Simplified bearing plate computations for post-tensioning anchorages

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Presents an analytical method, based on tests, for simplifying bearing plate computations under anchorages of post-tensioned concrete members.
A fully-worked numerical example demonstrates the design procedure.

Many post-tensioning anchorage systems today are using an independent mild steel bearing plate to transmit the prestressing force to the concrete member. The purpose of this plate is to distribute efficiently the high concentrated bearing stresses to the concrete.

Unfortunately, though, there is no simple analytical method available whereby the bearing plate stresses can be evaluated accurately.

In general, the plate area is governed by the concrete bearing stress, which is easy to establish if the average concrete stress is considered. However, the computation of the plate thickness is extremely difficult because of unknown factors such as load distribution, stress distribution, and arch effects.

In practice, the bearing plate thickness is usually based on tests or past experience.

This paper presents a simplified design procedure for calculating the bearing stresses of plates under post-tensioning anchorages. The method is based on the results of many tests, one of which is described in this paper.

Because of the complexity of the problem, several simplifying assumptions are made for both square and rectangular plates.

A numerical example, using a square plate, is included to show how this method can be used to calculate the bearing plate stresses.

Bearing Plate Test

Prior to actual use, it is customary to test the performance of a bearing plate anchorage assembly. The following is a description of one such test:

Fig. 1a shows a testing bed. The stressing jack on the extreme left bears on the concrete test block. Further to the right are two larger test bed blocks, one stationary and one sliding, with two rams in between. The tendon is threaded through the test block, through the two test bed blocks, and between the two rams. It is then anchored at a non-stressing "super" anchor block at the far right.

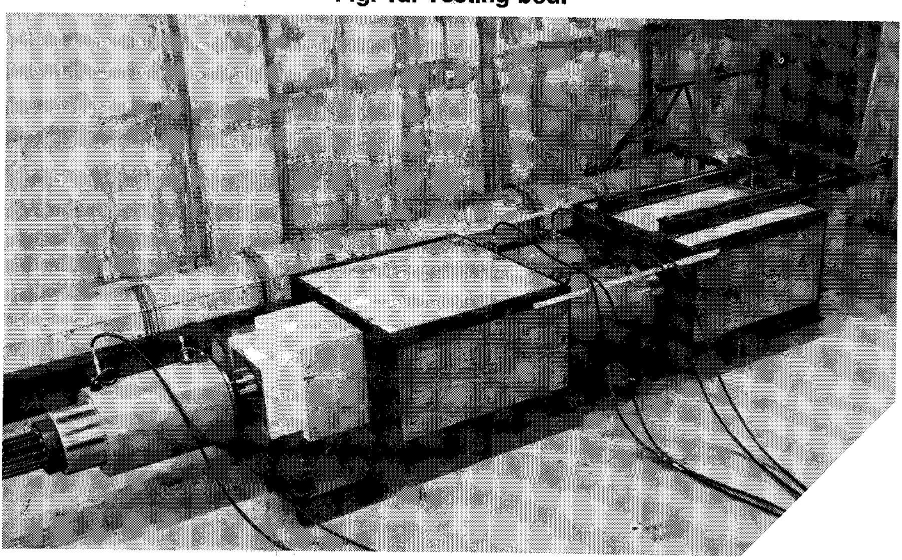
In general, the tendon is first stressed and anchored using the stressing jack. (A jacking force of 0.75 to 0.80 of ultimate is usually employed.) The stressing jack is then removed and the tendon is subsequently restressed in steps, to destruction, using the two rams.

Fig. 1b shows the test block consisting of the anchorage, embedded bearing plate, and concrete block. The plate cover, block strength, block reinforcement, and tendon enclosure is designed to duplicate actual site conditions.

Design criteria

The following design criteria, developed by the California Department-

Fig. 1a. Testing bed.



ment of Transportation, were used for the above test:

1. The final unit compressive stress on the concrete directly underneath the plate or assembly should not exceed 3000 psi.

