A compressor of the reciprocating type provides excitation frequencies from periodic pressure sources determined from the rotating speed (rpm) multiplied by the number of cylinders for simple action and twice the number of cylinders for double-action stages. The natural frequencies that should be avoided in restraining piping systems are half of the rpm, the rpm, and all multiples up to 5 times the rpm of the equipment. If these frequencies approach the natural frequencies of the connected piping systems, resonance in the form of large pressure sources can appear. The pressure surge loads can affect the pipes, supports, machinery, and adjacent building structures.

The best way to control vibration and its undesirable effects is to eliminate or isolate the source of vibration. Although elimination and isolation are desirable, in numerous applications these methods are not possible, so some vibration effects should be considered in the piping system design.

5.4 Expansion Loads

We demonstrated that as more restraints are added to a system, the pipe will be more effectively restrained for deadweight and occasional loads. However, most pipe, when operating, increases in temperature and expands. Piping which is too well restrained will not be able to expand, and large forces will develop at the points of lockup, causing large stresses to develop in the pipe.

The ideal restraint condition for thermal considerations is a total lack of restraint. Since this is not feasible, given other loads, some forces due to expansion will develop on restraints even in the most optimally supported system. In this section we show how these loads can be calculated. Additionally we show how to determine piping thermal movements for use in spring hanger selection and design of clearances in restraints.

5.4.1 Determination of thermal loads and stresses

Thermal expansion is of concern primarily parallel to the pipe length axis (axial direction). Thermal expansion can be computed by the following integral:

$$\Delta = L \int_{T_{cold}}^{T_{hot}} \alpha \, dT \tag{5.22}$$

where Δ = thermal expansion in direction specified by length, in (mm)
L = pipe length in direction of interest, in (mm)
α = coefficient of thermal expansion, in/(in · °F) [mm/(mm · °C)]
T = pipe temperature, °F(°C)

For most materials, the coefficient of thermal expansion varies with the temperature, so Eq. (5.22) is not easily applied. Because of this, the thermal expansion in inches per foot between an ambient temperature of 70°F (21°C) and various operating temperatures has been worked out for common piping materials. Values for some of these materials are given in Table 5.4.

The simplest method of calculating thermal loads on supports is by using the guided-cantilever method of modeling the piping system. This method assumes that the thermal growth of the axial runs are absorbed by the bending of pipe legs running perpendicular to the axial growth. Since the displacements force the leg intersections to translate, while rotations are limited by piping continuity, the pipe runs approximate the behavior of guided cantilevers. For a guided cantilever, an imposed displacement induces the following moment and force at each end:

$$M = \frac{6EI\Delta}{L^2} \tag{5.23}$$

$$P = \frac{12EI\Delta}{L^3} \tag{5.24}$$

where P = developed force, lb (N)
M = developed moment, in · lb (mm · N)
E = modulus of elasticity at installed temperature, psi (N/mm²)
I = moment of inertia of pipe, in⁴ (mm⁴)
Δ = imposed displacement, in (mm)
L = length of leg perpendicular to direction of growth, in (mm)

The amount of thermal growth absorbed (Δ) by each leg is inversely proportional to the ratio of the stiffness of the subject leg to the sum of the stiffnesses of all legs absorbing the thermal growth. The moments and forces developed must be resisted by the restraint system, whether it be directly by an anchor or by a force couple of two restraints.

Problem 5.6 The system in Fig. 5.26 is made of carbon steel and operates at 350°F (177°C). It uses 12-in (300-mm) standard schedule line with I = 279 in^4 (1.16 × 10^8 mm^4) and E = 27.7 × 10^6 psi (1.91 × 10^{11} N/m^2). From Table 5.4, the thermal expansion is found to be 0.0226 in/ft (1.883 mm/m). The system is restrained by two anchors (at points A and G) and two vertical restraints (at points D and E).

The first step in finding the thermal loads is to calculate the pipe expansions and determine the legs which will resist them. The resisting legs are all those perpendicular to the growth, except in those cases where the pipe is restrained at intermediate points. The summary of pipe movements and resisting legs is shown in Table 5.5.

