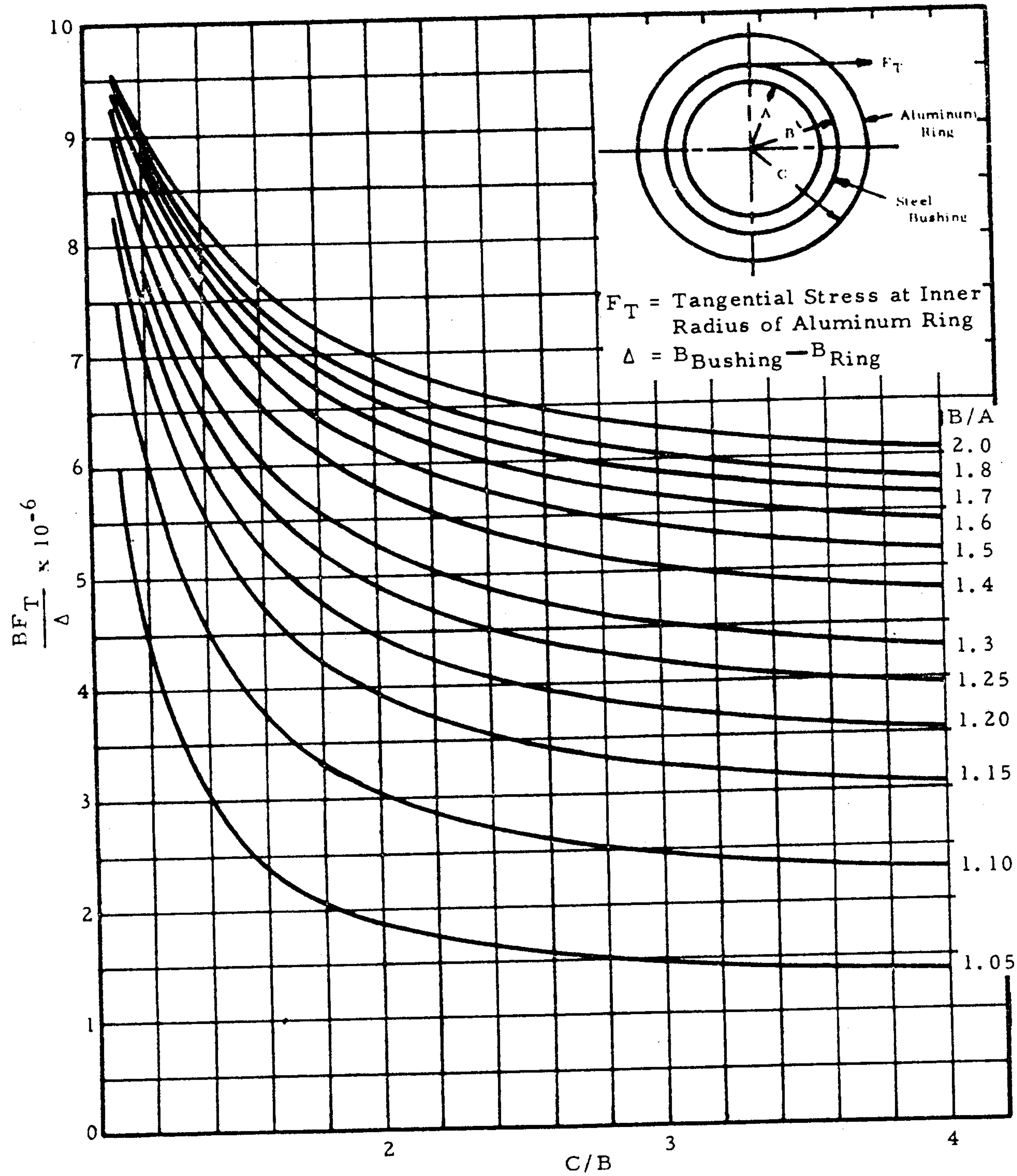


Figure 9-18. Tangential Stresses for Pressed Steel Bushings