```
clear
```
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```
axe:=5-a*x  // axe is error between a*x and 5
```
$5-a\,x$

```
bxe:=10-b*x // axe is error between b*x and 10
```
$10-b\,x$

```
aye:=15-a*y  // aye is error between a*y and 15
```
$15-a\,y$

```
bye:=20-b*y // bye is error between b*y and 20
```
$20-b\,y$

```
Z:=axe^2+bxe^2+aye^2+bye^2; // Z is sum of square errors
```
$(a\,x-5)^2+(b\,x-10)^2+(a\,y-15)^2+(b\,y-20)^2$

Set derivative of objective function wrt each coordinate to zero to find local max/min/saddle of Z

```
eq1:=diff(Z,x)=0
```
$2\,a\,(a\,x-5)+2\,b\,(b\,x-10)=0$

```
eq2:=diff(Z,y)=0;
```
$2\,a\,(a\,y-15)+2\,b\,(b\,y-20)=0$

```
eq3:=diff(Z,a)=0
```
$2\,x\,(a\,x-5)+2\,y\,(a\,y-15)=0$

```
eq4:=diff(Z,b)=0
```
$2\,x\,(b\,x-10)+2\,y\,(b\,y-20)=0$

```
soln:=solve({eq1,eq2,eq3,eq4},{x,y,a,b});
```
$$\begin{pmatrix} a\\ b\\ x\\ y\end{pmatrix}\in\left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}\cup\left\{\left.\begin{pmatrix}\frac{3315\,z-205\,\sqrt{221}\,z}{442\,z^2}\\ \frac{2210\,z-155\,\sqrt{221}\,z}{221\,z^2}\\ -\frac{10\,z}{11}-\frac{\sqrt{221}\,z}{11}\\ z\end{pmatrix}\right|\,z\in\mathbb{C}\setminus\{0\}\right\}\cup\left\{\left.\begin{pmatrix}\frac{3315\,z+205\,\sqrt{221}\,z}{442\,z^2}\\ \frac{2210\,z+155\,\sqrt{221}\,z}{221\,z^2}\\ \frac{\sqrt{221}\,z}{11}-\frac{10\,z}{11}\\ z\end{pmatrix}\right|\,z\in\mathbb{C}\setminus\{0\}\right\}$$

Above shows 0000 is local max/min/saddle... not interesting.
We have two solutions which we will call solution 1 and 2, both parameterized in terms of new variable z which can take on any real or complex value (l.c. z different han U.C. Z)

```
soln1:= soln[2][2]
```
$$\left\{\left.\begin{pmatrix}\frac{3315\,z-205\,\sqrt{221}\,z}{442\,z^2}\\ \frac{2210\,z-155\,\sqrt{221}\,z}{221\,z^2}\\ -\frac{10\,z}{11}-\frac{\sqrt{221}\,z}{11}\\ z\end{pmatrix}\right|\,z\in\mathbb{C}\setminus\{0\}\right\}$$

```
soln2:=soln[2][3]
```
$$\left\{\left.\begin{pmatrix}\frac{3315\,z+205\,\sqrt{221}\,z}{442\,z^2}\\ \frac{2210\,z+155\,\sqrt{221}\,z}{221\,z^2}\\ \frac{\sqrt{221}\,z}{11}-\frac{10\,z}{11}\\ z\end{pmatrix}\right|\,z\in\mathbb{C}\setminus\{0\}\right\}$$

```
subs1:=[a=op(soln1,1)[1],b=op(soln1,1)[2],x=op(soln1,1)[3],y=op(soln1,1)[4]]
```
$$\left[a=\frac{3315\,z-205\,\sqrt{221}\,z}{442\,z^2},\ b=\frac{2210\,z-155\,\sqrt{221}\,z}{221\,z^2},\ x=-\frac{10\,z}{11}-\frac{\sqrt{221}\,z}{11},\ y=z\right]$$

