BEHAVIOUR OF AN INDUCTION MOTOR WITH EQUALIZING CONNECTIONS IN THE STATOR WINDING

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SUMMARY

The paper deals with the behaviour of an induction motor with additional connections in the stator winding. These connections serve to generate a mmf which counteracts the asymmetrically distributed magnetic field in the air gap resulting from asymmetrical construction or assembly of the machine, especially the off-centre position of the rotor in the stator bore. Such an asymmetrical magnetic field induces mechanical vibration of the rotor and stator (with the attendant increased load on the bearings) as well as an unbearable noise. Equalizing currents between winding coils make for drastic reduction of radial forces and ensure almost vibration-free operation.

An analysis of the magnetic field in an asymmetrically constructed machine, without and with equalizing connections, is presented. The theoretical results were confirmed by experiments on a specially built laboratory set-up.

1 - Introduction

The problem of mechanical vibration and noise in induction motors is familiar to manufactures and users of these machines. This phenomenon is mostly associated with asymmetrical construction or assembly of the machine, especially the off-centre position of the rotor in the stator bore.

There are three types of rotor eccentricity: (a) static, where the rotor revolves around its axis, when the latter is shifted relative to that of the stator; (b) dynamic, where the axis of revolution coincides with the stator axis, but does not constitute the axis of symmetry of the rotor. (c) a and b combined.

The off-centre position of the rotor causes unidirectional magnetic pull (u.m.p.) which, as will be shown, can be reduced by means of equalizing connections, in analogy to the similar arrengement in the armature of a d.c. machine (equalizing winding).

2 - Magnetic field in induction motor with off-centre rotor

In the case of a place- and time-independent air gap, the flux density distribution in a three-phase induction motor with full pitch winding may be described by:

$$B(x,t) = \frac{3\sqrt{2}}{\pi}\,\frac{\mu_0 . N . I}{p . \delta}\Big[\pounds_1 \sin(wt-\pi x/\tau) + \frac{\pounds_5}{5}\sin(wt+5\pi x/\tau) + \frac{\pounds_7}{7}\sin(wt-7\pi x/\tau) + \ldots\Big]$$

where: $\pounds_v$ are the winding factors.
N is the phase-winding number of turns.
δ is air gap length.
τ is the pole pitch.
p is the number of pole pairs.

When the air gap varies along the circumference of the rotor, it may be described for static eccentricity by:

$$\delta(\alpha,t) = \delta_m\,[1 - \epsilon.\cos(\alpha - \phi_\epsilon)]$$

and for dynamic eccentricity by:

$$\delta(\alpha,t) = \delta_m\,[1 - \epsilon.\cos(\alpha - w_\epsilon t - \phi_\epsilon)]$$

where: δ_m is the mean value of the air gap.
α is the internal angle of the rotor.
ϕ_ϵ is the direction of the initial eccentricity.
w_ϵ is the change frequency of the air-gap at a determined point.

ϵ is the relative eccentricity, defined as the ratio of the absolute eccentricity e to the mean air gap δ_m.

The corresponding magnetic field distributions are:

$$B_s(\alpha,t) = B_0 \left\{1 + 2 \sum_{\beta=1}^{\infty} T^{\beta} \cos[\beta(\alpha - \phi_{\epsilon})]\right\} *$$

$$* \left[£_1 \sin(wt-p\alpha) + \frac{£_5}{5} \sin(wt+5p\alpha) + \right.$$

$$\left. + \frac{£_7}{7} \sin(wt-7p\alpha) + \ldots\right] \qquad \ldots\ldots \quad (1)$$

for static eccentricity, and

$$B_d(\alpha,t) = B_0 \left\{1 + 2 \sum_{\beta=1}^{\infty} T^{\beta} \cos[\beta(\alpha - w_{\epsilon}t - \phi_{\epsilon})]\right\} *$$

$$* \left[£_1 \sin(wt-p\alpha) + \frac{£_5}{5} \sin(wt+5p\alpha) + \right.$$

$$\left. + \frac{£_7}{7} \sin(wt-7p\alpha) + \ldots\right] \qquad \ldots\ldots \quad (2)$$

for dynamic eccentricity, where:

$$B_0 = \frac{3\sqrt{2}}{\pi} \frac{\mu_0 \cdot N.I}{p} \frac{k}{\delta_m (1-\epsilon^2)^{\frac{1}{2}}} \qquad \ldots\ldots \quad (3)$$

$$T = \frac{1 - (1-\epsilon^2)^{\frac{1}{2}}}{\epsilon}$$

3 - Currents in winding with equalizing connections

The effect of equalizing connections is examined for static eccentricity. Consider a phase winding of a four-pole induction motor, with two parallel branches comprising three series-connected coils each, Fig. 1.

