7 Thermal Calculations

Based on spacing calculated above considering header pipe P-208 of length segment 200ft. At both side of this expansion loop there are anchor supports and eight guided supports equally spaced at length 22.22 ft. This expansion loop will be further analyzed for thermal and static loads.

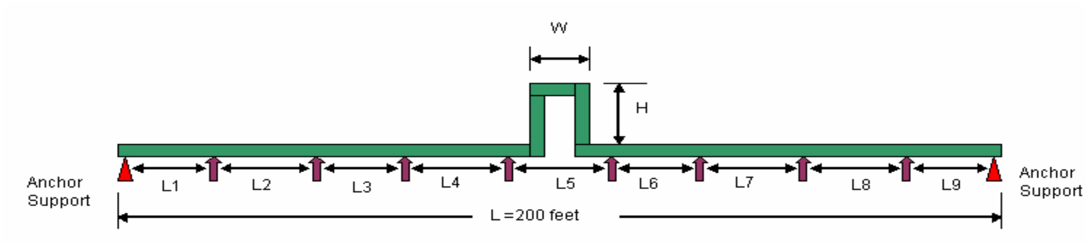

Figure 7-1 Header Pipe including an expansion loop

7.1 Thermal Analysis

For thermal analysis, using the data from Table 6-1, 6-11, Appendix A-2 and A-3, and arranging it in Table 7-1 given below.

Table 7-1 Input Data

Type of Input	Value
Modulus of Elasticity(E)	27.5 x 10^6 psi
Expansion rate (co-efficient)(α)	0.0226 in/ft
Moment of Inertia(I)	72.5 in^4
Section modulus(Z)	16.8 in^3

Note : (using appendix Table A3, A6 and A2)

Methodology

For thermal analysis in pipes we will use method of guided "cantilever method", in which thermal load and moments will be calculated as given below [3];

Thermal Load

$$F = \frac{12 \times E \times I \times \Delta}{L^3} \qquad (7.1)$$

Moment: $$M = \frac{6 \times E \times I \times \Delta}{L^2} \tag{7.2}$$

Where

Δ = Thermal Expansion, in

L = Length of segment under observation, in

E = Modulus of Elasticity, psi

I = Section modulus, in^3

Total Displacement absorbed by a section of pipe [3]:

$$\Delta_n = \frac{L_n^{\,3}\Delta_T}{\Sigma L_i^{\,3}} \tag{7.3}$$

Where

Δ_n = Displacement absorbed by leg n, in

L_n = length of leg n, ft

L_i = length of each leg resisting specified displacement, ft

Δ_T = Total displacement to be absorbed, in

Analysis

Considering 200 feet segment of pipe no. 208 and then taking its half symmetry for analysis by assuming the pipe segments to be straight and acts just a cantilever beam. As shown in figure the header pipe no. 208 has been divided into different sections. As this pipe has two main sections, one is the main line and the other is vertical leg which is perpendicular to the main line, so the nomenclature of the piping section as given below:

Main line including the segments A-B, B-C, C-D, D-E = 22.22 ft, E-F= 7.1 ft

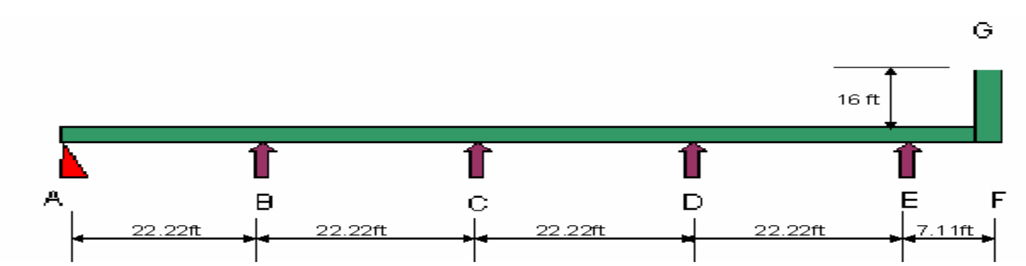

Figure 7-2 Header Pipe Sections

Segments perpendicular to the main line

F-G = 16 ft

For Main Line

Magnitude of expansion of each section = Expansion rate (0.0226in/ft) x Section length, these magnitudes and resisting segments are arranged in Table 7-2 and 7-3 below.