in Aluminum Rings

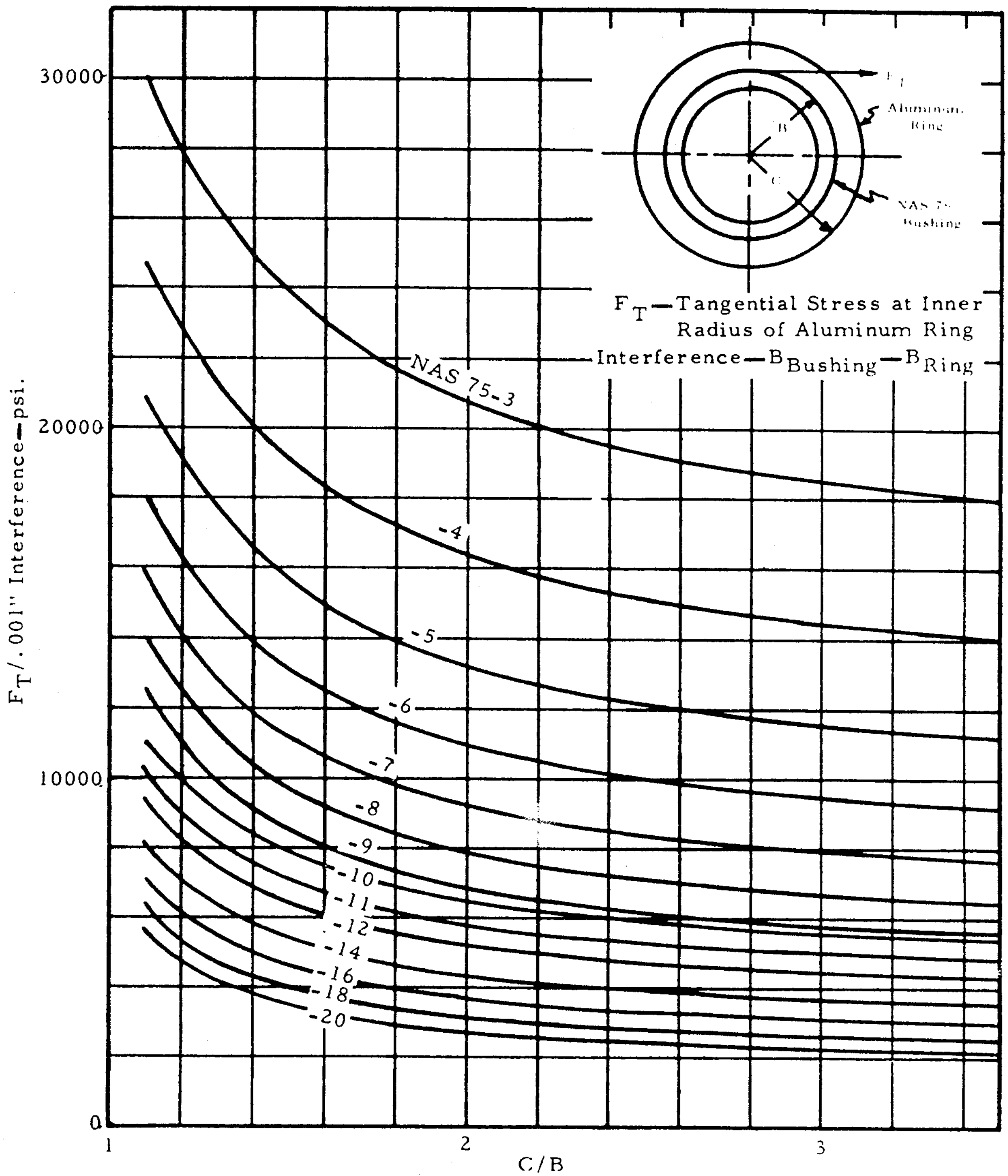
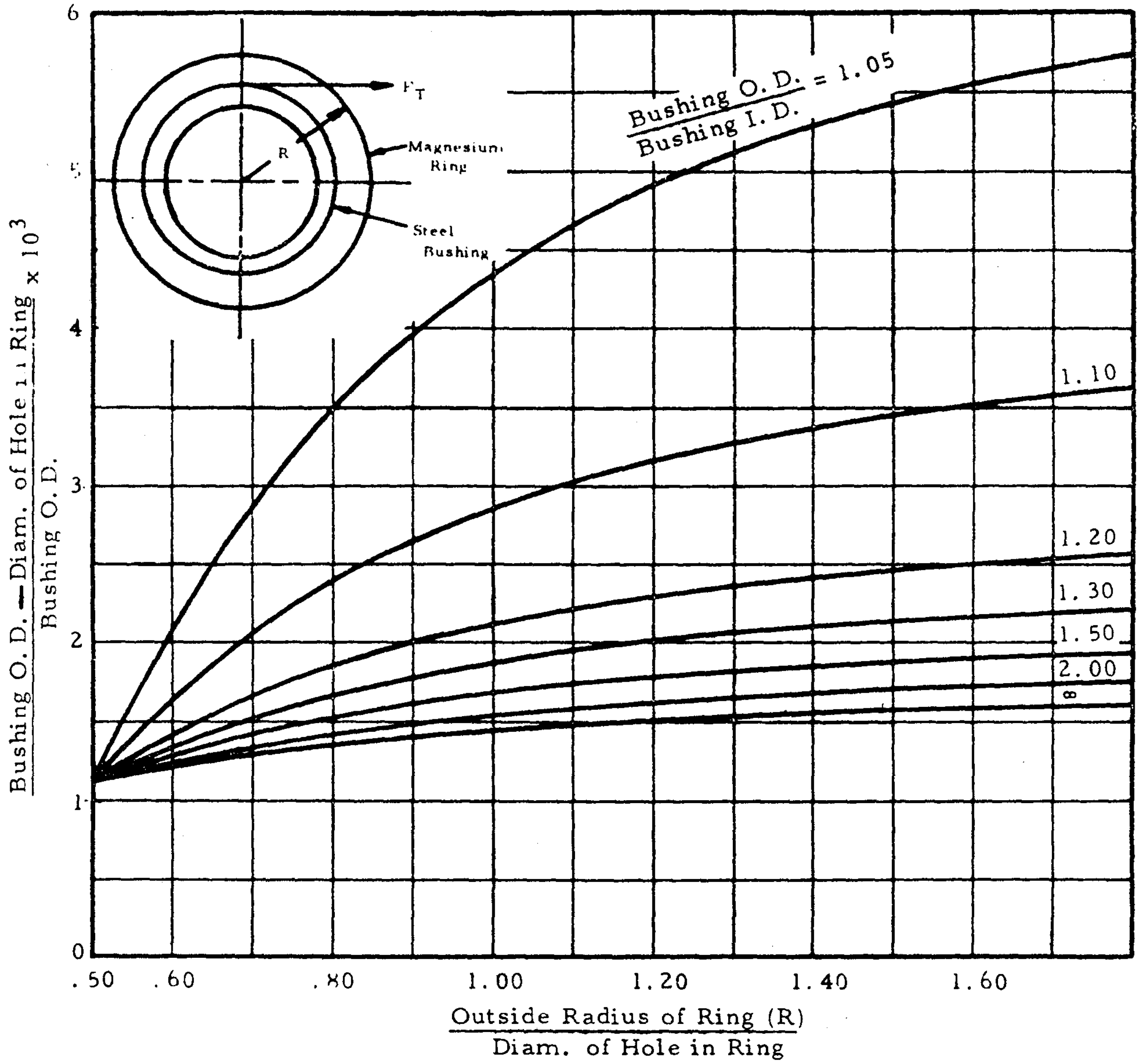