The additional connections (c-d and e-f) equalize the voltage drop in coil pairs shifted by 180°. The average values of the air gaps under the generic coil are given by:

$$\bar{\delta}_i = \frac{1}{\tau} \int_{\alpha_i-\tau/2}^{\alpha_i+\tau/2} \delta(\alpha) d\alpha$$

wich yields, for the coils belonging to the considered phase winding:

$$\bar{\delta}_1 = \delta m\left[1 - 2\sqrt{2}\ (\epsilon/\pi)\ \cos(\alpha_1)\right]$$

$$\bar{\delta}_2 = \delta m\left[1 - 2\sqrt{2}\ (\epsilon/\pi)\ \cos(\alpha_1-\pi/9)\right]$$

$$\bar{\delta}_6 = \delta m\left[1 + 2\sqrt{2}\ (\epsilon/\pi)\ \cos(\alpha_1+4\pi/9)\right]$$

$$\bar{\delta}_{10} = \delta m\left[1 + 2\sqrt{2}\ (\epsilon/\pi)\ \cos(\alpha_1)\right]$$

$$\bar{\delta}_{11} = \delta m\left[1 + 2\sqrt{2}\ (\epsilon/\pi)\ \cos(\alpha_1-\pi/9)\right]$$

$$\bar{\delta}_{15} = \delta m\left[1 - 2\sqrt{2}\ (\epsilon/\pi)\ \cos(\alpha_1+4\pi/9)\right]$$

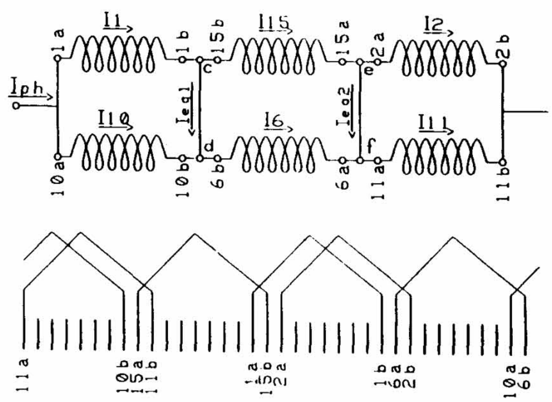

Fig. 1: Scheme of phase winding.

The coil currents are calculated bearing in mind that the reluctance related to a generic coil i, is proportional to the average length of the air gap under this coil. The coil reactance (X_{μ}) being inversely proportional to the average air gap, the magnetization susceptance (Y_{μ}) is proportional to the average air gap length:

$$R_{\mu i} = \frac{1}{\mu_0} \frac{\bar{\delta}_i}{A}$$

$$X_{\mu i} = wN'^2/R_{\mu i} = wN'^2 \mu_0 A/\bar{\delta}_i$$

$$Y_{\mu i} = X_{\mu i}^{-1} = \bar{\delta}_i/wN'^2 \mu_0 A$$

where N' is the number of coil turns.

The equivalent susceptance of a parallel-connected coil pair is:

$$Y_{\mu 1} + Y_{\mu 10} = Y_{\mu 2} + Y_{\mu 11} = Y_{\mu 6} + Y_{\mu 15} = 2Y_{\mu}m$$

Consequently, each coil pair is supplied by equal voltage, hence the magnetization currents in the coils are:

$$I\mu_1 = I\mu_b\left[1 - 2\sqrt{2}\ (\in/\pi)\ \cos(\alpha_1)\right]$$

$$I\mu_2 = I\mu_b\left[1 - 2\sqrt{2}\ (\in/\pi)\ \cos(\alpha_1 - \pi/9)\right]$$

$$I\mu_6 = I\mu_b\left[1 + 2\sqrt{2}\ (\in/\pi)\ \cos(\alpha_1 + 4\pi/9)\right]$$

$$I\mu_{10} = I\mu_b\left[1 + 2\sqrt{2}\ (\in/\pi)\ \cos(\alpha_1)\right]$$

$$I\mu_{11} = I\mu_b\left[1 + 2\sqrt{2}\ (\in/\pi)\ \cos(\alpha_1 - \pi/9)\right]$$

$$I\mu_{15} = I\mu_b\left[1 - 2\sqrt{2}\ (\in/\pi)\ \cos(\alpha_1 + 4\pi/9)\right]$$

The currents in the two equalizing connections are:

$$Ieq_1 = I\mu_1 - I\mu_{15} = -2\sqrt{2}\ (\in/\pi)\ I\mu_b\ *$$
$$*\ \{\cos\alpha_1\ [1 - \sin(\pi/18)] + \sin\alpha_1 \cos(\pi/18)\}$$

$$Ieq_2 = I\mu_{15} - I\mu_2 = -2\sqrt{2}\ (\in/\pi)\ I\mu_b\ *$$
$$*\ \{\cos\alpha_1\ [\cos(\pi/9) - \sin(\pi/18)] +$$
$$+ \sin\alpha_1\ [\sin(\pi/9) + \cos(\pi/18)]\}$$

the corresponding maxima being:

$$Ieq_1 max = -1.16 \cdot \in \cdot I\mu_b$$
$$Ieq_2 max = 1.38 \cdot \in \cdot I\mu_b$$