2. The bending stresses developed in the plates or assemblies induced by the pull of the prestressing steel should not exceed the yield point of the material or cause visible distortion in the anchorage plate when 95 percent of the specified ultimate tensile strength of the tendon is applied as determined by the engineer.

To conform with Criterion 1, the average (uniform) concrete stress underneath the plate is computed under final load, assumed to be 60 percent of the guaranteed ultimate tensile strength of the tendon (GUTS).

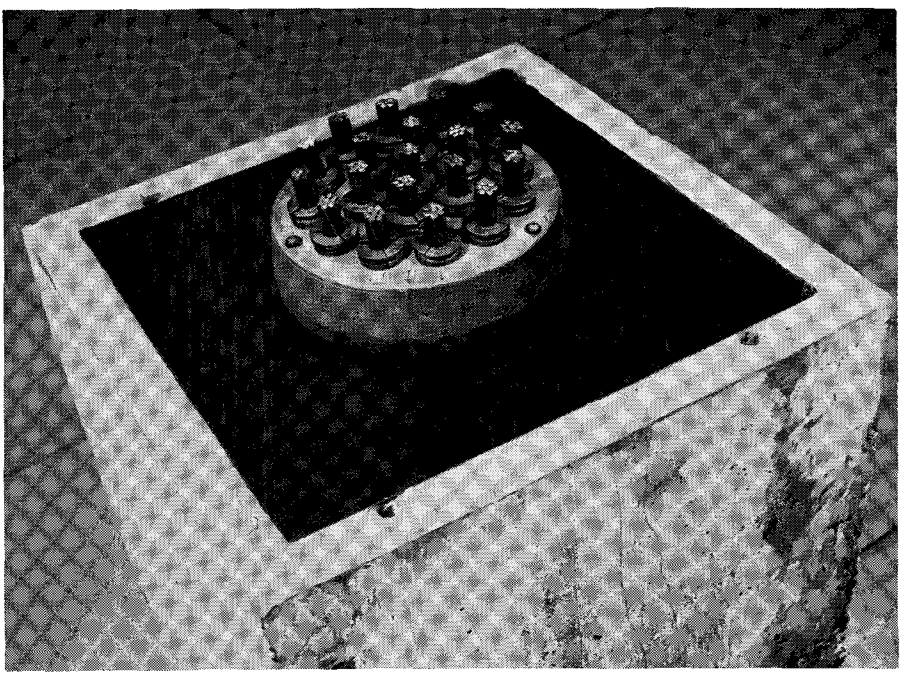
It should be noted that the 3000-psi concrete stress, recommended for general use, may be substantially increased in particular cases, especially when precast concrete is employed.

The steel bearing stress at the contact area between the anchorage and the plate should also be checked. It is suggested that this stress be limited, under final load, to 90 percent of the plate yield stress.

Simplified Bearing Plate Computations

This section discusses the range of applicability of this method, the design assumptions and design procedure for both square and rectangular plates and three typical stress block characteristics. Finally, the last section gives a fully worked numerical example to show how this method is used to compute the bearing plate stresses.

Fig. 1b. Test block detail.



Range of applicability of simplified method

It should be emphasized that this paper deals only with the bearing plate computations. The concrete strength, concrete cover on plate edges, and mild steel reinforcement must be properly selected to develop the required forces.

Due to the simplifying assumptions the calculations must be considered "empirical" and should not replace the basic post-tensioning anchorage plate testing of the assemblies. The proposed analytical approach is recommended for particular cases (e.g., tendon sizes and plate shapes falling in between the tested basic size tendons).

Also, the tendon manufacturer may adjust the design assumptions to correspond to the test results of his basic size tendons.

Design assumptions

The following design assumptions apply to both square and rectangular plates:

1. Use a stressing force of 0.95 of GUTS.

2. If the anchorage or hole in the plate is circular, use square shapes of an equivalent area. This will simplify the computations.

