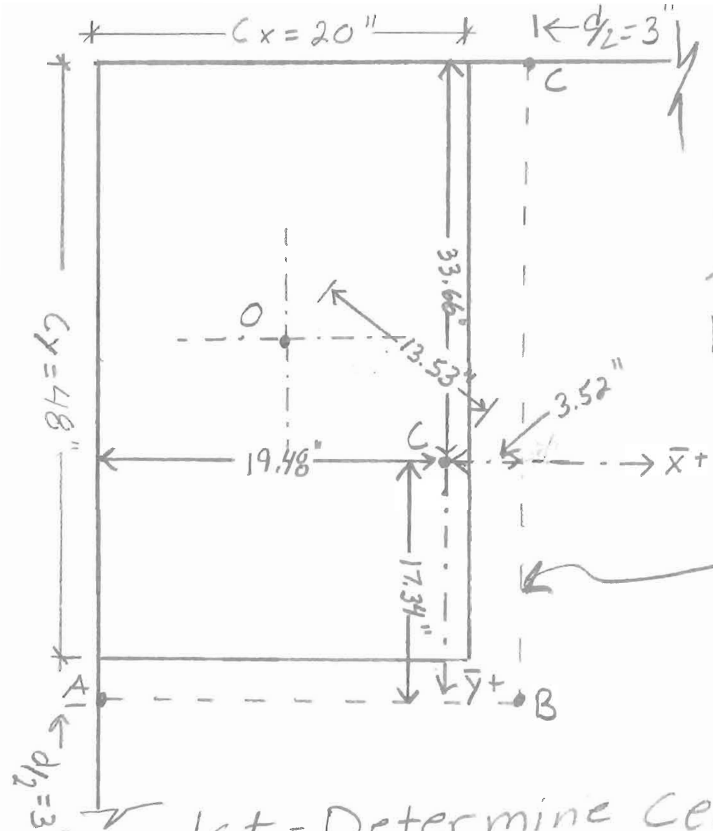
Punching Shear Example
Rectangular Corner Column



$l_{x_1} = 20" + 3" = 23"$ $\quad d = 6"$

$l_{y_1} = 48" + 3" = 51"$

$b_o = (23" + 51") = 74"$

1st - Determine Centroid Location of Critical Section

$C1 = \dfrac{l_{x_1}^2}{2b_o} = \dfrac{23^2}{2 \cdot 75"} = 3.52"$ $\qquad H1 = \dfrac{l_{y_1}^2}{2b_o} = \dfrac{51^2}{2 \cdot 75"} = 17.34"$

$Cprime1 = l_{x_1} - C1 = 23" - 3.52" = 19.48"$

$Hprime1 = l_{y_1} - H1 = 51' - 17.34" = 33.66"$

$\bar{X}_O = \dfrac{Cx}{2} - Cprime1 = \dfrac{20}{2} - 19.48" = -9.48"$

$\bar{Y}_O = \dfrac{Cy}{2} - Hprime1 = \dfrac{48}{2} - 33.66" = -9.66"$

$XO = \sqrt{-9.48^2 + -9.66^2} = 13.53"$

$\bar{X}_O$ & $\bar{Y}_O$ is the location of column origin with respect to critical section centroid

2nd - Determine $J\bar{x}, J\bar{y}, J\bar{x}\bar{y}, \theta$

- According to ACI 421.1R & ACI 421.2R, when the critical section has no axis of symmetry, the centroidal principal axes can be determined by the rotation of the centroidal nonprincipal $\bar{x}, \bar{y}$ axes an angle θ, given by

$$\tan 2\theta = \frac{-2J\bar{x}\bar{y}}{J\bar{x} - J\bar{y}} \qquad \text{(B-10)}$$

The absolute value of θ is less than $\pi/2$; when the value is positive, θ is measured in the clockwise direction. $J\bar{x}$ and $J\bar{y}$ can be calculated by Eq. (B-8) and (B-9), substituting $\bar{x}$ and $\bar{y}$ for x, y. $J\bar{x}\bar{y}$ is equal to d times the product of inertia of the perimeter of the critical section about the centroidal nonprincipal $\bar{x}$ and $\bar{y}$ axes.

Eq. (B-8) $J_x = d\sum\left[\frac{\ell}{3}(y_i^2 + y_i y_j + y_j^2)\right]$

Eq. (B-9) $J_y = d\sum\left[\frac{\ell}{3}(x_i^2 + x_i x_j + x_j^2)\right]$

Eq (B-11) $J\overline{xy} = d\sum\left[\frac{\ell}{6}(2\bar{x}_i\bar{y}_i + \bar{x}_i\bar{y}_j + \bar{x}_j\bar{y}_i + 2\bar{x}_j\bar{y}_j)\right]$

$\bar{x}, \bar{y}$ Coordinates of A, B, C

A(-19.48, 17.34) B(3.52, 17.34) C(3.52, -33.66)

- W.R.T. Nonprincipal axes

$J\bar{x} = {}_{A\text{-}B}\ \frac{23}{3}(17.34^2 + 17.34 \cdot 17.34 + 17.34^2) + {}_{B\text{-}C}\ \frac{51}{3}(17.34^2 + 17.34 \cdot {-33.66} + {-33.66}^2)$

$J\bar{x} = 6'' \times \sum(6915.54 + 14450.1) = \underline{128{,}194 \text{ in}^4}$

$J\bar{y} = {}_{A\text{-}B}\ \frac{23}{3}(-19.48^2 + -19.48 \cdot 3.52 + 3.52^2) + {}_{B\text{-}C}\ \frac{51}{3}(3.52^2 + 3.52 \cdot 3.52 + 3.52^2)$

$J\bar{y} = 6'' \times \sum(3529.97 + 631.91) = \underline{24{,}971 \text{ in}^4}$

$J\bar{x}\bar{y} = {}_{A\text{-}B}\ \frac{23}{6}(2 \cdot -19.48 \cdot 17.34 + -19.48 \cdot 17.34 + 3.52 \cdot 17.34 + 2 \cdot 3.52 \cdot 17.34) +$

${}_{B\text{-}C}\ \frac{51}{6}(2 \cdot 3.52 \cdot 17.34 + 3.52 \cdot -33.66 + 3.52 \cdot 17.34 + 2 \cdot 3.52 \cdot -33.66)$