The stiffness of each pipe segment is calculated as $K = 12EI/L^3$. Since the cross-sectional and material properties of all the pipes in this problem are the

TABLE 5.4 Expansion of Pipe, in Inches per Foot from 70°F Base

Temperature °F	Carbon steel carbon moly steel 2¼% Cr 1% moly	Intermediate alloy steels (5%–9% Cr–moly)	Austenitic stainless steels (304, 316, 347)	Copper	Brass	Aluminum
−200	0.0180		0.0281	0.0275	0.0287	0.0373
−150	0.0152		0.0236	0.0231	0.0241	0.0310
−100	0.0121		0.0187	0.0183	0.0190	0.0244
−50	0.0087		0.0134	0.0132	0.0137	0.0176
0	0.0051		0.0078	0.0079	0.0081	0.0104
50	0.0015	0.0022	0.0022	0.0023	0.0030	
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
100	0.0023	0.0022	0.0034	0.0034	0.0035	0.0046
125	0.0042	0.0040	0.0062	0.0063	0.0064	0.0084
150	0.0061	0.0058	0.0090	0.0091	0.0093	0.0123
175	0.0080	0.0076	0.0118	0.0121	0.0123	0.0162
200	0.0099	0.0094	0.0146	0.0151	0.0152	0.0200
225	0.0120	0.0113	0.0175	0.0179	0.0183	0.0242
250	0.0140	0.0132	0.0203	0.0208	0.0214	0.0283
275	0.0161	0.0152	0.0232	0.0238	0.0245	0.0325
300	0.0182	0.0171	0.0261	0.0267	0.0276	0.0366
325	0.0209	0.0191	0.0291	0.0297	0.0308	0.0409
350	0.0226	0.0210	0.0321	0.0327	0.0340	0.0452
375	0.0248	0.0230	0.0351	0.0358	0.0372	0.0496
400	0.0270	0.0250	0.0380	0.0388	0.0405	0.0539
425	0.0293	0.0271	0.0410	0.0419	0.0438	0.0584
450	0.0316	0.0292	0.0440	0.0449	0.0472	0.0628
475	0.0339	0.0314	0.0470	0.0481	0.0506	0.0673
500	0.0362	0.0335	0.0501	0.0512	0.0540	0.0717
525	0.0387	0.0357	0.0531	0.0543	0.0575	0.0764
550	0.0411	0.0379	0.0562	0.0574	0.0610	0.0810
575	0.0436	0.0402	0.0593	0.0607	0.0645	0.0857
600	0.0460	0.0424	0.0624	0.0639	0.0680	0.0903
625	0.0486	0.0447	0.0656	0.0671	0.0717	
650	0.0511	0.0469	0.0687	0.0703	0.0753	
675	0.0537	0.0491	0.0719	0.0736	0.0790	
700	0.0563	0.0514	0.0750	0.0768	0.0826	
725	0.0589	0.0538	0.0783	0.0801	0.0864	
750	0.0616	0.0562	0.0815	0.0834	0.0902	
775	0.0643	0.0586	0.0848	0.0867	0.0940	
800	0.0670	0.0610	0.0880	0.0900	0.0978	
825	0.0697	0.0634	0.0913	0.0934	0.1017	
850	0.0725	0.0658	0.0946	0.0967	0.1056	
875	0.0753	0.0683	0.0979	0.1002	0.1096	
900	0.0781	0.0707	0.1012	0.1037	0.1135	
925	0.0808	0.0732	0.1046	0.1071	0.1173	
950	0.0835	0.0756	0.1080	0.1105	0.1216	
975	0.0862	0.0781	0.1114	0.1140	0.1257	
1000	0.0889	0.0806	0.1148	0.1175	0.1298	
1005	0.0895	0.0811	0.1155			
1010	0.0900	0.0816	0.1162			
1015	0.0906	0.0821	0.1168			
1050	0.0946	0.0855	0.1216			
1100	0.1004	0.0905	0.1284			
1150	0.1057	0.0952	0.1352			
1200	0.1110	0.1000	0.1420			
1250	0.1166	0.1053	0.1488			
1300	0.1222	0.1106	0.1556			
1350	0.1278	0.1155	0.1624			
1400	0.1334	0.1205	0.1692			

NOTE: 1 in = 25.4 mm; 1 ft = 0.305 m; 70°F = 21°C.
SOURCE: Courtesy of Power Piping Co.