Table 7-2 For main line magnitude of expansion and directions

Segment	Length of section (ft)	Direction of expansion	Magnitude of expansion	Resisting segments
A-F	22.22	X	2.17 in	F-G
A-B	22.22	X	0.50 in	F-G
B-C	22.22	X	0.50 in	F-G
C-D	22.22	X	0.50 in	F-G
D-E	22.22	X	0.50 in	F-G
E-F	7.11	X	0.16 in	F-G

For Vertical Section of pipe

Table 7-3 Vertical section magnitude of expansion and direction

Segment	Length of section (ft)	Direction of expansion	Magnitude of expansion	Resisting segments
F-G	16	Y	0.361in	A-F

Thermal stress developed at on Anchor support A [2]

$$\sigma_t = E \times \alpha \times \Delta T \tag{7. 4}$$

Where

ΔT = Temperature variation = 169-0 = 169^0 C (From Table 6-1)

α = Thermal expansion co-efficient = 14.4 x 10^{-6}mm/mm.^{0}C (Appendix Table A6)

σ_t = Thermal stress, psi

$$\sigma_T = 27.5 \times 10^6 \times 14.4 \times 10^{-6} \times 169$$
$$\sigma_T = 69.25 Ksi$$

Δ_x absorbed by leg F-G, using Equation (7.3)

$$\Delta_n = L_n^3 \Delta_T / \Sigma L_i^3 \tag{7.3}$$

$L_n = L_{FG}$ = 16 ft (From Table 7.2)

L_i = 16 ft, Δ_T = 2.17 in (From Table 7-2)

$$\Delta x = \frac{16^3 \times 2.17}{16^3}$$
$$\Delta x = 2.17 in$$

F_x across F-G by using Equation (7.1)

$$\text{Fx}=\frac{12\text{EI}\Delta}{\text{L}^3} \tag{7.1}$$

Modulus of Elasticity (E) = 27.5 x 10^6 psi (Appendix Table A3)

Moment of Inertia (I) = 72.5 in^4 (Appendix Table A2)

Length of pipe segment F-G, L = 16 ft (From Table 7-2)

Δ = 2.17 in

$$\text{Fx}=\frac{12\times 27.5\times 10^6\times 72.5\times 2.17}{(16\times 12)^3}$$
$$\text{Fx}=7335\text{lb}$$

Δ_y absorbed by leg A-F using equation (7.3)

$L_n = L_{AF}$ = 96 ft

L_i = 96 ft,

Δ_T = 0.3616 in

$$\Delta y = \frac{96^3 \times 0.3616}{96^3}$$
$$\Delta y = 0.3616 in$$

Fy across A-F

Modulus of Elasticity (E) = 27.5 x 10^6 psi

Moment of Inertia (I) = 72.5 in^4

Length of pipe segment F-G, L = 96 ft,

Δ_y = 0.3616 in

For force in Y-direction across A-F, using Equation (7.1)

$$F = \frac{12 \times E \times I \times \Delta}{L^3} \tag{7.1}$$

$$Fy=\frac{12\times27.5\times10^6\times72.5\times.3616}{(96\times12)^3}$$

Fy=5.65lb

Moment about Z-axis by using Equation (7.2)

$$Mz=\frac{6EI\Delta}{L^2} \tag{7.2}$$

$$Mz=\frac{6\times27.5\times10^6\times72.5\times0.3616}{(96\times12)^2}$$

Mz=3259.5lb-in

Loads on Supports in Y-direction

For section A-B

Thermal expansion produced in section AB using Equation (7.3)

$$\Delta y,_{(A\text{-}B)} = (\Delta y,_{total} \text{ x } L_{AB})/L_{AF} \tag{7.3}$$