$6'' + \sum(-3182.58 + -1464.88) = -27{,}885 \text{ in}^4$

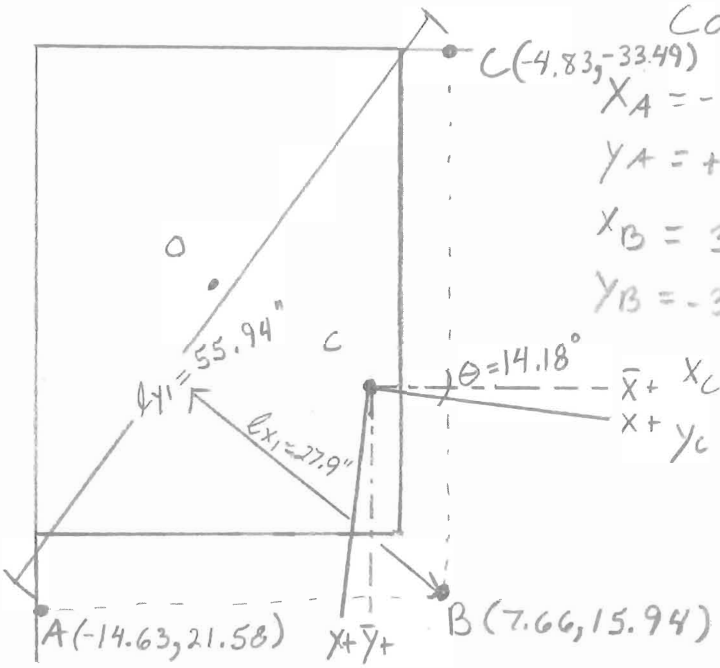
$J\overline{xy} = \underline{-27{,}885 \text{ in}^4}$

Now solve for θ, the axis rotation

$$\tan 2\theta = \frac{-2J_{\bar{x}\bar{y}}}{J_{\bar{x}} - J_{\bar{y}}}$$

$$\frac{-2 \cdot -27,885}{128,194 - 24,971} = 0.540$$

$$\theta = \frac{1}{2}\tan^{-1}(0.540) = 14.18°$$



Coordinates

$X_A = -19.48\cos(14.18) + 17.34\sin(14.18) = -14.63''$

$Y_A = +19.48\sin(14.18) + 17.34\cos(14.18) = 21.58''$

$X_B = 3.52\cos(14.18) + 17.34\sin(14.18) = 7.66''$

$Y_B = -3.52\sin(14.18) + 17.34\cos(14.18) = 15.94''$

$X_C = 3.52\cos(14.18) + -33.66\sin(14.18) = -4.83''$

$Y_C = -3.52\sin(14.18) + -33.66\cos(14.18) = -33.49$

Now determine J_x & J_y W.R.T. Centroidal axes

$$J_x = {}_{A-B}\frac{23}{3}(21.58^2 + 21.58 \cdot 15.94 + 15.94^2) + \overset{B-C}{\frac{51}{3}}(15.94^2 + 15.94 \cdot -33.49 + -33.49^2)$$

$$J_x = 6 \cdot \sum(8155 + 14,311) = \underline{134,797 \text{ in}^4}$$

$$J_y = {}_{A-B}\frac{23}{3}(-14.63^2 + -14.63 \cdot 7.66 + 7.66^2) + \overset{B-C}{\frac{51}{3}}(7.66^2 + 7.66 \cdot -4.83 + -4.83^2)$$

$$J_y = 6 \cdot \sum(1231 + 765) = \underline{11,976 \text{ in}^4}$$

Now determine γ_{vx} & γ_{vy}

From code → $\gamma_{vx} = 0.4$

$$\gamma_{vy} = 1 - \frac{1}{1+\frac{2}{3}\sqrt{\frac{\ell_x}{\ell_y}-0.2}} = 1 - \frac{1}{1+\frac{2}{3}\sqrt{\frac{27.9}{55.9}-0.2}} = 0.267$$

Example Loading on Column

$V_u = 100$ kips $M_{ux} = 20$ k-ft (240 k-in) $M_{uy} = 30$ k-ft (360 k-in)

According to ACI 421.1R & ACI 421.2R: when the centroid of the the shear-critical section does not coincide with, O, the centroid of the column, the unbalanced moment M_{ux} or M_{uy} about x- or y-axis through the centroid of shear-critical section is related to the unbalanced moment M_{uOx} or M_{uOy} about the x- or y-axis through O by

Eq. (4-5) $M_{ux} = M_{uOx} + V_u y_o$; $M_{uy} = M_{uOy} + V_u X_o$

(X_o, y_o) are the coordinates of O W.R.T. the centroid of the shear-critical section along the centroidal principal axis x, y.

For the shear-critical section, the moments about the centroidal nonprincipal axes $\bar{x}$ and $\bar{y}$ ($M_{u\bar{x}}$, $M_{u\bar{y}}$) are equivalent to the moments about the x and y axes (M_{ux} and M_{uy}) that are given by Eq. (4-6)

Location of O, W.R.T. Centroidal principal axes
O (-11.55, -7.04)

$$M_{ux} = M_{u\bar{x}} \cos\theta - M_{u\bar{y}} \sin\theta$$

$$M_{uy} = M_{u\bar{x}} \sin\theta + M_{u\bar{y}} \cos\theta$$, θ is angle of rotation of the axes $\bar{x}, \bar{y}$ to coincide with principal axes.

Use Eq. (4-5)

$$M_{ux} = M_{uox} + V_u y_o = 240\text{ k-in} + 100\text{ k} \cdot (-7.04'') = -464\text{ kip-in}$$

$$M_{uy} = M_{uoy} + V_u x_o = 360\text{ k-in} + 100\text{ k}(-11.55'') = -795\text{ kip-in}$$

Use Eq. (4-6)

$$M_{ux} = -464 \cos(14.18) + 795 \sin(14.18) = -255\text{ kip-in} = -18.75\text{ k-ft}$$

$$M_{uy} = -464 \sin(14.18) - 795 \cos(14.18) = -884\text{ kip-in} = -73.6\text{ k-ft}$$

Are the moments Mux & Muy correct?