3. Due to the very large anchorage stiffness, assume a uniform bearing stress, using a 45-deg arch effect, as shown in Fig. 2.

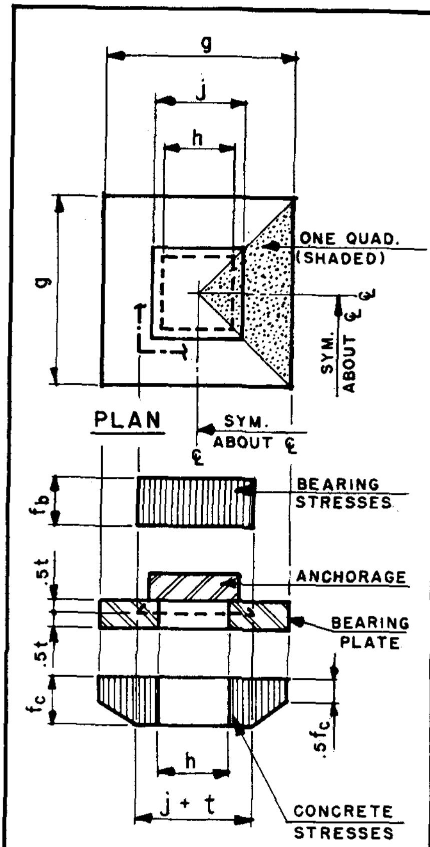
4. Due to plate flexibility, assume the concrete bearing stress to be variable as shown in Fig. 2. This assumption approximates a uniform stress under the anchorage with a parabolic fade-out under the overhangs.

5. Assume that the anchorage and the plate act as one homogeneous body.

6. In computing the section properties, consider the opening in the plate but disregard the holes in the anchorage.

7. In general, a computation of the maximum stresses due to combined biaxial bending and shear is not necessary.

8. At contact area between anchorage and plate, the ratio of horizontal shear to prestressing force should not exceed 0.5.