$(\Delta y,_{total}) = 0.3616$ in

$L_{AB} = 22.22$ ft $= 266.64$ in

$L_{AF} = 96$ ft

$$\Delta y,_{A-B} = \frac{0.3616 \times 22.22}{96}$$

$$\Delta y,_{A-B} = 0.0837 in$$

Force and thermal moment in section AB using Equation (7.1) and (7.2)

$$Fy,_{A\text{-}B}=\frac{12\times27.5\times10^6\times72.5\times0.0837}{266.64^3}$$

$$Fy,_{A\text{-}B}=105.633lb$$

$$Mz,_{A\text{-}B}=14.083\times10^3 lb\text{-}in$$

Similarly for the rest of sections, B-C, C-D, D-E and E-F by using Equations (7.1), (7.2) and (7.3) in the same way as above,

For section B-C

Thermal expansion produced in section B-C using Equation (7.3)

$$\Delta y,_{(B\text{-}C)} = (\Delta y,_{total} \text{x} L_{AC})/L_{AF} \tag{7.3}$$

$(\Delta y,_{total}) = 0.3616$ in

$L_{AC} = 44.44$ ft

L_{AF} = 96 ft

$$\Delta y,_{B-C} = \frac{0.36166 \times 44.44}{96}$$

$$\Delta y,_{B-C} = 0.167 in$$

For force in section B-C using Equation (7.1)

$$F = \frac{12 \times E \times I \times \Delta}{L^3} \quad (7.1)$$

Modulus of Elasticity (E) = 27.5 x 10^6 psi

Moment of Inertia (I) = 72.5 in^4

L = 22.22 ft = 266.64 in

$$Fy,_{B\text{-}C} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 0.167}{266.64^3}$$

$$Fy,_{B\text{-}C} = 210.76 lb$$

For thermal moment in section B-C using Equation (7.2)

$$M = \frac{6 \times E \times I \times \Delta}{L^2} \quad (7.2)$$

$$Mz,_{B\text{-}C} = 28.098 \times 10^3 lb\text{-}in$$

For section C-D

Thermal expansion produced in section C-D using Equation (7.3)

$\Delta y,_{(C\text{-}D)} = (\Delta y,_{total} \text{ x } L_{AD})/L_{AF}$

$(\Delta y,_{total}) = 0.3616$ in

L_{AD} = 66.66 ft

L_{AF} = 96 ft

$$\Delta y,_{C-D} = \frac{0.36166 \times 66.66}{96}$$

$$\Delta y,_{C-D} = 0.251 in$$

For force in section C-D using Equation (7.1)

$$F = \frac{12 \times E \times I \times \Delta}{L^3} \quad (7.1)$$

Modulus of Elasticity (E) = 27.5 x 10^6 psi

Moment of Inertia (I) = 72.5 in^4

$L = 22.22$ ft $= 266.64$ in

$$Fy,_{C\text{-}D} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 0.251}{266.64^3}$$

$$Fy,_{C\text{-}D} = 316.78 lb$$

For thermal moment in section C-D using Equation (7.2)

$$M = \frac{6 \times E \times I \times \Delta}{L^2} \tag{7.2}$$

$$Mz,_{C\text{-}D} = 42.232 \times 10^3 lb\text{-}in$$

For section D-E

Thermal expansion produced in section D-E using Equation (7.3)

$$\Delta y,_{(D\text{-}E)} = (\Delta y,_{total} \text{ x } L_{AE})/L_{AF} \tag{7.3}$$