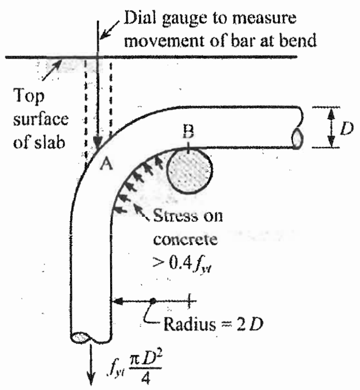


Fig. 3.1—Geometrical and stress conditions at bend of shear reinforcing bar.

occurs at the bends of shear reinforcement, at Point A of Fig. 3.1, before the yield strength can be reached in the shear reinforcement, causing a loss of tension. Furthermore, the concrete within the bend in the stirrups is subjected to stresses that could potentially exceed 0.4 times the stirrup's yield strength f_{yt}, causing concrete crushing. If f_{yt} is 60 ksi (414 MPa), the average compressive stress on the concrete under the bend has to reach $0.4f_{yt}$ for equilibrium. Because this high stress can crush the concrete, however, slip occurs before the development of the full f_{yt} in the leg of the stirrup at its connection with the bend. These difficulties, including the consequences of improper stirrup details, were also discussed by others (Marti 1990; Joint ACI-ASCE Committee 426 1974; Hawkins 1974; Hawkins et al. 1975). The movement at the end of the vertical leg of a stirrup can be reduced by attachment to a flexural reinforcement bar, as shown at Point B of Fig. 3.1. The flexural reinforcing bar, however, cannot be placed any closer to the vertical leg of the stirrup without reducing the effective slab depth d. Flexural reinforcing bars can provide such improvement to shear reinforcement anchorage only if attachment and direct contact exists at the intersection of the bars (Point B of Fig. 3.1). Under normal construction, however, it is very difficult to ensure such conditions for all stirrups. Thus, such support is normally not fully effective, and the end of the vertical leg of the stirrup can move. The amount of movement is the same for a short or long shear-reinforcing bar. Therefore, the loss in tension is important, and the stress is unlikely to reach yield in short shear reinforcement (in thin slabs). These problems are largely avoided if shear reinforcement is provided with mechanical anchorage.

CHAPTER 4—PUNCHING SHEAR DESIGN EQUATIONS

4.1—Strength requirement

This chapter presents the design procedure of ACI 318 when stirrups or headed studs are required in the slab in the vicinity of a column transferring moment and shear. The equations of Sections 4.3.2 and 4.3.3 apply when stirrups and headed studs are used, respectively.

Design of critical slab sections perpendicular to the plane of a slab should be based on

$$v_u \le \phi v_n \tag{4-1}$$

in which v_u is the shear stress in the critical section caused by the transfer, between the slab and the column, of factored shearing force or factored shearing force combined with moment; v_n is the nominal shear strength (psi or MPa); and ϕ is the strength reduction factor.

Equation (4-1) should be satisfied at a critical section perpendicular to the plane of the slab at a distance $d/2$ from the column perimeter and located so that its perimeter b_o is minimum (Fig. 4.1(a)). It should also be satisfied at a critical section at $d/2$ from the outermost peripheral line of the shear reinforcement (Fig. 4.1(b)), where d is the average of distances from extreme compression fiber to the centroids of the tension reinforcements running in two orthogonal directions. Figure 4.1(a) indicates the positive directions of the internal force V_u and moments M_{ux} and M_{uy} that the column exerts on the slab.

4.2—Calculation of factored shear stress v_u

ACI 318 requires that the shear stress resulting from moment transfer by eccentricity of shear be assumed to vary linearly about the centroid of the shear-critical section. The shear stress distribution, expressed by Eq. (4-2), satisfies this requirement. The maximum factored shear stress v_u at a critical section produced by the combination of factored shear force V_u and unbalanced moments M_{ux} and M_{uy} is

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_{vx} M_{ux} y}{J_x} + \frac{\gamma_{vy} M_{uy} x}{J_y} \tag{4-2}$$

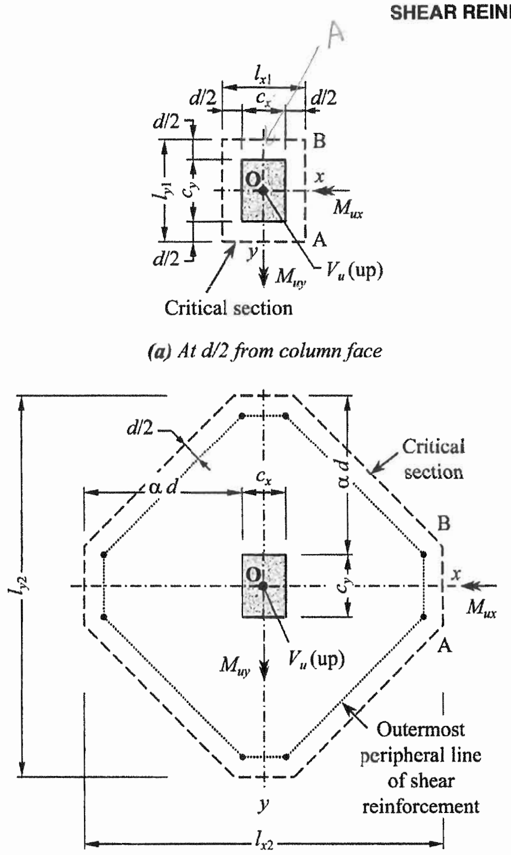
The coefficients γ_{vx} and γ_{vy} are given by

$$\left.\begin{aligned} \gamma_{vx} &= 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_{y1}/l_{x1}}} \\ \gamma_{vy} &= 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_{x1}/l_{y1}}} \end{aligned}\right\} \tag{4-3}$$

where l_{x1} and l_{y1} are lengths of the sides in the x and y directions of a rectangular critical section at $d/2$ from the column face (Fig. 4.1(a)). Appendix B gives equations for J_x, J_y, γ_{vx}, and γ_{vy} for a shear-critical section of any shape. For a shear-critical section in the shape of a closed rectangle, the shear stress due to V_u combined with M_{uy}, ACI 318 gives Eq. (4-2) with $M_{ux} = 0$ and J_y replaced by J_c, which is defined as property of assumed critical section "analogous to polar moment of inertia." For the closed rectangle in Fig. 4.1(a), ACI 318 gives

$$J_c = d\left[\frac{l_{x1}^3}{6} + \frac{l_{y1} l_{x1}^2}{2}\right] + \frac{l_{x1} d^3}{6} \tag{4-4}$$

The first term on the right-hand side of this equation is equal to J_y; the ratio of the second term to the first is commonly less



(a) At d/2 from column face

(b) At d/2 from outermost peripheral line of shear reinforcement

Fig. 4.1—Critical sections for shear in slab in vicinity of interior column. Positive directions for V_u, M_{ux}, *and* M_{uy} *are indicated.*

than 3%. The value of v_u obtained by the use of J_y in Eq. (4-2) differs on the safe side from the value obtained with J_c.

