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Design of Optimum Plate-Fin Natural Convective Heat Sinks

The effort described herein extends the use of least-material single rectangular plate-fin analysis to multiple fin arrays, using a composite Nusselt number correlation. The optimally spaced least-material array was also found to be the globally best thermal design. Comparisons of the thermal capability of these optimum arrays, on the basis of total heat dissipation, heat dissipation per unit mass, and space claim specific heat dissipation, are provided for several potential heat sink materials. The impact of manufacturability constraints on the design and performance of these heat sinks is briefly discussed.
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Introduction

The exponential increase in the power density of microelectronic components, resulting from rapid advances in semiconductor technology [1], continues to fuel considerable interest in advanced thermal management of electronic equipment. While the use of fin structures to enhance air cooling is a simple and attractive thermal packaging technique, significant improvements in performance and reduction in resource consumption will be needed if air-cooled heat sinks are to meet the demands of the CMOS chips currently under development. The use of passive, natural convection-cooled rectangular plate-fin heat sinks offers substantial advantages in cost and reliability, but is often accompanied by relatively low heat transfer rates. The development of techniques for the enhancement of heat dissipation from such natural convection heat sinks, on the basis of projected area, heat sink mass, and heat sink volume, is, thus, essential if this thermal management technique is to meet the expectations of the packaging community.

The effort described herein builds on the strong theoretical base of the Kraus single plate-fin design and optimization [2] and extends the benefits of "least-material" single fins to the multiple fin arrays that constitute most heat sinks. In order to concretize the benefits of such "least-material" heat sink designs, the proposed modeling and optimization techniques will be applied to an advanced heat sink configuration, considered suitable for the cooling of a high-end microprocessor, later this decade. Thus, many of the results are derived for an aluminum heat sink on a 10×10 cm base, operating at an excess temperature of 25 K. In consideration of design and manufacturability constraints, the design space was chosen to include plate fins, between 1 and 15 mm in thickness, as well as fin lateral spacing from 1 to 15 mm. Further parametric analysis was performed on arrays of different base area (5×5 cm and 15×15 cm) yielding trends consistent with the present findings and making it possible to generalize these findings to the thermal optimization of a broad range of fin arrays [3].

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Heat Transfer From Plate-Fin Arrays

Basic Relations. The present study attached to an isothermal heat sink base, as may occur with the use of a thick metal base, heat pipe or vapor chamber base plate [4,5], focuses on the thermal behavior of fins. The readers' attention is directed to a parallel study, performed at this laboratory [6] in which the base of the heat sink was analyzed and optimized. Heat transfer from a fin array depicted in Fig. 1(*a*), is the sum of heat dissipation from the fins and from the exposed, unfinned area on the array base.

For an array with a large number of fins, and an isothermal base, the heat transfer, q_T can be approximated by

$$q_T = \mathrm{n}_{\mathrm{fin}} q_{\mathrm{fin}} + h_{\mathrm{base}} A_b \theta_b \tag{1}$$

where n_{fin} is the total number of fins, q_{fin} is the heat transfer rate from a single fin, h_{base} is the average heat transfer coefficient for the unfinned base area, A_b, and θ_b is the array base-to-ambient temperature difference. It is convenient to divide the base area into "unit cells," around each fin, $A_{b,f}$, and to re-express Eq. (1) in the form

$$q_T = n_{\mathrm{fin}}(q_{\mathrm{fin}} + h_{\mathrm{base}} A_{b,f} \theta_b) \tag{2}$$

The base area and fin area, available for heat transfer from each fin, can be found using

$$A_{b,f} = Ls \tag{3}$$

$$A_{\mathrm{fin}} = 2(LH + Ht + Lt/2) \tag{4}$$

where n_{fin} is the number of plate-fins on the array, given by

$$n_{\mathrm{fin}} = W/(s+t) \tag{5}$$

and assumed equal to the number of interfin spaces.

The total plate-fin volume is

$$V_{\mathrm{fin}} = n_{\mathrm{fin}} HLt \tag{6}$$

Applying the Murray-Gardner assumptions provided in [2], and assuming an insulated tip, the heat dissipation capability of a single fin can be expressed as

$$q_{\mathrm{fin}} = Lk_{\mathrm{fin}} t \theta_b m(\tanh mH) \tag{7}$$

In this relation, m is the fin parameter equal to

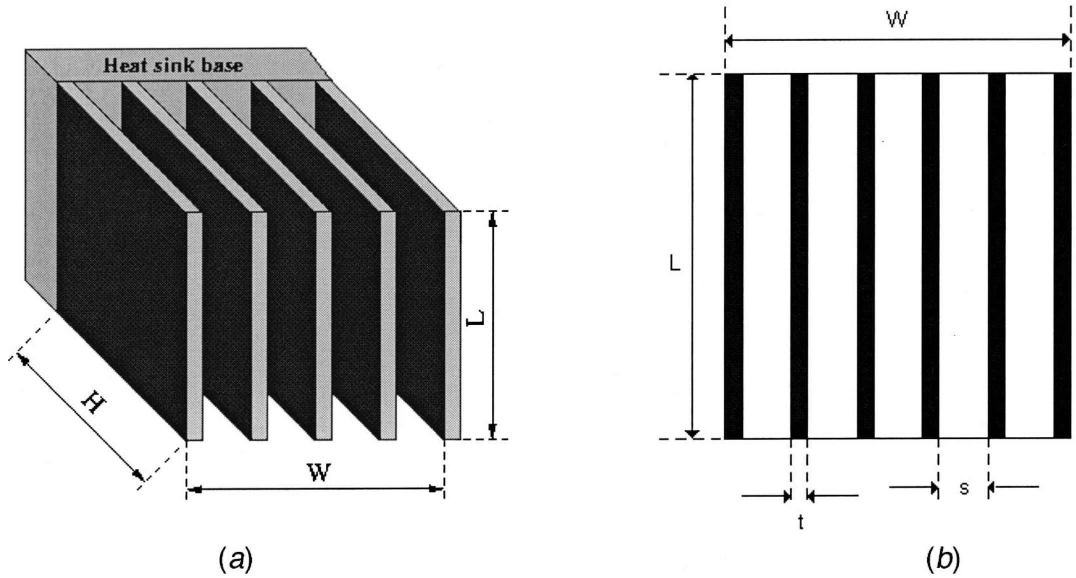


Fig. 1 Rectangular plate-fin heat sink array—(*a*) vertical configuration, (*b*) 2-D plate-fin schematic

$$m=(2h_{\text{fin}}/k_{\text{fin}}t)^{1/2} \tag{8}$$

where k_{fin} is the thermal conductivity of the fin, t is the thickness of a plate fin, and h_{fin} is the average fin area heat transfer coefficient.

For such a rectangular longitudinal fin, the fin efficiency is calculated using

$$\eta_{\text{fin}}=(\tanh mH)/mH \tag{9}$$

The geometric parameters for the plate fin arrays are illustrated in Fig. 1.