```
Z1:=Z|subs1
```

$$\left(\frac{\sigma_2}{221\,z}-20\right)^2+\left(\frac{\sigma_3}{442\,z}-15\right)^2+\left(\frac{\sigma_1\,\sigma_2}{221\,z^2}+10\right)^2+\left(\frac{\sigma_1\,\sigma_3}{442\,z^2}+5\right)^2$$

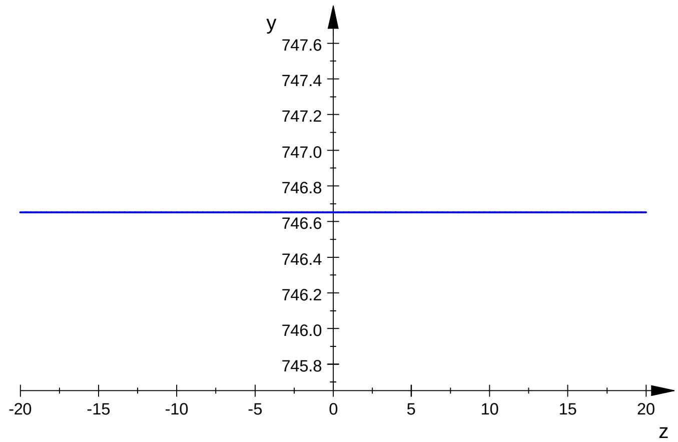
where

$$\sigma_1=\frac{10\,z}{11}+\frac{\sqrt{221}\,z}{11}$$

$$\sigma_2=2210\,z-155\,\sqrt{221}\,z$$

$$\sigma_3=3315\,z-205\,\sqrt{221}\,z$$

```
plot(Z1,z=-20..20)
```

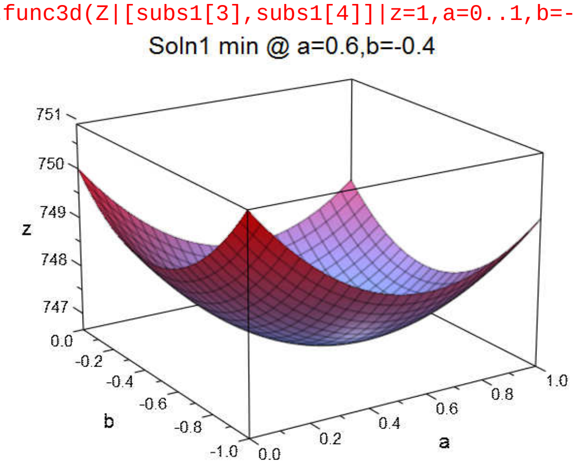



```
float(subs1|z=1)
```

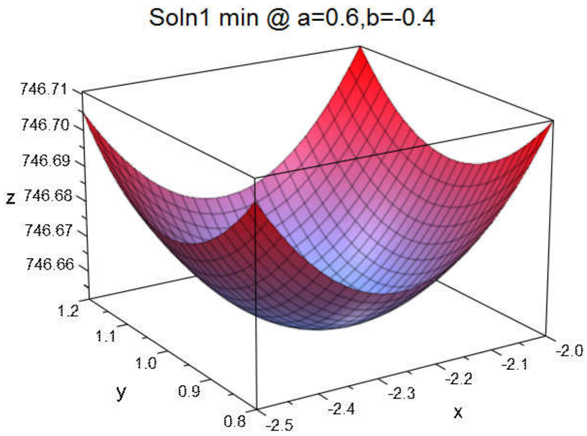
$[a=0.6051038615,\ b=-0.4264283069,\ x=-2.260551704,\ y=1.0]$

```
use(plot)  // Load plotting routines
Warning: 'hull' already has a value, not exported. [use]
Warning: 'Integral' seems to be proteced; not exported. [use]
Warning: 'Pyramid' seems to be proteced; not exported. [use]
```

```
plotfunc3d(Z|[subs1[3],subs1[4]]|z=1,a=0..1,b=-1..0,Header="Soln1 min @ a=0.6,b=-0.4")
```




```
plotfunc3d(Z|[subs1[1],subs1[2]]|z=1,x=-2.5..-2,y=0.8..1.2,Header="Soln1 min @ a=0.6,b=-0.4")
```




```
subs2:=[a=op(soln2,1)[1],b=op(soln2,1)[2],x=op(soln2,1)[3],y=op(soln2,1)[4]]
```

$$\left[a = \frac{3315\,z + 205\,\sqrt{221}\,z}{442\,z^2},\ b = \frac{2210\,z + 155\,\sqrt{221}\,z}{221\,z^2},\ x = \frac{\sqrt{221}\,z}{11} - \frac{10\,z}{11},\ y = z\right]$$