CROSS SECTION
WITH CORRESPONDING STRESSES

FIG. 2

Discussion of assumptions

Assumptions 1, 2, and 3 are self-ex-

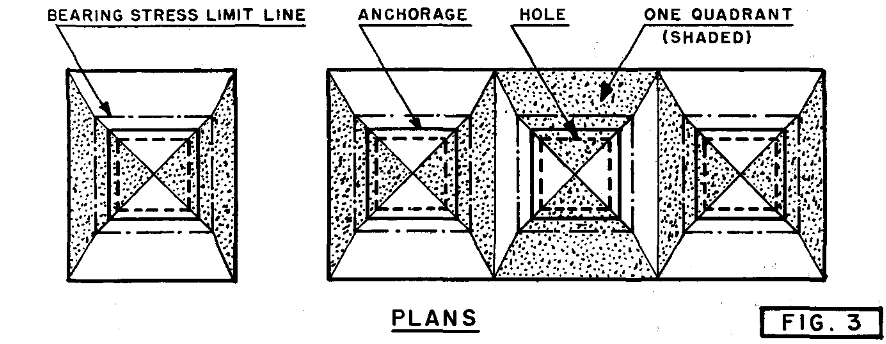


FIG. 3

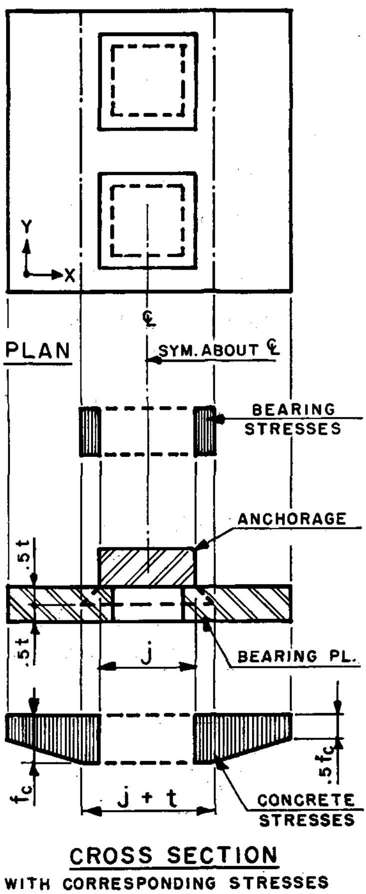


FIG. 4

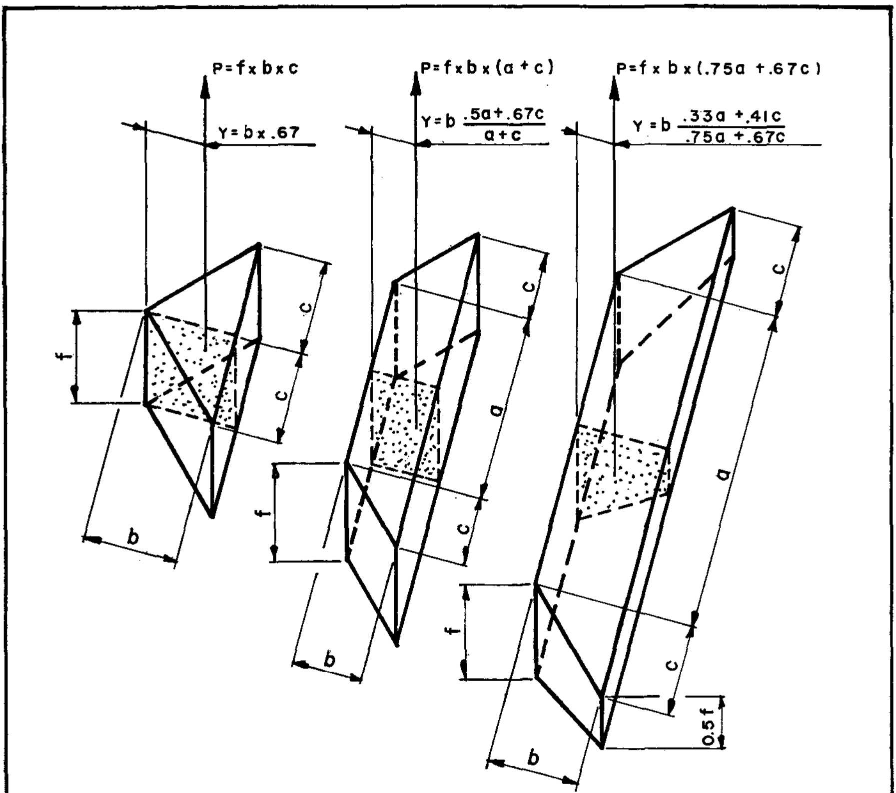
planatory. A commentary of the remaining assumptions is given below.

4. During a test under 0.95 of GUTS, the plate deflects in a dish-like shape. The maximum deflection of the plate is at the center whereas the plate edges sink into the concrete. (For example, in the case shown on Figs. 1b and 6, a deflection of 1/16 in. is observed at the center of the plate and a sinking of 1/32 in. at the edges.) After the load is released the plate will go back to its original shape if it has been properly dimensioned.

The tendon manufacturer may also adjust the shape of the bearing stress blocks in accordance with his basic tendon test results.

5. A visual test can show convincingly that the anchorage and plate assembly perform homogeneously. This is because there exists a large prestressing load which forces the anchorage against the plate. See also the discussion below under Assumption 8.

6. The anchorage is made out of a high quality steel. In the case of a strand anchorage, using a male plug or individual strand jaws, the anchorage is subjected to large tensile hoop stresses, and the holes are filled with strands, jaws, or a plug. The resulting compres-



ISOMETRIC VIEW OF TYP. STRESS BLOCKS

FIG. 5

sion stress due to bending is actually advantageous.