$(\Delta y,_{total}) = 0.3616$ in

$L_{AE} = 88.88$ft

$L_{AF} = 96$ ft

$$\Delta y,_{D\text{-}E} = \frac{0.3616 \times 88.88}{96}$$

$$\Delta y,_{D\text{-}E} = 0.3348 in$$

For force in section C-D using Equation (7.1)

$$F = \frac{12 \times E \times I \times \Delta}{L^3}$$

Modulus of Elasticity (E) = 27.5×10^6 psi

Moment of Inertia (I) = 72.5 in^4

$L = 22.22$ ft $= 266.64$ in

$$Fy,_{D\text{-}E} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 0.3348}{266.64^3}$$

$$Fy,_{D\text{-}E} = 421.71 lb$$

For thermal moment in section C-D using Equation (7.2)

$$M = \frac{6 \times E \times I \times \Delta}{L^2} \tag{7.2}$$

$$Mz,_{D\text{-}E} = 56.2 \times 10^3 lb\text{-}in$$

For section E-F

Thermal expansion produced in section E-F using Equation (7.3)

$$\Delta y,_{(E\text{-}F)} = (\Delta y,_{total} \text{ x } L_{A\text{-}F})/L_{A\text{-}F} \tag{7.3}$$

$(\Delta y,_{total}) = 0.3616$ in

$L_{AE} = 96$ft

$L_{AF} = 96$ ft

$$\Delta y,_{E-F} = \frac{0.3616 \times 96}{96}$$

$$\Delta y,_{E-F} = 0.3616 in$$

And similarly using Equation (7.1) for force in section E-F and Equation (7.2) for thermal moment

$$\text{Fy},_{\text{E-F}} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 0.3616}{266.64^3}$$

$$\text{Fy},_{\text{E-F}} = 455.6\text{lb}$$

$$\text{Mz},_{\text{E-F}} = 60.74 \times 10^3 \text{lb-in}$$

Vertical force on support A, B, C, D and E

$$\text{Fy},_{\text{A}} = 105.633\text{lb}$$

$$\text{Fy},_{\text{B}} = \text{Fy},_{\text{A}} + \frac{\text{Mz}_{\text{AB}}}{\text{L}_{\text{AB}}} + \frac{\text{Mz}_{\text{BC}}}{\text{L}_{\text{BC}}} \tag{7.5}$$

$$\text{Fy},_{\text{B}} = 105.633 + \frac{14082}{266.64} + \frac{28098}{266.64}$$

$$\text{Fy},_{\text{B}} = 264.62\text{lb}$$

Similarly using Equation (7.5) for supports C, D, and E in the same way as above,

$$\text{Fy},_{\text{C}} = 210.76 + \frac{28098}{266.64} + \frac{42232}{266.64}$$

$$\text{Fy},_{\text{C}} = 474.52\text{lb}$$

$$\text{Fy},_{\text{D}} = 421.52 + \frac{42232}{266.64} + \frac{56200}{266.64}$$

$$\text{Fy},_{\text{D}} = 792.26\text{lb}$$

$$\text{Fy},_{\text{E}} = 455.6 + \frac{56200}{266.64} + \frac{60740}{266.64}$$

$$\text{Fy},_{\text{E}} = 895.78\text{lb}$$

Loads on Supports in x-direction

Using Equation (7.3) for deflection across AF

$$\Delta x,_{(A\text{-}F)} = (\Delta x,_{total} \text{ x } L_{FG})/L_{FG} \tag{7.3}$$

$(\Delta x,_{total}) = 2.17$ in

$L_{AE} = 16$ft

$L_{AF} = 16$ ft

$$\Delta x \text{ across AF} = \frac{16^3 \times 2.17in}{16^3}$$
$$= 2.17in$$

For Section A-B

Thermal expansion produced in section A-B using Equation (7.3)

$$\Delta x,_{(A\text{-}B)} = (\Delta y,_{total} \text{ x } L_{AB})/L_{AF}$$

$(\Delta y,_{total}) = 0.3616$ in

$L_{AB} = 22.22$ ft

$L_{AF} = 96$ ft

$$\Delta x,_{A-B} = \frac{2.17 \times 22.22}{96}$$
$$\Delta x,_{A-B} = 0.50in$$

For force in section A-B using Equation (7.1)

$$F = \frac{12 \times E \times I \times \Delta}{L^3} \tag{7.1}$$

Modulus of Elasticity (E) = $27.5 \text{ x } 10^6$ psi

Moment of Inertia (I) = 72.5 in^4

L = 22.22 ft = 266.64 in

$$Fx,_{A\text{-}B} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 2.17}{266.64^3}.$$
$$Fx,_{A\text{-}B} = 631lb$$