When the centroid of the shear-critical section does not coincide with O, the centroid of the column (Fig. 4.2(b) and (c)), the unbalanced moment M_{ux} or M_{uy} about the x- or y-axis through the centroid of shear-critical section is related to the unbalanced moment M_{uOx} or M_{uOy} about the x- or y-axis through O by

$$M_{ux} = M_{uOx} + V_u y_O; \quad M_{uy} = M_{uOy} + V_u x_O \qquad (4\text{-}5)$$

where (x_O, y_O) are the coordinates of O with respect to the centroid of the shear-critical section along the centroidal principal x and y axes.

For the shear-critical section in Fig. 4.2(c), the moments about the centroidal nonprincipal axes $\bar{x}$ and $\bar{y}$ ($M_{u\bar{x}}$ and $M_{u\bar{y}}$) are equivalent to the moments about the x and y axes (M_{ux} and M_{uy}) that are given by Eq. (4-6).

$$M_{ux} = M_{u\bar{x}}\cos\theta - M_{u\bar{y}}\sin\theta; \; M_{uy} = M_{u\bar{x}}\sin\theta + M_{u\bar{y}}\cos\theta \quad (4\text{-}6)$$

where θ is the angle of rotation of the axes $\bar{x}$ and $\bar{y}$ to coincide with the principal axes.

4.3—Calculation of shear strength v_n

Whenever the specified compressive strength of concrete f'_c is used in Eq. (4-7a), (4-8a), (4-9a), (4-10a), and (4-12a), its value is in pounds per square inch; when f'_c is in MPa, Eq. (4-7b), (4-8b), (4-9b), (4-10b) and (4-12b) are used. For prestressed slabs, refer to Chapter 5.

4.3.1 *Shear strength without shear reinforcement*—For nonprestressed slabs, the shear strength of concrete at a critical section at $d/2$ from column face, where shear reinforcement is not provided, should be the smallest of

$$v_n = \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f'_c} \quad \text{(in.-lb units)} \qquad (4\text{-}7a)$$

$$v_n = \left(2 + \frac{4}{\beta}\right)\lambda\frac{\sqrt{f'_c}}{12} \quad \text{(SI units)} \qquad (4\text{-}7b)$$

where β is the ratio of long side to short side of the column cross section

$$v_n = \left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f'_c} \quad \text{(in.-lb units)} \qquad (4\text{-}8a)$$

$$v_n = \left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\frac{\sqrt{f'_c}}{12} \quad \text{(SI units)} \qquad (4\text{-}8b)$$

where α_s is 40 for interior columns, 30 for edge columns or 20 for corner columns, and

$$v_n = 4\lambda\sqrt{f'_c} \quad \text{(in.-lb units)} \qquad (4\text{-}9a)$$

$$v_n = \lambda\sqrt{f'_c}/3 \quad \text{(SI units)} \qquad (4\text{-}9b)$$

At a critical section outside the shear-reinforced zone

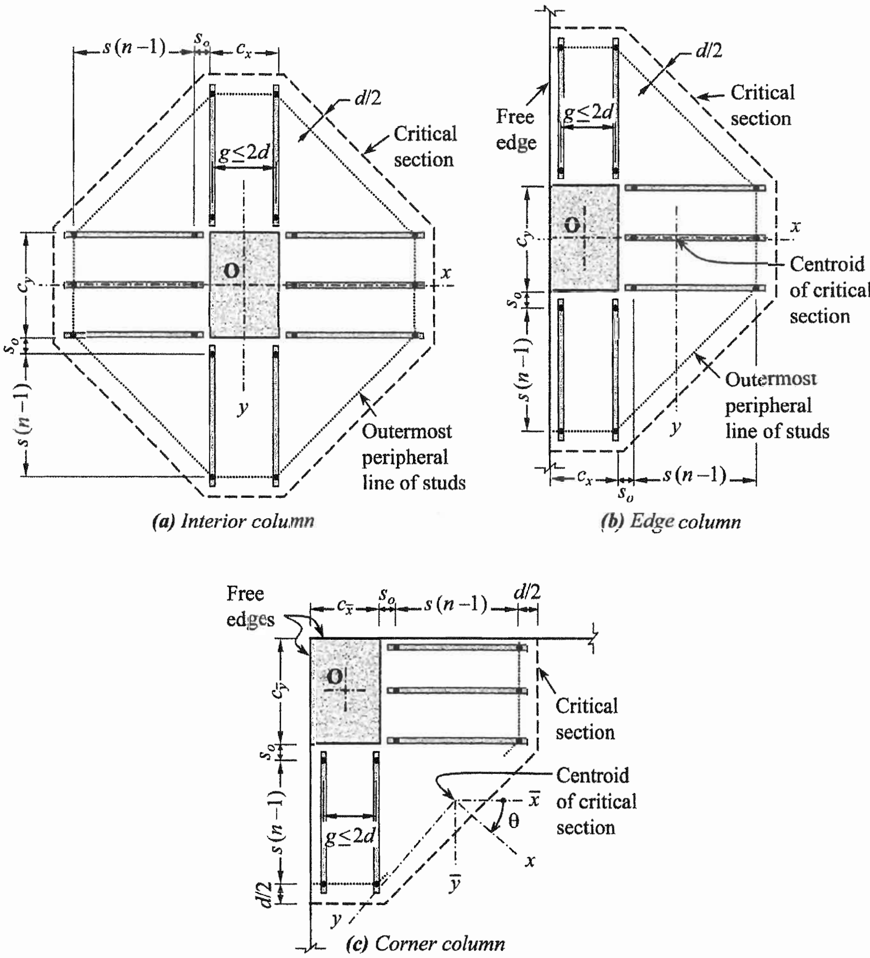
$$v_n = 2\lambda\sqrt{f'_c} \quad \text{(in.-lb units)} \qquad (4\text{-}10a)$$

$$v_n = \lambda\sqrt{f'_c}/6 \quad \text{(SI units)} \qquad (4\text{-}10b)$$

Equation (4-1) should be checked first at a critical section at $d/2$ from the column face (Fig. 4.1(a)). If Eq. (4-1) is not satisfied, shear reinforcement is required.