7. The maximum stresses due to biaxial bending and shear combined is found from the following equations:

$$f_b = \tfrac{1}{2}(f_{b1} + f_{b2}) \pm \{[\tfrac{1}{2}(f_{b1} - f_{b2})]^2 + f_v^2\}^{1/2}$$

and

$$f_v = \{[\tfrac{1}{2}(f_{b1} - f_{b2})]^2 + f_v^2\}^{1/2}$$

where f_{b1} and f_{b2} are bending stresses in the two directions.

This condition will occur at or near the 45-deg lines between quadrants. However, at the critical points, the bending stresses in the two directions are equal. Also, the shear stress is zero at the location of the maximum bending stress. Therefore the maximum f_b and f_v stresses will not exceed the stresses computed in one direction and need not to be checked.

8. The contact area between the anchorage and the bearing plate is usually minimal. The governing stress in this location is the horizontal shear, which is subject to redistribution. Also, it should be recognized that Assumption 2 (square anchorage and hole) and the fact that the plan area will be divided into four quadrants (see the "design procedure" which follows) greatly influence the magnitude of the horizontal

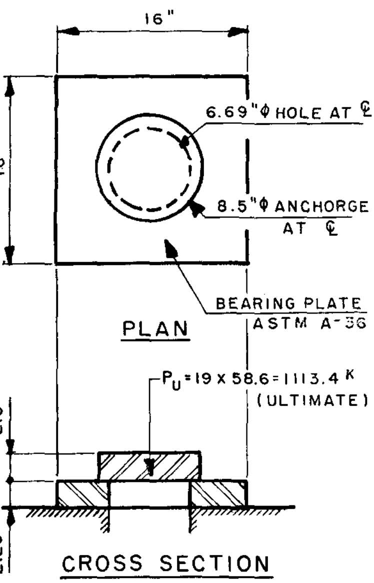


FIG. 6

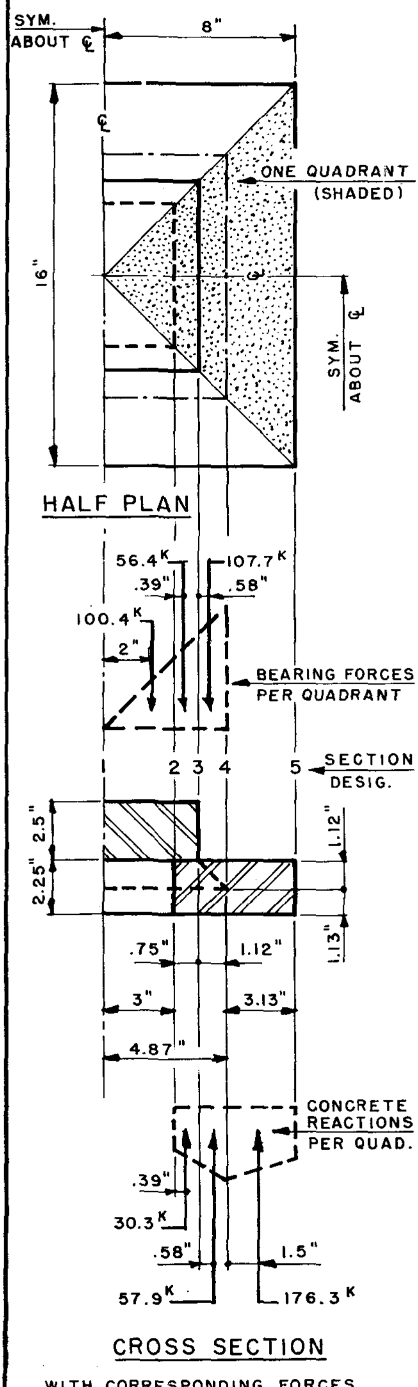


FIG. 7

shear stress. Nevertheless, the total shear force per quadrant over one-quarter of the stressing force will give a reasonable value of the friction coefficient which prevents sliding of the anchorage on the plate face.