same, the relative stiffness of each leg is given by $1/L^3$. Therefore, the proportion of the total displacement absorbed by any given leg n is defined as

$$\Delta_n = \frac{L_n^3}{\Sigma L_i^3}\Delta_T$$

whereas Δ_n = displacement absorbed by leg n, in (mm)
L_n = length of leg n, ft (m)
L_i = length of each other leg resisting specified displacement, ft (m)
Δ_T = total displacement to be absorbed, in (mm)

Once the displacement absorbed by the pipe segment is known, the shear forces and moments can be found by substitution. In this way, the forces and moments on the restraints in Fig. 5.26 can be found:

$$\Delta_X \text{ absorbed by } B\text{-}C = \frac{0.34(30)^3}{30^3 + 60^3 + 30^3} = 0.034 \text{ in}$$

$$= \frac{8.6(9.15)^3}{9.15^3 + 18.3^3 + 9.15^3} = 0.864 \text{ mm}$$

From Eq. (5.24),

$$F_X \text{ across } B\text{-}C = \frac{12(27.7 \times 10^6)(279)(0.034)}{360^3} = 66 \text{ lb}$$

$$= \frac{12(1.91 \times 10^5)(1.16 \times 10^8)(0.864)}{9150^3} = 300 \text{ N}$$

and

$$\Delta_Y \text{ across } A\text{-}B = \frac{0.68(15)^3}{15^3 + 20^3} = 0.202 \text{ in (5.1 mm)}$$

$$F_Y \text{ across } A\text{-}B = \frac{12(27.7 \times 10^6)(279)(0.202)}{180^3} = 3210 \text{ lb (14,285 N)}$$

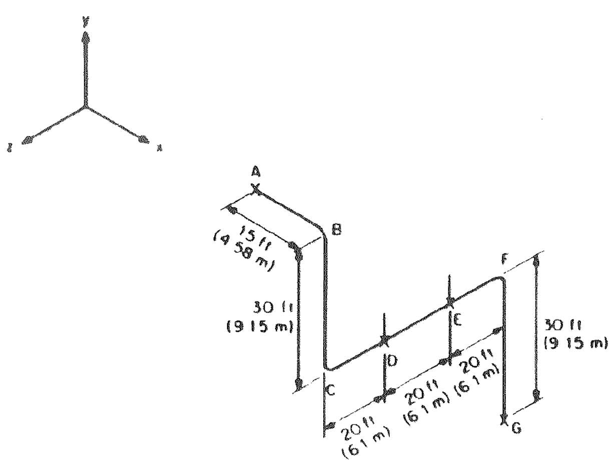

Figure 5.26 Problem 5.6.

TABLE 5.5 Summary of Pipe Movements

From segment	Direction	Magnitude	Resisted by
A-B	X	(0.0226 × 15) = 0.34 in (8.6 mm)	B-C, C-F, F-G
B-C	Y	(0.0226 × 30) = 0.68 in (17.3 mm)	A-B, C-D
C-F	Z	(0.0226 × 60) = 1.36 in (34.5 mm)	A-B, B-C, F-G
F-G	Y	(0.0226 × 30) = 0.68 in (17.3 mm)	E-F

From Eq. (5.23), for segment A-B,

$$M_Z = \frac{6(27.7 \times 10^6)(279)(0.202)}{180^2} = 289{,}096 \text{ in lb (32,697 m N)}$$

$$\Delta_Z = \frac{1.36(15^3)}{15^3 + 30^3 + 30^3} = 0.08 \text{ in (2.03 mm)}$$

$$F_Z = \frac{2(27.7 \times 10^6)(279)(0.08)}{180^3} = 1272 \text{ lb (5661 N)}$$

$$M_Y = \frac{6(27.7 \times 10^6)(279)(0.08)}{180^2} = 114{,}493 \text{ in lb (12,950 m N)}$$

and

$$M_{X\,\text{anchor}\,A} = M_{X,B\text{-}C} = \frac{1.36(30^3)}{15^3 + 30^3 + 30^3}\,\frac{6(27.7 \times 10^6)(279)}{360^2}$$

$$= 228{,}987 \text{ in lb (25,899 m N)}$$

Therefore the approximate thermal loads on the anchor at point A are

F_X = 66 lb (300 N) $\qquad M_X$ = 228,987 in lb (25,899 m N)
F_Y = 3210 lb (14,285 N) $\qquad M_Y$ = 114,493 in lb (1290 m N)
F_Z = 1272 lb (5661 N) $\qquad M_Z$ = 289,096 in lb (32,697 m N)