4.3.2 *Shear strength with stirrups*—ACI 318 permits the use of stirrups as shear reinforcement when $d \geq 6$ in. (152 mm), but not less than 16 times the diameter of the stirrups. When stirrup shear reinforcement is used, ACI 318 requires that the maximum factored shear stress at $d/2$ from column face satisfy: $v_u \leq 6\phi\sqrt{f'_c}$ (in.-lb units) ($\phi\sqrt{f'_c}/2$ [SI units]). The shear strength at a critical section within the shear-reinforced zone should be computed by

$$v_n = v_c + v_s \qquad (4\text{-}11)$$



Note: n = number of equally spaced studs in two orthogonal directions. Only the innermost and outermost studs are shown.

Fig. 4.2—Typical arrangement of shear studs and critical sections outside shear-reinforced zone.

in which

$$v_c = 2\lambda\sqrt{f'_c} \quad \text{(in.-lb units)} \tag{4-12a}$$

$$v_c = 0.17\lambda\sqrt{f'_c} \quad \text{(SI units)} \tag{4-12b}$$

and

$$v_s = \frac{A_v f_{yt}}{b_o s} \tag{4-13}$$

where A_v is the cross-sectional area of the shear reinforcement legs on one peripheral line parallel to the perimeter of the column section, and s is the spacing between peripheral lines of shear reinforcement.

The upper limits, permitted by ACI 318, of s_o and the spacing s between the peripheral lines are

$$s_o \le 0.5d \tag{4-14}$$

$$s \le 0.5d \tag{4-15}$$

where s_o is the distance between the first peripheral line of shear reinforcement and the column face. The upper limit of s_o is intended to eliminate the possibility of shear failure between the column face and the innermost peripheral line of shear reinforcement. Similarly, the upper limit of s is to avoid failure between consecutive peripheral lines of stirrups. A line of stirrups too close to the column can be ineffective in intercepting shear cracks; thus, s_o should not be smaller than $0.35d$.

The shear reinforcement should extend away from the column face so that the shear stress v_u at a critical section at $d/2$ from outermost peripheral line of shear reinforcement (Fig. 4.1(b) and 4.2) does not exceed ϕv_n, where v_n is calculated using Eq. (4-10a) or (4-10b).

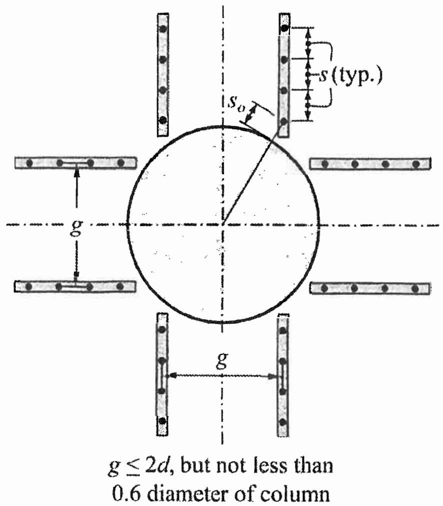


Fig. A.2—Shear headed stud reinforcement arrangement for circular columns.

lines of shear studs should not exceed $2d$, where d is the effective depth of the slab. When stirrups are used, the same limit for g should be observed (Fig. A.1(a)).

The stud arrangement for circular columns is shown in Fig. A.2. The minimum number of peripheral lines of shear studs, in the vicinity of rectangular and circular columns, is two.

A.3—Stud length

The studs are most effective when their anchors are as close as possible to the top and bottom surfaces of the slab. Unless otherwise protected, the minimum concrete cover of the anchors should be as required by ACI 318. The cover of the anchors should not exceed the minimum cover plus one-half bar diameter of flexural reinforcement (Fig. 6.1). The mechanical anchors should be placed in the forms above reinforcement supports, which ensure the specified concrete cover.

APPENDIX B—PROPERTIES OF CRITICAL SECTIONS OF GENERAL SHAPE

Figure B.1 shows the top view of critical sections for shear in slabs. The centroidal principal x and y axes of the critical sections, V_u, M_{ux}, and M_{uy} are shown in their positive directions. The shear force V_u acts at the column centroid; V_u, M_{ux}, and M_{uy} represent the effects of the column on the slab. l_x and l_y are projections of the shear-critical sections on directions of principal x and y axes.

The coefficients γ_{vx} and γ_{vy} are given by Eq. (B-1) to (B-6). ACI 318-08 gives Eq. (B-1) and (B-2); Eq. (B-3) to (B-6) are based on finite-element studies (Elgabry and Ghali 1996; Megally and Ghali 1996).

Interior column-slab connections (Fig. B.1(a))

$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_y/l_x}} \qquad \text{(B-1)}$$

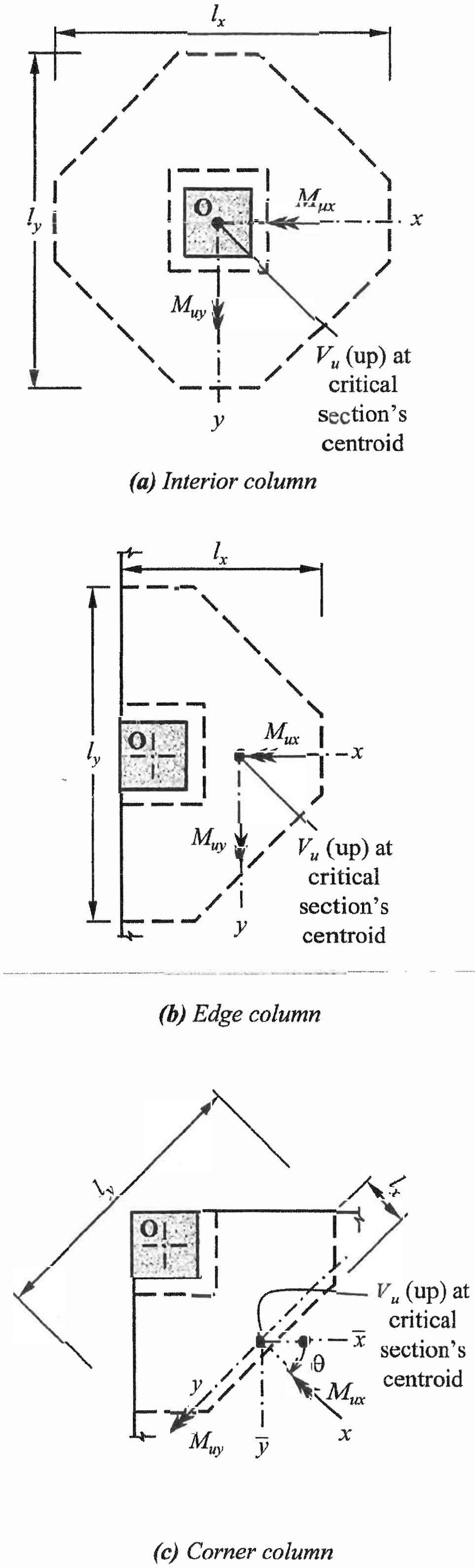


Fig. B.1—Shear-critical sections outside shear-reinforced zones and sign convention of factored internal forces transferred from columns to slabs.