Design procedure for square plates

Using the design assumptions listed previously:

1. Divide the plan loading area into four quadrants as shown in Fig. 2. This procedure is to prevent the same forces being considered twice in two directions.

2. Determine the average concrete stress underneath the plate and the steel bearing stress on the plate and check the stresses with the allowable values.

3. Compute the section properties in the critical location. Use the full anchorage and plate widths. Take into

consideration the hole in the plate but disregard the holes in the anchorage.

4. Determine the shears and bending moments per quadrant. Compute the vertical shear and bending stresses and check these values with the allowable stresses.

5. Calculate the horizontal shear force per quadrant at the contact area between the anchorage and the plate. The ratio of this horizontal force to the vertical prestressing force per quadrant should not exceed 0.5.

Design procedure for rectangular plates

The design assumptions listed for square plates apply also to rectangular plates. However, in the design procedure (see Step 1 in previous section) the quadrant should be laid out as shown in Fig. 3. In a rectangular arrangement there are two different quadrants per plate. However, only the one with a larger plate overhang will govern.

In the case of a common plate accommodating several anchorages closely spaced, the computations may be simplified as shown in Fig. 4.

In the y direction, compute the bearing stresses of the plate assembly under uniform load within the strip "j". However, in the x direction which governs, compute the bearing stresses of the plates under the loads shown in the cross section.

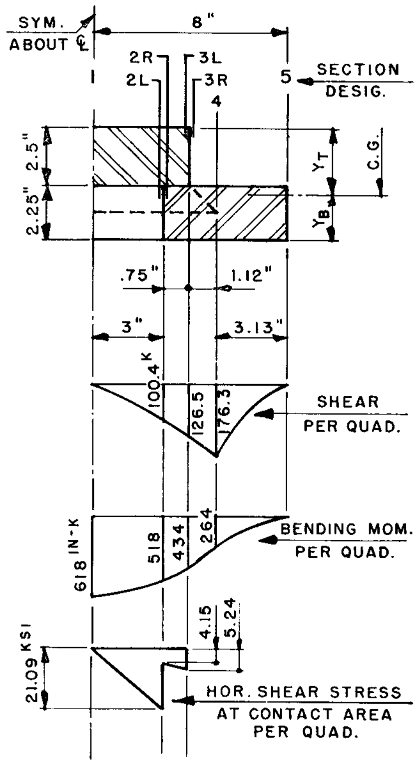
Stress block characteristics

The characteristics of three typical stress blocks employed in this paper are shown in Fig. 5. The following notation is used:

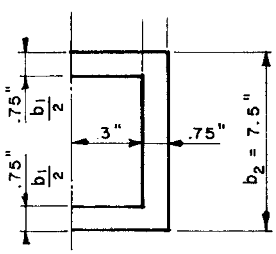
P = total force per stress block, kips
f = stress, ksi
a, b, c = distances, in.
y = location of center of gravity, in.

SECTION PROPERTIES
16" WIDE SECTION

SECTION	Y_T IN	Y_B IN	I IN^4	Q IN^3
1,2L	2.54	2.21	76.8	24.2
2R,3L	2.81	1.94	94.4	29.3
3R,4	1.125	1.125	15.2	



CROSS SECTION
WITH CORRESPONDING DIAGRAMS



PLAN OF CONTACT AREA
SUBJECTED TO HOR. SHEAR STRESS

FIG. 8

Design Example

Assume we have a round anchorage for nineteen 0.6-in. diameter strands with a guaranteed ultimate force of 58.6 kips per strand (or $P_u = 1113.4$ kips per anchorage).

Let the prestressing force be distributed on an ASTM A36 square plate as shown in Fig. 6 and Fig. 1b. The dimensions of the plate are 16 x 16 in. square and 2¼ in. thick In the middle of the plate there is a 6.69 in. diameter opening.