Figure 9-19. Tangential Stresses for Pressed NAS 75 Bushings



O. D. of the bushing is the after-plating diameter of the bushing.

The curves are based upon a maximum allowable interference tangential stress of 8000 psi.

Figure 9-20. Maximum Interference Fits of Steel Bushings in Magnesium Alloy Rings

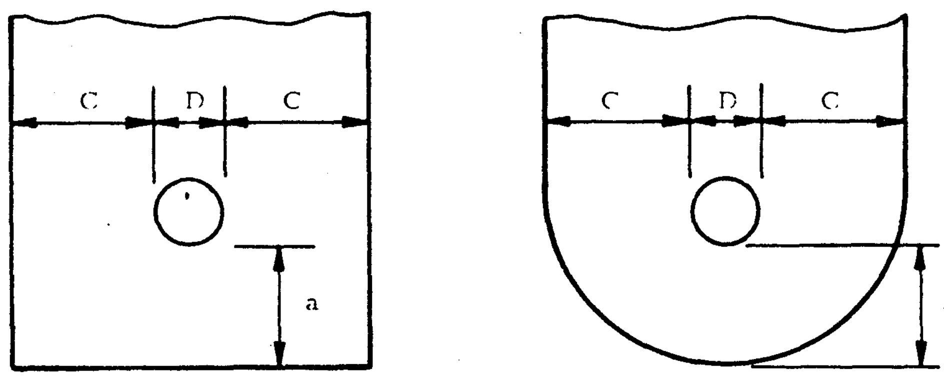


Figure 9-21. Lug Geometry for Fatigue Analysis

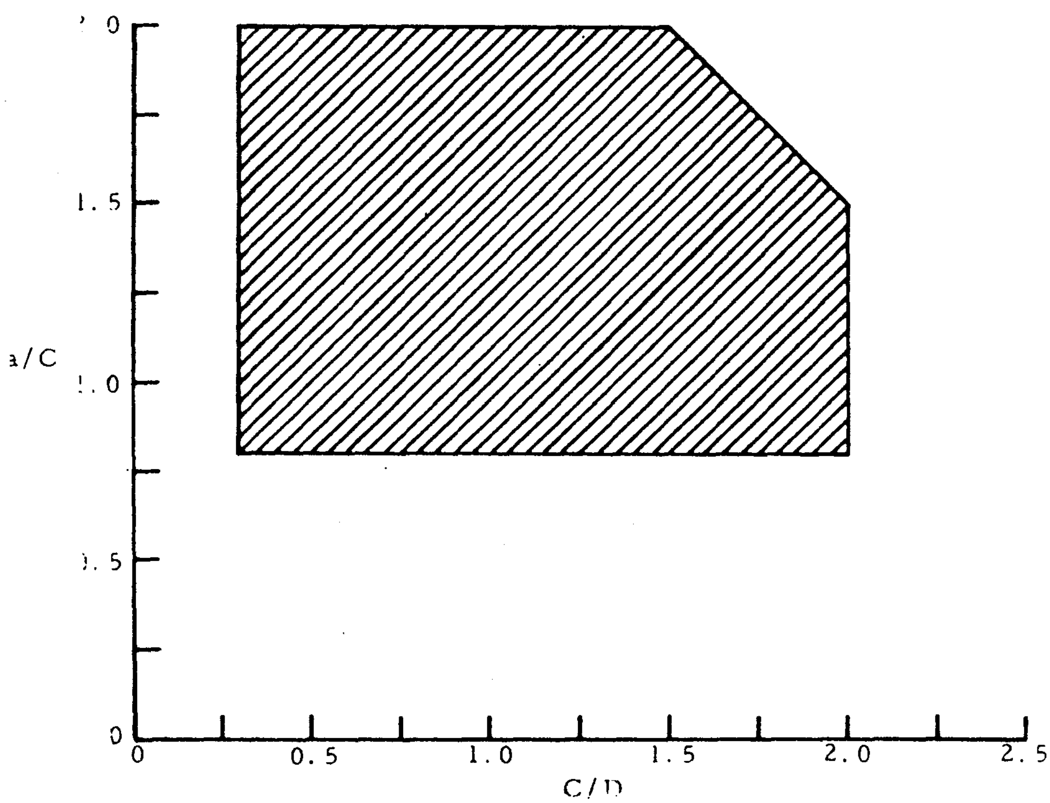


Figure 9-22. Region of Lug Geometries Covered by Fatigue Prediction Method

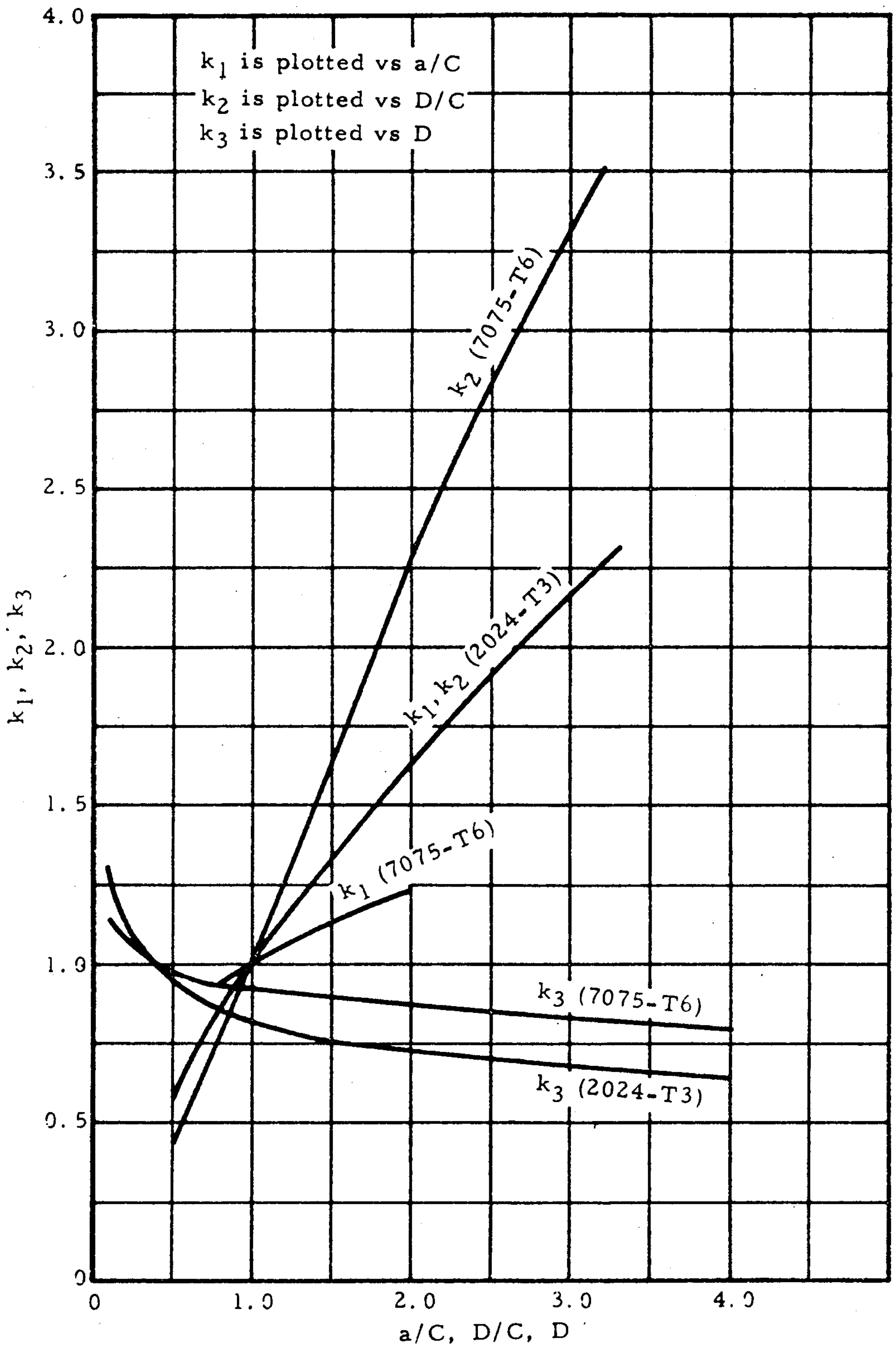


Figure 9-23. Parameters To Be Used in Figure 9-24 for Lug Fatigue Analysis

To find the allowable life knowing the applied stresses and lug dimensions, or to find the allowable stresses knowing the life, R value ($R = f_{min}/f_{max}$) and, lug dimensions, use the following procedure:

(1) Enter Figure 9-22 to check that the lug dimensional ratios fall within the region covered by the method. Enter Figure 9-23 and read k_1, k_2, and k_3; calculate the product $k_1k_2k_3$.

(2) Calculate the allowable net-tension static stress for the lug, F_{nu_L}, according to the method described in Section 9. 3. 2.