Thermal expansion produced in section B-C using Equation (7.3)

$$\Delta x,_{(B\text{-}C)} = (\Delta y,_{total} \text{ x } L_{AC})/L_{AF} \tag{7.3}$$

$(\Delta y,_{total}) = 0.3616$ in

$L_{AC} = 44.44$ ft

$L_{AF} = 96$ ft

For Section B-C

$$\Delta x,_{B-C} = \frac{2.17 \times 44.44}{96}$$
$$\Delta x,_{B-C} = 1.004 in$$

For force in section B-C using Equation (7.1)

$$F = \frac{12 \times E \times I \times \Delta}{L^3} \quad (7.1)$$

Modulus of Elasticity (E) = 27.5 x 10^6 psi

Moment of Inertia (I) = 72.5 in^4

L = 22.22 ft = 266.64 in

$$Fx,_{B\text{-}C} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 1.004}{266.64^3}$$
$$Fx,_{B\text{-}C} = 1267 lb$$

Thermal expansion produced in section C-D using Equation (7.3)

$$\Delta x,_{(C\text{-}D)} = (\Delta y,_{total} \text{ x } L_{AD})/L_{AF} \quad (7.3)$$

$(\Delta y,_{total}) = 0.3616$ in

$L_{AD} = 66.66$ ft

$L_{AF} = 96$ ft

For section C-D

$$\Delta x,_{C-D} = \frac{2.17 \times 66.66}{96}$$
$$\Delta x,_{C-D} = 1.5 in$$

For force in section C-D using equation given below

$$F = \frac{12 \times E \times I \times \Delta}{L^3} \quad (7.1)$$

Modulus of Elasticity (E) = 27.5 x 10^6 psi

Moment of Inertia (I) = 72.5 in^4

L = 22.22 ft = 266.64 in

$$Fx,_{C\text{-}D} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 1.5}{266.64^3}$$
$$Fx,_{C\text{-}D} = 1893 lb$$

And similarly for section D-E and E-F using Equation (7.1) for force and Equation (7.3) for thermal expansion

For section D-E

$$\Delta x,_{D\text{-}E} = \frac{2.17 \times 88.88}{96}$$

$$\Delta x,_{D\text{-}E} = 2.00 in$$

$$Fx,_{D\text{-}E} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 2}{266.64^3}$$

$$Fx,_{D\text{-}E} = 2524.1 lb$$

For section D-E

$$\Delta x,_{E\text{-}F} = \frac{2.17 \times 96}{96}$$

$$\Delta x,_{E\text{-}F} = 2.17 in$$

$$Fx,_{E\text{-}F} = \frac{12 \times 27.5 \times 10^6 \times 72.5 \times 2.17}{266.64^3}$$

$$Fx,_{E\text{-}F} = 2738.645 lb$$

Axial forces on every support A, B, C, D, and E separately are:

$Fx,_A = 631 lb$ (From above calculation)

For every support in the middle of other support following equation is used [3].

$$Fx,_B = \left(F_{A\text{-}B} + F_{B\text{-}C}\right) / 2 \tag{7.6}$$

$$Fx,_B = (631+1267)/2 = 949 lb$$

And similarly for support C, D and E using Equation (7.6)

$$Fx,_C = (1267+1893)/2 = 1580 lb$$

$$Fx,_D = (1893+2524.1)/2 = 2208.5 lb$$

$$Fx,_E = (2524.1+2738.643)/2 = 2631.4 lb$$

All the resultants loads are arranged in Table 7-4 below,

Table 7-4 Summary of all Loads due to Thermal expansion

Support	Fx, lb	Fy, lb	Mz, lb-in
Anchor A	631	105.63	14.08×10^3
Support B	949	264.62	28.08×10^3
Support C	1580	474.96	42.23×10^3
Support D	2208.1	792.26	56.20×10^3
Support E	2631.14	895.78	60.74×10^3