First, check under the final load of $0.6P_u$, the average concrete stress underneath the plate (f_{cf}), and the steel bearing stress on the plate (f_{pf}).

$$f_{cf} = \frac{0.6 \times 1113.4}{16 \times 16 - 0.25\,(\pi)\,6.69^2}$$
$$= 3.00 \text{ ksi (ok)}$$

$$f_{pf} = \frac{0.6 \times 1113.4}{0.25\,(\pi)\,(8.5^2 - 6.69^2)}$$
$$= 30.95 \text{ ksi} < 0.9 \times 36 \text{ (ok)}$$

Compute the bending and shear stresses (see Fig. 7) under a load of $0.95\ P_u$, making use of the assumptions listed previously.

Total force:

$$P = 0.95 \times 1113.4 = 1058 \text{ kips}$$

Bearing stress:

$$f_p = 1058/(4.87 \times 2)^2 = 11.15 \text{ ksi}$$

Concrete stress, using stress block formulas:

$$1058/4 = f_c[0.75(6 + 0.75) + 1.12\,(7.5 + 1.12) + 3.13z]$$

where $z = 0.75 \times 9.74 + 0.67 \times 3.13 = 9.40$

from which $f_c = 5.99$ ksi.

Bearing forces per quadrant:

$$P_{p12} = 11.15(3)3 = 100.4 \text{ kips}$$

$$P_{p23} = 11.15(0.75)(6 + 0.75) = 56.4 \text{ kips}$$

$$P_{p34} = 11.15(1.12)(7.5 + 1.12) = 107.7 \text{ kips}$$

Concrete reaction forces per quadrant:

$$P_{c23} = 5.99(0.75)(6 + 0.75) = 30.3 \text{ kips}$$

$$P_{c34} = 5.99(1.12)(7.5 + 1.12) = 57.9 \text{ kips}$$

$$P_{c45} = 5.99(3.13)9.40 = 176.3 \text{ kips}$$

Check the sum of forces per quadrant:

$$\Sigma p_p = \Sigma P_c = 0.25P = 264.5 \text{ kips}$$

The location distances y of all forces are shown in Fig. 7. These values were computed using the stress block formulas.

The section properties for the 16-in. wide plate is shown in Fig. 8. The values Q represent the statical moment of the 7.5 x 2.5 in. area around the center of gravity axis.

The shears and bending moment of one quadrant are shown in Fig. 8.

Vertical shear stress at Section 3R:

$$f_v = (3/2)\,[126.5/(16 \times 2.25)] = 5.3 \text{ ksi}$$

Bending stresses at Section 3R:

$$f_b = \pm(434/15.2)1.125 = \pm 32.1 \text{ ksi}$$
$$(< 36 \text{ ksi, ok})$$

Bending stresses at Section 1:

$$f_{bt} = +(618/76.8)2.54 = +20.4 \text{ ksi}$$

$$f_{bb} = -(618/76.8)2.21 = -17.8 \text{ ksi}$$

Horizontal shear stresses at contact area:

$f_h = VQ/bI$

$f_{h2l} = (100.4 \text{ x } 24.2)/(1.5 \times 76.8)$
$= 21.09$ ksi

$f_{h2r} = (100.4 \times 29.3)/(7.5 \times 94.4)$
$= 4.15$ ksi

$f_{h3l} = (126.5 \times 29.3)/(7.5 \times 94.4)$
$= 5.24$ ksi

The sum of f_h is the total horizontal force per quadrant:

$$\Sigma f_h = (\tfrac{1}{2} \times 3 \times 1.5 \times 21.09) + [0.75 \times 7.5(4.15 + 5.24)/2] = 73.9 \text{ kips}$$

Friction coefficient at contact area:

$$C_f = 73.9/(0.25 \times 1058) = 0.28 < 0.5 \text{ (ok)}$$

For biaxial stresses and horizontal shear magnitude see the commentary on Assumptions 7 and 8 in the "Design Assumptions" section.

Discussion of this paper is invited. Please forward your discussion to PCI Headquarters by December 1, 1975.