Heat Transfer Coefficients. Elenbaas [7] was the first to examine natural convective heat transfer between isothermal vertical flat plates and to document the variation of the heat transfer coefficient with plate spacing. For wide spacings, the coefficient was found to approach values associated with isolated plates, whereas for closely spaced plates the heat transfer coefficient decreased to values associated with fully developed, laminar flow. Widely spaced limit: for typical heat sink sizes and temperatures differences, laminar flow can be expected to occur, justifying the use of the commonly used form for the laminar flow Nusselt number correlation for the isolated plate heat transfer coefficient

$$\text{Nu}_L=[C\text{Ra}_L^{1/4}] \tag{10}$$

Or, extracting the heat transfer coefficient from the Nusselt number

$$h_{\text{base}}=[C\text{Ra}_L^{1/4}]k_f/L \tag{11}$$

where k_f is the fluid thermal conductivity, and Ra_L is the Raleigh Number based on the length of the heat sink base, L, and is given by,

$$\text{Ra}_L=(g\beta\theta_b\text{Pr}L^3)/\nu^2 \tag{12}$$

The heat transfer literature is rich in studies of laminar natural convection from vertical, isothermal plates. Nearly all of the available results can be correlated in the form of Eq. (10). The value of C has been reported to be in the range 0.515–0.59 in [8–10]. More complex expressions for C, recognizing the dependence on the Prandtl number, Pr, are provided in [10–13], which yield values of 0.515–0.55, on assuming a Prandl number of 0.71. However, the values for C obtained in widely spaced parallel plate channels most relevant to plate-fin array heat transfer, are found to be in the range 0.59–0.62 [7,14,15]. In this study, a C value of 0.59 was chosen for the isolated plate asymptote, and used, as well, to calculate the heat transfer coefficient prevailing along the exposed area of the heat sink base. Models for widely spaced plate arrays have resulted from research by Yovanovich and co-workers at the University of Waterloo [16–19]. This subsequently led to the development of the META code [19], which utilizes correlations based on the square root of the fluid wetted area as the characteristic dimension. A more comprehensive annular heat sink model can be found in Wang et al. [20,21], and subsequent extension to rectangular plate-fin array characterization is currently under development at the University of Waterloo [22].

Channel Values. For typical heat sink fin spacings, the prevailing heat transfer coefficients are intermediate between the isolated plate and fully developed limits, and can be found from correlations for natural convection in parallel, vertical plate channels [14,15]. Bar-Cohen and Rohsenow [14] extended the pioneering Elenbass correlation to a variety of boundary conditions. When applied to nonisothermal plates, as encountered in rectangular plate-fin arrays, this composite Nusselt number correlation takes the form

$$\text{Nu}_{\text{fin}}=h_{\text{fin}}s/k_f=[576/(\eta_{\text{fin}}\text{El})^2+2.873/(\eta_{\text{fin}}\text{El})^{1/2}]^{1/2} \tag{13}$$

where El is the Elenbaas number given by

$$\text{El}=(\text{g}\beta\theta_{\text{b}}\text{Prs}^4)/\text{L}\nu^2 \tag{14}$$

and where η_{fin} is used to relate the Nusselt number to the average temperature of the fin surface, i.e., $\theta=\eta_{\text{fin}}\theta_b$. Equation (13) represents a smoothly varying Nusselt number relation, where the first term $[576/(\eta_{\text{fin}}\text{El})^2]$ represents the closely spaced channel condition, while the second component $2.873/(\eta_{\text{fin}}\text{El})^{1/2}$ characterizes the isolated plate limit. This relation can be used to determine the Nusselt number and heat transfer coefficient for any channel spacing.

Fin Array Metrics. In evaluating and characterizing the cooling capacity of a heat sink, it is important to recognize the existence of several distinct fin array metrics. Often heat sinks are characterized simply by their thermal resistance, R_{hs}, expressed as

$$R_{hs}=\theta_b/q_T \tag{15}$$

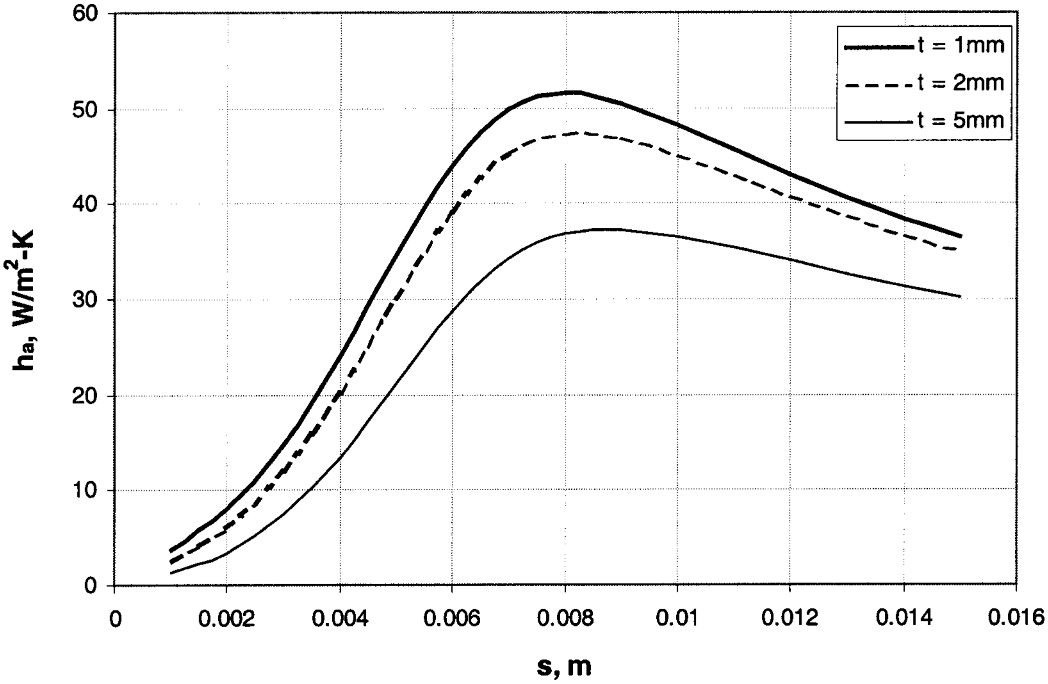


Fig. 2 Array heat transfer coefficient, h_a, W/m^2-K, plate-fin arrays, H=4.5 cm, L=W=10 m, k_{fin}=200 W/m-K, θ_b=25 K

While system designers may find R_{hs} the most useful of the heat sink metrics, its use masks the effect of heat sink area, as well as volume and material choice, on its thermal performance.

The effect of base area can be best captured in an array heat transfer coefficient, h_a, referenced to the area and excess temperature of the base, as

$$h_a = q_T / LW\theta_b \tag{16}$$

Comparison of h_a values to those normally associated with natural or forced convection on a bare surface can serve to identify the thermal enhancement provided by the use of the fins.