(3) Determine the value 0.4 F_{nu_L}. This is the alternating stress corresponding to a maximum stress value of 0.8 F_{nu_L} when $f_{min} = 0$. 0.8 F_{nu_L} was chosen as an average yield stress value for 2024 and 7075 aluminum alloy lugs.

(4) Using the value 0.4 F_{nu_L} as an alternating stress, draw a straight line between the intersection of this value and the appropriate $k_1k_2k_3$ curve on Figures 9-24 or 9-25, and the point 0.5 F_{nu_L} at 1 cycle. This extends the $k_1k_2k_3$ curve to cover the entire life range to static failure.

(5) Enter Figure 9-24 or 9-25 (lug fatigue curves for the case where R = 0) with $k_1k_2k_3$. For values of life, $N = 10^3$, 3×10^3, 10^4, etc., or any other convenient values, determine the corresponding values of f_a, the stress amplitude causing fatigue failure when R = 0.

(6) Plot the values of f_a found in Step 5 along the R = 0 line in a Goodman diagram such as shown in Figure 9-26 ($f_m = f_a$ when R = 0). The Goodman diagram shown in Figure 9-27 applies to a particular 7075-T6 lug for which $k_1k_2k_3 = 1.32$ (see example problem 1), but is typical of all such diagrams.

(7) Plot the allowable net-tension static stress found in Step 2 as f_m at the point (f_m, 0) of the Goodman diagram ($f_m = f_{max}$ when $f_a = 0$). For the case considered in Figure 9-26, this point is plotted as (f_m = 70,000 psi, $f_a = 0$).

(8) Connect the point plotted in Step 7 with each of the points plotted in Step 6 by straight lines. These are the constant life lines for the particular lug being analyzed. The Goodman diagram is now complete and may be used to determine a life for any given applied stresses, or to determine allowable stresses knowing the life and R value.

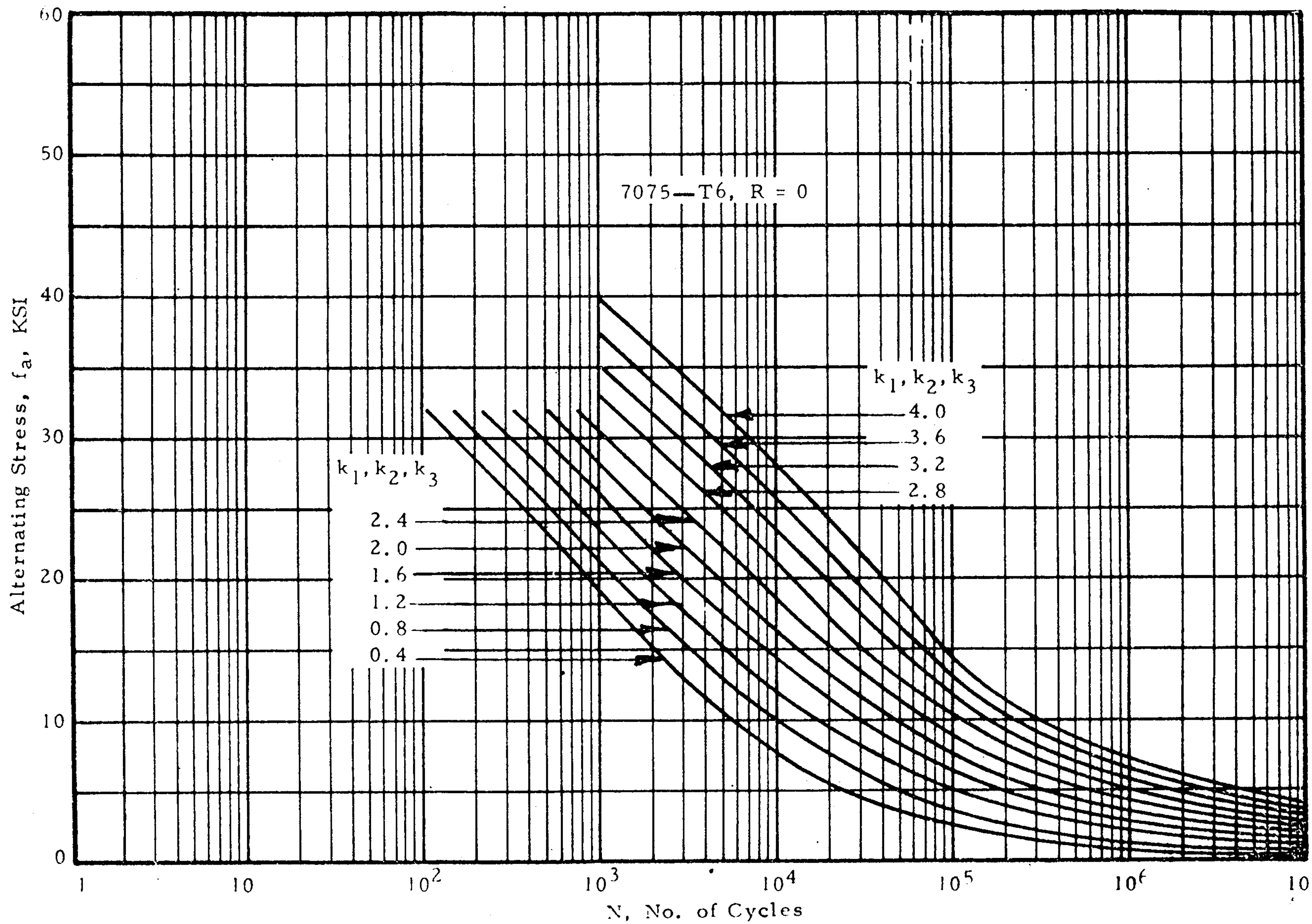


Figure 9-24. Fatigue Life Curves for Constant k_1, k_2, k_3 Values; 7075—T6, R = 0