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_x/l_y}} \quad \text{(B-2)}$$

Edge column-slab connections (Fig. B.1(b))

$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_y/l_x}} \quad \text{(B-3)}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{l_x}{l_y} - 0.2}} \text{ but } \gamma_{vy} = 0 \text{ when } \frac{l_x}{l_y} < 0.2 \quad \text{(B-4)}$$

Corner column-slab connections (Fig. B.1(c))

$$\gamma_{vx} = 0.4 \quad \text{(B-5)}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{l_x}{l_y} - 0.2}} \text{ but } \gamma_{vy} = 0 \text{ when } \frac{l_x}{l_y} < 0.2 \quad \text{(B-6)}$$

Equations (B-7) to (B-9) give the values of A_c, J_x, and J_y that determine by Eq. (4-2) the distribution of shear stress v_u, whose resultant components are exactly V_u, $\gamma_{vx}M_{ux}$, and $\gamma_{vy}M_{uy}$. Generally, the critical section perimeter can be considered as composed of straight segments. The values of A_c, J_x, and J_y can be determined by summation of the contribution of the segments

$$A_c = d\sum l \quad \text{(B-7)}$$

$$J_x = d\sum\left[\frac{l}{3}(y_i^2 + y_iy_j + y_j^2)\right] \quad \text{(B-8)}$$

$$J_x = d\sum\left[\frac{l}{3}(x_i^2 + x_ix_j + x_j^2)\right] \quad \text{(B-9)}$$

where x_i, y_i, x_j, and y_j are coordinates of points i and j at the extremities of a typical segment whose length is l. For a circular shear-critical section, $A_c = 2\pi d$ (radius) and $J_x = J_y = \pi d$ (radius)^3.

When the critical section has no axis of symmetry, such as in Fig. 4.2(c), the centroidal principal axes can be determined by the rotation of the centroidal nonprincipal $\bar{x}$ and $\bar{y}$ axes an angle θ, given by

$$\tan 2\theta = \frac{-2J_{\overline{xy}}}{J_{\bar{x}} - J_{\bar{y}}} \quad \text{(B-10)}$$

The absolute value of θ is less than $\pi/2$; when the value is positive, θ is measured in the clockwise direction. $J_{\bar{x}}$ and $J_{\bar{y}}$ can be calculated by Eq. (B-8) and (B-9), substituting $\bar{x}$ and $\bar{y}$ for x and y. $J_{\overline{xy}}$ is equal to d times the product of inertia of the perimeter of the critical section about the centroidal nonprincipal $\bar{x}$ and $\bar{y}$ axes

$$J_{\overline{xy}} = d\sum\left[\frac{l}{6}(2\bar{x}_i\bar{y}_i + \bar{x}_i\bar{y}_j + \bar{x}_j\bar{y}_i + 2\bar{x}_j\bar{y}_j)\right] \quad \text{(B-11)}$$

The coordinates of any point on the perimeter of the critical section with respect to the centroidal principal axes can be calculated by Eq. (B-12) and (B-13)

$$x = \bar{x}\cos\theta + \bar{y}\sin\theta \quad \text{(B-12)}$$

$$y = -\bar{x}\sin\theta + \bar{y}\cos\theta \quad \text{(B-13)}$$

The x and y coordinates, determined by Eq. (B-12) and (B-13), can now be substituted in Eq. (B-8) and (B-9) to give the values of J_x and J_y.

When the maximum v_u occurs at a single point on the critical section, rather than on a side, the peak value of v_u does not govern the strength due to stress redistribution (Brown and Dilger 1994). In this case, v_u may be investigated at a point located at a distance $0.4d$ from the peak point. This will give a reduced v_u value compared with the peak value; the reduction should not be allowed to exceed 15%.

APPENDIX C—VALUES OF v_c WITHIN SHEAR-REINFORCED ZONE

This design procedure of the shear reinforcement requires calculation of $v_n = v_c + v_s$ at the critical section at $d/2$ from the column face. The value allowed for v_c is $2\sqrt{f_c'}$ (in.-lb units) ($\sqrt{f_c'}/6$ [SI units]) when stirrups are used, and $3\sqrt{f_c'}$ (in.-lb units) ($\sqrt{f_c'}/4$ [SI units]) when headed shear studs are used. The reason for the higher value of v_c for slabs with headed shear stud reinforcement is the almost slip-free anchorage of the studs. In structural elements reinforced with conventional stirrups, the anchorage by hooks or 90-degree bends is subject to slip, which can be as high as 0.04 in. (1 mm) when the stress in the stirrup leg approaches its yield strength (Leonhardt and Walther 1965). This slip is detrimental to the effectiveness of stirrups in slabs because of their relative small depth compared with beams. The influence of the slip is manifold:

- Increase in width of the shear crack;
- Extension of the shear crack into the compression zone;
- Reduction of the shear resistance of the compression zone; and
- Reduction of the shear friction across the crack.

All of these effects reduce the shear capacity of the concrete in slabs with stirrups. To reflect the stirrup slip in the shear resistance equations, refinement of the shear failure model is required. The empirical equation $v_n = v_c + v_s$, adopted in almost all codes, is not the ideal approach to solve the shear design problem. A mechanics-based model that is acceptable for codes is not presently available. There is, however, enough experimental evidence that use of the empirical equation $v_n = v_c + v_s$ with $v_c = 3\sqrt{f_c'}$ (in.-lb units)

Table D.1—Shear stresses* (psi) due to factored loads; edge column-slab connection (Fig. D.2)

Shear-critical section	Case I		Case II	
At $d/2$ from column face	$(v_u)_A$	$(v_u)_B$	$(v_u)_A$	$(v_u)_B$
	–338	302	299	–100
At $d/2$ from outermost peripheral line of studs	$(v_u)_C$	$(v_u)_D$	$(v_u)_C$	$(v_u)_D$
	–13	87	77	–27

*v_u represents stress exerted by column on slab, with positive sign indicating upward stress.
Note: 1 MPa = 145 psi.