Figure 2 displays the variation of h_a for arrays of plate-fins in the configurations shown in Fig. 1, placed on a 10×10 cm base operating at an excess temperature of 25 K. The h_a for the plate-fin array rises with increasing fin spacing, attaining a maximum value of approximately 52.4 W/m^{2}K, for 1 mm thick fins, or nearly an order of magnitude above natural convection on the base surface, at a spacing of approximately 8 mm. It then decreases more gently as the spacing continues to widen towards 15 mm.

Elenbass [7] was the first to suggest that such an optimum spacing, at which the product of the plate area and plate heat transfer coefficient, constituting the overall array heat transfer rate, was a maximum, existed for each array. The optimum fin array Nusselt number value was found to be equal to 1.25. Based on Bar-Cohen [23], an expression for the optimum spacing, s_{opt}, is as follows:

$$s_{\text{opt}} = 2.66(L\nu^2/g\beta\eta_{\text{fin}}\theta_b \text{Pr})^{1/4} \tag{17}$$

It may be noted that modification of Eq. (13), to incorporate a lower C value of 0.515, and recorrelation of Eq. (17) yields a nearly identical expression for the optimum spacing, $s_{\text{opt}}[s_{\text{opt}} = 2.677(L\nu^2/g\beta\eta_{\text{fin}}\theta_b \text{Pr})^{1/4}]$, indicating only a minor dependence of the optimal design on the isolated plate C value.

The effectiveness with which fin material is utilized in the promotion of heat transfer can be characterized by the mass specific heat transfer coefficient, which is given by

$$h_m = q_T / \rho_{\text{fin}} V_{\text{fin}} \theta_b \tag{18}$$

where ρ_{fin} and V_{fin} are the mass density and volume of the fin material in the array, respectively.

The "space claim" heat transfer coefficient, h_{sc}, which represents the utilization of the volume occupied by the heat sink ($L \times W \times H$) for heat dissipation, can be calculated using

$$h_{sc} = q_T / LWH\theta_b \tag{19}$$

Least-Material Plate-Fin Arrays. While Eq. (7) can be used to determine heat transfer from any plate-fin geometry, it has been found possible to determine the fin aspect ratio (t/H) of the so-called "least-material" geometry, which maximizes the heat transfer rate for a given fin volume and mass [2,24–26]. In their classic work, Kern and Kraus [2] showed that the least-material plate fin is characterized by a unique value of the mH product, equaling 1.4192. Since the efficiency of such a fin is solely dependent on the mH product, the efficiency of the least-material plate fin is a fixed value, equal to 0.626. Subsequently, geometric optimizations of single fins, taking into consideration the effects of variable thermal conductivity, heat loss from fin tip, and internal heat generation, have been provided by Aziz [27] and in Kraus et al. [26].

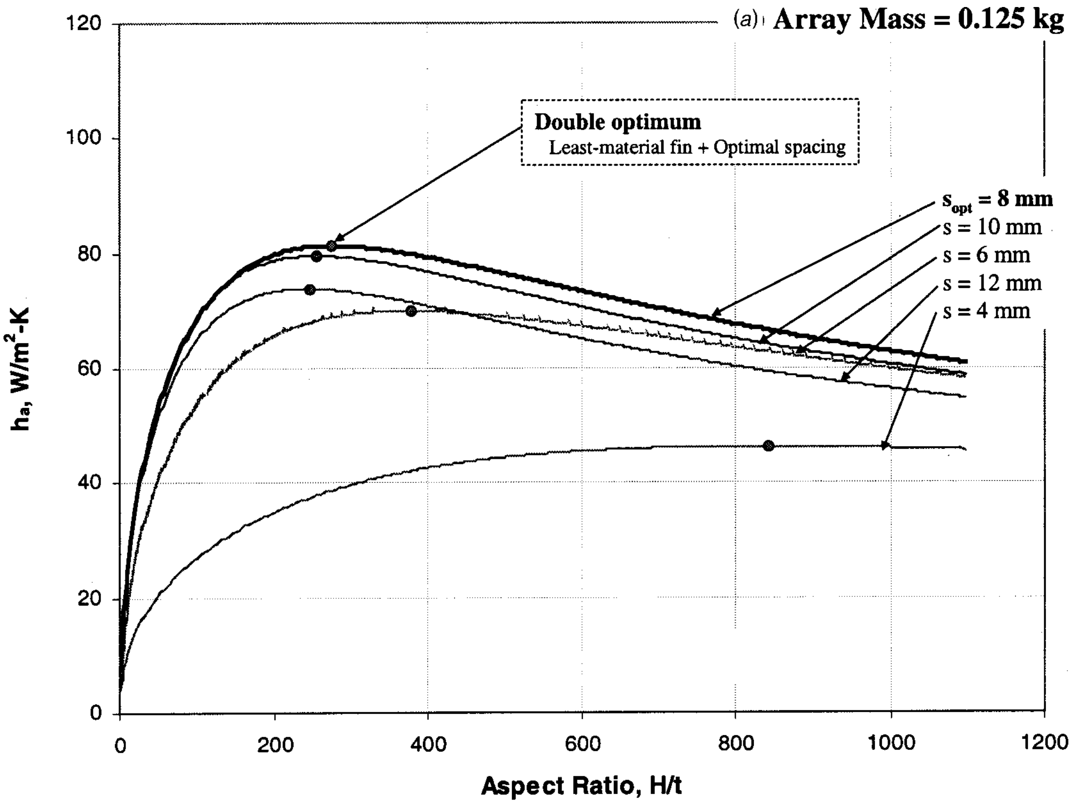
The aspect ratio and the heat dissipation of the least-material plate fin can be found from the following expressions [25]:

$$t = 0.993 h_{\text{fin}} H^2 / k_{\text{fin}} \tag{20}$$

$$q_{\text{fin}} = 1.25 L\theta_b (h_{\text{fin}}{}^2 t H k_{\text{fin}})^{1/3} \tag{21}$$

Least-Material Array Optimization

The preceding has revealed that there exists a fin spacing in plate fin arrays, which maximizes the heat transfer rate from the array. Moreover, relations have been presented for determining the aspect ratio of each of the fin shapes, which maximizes heat transfer for a specified fin mass. It appears that the desire to minimize the cost and weight of commercial heat sinks, while substantially enhancing their natural convection cooling rate, can be best ad-