Similarly, the loads can be found on the vertical restraints at points D and E:

$$\Delta_{YCD} = \frac{0.68(20)^3}{15^3 + 20^3} = 0.478 \text{ in (12.1 mm)}$$

$$F_{YCD} = \frac{12(27.7 \times 10^6)(279)(0.478)}{240^3} = 3210 \text{ lb (14,285 N)}$$

$$M_{ZCD} = \frac{6(27.7 \times 10^6)(279)(0.478)}{240^2} = 384{,}804 \text{ in lb (43,523 m N)}$$

$$\Delta_{YEF} = 0.68 \text{ in (17.3 mm)}$$

$$F_{YEF} = \frac{12(27.7 \times 10^6)(279)(0.68)}{240^3} = 4567 \text{ lb (20,321 N)}$$

$$M_{ZEF} = \frac{6(27.7 \times 10^6)(279)(0.68)}{240^2} = 547{,}421 \text{ in lb (61,916 m N)}$$

The moments at points D and E are resisted by force couples between the two restraints. The total forces can be found by:

$$F_{YD} = -3210 - \frac{384{,}805}{240} - \frac{547{,}421}{240} = -7094 \text{ lb (31,570 N) down}$$

$$F_{YE} = 4567 + \frac{384{,}805}{240} + \frac{547{,}421}{240} = 8451 \text{ lb (37,608 N) up}$$

Similarly, the forces and moments can be calculated at the anchor at point G

The guided-cantilever method does not provide fully accurate results because the leg intersection points are actually free to rotate to some degree, thus redistributing the developed moment. In most cases this method yields conservative results: however, in many cases greater accuracy is desired.

Charts have been developed for the purpose of providing the design engineer with accurate load calculations for a wide range of piping configurations. Table 5.6 presents a series of charts developed by the ITT Grinnell Company and reprinted here with their permission. The full series are available in ITT Grinnell's *Piping Design and Engineering*. Sufficient piping configurations are provided to permit the engineer to break down most systems into segments which can be solved easily either by the charts or by the guided-cantilever method.

Most of the Grinnell configurations are of expansion loops. Through judicial use of loops and proper placement of supports, excessive thermal stresses may be reduced. By selecting support locations which direct the expansion to areas of greater flexibility, expansion stresses can be controlled. Expansion loops such as those shown in Fig. 5.27 are used to increase the flexibility of hot piping systems. These loops are designed to accommodate the thermal expansion anticipated during piping operation.

Figure 5.27*a* illustrates the free thermal movement of the piping, with the terminal locations of piping representing equipment nozzles. The uncontrolled expansion of the pipe can cause locations *A* and *G* to become overstressed or overloaded by excessive moments and forces on the nozzles. By providing guides at locations *C* and *F* and a limit stop or rigid restraint at location *B*, as shown in Fig. 5.27*b*, the thermal expansion can be directed into the expansion loop. Thus the loading on the terminal connections can be reduced significantly.

5.4.2 Determination of Thermal Movements

Piping thermal movements may be estimated at intermediate points in a system by assuming a linear variation between points of known displacement. For example, if the movement of a point located 5 ft (1.5 m) from point *A* (toward *B*) in Fig. 5.26 were desired, it could be computed by the following ratios (using the displacements at point *B* determined in Problem 5.6):

$$\Delta X = \tfrac{5}{15}(0.34) = 0.11 \text{ in } (2.8 \text{ mm})$$

$$\Delta Y = \tfrac{5}{15}(0.202) = 0.067 \text{ in } (1.7 \text{ mm})$$

$$\Delta Z = \tfrac{5}{15}(0.08) = 0.027 \text{ in } (0.69 \text{ mm})$$

These movements are most often required in the *y* direction for use in spring hanger selection. An example demonstrating the method for determining vertical displacements in a piping system follows.