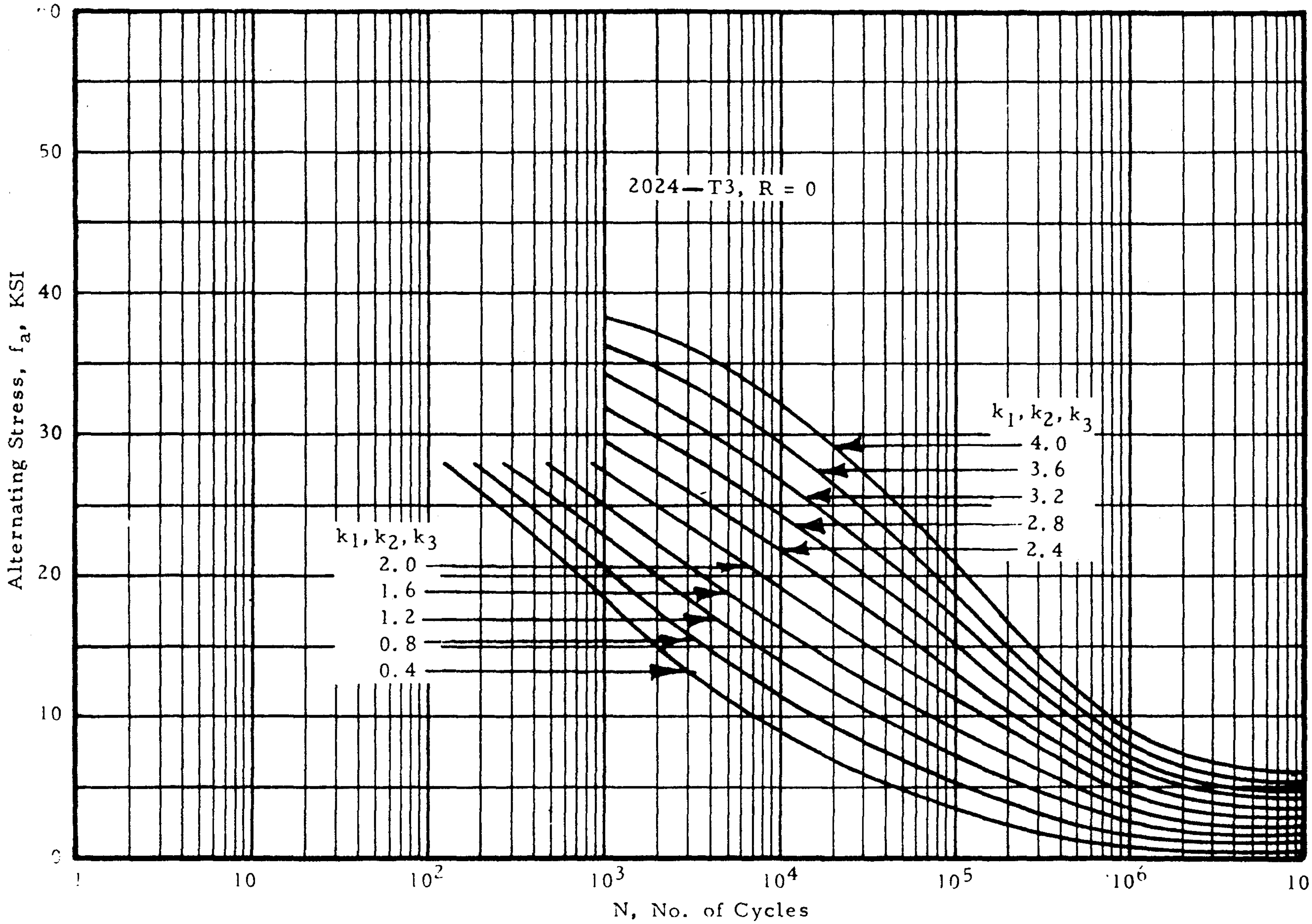


Figure 9-25. Fatigue Life Curves for Constant k_1, k_2, k_3 Values; 2024—T3, R = 0