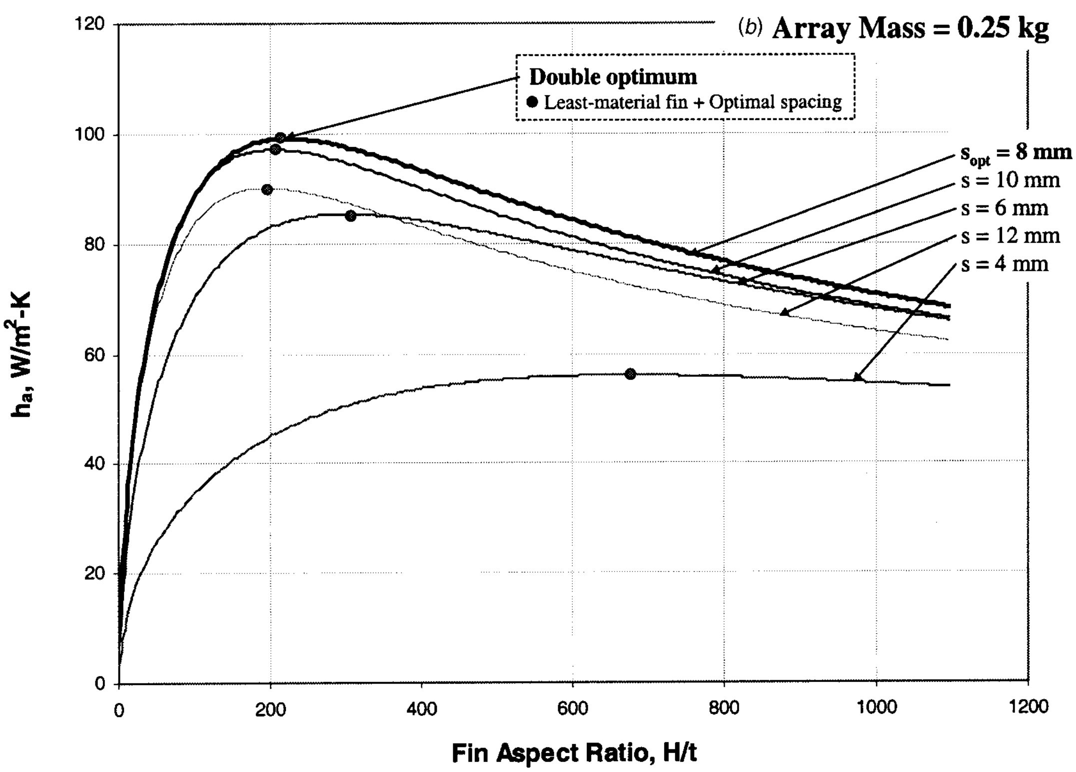


Fig. 3 Array heat transfer coefficient, h_a, W/m^2-K, plate-fin arrays— (*a*) array mass=0.125 kg, (*b*) array mass=0.25 kg, (*c*) array mass=0.375 kg, $L=W=10$ cm, aluminum, $k_{fin}=200$ W/m-K, $\theta_b=25$ K

dressed by the design of fin arrays that combine these two features, using least-material fins that are optimally spaced from each other.

Doubly Optimum Arrays. This concept is illustrated in Fig. 3, where the variation of the array heat transfer coefficient, h_a, with fin aspect ratio and fin lateral spacing, is plotted for a fixed array mass. Examining Fig. 3, it may be seen that of the configuration explored, the doubly optimum array consistently yielded

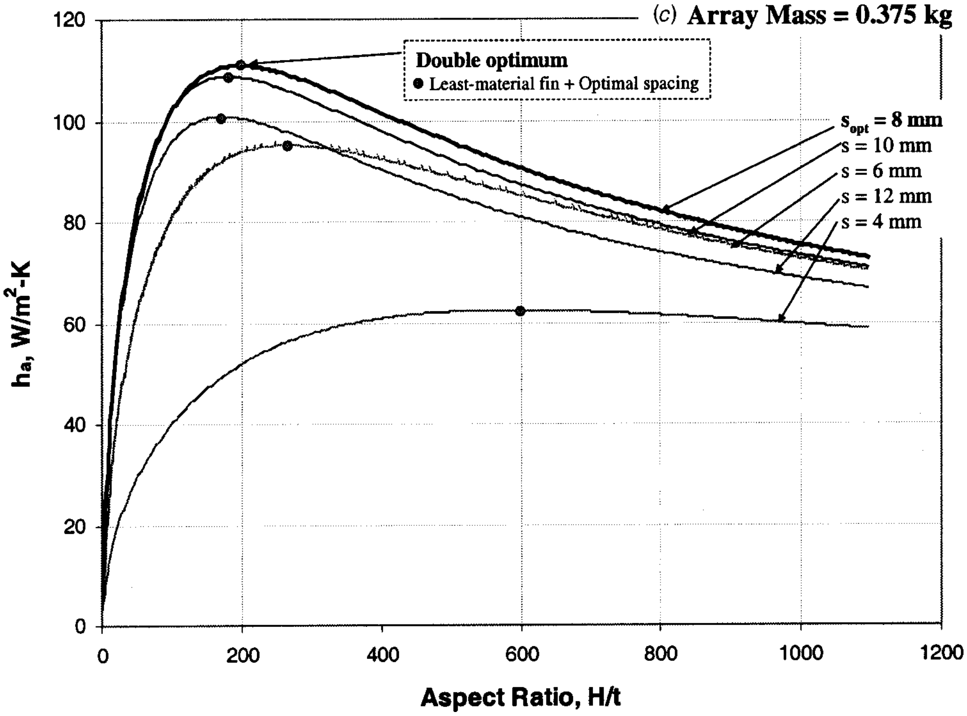


Fig. 3 (*continued*)

the highest heat transfer rates, with minimum thermal resistances of 1.23, 1.0, and 0.9 K/W for aluminum fin mass of 0.125, 0.25, and 0.375 kg, respectively.

Succeeding sections of this manuscript provide the results of a large number of computations aimed at quantifying the thermal performance of such "doubly-optimum" natural convection arrays. The least-material formulations provide a functional dependence between the fin dimension, t and fin height, H, thus eliminating the fin height as an independent variable, and considerably reducing the number of heat sink configurations that need to be evaluated. As a consequence, the computation time required to identify the most desirable configurations is dramatically shortened.

Array Heat Transfer Coefficient. As anticipated from Bar-Cohen and Jelinek [25], Fig. 4 reveals that the fin thickness which maximizes array heat transfer is found to equal the optimal clear horizontal spacing. For the conditions typical of this advanced heat sink, the dimensions are found to equal 9 mm, maximizing the array heat transfer coefficient at 198 W/m^2-K. Interestingly for

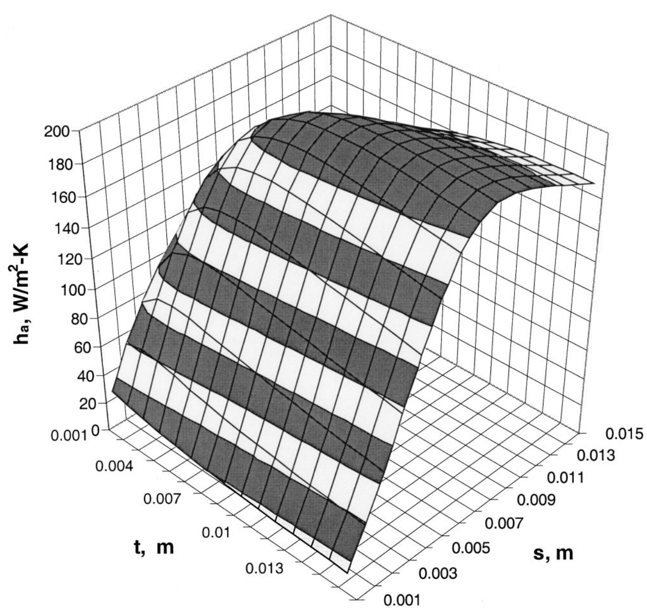


Fig. 4 Array heat transfer coefficient, W/m^2-K, least-material plate-fin arrays, $L = W = 10$ cm, aluminum, $k_{fin} = 200$ W/m-K, $\theta_b = 25$ K