Step 2—Because the value (v_u/ϕ) exceeds v_n, shear reinforcement is required; the same quantity is less than the upper limit $v_n = 8\sqrt{f_c'}$, psi ($2\sqrt{f_c'}/3$, MPa), indicating that the slab thickness is adequate.

The shear stress resisted by concrete in presence of headed studs at the shear-critical section at $d/2$ from the column face is

$$v_c = 3\sqrt{f_c'} = 190 \text{ psi } (\sqrt{f_c'}/4 = 1.31 \text{ MPa})$$

Use of Eq. (4-1), (4-11), and (4-13) gives

$$v_s \geq \frac{v_u}{\phi} - v_c = 451 - 190 = 261 \text{ psi (180 MPa)}$$

$$\frac{A_v}{s} \geq \frac{v_s b_o}{f_{yt}} = \frac{261(65.25)}{51{,}000} = 0.33 \text{ in. (8.5 mm)}$$

Step 3

$$s_o \leq 0.5d = 2.8 \text{ in. (71 mm)}; \; s \leq 0.5d = 2.8 \text{ in. (71 mm)}$$

Using 3/8 in. (9.5 mm) diameter studs, arranged as shown in Fig. D.2(b), with s_o = 2.25 in. (57 mm) and s = 2.75 in. (70 mm) gives: (A_v/s) = 9(0.11)/2.75 = 0.36 in. (9.1 mm). This value is greater than 0.33 in. (8.5 mm), indicating that the choice of studs and their spacing are adequate.

Step 4—Try nine peripheral lines of studs; the properties of the shear-critical section at $d/2$ from the outermost peripheral line of studs are:

b_o = 132 in. (3353 mm); A_c = 742 in.^{2} (479 × 10^3 mm^2); J_y = 142.9 × 10^3 in.^{4} (59.48 × 10^9 mm^4);

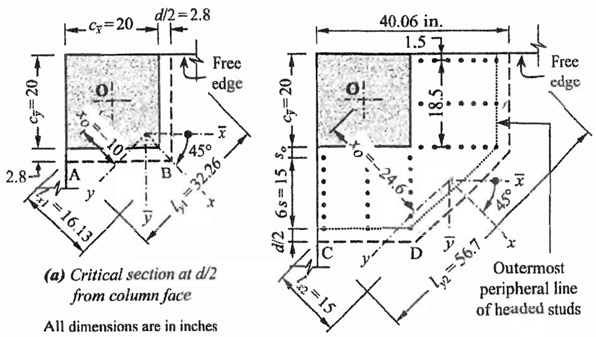
l_{x2} = 45 in. (1143 mm); l_{y2} = 72 in. (1829 mm); γ_{vy} = 0.30 (Eq. (B-4));

x_C = –27.6 in. (–701 mm); x_D = 17.4 in. (445 mm); x_O = –18.6 in. (–472 mm).

The factored shearing force and unbalanced moment at an axis, passing through the centroid of the shear-critical section outside the shear-reinforced zone, are (Eq. (4-5)):

$$\textit{Case I: } V_u = 36 \text{ kips (160 kN)}; \; M_{uy} = 1720 + 36(-18.6) = 1050 \text{ kip-in. (118 kN-m)}$$

$$\textit{Case II: } V_u = 10 \text{ kips (44 kN)}; \; M_{uy} = -900 + 10(-18.6) = -1090 \text{ kip-in. (–123 kN-m)}$$



(a) Critical section at d/2 from column face

All dimensions are in inches

(b) Critical section outside the shear-reinforced zone

Fig. D.3—Example of corner column-slab connection: shear-critical sections and stud arrangement. (Note: 1 in. = 25.4 mm.)

Equation (4-2) gives the shear stresses at Points C and D, listed in Table D.1 for Cases I and II.

The maximum shear stress, in absolute value, occurs at Point D (Case I) and $|(v_u/\phi)_D|$ = 87/0.75 = 116 psi = $1.8\sqrt{f_c'}$ (0.80 MPa = $0.15\sqrt{f_c'}$). The nominal shear strength outside the shear-reinforced zone, $v_n = 2\sqrt{f_c'}$ = 126 psi ($0.17\sqrt{f_c'}$ = 0.87 MPa).

Step 5—The value of (v_u/ϕ) is less than v_n, indicating that the extent of the shear-reinforced zone, as shown in Fig. D.2(b), is adequate.

D.3—Corner column-slab connection

The corner column-slab connection in Fig. D.3(a) is designed for gravity loads combined with wind load in positive or negative $\bar{x}$-direction. The cross-sectional dimensions of the column are $c_{\bar{x}} = c_{\bar{y}}$ = 20 in. (508 mm) (Fig. D.3(a)). The same values of: h, c_t, d, f_c', f_{yt}, D, and d_b, in Section D.1 apply in this example. Two cases (I and II) are considered, producing extreme shear stresses at Points A and B of the shear-critical section at $d/2$ from the column or at C and D of the shear-critical section at $d/2$ from the outermost peripheral line of studs (Fig. D.3(a) and (b)). The factored forces, due to gravity loads combined with wind load, are given.

Case I—Wind load in positive $\bar{x}$-direction

$$V_u = 6 \text{ kips (27 kN)}; \; M_{uO\bar{y}} = -338 \text{ kip-in. (–38 kN-m)}; \; M_{uO\bar{x}} = 238 \text{ kip-in. (27 kN-m)}$$

For the shear-critical section at $d/2$ from column face, $\bar{x}_O = \bar{y}_O$ = –7.11 in. (–181 mm) and θ = 45 degrees; thus, Eq. (4-5) and (4-6) give

$$M_{u\bar{y}} = -338 + 6(-7.11) = -381 \text{ kip-in.}; \; M_{u\bar{x}} = 238 + 6(-7.11) = 195 \text{ kip-in.}$$

$$M_{uy} = -132 \text{ kip-in. (–15 kN-m)}; \; M_{ux} = 407 \text{ kip-in. (46 kN-m)}$$

Case II—Wind load in negative $\bar{x}$-direction

$$V_u = 22 \text{ kips (97 kN)}; M_{uO\bar{y}} = 953 \text{ kip-in. (108 kN-m)}; \quad M_{uO\bar{x}} = 377 \text{ kip-in. (43 kN-m)}$$

$$M_{u\bar{y}} = 953 + 22(-7.11) = 797 \text{ kip-in.}; \quad M_{u\bar{x}} = 377 + 22(-7.11) = 221 \text{ kip-in.}$$

$$M_{uy} = 720 \text{ kip-in. (81 kN-m)}; \quad M_{ux} = -407 \text{ kip-in. (-46 kN-m)}$$

The five steps of design, outlined in Section 4.4, are followed.