```
Z2:=Z|subs2
```

$$\left(\frac{\sigma_2}{221\,z} - 20\right)^2 + \left(\frac{\sigma_3}{442\,z} - 15\right)^2 + \left(\frac{\sigma_1\,\sigma_2}{221\,z^2} + 10\right)^2 + \left(\frac{\sigma_1\,\sigma_3}{442\,z^2} + 5\right)^2$$

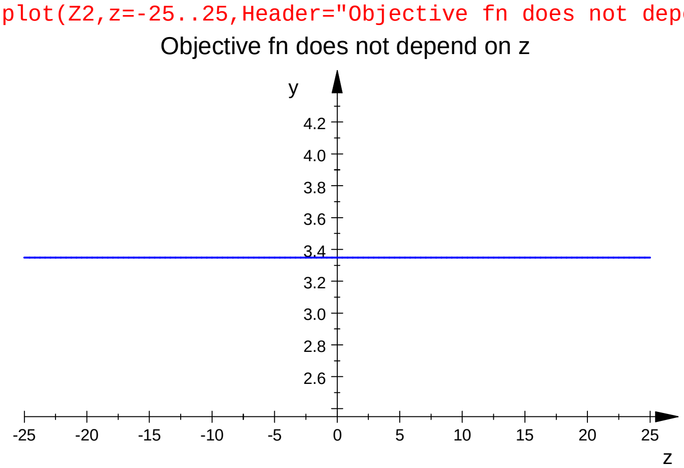
where

$$\sigma_1 = \frac{10\,z}{11} - \frac{\sqrt{221}\,z}{11}$$

$$\sigma_2 = 2210\,z + 155\,\sqrt{221}\,z$$

$$\sigma_3 = 3315\,z + 205\,\sqrt{221}\,z$$

```
plot(Z2,z=-25..25,Header="Objective fn does not depend on z")
```



From above plots we suspect solution 2 is the local minimum.
We also note that the objective function Z is constant wrt z (flat graph) for both solution 1 and solution 2

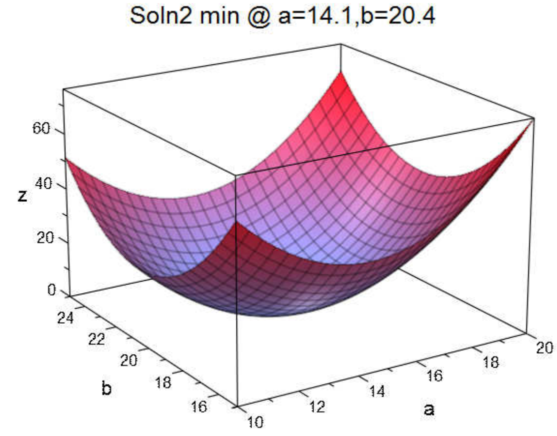
Here is solution 2 values:

```
float(subs2|z=1)
```

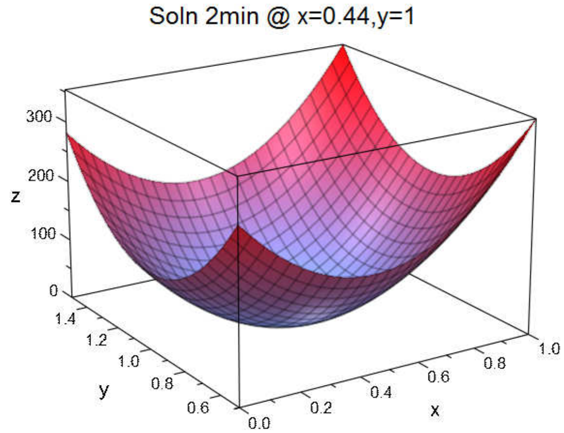
$$[a = 14.39489614,\ b = 20.42642831,\ x = 0.4423698861,\ y = 1.0]$$

```
Warning: 'hull' already has a value, not exported. [use]
Warning: 'Integral' seems to be proteced; not exported. [use]
Warning: 'Pyramid' seems to be proteced; not exported. [use]
```

```
plotfunc3d(Z|[subs2[3],subs2[4]]|z=1,a=10..20,b=15..25,Header="Soln2 min @ a=14.1,b=20.4")
```




```
plotfunc3d(Z|[subs2[1],subs2[2]]|z=1,x=0..1,y=0.5..1.5,Header="Soln 2min @ x=0.44,y=1")
```



Approach: Developed objective function as SRSS

Conclusions:
2 classes of local minima:
Solution 1:[a = (3315*z - 205*221^(1/2)*z)/(442*z^2), b = (2210*z - 155*221^(1/2)*z)/(221*z^2), x = - (10*z)/11 - (221^(1/2)*z)/11, y = z]

Solution 2: [a = (3315*z + 205*221^(1/2)*z)/(442*z^2), b = (2210*z + 155*221^(1/2)*z)/(221*z^2), x = (221^(1/2)*z)/11 - (10*z)/11, y = z]

Both solutions are parameterized by lower case "z". Experimentation revealed objective function does not depend on z, so we are free to choose any value and the solution is equally good wrt objective function. We choose z=1, which gives:

solution 1: [a = 0.6051038615, b = -0.4264283069, x = -2.260551704, y = 1.0]
Objective function for soln1 is 746.6.

solution 2: [a = 14.39489614, b = 20.42642831, x = 0.4423698861, y = 1.0]
Objective function for soln2 is 3.348281317