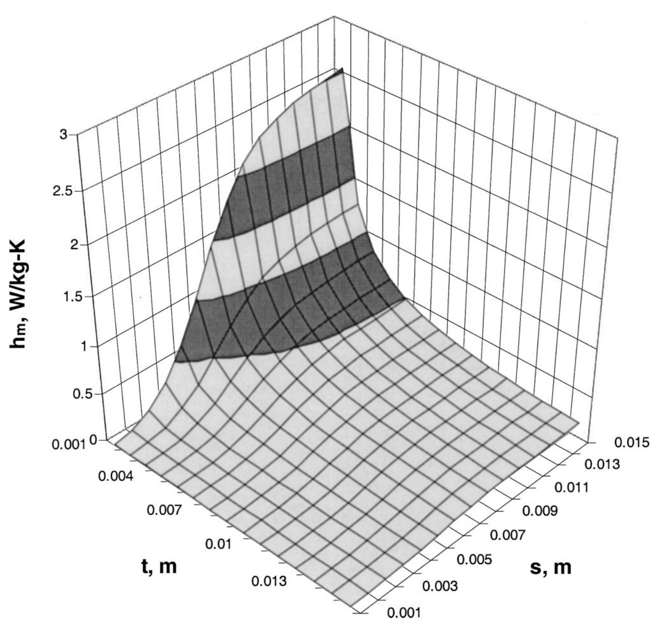


Fig. 5 Mass specific heat transfer coefficient, h_m, W/kg-K, least-material plate-fin arrays, $L = W = 10$ cm, aluminum, $\rho_{fin} = 2700$ kg/m^3, $k_{fin} = 200$ W/m-K, $\theta_b = 25$ K

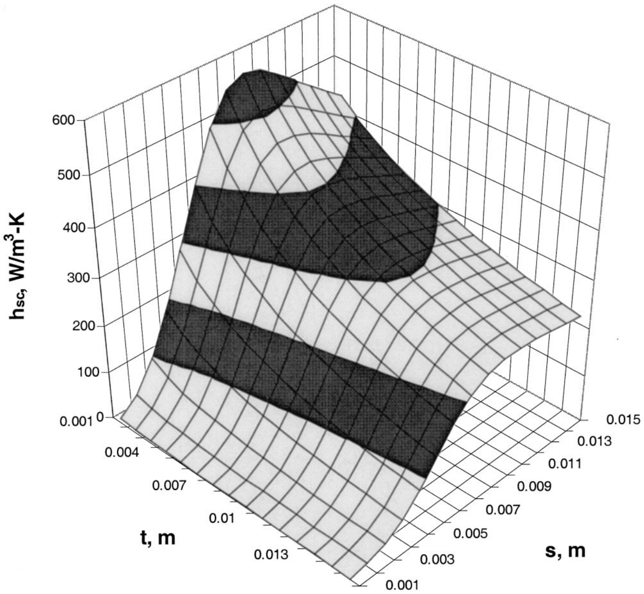


Fig. 6 Space claim heat transfer coefficient, h_{sc}, W/m^3-K, least-material plate-fin arrays, $L=W=10$ cm, aluminum, $k_{\text{fin}}=200$ W/m-K, $\theta_b=25$ K

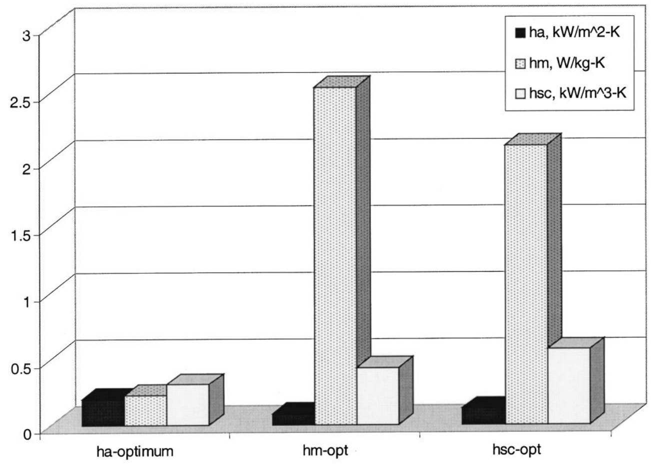


Fig. 7 Comparison of metrics for optimum designs, optimal least-material plate-fin arrays, $L=W=10$ cm, aluminum, $k_{\text{fin}}=200$ W/m-K, $\theta_b=25$ K

a broad range of fin thickness between 6 and 15 mm, and spacings between 8 and 10 mm, the array heat transfer coefficient is seen to fall only 5% below the peak value. For the conditions shown, this base area can thus transfer up to some 50 W in natural convection. An upper bound for maximum heat dissipation can be estimated to be 222 W/m^2-K, by extending the optimal plate-fin geometry to infinite height [24].

It may be noted that, for a base temperature of 70 °C and a ambient temperature of 45 °C, the heat transfer via radiation, estimated by the Bilitzky [24,28] effective channel emittance values, yielded a negligible h_a contribution of approximately 0.21 W/m^2-K for the optimum configuration.

Mass Specific Heat Transfer Coefficient. The mass-specific volumetric heat dissipation for the specified plate-fin arrays is plotted in Fig. 5, showing the maximum value of 2.5 W/kg-K to occur at largest horizontal spacing of 15 mm for the smallest fin thickness of 1 mm. Comparison of Figs. 4 and 5 reveal that maximum thermal performance occurs at the expense of material utilization, providing only 0.23 W/kg-K. Conversely, the geometry yielding the highest value of h_m is only capable of transferring some 85 W/m^2-K, far below the best value of nearly 200 W/m^2-K. This "best" h_m array thus uses some 335 g to transfer 21 W.

Space Claim Heat Transfer Coefficient. In Fig. 6, which plots the least-material space claim heat transfer coefficient for the rectangular plate heat sink against the array geometry, the maximum h_{sc} of 566 W/m^3-K, is found to occur at a fin thickness of 1 mm and a lateral spacing of 9 mm, the previously recognized optimum spacing. Consideration of volumetric efficiency thus provide a design with 67% shorter fins than desired for the maximum h_a value, with a drop in heat dissipation of 38%. Interestingly, both the h_a peak and the h_m peak fall in regions of relatively high space claim heat transfer coefficient, at 306 and 428 W/m^3-K, respectively.

