```
clear
```

clear

```
axe:=5-a*x  // axe is error between a*x and 5
```

$5 - a\,x$

```
bxe:=10-b*x // axe is error between b*x and 10
```

$10 - b\,x$

```
aye:=15-a*y  // aye is error between a*y and 15
```

$15 - a\,y$

```
bye:=20-b*y // bye is error between b*y and 20
```

$20 - b\,y$

```
Z:=axe^2+bxe^2+aye^2+bye^2; // Z is sum of square errors
```

$(a\,x-5)^2+(b\,x-10)^2+(a\,y-15)^2+(b\,y-20)^2$

Set derivative of objective function wrt each coordinate to zero to find local max/min/saddle of Z

```
eq1:=diff(Z,x)=0
```

$2\,a\,(a\,x-5)+2\,b\,(b\,x-10)=0$

```
eq2:=diff(Z,y)=0;
```

$2\,a\,(a\,y-15)+2\,b\,(b\,y-20)=0$

```
eq3:=diff(Z,a)=0
```

$2\,x\,(a\,x-5)+2\,y\,(a\,y-15)=0$

```
eq4:=diff(Z,b)=0
```

$2\,x\,(b\,x-10)+2\,y\,(b\,y-20)=0$

```
soln:=solve({eq1,eq2,eq3,eq4},{x,y,a,b});
```

$$\begin{pmatrix} a \\ b \\ x \\ y \end{pmatrix} \in \left\{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\right\} \cup \left\{\begin{pmatrix} \frac{3315\,z-205\,\sqrt{221}\,z}{442\,z^2} \\ \frac{2210\,z-155\,\sqrt{221}\,z}{221\,z^2} \\ -\frac{10\,z}{11}-\frac{\sqrt{221}\,z}{11} \\ z \end{pmatrix} \middle| z \in \mathbb{C}\setminus\{0\}\right\} \cup \left\{\begin{pmatrix} \frac{3315\,z+205\,\sqrt{221}\,z}{442\,z^2} \\ \frac{2210\,z+155\,\sqrt{221}\,z}{221\,z^2} \\ \frac{\sqrt{221}\,z}{11}-\frac{10\,z}{11} \\ z \end{pmatrix} \middle| z \in \mathbb{C}\setminus\{0\}\right\}$$

Above shows 0000 is local max/min/saddle... not interesting.
We have two solutions which we will call solution 1 and 2, both parameterized in terms of new variable z which can take on any real or complex value (l.c. z different han U.C. Z)

```
soln1:= soln[2][2]
```

$$\left\{\begin{pmatrix} \frac{3315\,z-205\,\sqrt{221}\,z}{442\,z^2} \\ \frac{2210\,z-155\,\sqrt{221}\,z}{221\,z^2} \\ -\frac{10\,z}{11}-\frac{\sqrt{221}\,z}{11} \\ z \end{pmatrix} \middle| z \in \mathbb{C}\setminus\{0\}\right\}$$

```
soln2:=soln[2][3]
```

$$\left\{\begin{pmatrix} \frac{3315\,z+205\,\sqrt{221}\,z}{442\,z^2} \\ \frac{2210\,z+155\,\sqrt{221}\,z}{221\,z^2} \\ \frac{\sqrt{221}\,z}{11}-\frac{10\,z}{11} \\ z \end{pmatrix} \middle| z \in \mathbb{C}\setminus\{0\}\right\}$$

```
subs1:=[a=op(soln1,1)[1],b=op(soln1,1)[2],x=op(soln1,1)[3],y=op(soln1,1)[4]]
```

$$\left[a=\frac{3315\,z-205\,\sqrt{221}\,z}{442\,z^2},\ b=\frac{2210\,z-155\,\sqrt{221}\,z}{221\,z^2},\ x=-\frac{10\,z}{11}-\frac{\sqrt{221}\,z}{11},\ y=z\right]$$

```
Z1:=Z|subs1
```