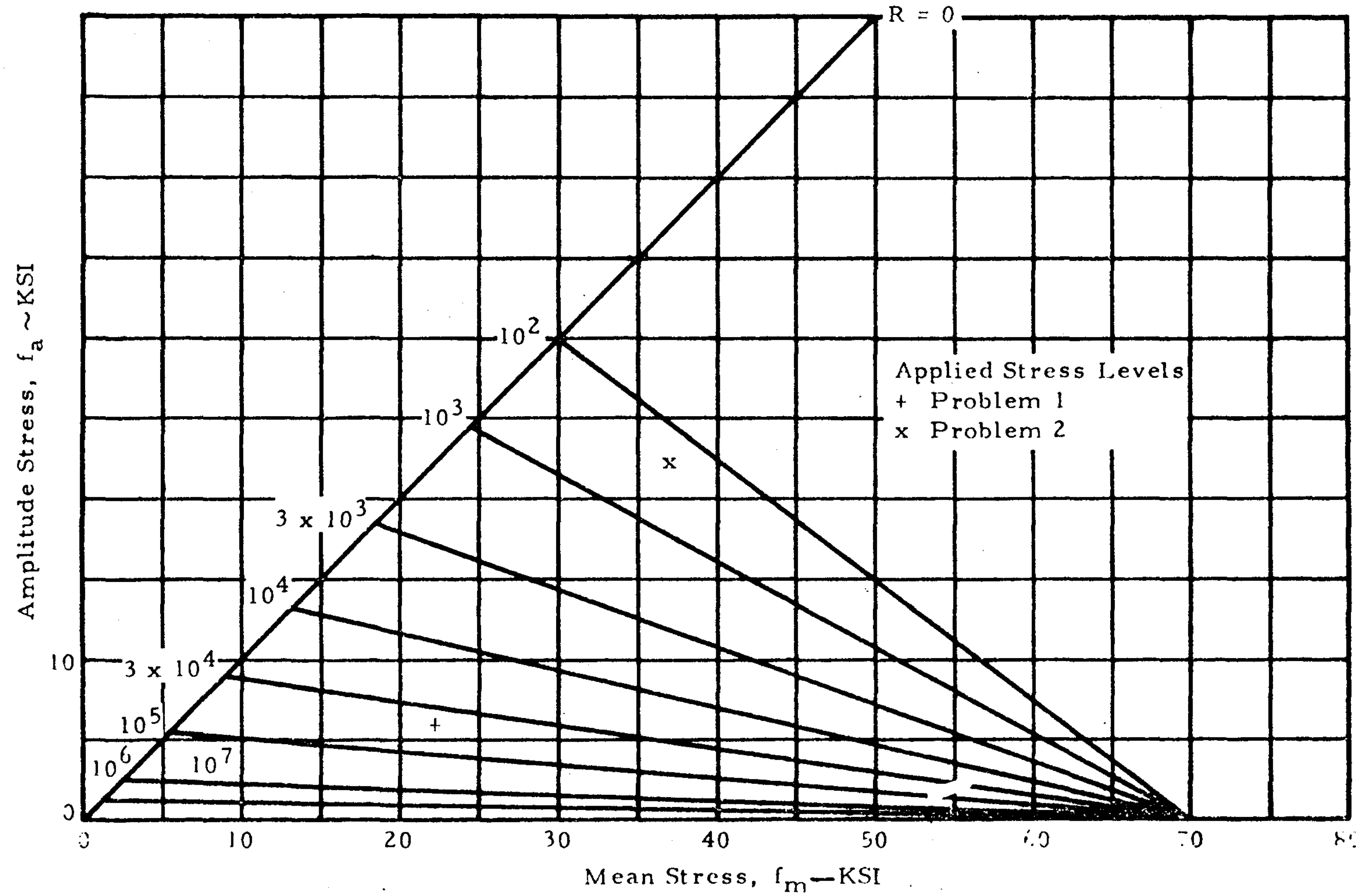


Figure 9-26. Goodman Diagram for Example Problems

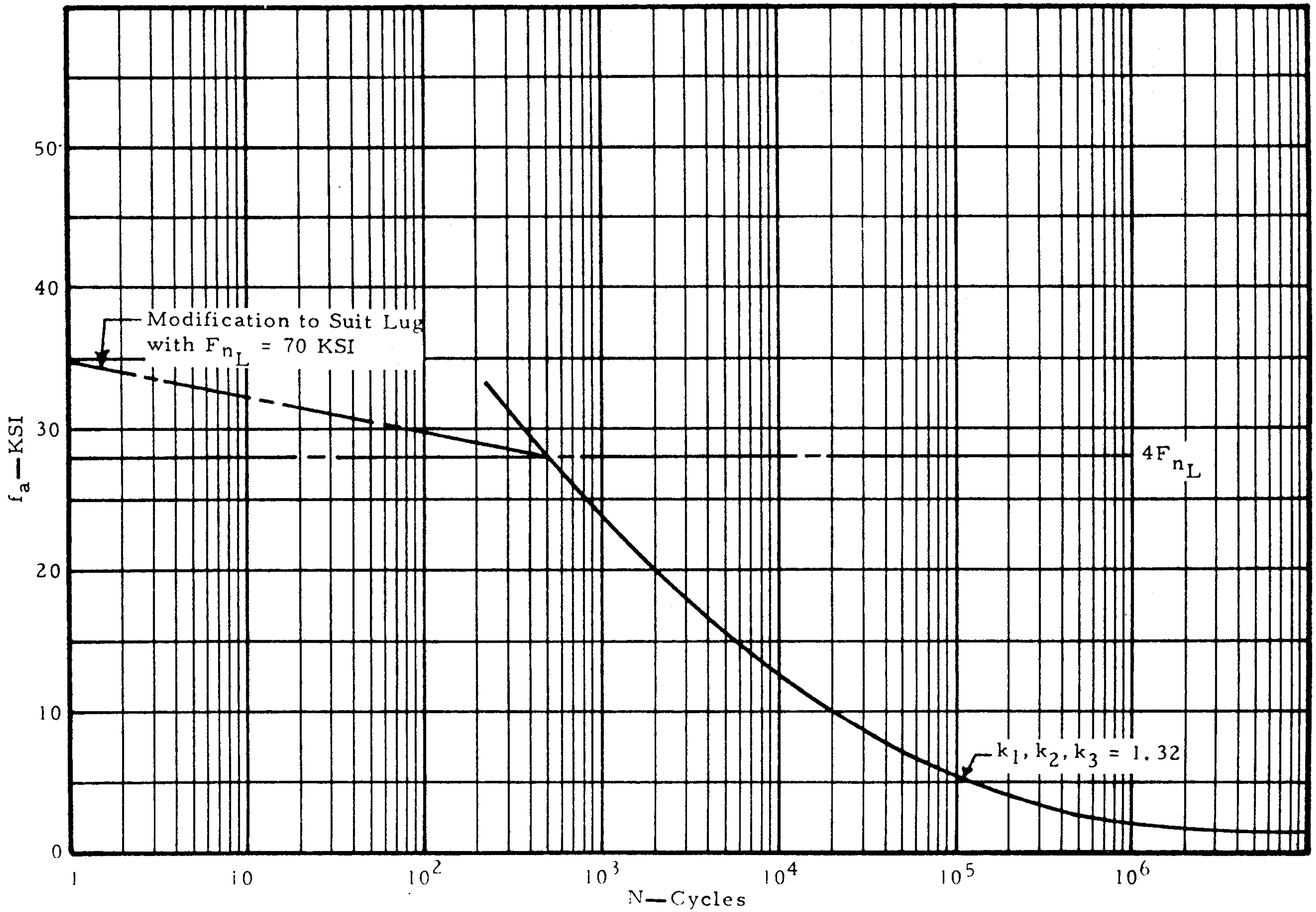


Figure 9-27. Modification to Curve k_1, k_2, k_3 = 1.32

9.18 Example Problem of Lug Fatigue Analysis

Given a concentric 7075-T6 aluminum lug as shown in Figure 9-21, with the following dimensions: a = 0.344 in, c = 0.3444 in, and D = 0.437 in. If the lug is subjected to a cycle axial load such that the maximum net-section stress is 27,000 psi and the minimum net section stress is 18,470 psi, find the fatigue life.

From the lug dimensions,

$a/c = 1.0 \quad c/D = 0.787 \quad (D/c = 1.27)$

(1) Figure 9-22 indicates that the lug may be analyzed using this method. From Figure 9-23,

$k_1 = 1.0;\ k_2 = 1.33;\ k_3 = 0.99;\ k_1k_2k_3 = 1.32$

(2) Calculate the allowable net-section tensile ultimate stress, F_{nu_L}, for Equation (9-4) in Section 9.3.2. For the given lug, F_{nu_L} = 70,000 psi.

(3) $0.4\ F_{nu_L} = 0.4 \times 70,000 = 28,000$ psi.

(4) Draw a light pencil line on Figure 9-24 from the point (f_a = 28,000 psi on $k_1k_2k_3$ = 1.32) to the point (f_a = 35,000, N = 1 cycle) (This is illustrated, for clarity, on Figure 9-27).