Metric Comparison. The development and optimization of a particular heat sink design requires guidelines regarding the relative importance of the various thermal metrics. However, Fig. 7 shows the maximum h_{sc} design to be the most satisfactory, when considering heat dissipation capability, in conjunction with material and space claim utilization. The maximum h_{sc} design yields h_a and h_m values that are 62 and 83%, respectively, of their corresponding maximum values. Alternatively, the optimized h_a and h_m designs perform poorly with respect to mass utilization and total heat dissipation, respectively.

Material Selection. The thermal optimization discussed thus far was carried out for aluminum, the most common of heat sink materials. Following [2] the ratio of $k_{\text{fin}}/\rho_{\text{fin}}$ can serve to guide the selection of heat sink materials with the highest value material yielding the best heat transfer capability per unit mass. A summary of metric values obtained for aluminum (Al), magnesium (Mg), and copper (Cu), are detailed in Fig. 8 and Table 1. While aluminum and copper, with $k_{\text{fin}}/\rho_{\text{fin}}$ ratios of 0.074 and 0.045, respectively, are commonly used heat sink materials, Kraus and Morales [29] were the first to clearly describe the advantage of magnesium, with a $k_{\text{fin}}/\rho_{\text{fin}}$ ratio of 0.09. A more recent study on the use of magnesium fin arrays, can be found in Brown et al. [30]. The magnesium M1A alloy used in this study [30] possessed a thermal conductivity, k_{fin}, of 138 W/m-K and a density, ρ_{fin}, of 1760 kg/m^3, which deviates mildly from the ideal values presented in Table 1. Looking closely at the maximum optimum columns of Table 1, it can be seen, as anticipated, that the copper arrays show higher values for h_a than the magnesium and aluminum heat sinks by a factor of 1.6 and 1.4, respectively. The Mg and Al designs are considerably better than copper with respect to h_m, by a factor of 5 and 3.3, respectively. Comparisons for height constrained least-material designs, provided in Table 1, show the different fin materials to have comparable heat dissipation capability and thus space claim utilization. However, with respect to array mass utilization, the magnesium and aluminum designs outperform the copper arrays by a factor of 1.7 and 2, respectively.

General Optimization for Fixed Base Area

The optimal fin dimension and horizontal spacing resulting from the least-material optimization can be considered to be a starting point for array optimization. The use of Eqs. (20) and (21) assure that each fin will have the lowest mass possible for the resulting value of q_{fin}, i.e., it will constitute an individually optimum fin. A more comprehensive search of the relevant parametric space is needed to establish the fin geometry that yields the true array optimum. Typical results of the global plate-fin array optimization are illustrated in Fig. 9, with reference to seven distinct configurations. The design space explored for the horizontal spacing, s, and the fin thickness, t, is the same as that for the least-material analysis, and the fin height is varied from 0.05 m to 1 m.

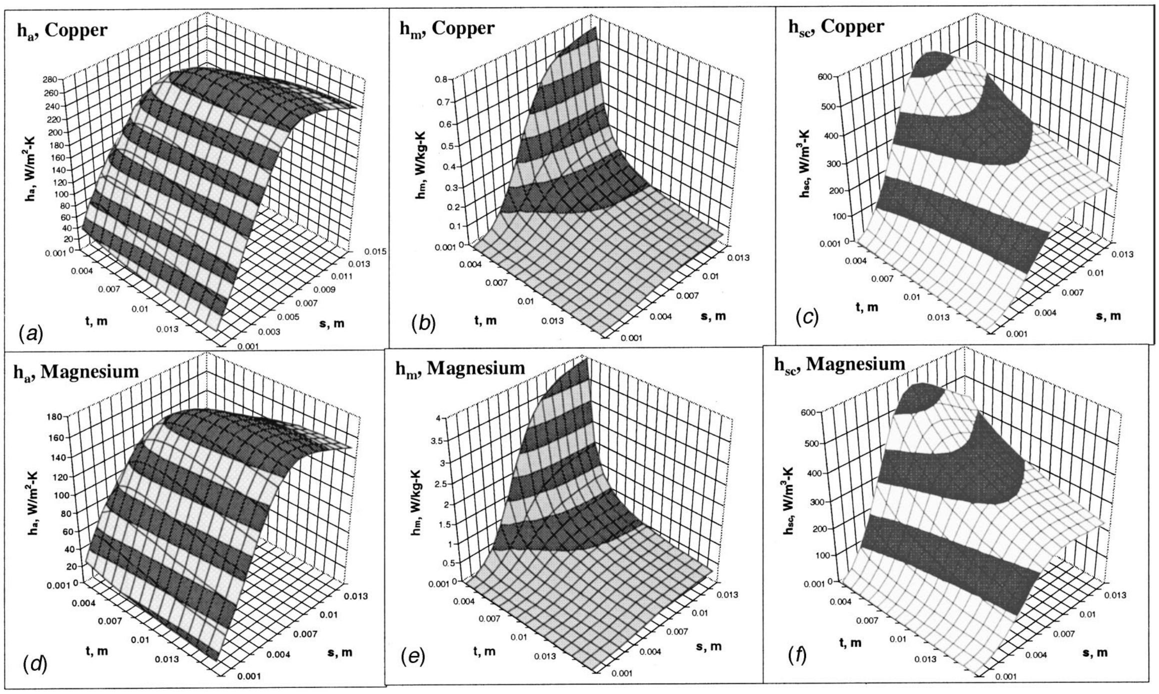


Fig. 8 Thermal performance metrics, least-material plate-fin arrays, copper—(*a*) h_a, W/m^2-K, (*b*) h_m, W/kg-K, (*c*) h_{sc}; W/m^3-K, magnesium: (*d*) h_a, W/m^2-K, (*e*) h_m, W/kg-K, (*f*) h_{sc}, W/m^3-K; $L=W=10$ cm, $\theta_b=25$ K

Table 1 Summary: maximum heat transfer and height constrained designs, least-material optimization, $L=W=10$ cm, $\theta_b=25$ K

Parameter	Maximum-optimum			Fin height=100 mm			Fin height=50 mm		
	Al	Cu	Mg	Al	Cu	Mg	Al	Cu	Mg
$\rho_{\rm fin}$, kg/m^3	2700	8933	1740	2700	8933	1740	2700	8933	1740
$k_{\rm fin}$, W/m-K	200	400	156	200	400	156	200	400	156
Geometry									
t, mm	9	9	9	0.21	0.11	0.3	0.05	0.03	0.07
s, mm	9	9	9	9	9	9	9	9	9
H, mm	648	917	573	100	100	100	50	50	50
Array mass, kg	8.6	41	5	0.063	0.11	0.052	0.008	0.013	0.0065
Fin number, N	6	6	6	11	11	11	11	11	11
Aspect ratio, H/t	72	102	64	466	934	364	934	1867	728
Spacing ratio, H/s	72	102	64	11	11	11	6	6	6
Metrics									
Q, W	49.5	69.7	43.8	16.1	16.3	16	8.93	8.95	8.91
h_a, W/m^2-K	198	279	175	64.5	65.3	64.1	35.71	35.82	35.65
h_m, W/kg-K	0.23	0.07	0.35	10.3	6.2	12.4	45	27	54
h_{sc}, W/m^3-K	305.6	279	306.2	645	653	641	714	716	713
$\eta_{\rm fin}$	0.626	0.626	0.626	0.626	0.626	0.626	0.626	0.626	0.626

It should be noted that the range of fin heights, H, has been deliberately extended to uncommonly large values to aid in making the appropriate comparisons.