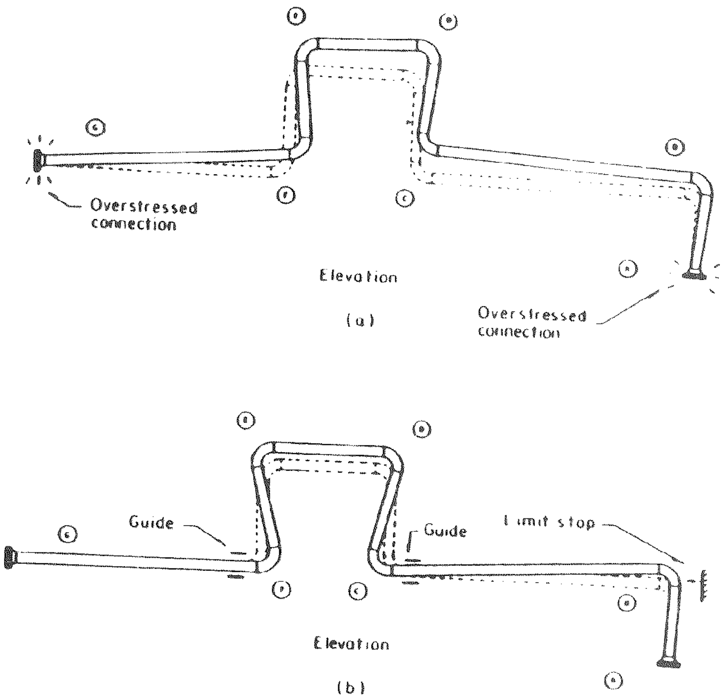

Figure 5.27 Use of supports to direct thermal growth; (*a*) loop without directed thermal growth; (*b*) loop with directed thermal movement.

Problem 5.7 The engineer should first generate an isometric drawing such as that in Fig. 5.28, showing all known vertical movements such as nozzle movements of the piping from the cold to hot positions as well as any points of zero vertical movement (due to restraints):

Point *A*:	2 in (50.8 mm) up, cold to hot
Point *C*:	0 in
Point *F*:	4 in (101.6 mm) down, cold to hot
Point *K*:	1 in (25.4 mm) up, cold to hot
Point *L*:	0 in
Point *M*:	0 in

Other sources of vertical movement are the risers *B-D* and *I-J*, which expand axially. From Table 5.4 we see that the expansion is = 0.0707 in/ft (0.0059 mm/m), so the riser expansions are computed as follows:

$$L_{BC} = (0.0707)(15) = 1.06 \text{ in } (26.9 \text{ mm}) \text{ up}$$

$$L_{CD} = (0.0707)(30) = 2.12 \text{ in } (53.8 \text{ mm}) \text{ down}$$

$$L_{IJ} = (0.0707)(10) = 0.707 \text{ in } (18.0 \text{ mm})$$

TABLE 5.6 Piping Expansion Stresses (Expansion factor c)

Temperature, T, °F	Carbon steel, C ≤ 0.30%	Carbon steel, C > 0.30%	C-moly and Low Cr.-moly Cr. ≤ 3%	Cr.-moly 5% ≤ Cr. Mo ≤ 9%	Austenitic stainless steels	Cr. stainless steels 12 Cr., 17 Cr., and 27 Cr.	25 Cr.–20 Ni	Wrough iron
70	0	0	0	0	0	0	0	0
100	37	40	40	35	54	34	47	44
150	98	106	106	92	143	90	125	120
200	160	171	171	149	232	145	204	195
250	228	244	244	212	323	204	287	273
300	294	315	315	271	414	264	368	352
350	305	391	391	335	509	326	455	434
400	436	467	467	396	603	389	541	514
450	510	547	547	465	699	455	629	598
500	584	626	626	531	794	520	716	681
550	664	711	711	603	893	590	809	768
600	743	796	796	672	989	659	901	855
650	827	886	886	714	1089	730	995	946
700	909	974	974	815	1189	799	1088	1035
750	996	1068	1068	891	1292	874	1186	1125
775	1038	1113	1113	929	1344	909	1235	1171
800			1159	967	1395	946	1284	1216
825			1208	1005	1448	983	1335	
850			1256	1043	1500	1022	1384	
875			1303	1081	1552	1061	1435	
900			1351	1121	1605	1097	1484	
925			1398	1161	1659	1134	1533	
950			1445	1200	1713	1174	1585	
975			1492	1240	1766	1212	1634	
1000			1538	1278	1820	1250	1681	
1050			1639	1357	1928	1328	1781	
1100			1737	1435	2036	1404	1879	
1150				1511	2144	1480	1980	

Expansion factor $c = \dfrac{\text{expansion in inches per 100 ft} \times E_c}{1728 \times 100}$

D = outside diameter I_P = moment of inertia of pipe