(5) Enter Figure 9-24 and read values of f_a for various numbers of life cycles, using the line $k_1k_2k_3$ = 1.32. These numbers are as follows:

N	10^2	10^3	3×10^3	10^4	3×10^4	10^5	10^6	10^7
f_a	30KSI	24.5	18.8	13.5	8.88	5.70	2.34	1.30

(6) Plot the values of f_a along the R = 0 line of the Goodman diagram. (Refer to Figure 9-26.)

(7) Plot F_{n_L} = 70,000 psi, as f_m at the point (f_m, 0) of the Goodman diagram. (Refer to Figure 9-26.)

(8) Connect the points plotted in Step 6 with the point plotted in Step 7 by straight lines. The Goodman diagram is now complete.

(9) Enter the Goodman diagram with values of $f_a = \frac{27,700 - 18,470}{2} =$ 4,615 psi and $f_m = \frac{27,700 + 18,470}{2} = 23,085$ psi, and read the fatigue life, $N = 8 \times 10^4$ cycles, by interpolation (test results show $N = 8.6 \times 10^4$ cycles).

If the known quantities are life and R value, e.g., $N = 10^4$ cycles and $R = 0$, the allowable stresses can be obtained by using the same Goodman diagram. Enter the completed Goodman diagram at $R = 0$ and $N = 10^4$ cycles and read the amplitude and mean stresses (in this case $f_a = f_m = 13,500$ psi).

Only if the lug dimensions are changed, must a new Goodman diagram be drawn.