Step 1—Properties of the shear-critical section in Fig. D.3(a) are: $b_o = 45.63$ in. (1159 mm); $A_c = 257$ in.^{2} (166×10^3 mm^2); $J_x = 22.26 \times 10^3$ in.^{4} (9.27×10^9 mm^4) and $J_y = 5.57 \times 10^3$ in.^{4} (2.32×10^9 mm^4). The projections of the critical section on the x and y axes are: $l_{x1} = 16.13$ in. (410 mm); and $l_{y1} = 32.26$ in. (820 mm). The fractions of unbalanced moments transferred by shear are (Eq. (B-5) and (B-6))

$$\gamma_{vy} = 1 - \frac{1}{1 + (2/3)\sqrt{(l_{x1}/l_{y1})} - 0.2} = 0.267 \text{ ; } \gamma_{vx} = 0.4$$

The factored shear stress at Point A (–8.07, 16.13 in.) in Case I is (Eq. (4-2))

$$(v_u)_A = \frac{6 \times 10^3}{257} + \frac{0.4(407 \times 10^3)16.13}{22.26 \times 10^3} + \frac{0.267(-132 \times 10^3)(-8.07)}{5.57 \times 10^3}$$

$$= 192 \text{ psi (1.33 MPa)}$$

Similar calculations give the values of v_u at Points A and B (8.07, 0 in.) for Cases I and II, which are listed in Table D.2.

The maximum shear stress, in absolute value, occurs at Point B (Case II) and $|(v_u/\phi)_B| = 364/0.75 = 485$ psi $= 7.7\sqrt{f'_c}$ (3.35 MPa $= 0.64\sqrt{f'_c}$). The nominal shear stress that can be resisted without shear reinforcement at the shear-critical section, $v_n = 4\sqrt{f'_c} = 253$ psi ($\sqrt{f'_c}/3 = 1.74$ MPa) (Eq. (4-7) to (4-9)).

Step 2—Because the value (v_u/ϕ) exceeds v_n, shear reinforcement is required; the same quantity is less than the upper limit, $v_n = 8\sqrt{f'_c}$ (in.-lb units) ($2\sqrt{f'_c}/3$ [SI units]), indicating that the slab thickness is adequate.

The shear stress resisted by concrete in the presence of headed studs at the shear-critical section at $d/2$ from the column face is

$$v_c = 3\sqrt{f'_c} = 190 \text{ psi } (\sqrt{f'_c}/4 = 1.31 \text{ MPa})$$

Use of Eq. (4-1), (4-11) and (4-13) gives

$$v_s \geq \frac{v_u}{\phi} - v_c = 485 - 190 = 295 \text{ psi (2.03 MPa)}$$

$$\frac{A_v}{s} \geq \frac{v_s b_o}{f_{yt}} = \frac{295(45.63)}{51{,}000} = 0.26 \text{ in. (6.7 mm)}$$

Table D.2—Shear stresses* (psi) due to factored loads; corner column-slab connection (Fig. D.3)

Shear-critical section	Case I		Case II	
At $d/2$ from column face	$(v_u)_A$	$(v_u)_B$	$(v_u)_A$	$(v_u)_B$
	192	–28	–312	364
At $d/2$ from outermost peripheral line of studs	$(v_u)_C$	$(v_u)_D$	$(v_u)_C$	$(v_u)_D$
	89	19	–46	65

*v_u represents stress exerted by column on slab, with positive sign indicating upward stress.
Note: 1 MPa = 145 psi.

Step 3

$$s_o \leq 0.5d = 2.8 \text{ in. (71 mm)}; s \leq 0.5d = 2.8 \text{ in. (71 mm)}$$

Using 3/8 in. (9.5 mm) diameter studs, arranged as shown in Fig. D.3(b), with $s_o = 2.25$ in. (57 mm) and $s = 2.5$ in. (64 mm) gives: $(A_v/s) = 6(0.11)/2.5 = 0.26$ in. (6.7 mm). This value is the same as that calculated in Step 2, indicating that the choice of studs and their spacing are adequate.

Step 4—Try seven peripheral lines of studs; the properties of the shear-critical section at $d/2$ from the outermost peripheral line of studs (Fig. D.3(b)) are:
$\bar{x}_O = \bar{y}_O = -17.37$ in. (–441 mm); $\theta = 45$ degrees;
$b_o = 69$ in. (1754 mm); $A_c = 388$ in.^{2} (251×10^3 mm^2);
$J_x = 116.9 \times 10^3$ in.^{4} (48.64×10^9 mm^4); $J_y = 9.60 \times 10^3$ in.^{4} (4.00×10^9 mm^4);
$l_{x2} = 15.0$ in. (380 mm); $l_{y2} = 56.7$ in. (1439 mm); $\gamma_{vx} = 0.40$ (Eq. (B-5)); $\gamma_{vy} = 0.14$ (Eq. (B-6)).

The factored shearing force and unbalanced moment about the centroidal principal axes of the shear-critical section outside the shear-reinforced zone (Eq. (4-5) and (4-6)), are:

Case I:

$$V_u = 6 \text{ kips (27 kN)}; M_{ux} = 407 \text{ kip-in. (46 kN-m)}; \quad M_{uy} = -218 \text{ kip-in. (-25 kN-m)}$$

Case II:

$$V_u = 22 \text{ kips (97 kN)}; M_{ux} = -407 \text{ kip-in. (-46 kN-m)}; \quad M_{uy} = 402 \text{ kip-in. (45 kN-m)}$$

Use of Eq. (4-2) gives the values of v_u at Points C (–10.38, 28.33 in.) and D (4.59, 13.36 in.) for Cases I and II, listed in Table D.2.

The maximum shear stress, in absolute value, occurs at Point C (Case I) and $|(v_u/\phi)_C| = 89/0.75 = 119$ psi $= 1.88\sqrt{f'_c}$ (0.82 MPa $= 0.16\sqrt{f'_c}$). The nominal shear stress outside the shear-reinforced zone, $v_n = 2\sqrt{f'_c} = 126$ psi ($0.17\sqrt{f'_c} = 0.87$ MPa).

Step 5—The value of (v_u/ϕ) is less than v_n, indicating that the extent of the shear-reinforced zone, as shown in Fig. D.3(b), is sufficient.

D.4—Prestressed slab-column connection

Design the shear reinforcement required for an interior column, transferring $V_u = 110$ kips (490 kN) combined with