Examination of Fig. 9 reveals that a "maximizing" fin height exists for each fin thickness-fin spacing configuration. The array heat transfer coefficient appears to increase steeply, as this fin height is approached, and to decrease in a gradual fashion, as this value of H is exceeded. This drop-off in fin performance appears to result from the decrease in fin efficiency, accompanied by a reduction in the fin heat transfer coefficient, for fins of progressively greater height. Despite this variation, it is easy to see that more than a factor of two separates the performance of the "best" and the "worst" arrays considered, thus justifying the effort required to optimize heat sink designs for electronic cooling applications.

It can be seen from Fig. 9, that the best performing array has the same lateral spacing and fin thickness as for the least-material optimal array, justifying the use of the least-material approach to design fin arrays. The "best" performing array improves on the

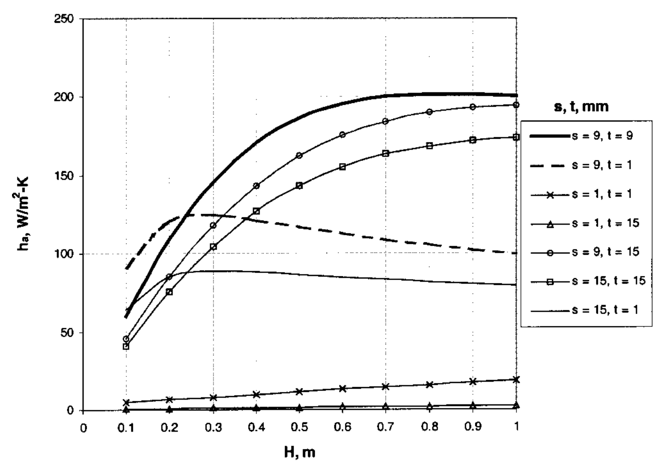


Fig. 9 Array heat transfer coefficient, h_a, plate-fin arrays, $L=W=10$ cm, $k_{\rm fin}=200$ W/m-K, $\theta_b=25$ K

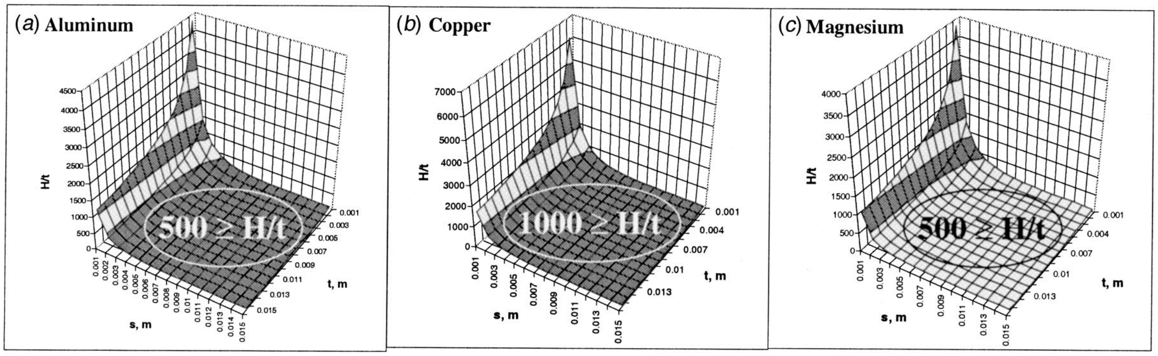


Fig. 10 Plate aspect ratio, H/t, least-material plate-fin arrays, (a) aluminum, (b) copper, (c) magnesium, $L=W=10$ cm, $\theta_b=25$ K

thermal performance of the least-material optimal array by only 2%, thus indicating excellent estimation of the maximum heat dissipation by using the least-material procedure. This improvement is as a result of extending plate height of the optimal "least-material" geometry from 0.648 m to 0.8 m, increasing the array mass by 23%.

Manufacturability Considerations

The financial constraints at work in the electronic industry make it essential that specific cooling requirements be achieved with the lowest cost solution. The material and manufacturing costs are the major factors influencing the use of plate fin heat sinks, especially in micro-electronic applications. In large volume production, the "least-material" methodology when utilized in conjunction with manufacturing consideration, will nearly always provide the lowest cost solution. Thus, in the coming years the designers will be required to work closely with the heat sink manufacturers, to generate high-performance cooling solutions, at the lowest cost possible. Manufacturability constraints, when applied to the thermal design, will result in heat sinks with reduced thermal performance. In the current study, maximum volumetric heat transfer coefficient is found to occur at largest horizontal spacing of 15 mm. However, Figs. 4, 5, and 8 indicate a plateau for h_m, for higher value of h_a, allowing the designer to ignore the geometry effects on volumetric heat dissipation, when designing in these ranges of h_a, thereby also relaxing the tolerance required for their manufacture. Existence of design plateau also allows for flexibility in choice of manufacturing process.

The fin heights recommended by the least-material optimization are often out of the range of existing manufacturing technology. Figure 10 shows the variation of the plate aspect ratio with array geometry. A close comparison of the aspect ratios, H/t, for the different materials, indicates the magnesium arrays to be easier to manufacture than the other arrays.

The trends observed in this section, suggesting the improved thermal performance for arrays of large aspect ratios and fin heights, provides incentive for the development of innovative manufacturing techniques, that allow the manufacture of thermally optimal fin arrays.

Conclusion

A least-material optimization procedure has been successfully demonstrated for vertical rectangular longitudinal plate fin arrays in natural convective heat transfer. The optimally spaced least-material array was also found to be the globally superior thermal design, showing the least-material approach to be a suitable optimization design heuristic. A comparison of the thermal performance for different materials shows magnesium to be the most efficient in material utilization. The fin aspect ratios of these optimum arrays appear to be largely out of the range of conventional manufacturing techniques, providing an incentive for the development of innovative manufacturing techniques, that allow the manufacture of thermally optimal fin arrays.

Nomenclature

A = heat transfer area
El = $=g\beta \mathrm{Pr}\theta_b s^3(s/L)/\nu^2$, Elenbaas no. based on s
H = fin height, m
L = length of array, m
Pr = Prandtl no., $=\mu c_p/k_f$
Ra_L = $=g\beta \mathrm{Pr}\theta_b L^3/\nu^2$, Raleigh's no. based on L
R_{hs} = heat sink thermal resistance, K/W
V_{fin} = total plate-fin array volume, m^3
W = width of array
c_p = fluid specific heat, J/kg-K
g = gravitational acceleration, m/s^2
h_a = array heat transfer coefficient based on base area, W/m^2-K
h_{base} = avg. exposed base area heat transfer coefficient, W/m^2-K
h_{sc} = space claim heat transfer coefficient, W/m^3-K
h_{fin} = average fin heat transfer coefficient, W/m^3-K
h_m = material specific heat transfer coefficient, W/kg-K
k_{fin} = thermal conductivity of heat sink material, W/m-k
k_f = thermal conductivity of fluid at base of array, W/m-k
m = fin parameter
n_{fin} = total no. of fins in array
q_{fin} = heat dissipation from single fin, W
q_T = total heat transfer from fin array, W
s = lateral clear spacing in fin arrays, m
t = fin thickness, m
β = thermal coefficient of expansion, K^{-1}
η_{fin} = fin efficiency
μ = mean dynamic viscosity of fluid
ρ_{fin} = fin material density, kg/m^3
ν = mean kinematic viscosity of fluid
θ_b = array base excess temperature, K

Subscripts

b, base = fin array base
hs = heat sink
fin, f = plate-fin
m = mass
opt = optimal, optimum
sc = space claim

References

[1] Bar-Cohen, A., 1992, "State-Of-The-Art and Trends in the Thermal Packaging of Electronic Equipment," ASME J. Electron. Packag., **114**, pp. 257–270.

[2] Kern, D. Q., and Kraus, A. D., 1972, *Extended Surface Heat Transfer*, McGraw-Hill, New York, NY.
[3] Iyengar, M., 2003, Ph.D. thesis in preparation, Department of Mechanical Engineering, University of Minnesota, Minneapolis, MN.
[4] Zhao, Z., and Avedisian, C. T., 1997, ''Enhancing Forced Air Convection Heat Transfer from an Array of Parallel Plate Fins Using a Heat Pipe,'' Int. J. Heat Mass Transf., **40**(13), pp. 3135–3147.
[5] Garner, S. D., and Toth, J. E., 1997, ''Heat Pipes: A Practical and Cost Effective Method for Maximizing Heat Sink Effectiveness,'' ASME EEP-Vol. 19-2, pp. 1897–1902.
[6] Ben Achour, M. F., and Bar-Cohen, A., 1999, ''Heat Sink Optimization for Maximum Performance and Minimum Mass,'' ASME EEP-Vol. 26-1, pp. 737–744.
[7] Elenbaas, W., 1942, ''Heat Dissipation of Parallel Plates by Free Convection,'' Physica (Utrecht), **9**(1), pp. 665–671.
[8] McAdams, W. H., 1954, *Heat Transmission*, McGraw-Hill, New York, NY.
[9] Karagiozis, A., Raithby, G. D., and Hollands, K. G. T., 1994, ''Natural Convection Heat Transfer from Arrays of Isothermal Triangular Fins in Air,'' ASME J. Heat Transfer, **116**, pp. 105–111.
[10] Burmeister, L., 1993, *Convective Heat Transfer*, Second Edition, John Wiley & Sons, New York, NY.
[11] Incropera, F., and DeWitt, W., 1996, *Introduction to Heat Transfer*, Third Edition, John Wiley & Sons, New York, NY.
[12] Churchill, S. W., and Chu, H. H. S., 1975, ''Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate,'' Int. J. Heat Mass Transf., **18**, pp. 1323–1329.
[13] Kays, W. M., and Crawford, M. E., 1993, *Convective Heat and Mass Transfer*, Third Edition, McGraw-Hill, New York, NY.
[14] Bar-Cohen, A., and Rohsenow, W., 1984, ''Thermally Optimum Spacing of Vertical, Natural Convection Cooled, Parallel Plates,'' ASME J. Heat Transfer, **106**, pp. 116–122.
[15] Raithby, G. D., and Hollands, K. G. T., 1998, ''Natural Convection,'' *Handbook of Heat Transfer*, eds., W. M. Rohsenow, J. P. Hartnett, and Y. I. Cho, McGraw-Hill, New York, NY, Chap. 4, pp. 33–34.
[16] Yovanovich, M. M., 1987, ''Natural Convection from Isothermal Spheroids in the Conductive to Laminar Flow Regimes,'' AIAA Pap., AIAA-87-1587.
[17] Yovanovich, M. M., 1987, ''On the Effect of Shape, Aspect Ratio, and Orientation upon Natural Convection from Isothermal Bodies,'' ASME HTD-Vol. 82.
[18] Lee, S., Yovanovich, M. M., and Jafarpur, K., 1991, ''Effects of Geometry and Orientation on Laminar Natural Convection from Isothermal Bodies,'' J. Thermophys. Heat Transfer, **5**, pp. 208–216.
[19] Culham, J. R., Yovanovich, M. M., and Lee, S., 1994, ''Thermal Modeling of Isothermal Cuboids and Rectangular Heat Sinks Cooled by Natural Convection,'' InterSociety Conference on Thermal Phenomena (ITHERM), pp. 73–82.
[20] Wang, C. S., Yovanovich, M. M., and Culham, J. R., 1999, ''General Model for Natural Convection: Application to Annular-Fin Heat Sinks,'' ASME J. Electron. Packag., **121**(1), pp. 44–49.
[21] Microelectronics Heat Transfer Laboratory Website (Online Tools, Circular and Square Annular Heat Sinks): http://www.mhtl.uwaterloo.ca/old/onlinetools/nat_hs/ann_hs/input.html
[22] Microelectronics Heat Transfer Laboratory Website (Online Tools, Rectangular Plate-Fin Heat Sinks): http://www.mhtl.uwaterloo.ca/old/onlinetools/nat_hs/rect_hs/input.html
[23] Bar-Cohen, A., 1979, ''Fin Thickness for an Optimized Natural Convection Array of Rectangular Fins,'' ASME J. Heat Transfer, **101**, pp. 564–566.
[24] Kraus, A. D., and Bar-Cohen, A., 1995, *Design and Analysis of Heat Sinks*, John Wiley & Sons, New York, NY.
[25] Bar-Cohen, A., and Jelinek, M., 1986, ''Optimum Arrays of Longitudinal, Rectangular Fins in Convective Heat Transfer,'' Heat Transfer Eng., **6**(3), pp. 68–78.
[26] Kraus, A. D., Aziz, A., and Welty, J., 2001, *Extended Surface Heat Transfer*, John Wiley & Sons, New York, NY.
[27] Aziz, A., 1992, ''Optimum Dimensions of Extended Surfaces Operating in a Convective Environment,'' Appl. Mech. Rev., **45**(5), pp. 155–173.
[28] Bilitzky, A., 1986, ''The Effect of Geometry on Heat Transfer by Free Convection from a Fin Array,'' MS thesis, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva, Israel.
[29] Kraus, A. D., and Morales, H. J., 1983, ''Case for the Magnesium Heat Sink, Proceedings of the Technical Program—National Electronic Packaging and Production,'' Cahners Exposition Group, Des Plaines, IL, pp. 102–112.
[30] Brown, J. F., Riopelle, L., and Shirazi, S. A., 1993, ''Magnesium Heat Sink Evaluations, Magnesium Properties and Applications for Automobiles,'' SAE Special Publications, Warrendale, PA, no. 962, pp. 27–36.