4 - Magnetic field

The current in each coil generates its own mmf and, magnetic field. The total magnetic field in the air gap is a result of the combined action of all coils in the winding. For a three-phase winding we obtain:

$$\Sigma F\Big|_{v\leq 14} = \frac{3\sqrt{2}N\ I\mu_b}{\pi}\Big\{(£_{q1}/2)\sin(wt-2\alpha+2\alpha_1-\pi/9) +$$
$$+ (£_{q5}/10)\sin(wt+10\alpha-10\alpha_1+5\pi/9) +$$
$$+ (£_{q7}/14)\sin(wt-14\alpha+14\alpha_1-7\pi/9) -$$
$$-(\in/\pi)[\ £_{q1}\ \sin(wt-\alpha+2\alpha_1-\pi/9) +$$
$$+ (£_{q1}/3)\ \sin(wt-3\alpha+2\alpha_1-\pi/9) +$$
$$+ (£_{q7}/3)\ \sin(wt+3\alpha-4\alpha_1+2\pi/9) -$$
$$- (£_{q7}/5)\ \sin(wt+5\alpha-4\alpha_1+2\pi/9) -$$
$$- (£_{q5}/7)\ \sin(wt-7\alpha+8\alpha_1+5\pi/9) +$$
$$+ (£_{q5}/9)\ \sin(wt+9\alpha-10\alpha_1+5\pi/9) +$$
$$+ (£_{q5}/9)\ \sin(wt-9\alpha+8\alpha_1+5\pi/9) +$$
$$+ (£_{q5}/11)\ \sin(wt+11\alpha-10\alpha_1+5\pi/9) -$$
$$- (£_{q7}/13)\ \sin\ (wt-13\alpha+14\alpha_1+2\pi/9)]\Big\}$$

where v=2 for the main harmonic of the mmf.

We confined ourselves to the first 7 harmonics in the mmf distribution ($v\leq 14$). The magnetic field distribution is obtained by multiplying the mmf by the air gap permeance, namely:

$$B(\alpha,t) = 2B_0\ \{£_{q1}\sin(wt-2\alpha) +$$
$$+ (£_{q5}/5)\ \sin(wt+10\alpha) + (£_{q7}/7)\ \sin(wt-14\alpha) -$$
$$- (2\in/\pi)[\ £_{q1}\ \sin(wt-\alpha) + (£_{q1}/3)\ \sin(wt-3\alpha) +$$
$$+ (£_{q7}/3)\ \sin(wt+3\alpha) - (£_{q7}/5)\ \sin(wt+5\alpha) +$$
$$+ (£_{q5}/7)\ \sin(wt-7\alpha) + (£_{q5}/9)\ \sin(wt+9\alpha) -$$
$$- (£_{q5}/9)\ \sin(wt-9\alpha) + (£_{q5}/11)\ \sin(wt+11\alpha) +$$
$$+ (£_{q7}/13)\ \sin(wt-13\alpha)]\}\ *$$
$$*\ [1 + 2T\cos(\alpha) + 2T^2\cos(2\alpha)] =$$
$$= B_0 \sum_{i=-16}^{12} G_i(\alpha,t) \qquad \ldots\ldots \quad (4)$$

where B_0 is as per eq.3.

It can be proved that among all harmonics of the magnetic field, the following of significance:

$G_{-3}(\alpha,t) = £_1(T - 2\in/3\pi)\ \sin(wt-3\alpha)$

$G_{-2}(\alpha,t) = £_1(1 - 8\in T/3\pi)\ \sin(wt-2\alpha)$

$G_{-1}(\alpha,t) = £_1(T - 2\in/\pi)\ \sin(wt-\alpha)$

$G_{+3}(\alpha,t) = -\ (2\in £_7/3\pi)\ \sin(wt+3\alpha)$

$G_{+5}(\alpha,t) = (2\in £_7/5\pi)\ \sin(wt+5\alpha)$

Comparison between eq.1 and eq.4 can be performed separately for each harmonic. The first harmonic (one single period around the rotor) is of greatest importance. The approximated values of the first harmonics amplitude are $B_0£_1T$ in eq. 1 and $B_0£_1(T-2\in/\pi)$ in eq.4. The factor T equals approximately to $\in/2$.

It can be seen that the first harmonic is drastically reduced when the stator is connected with equalizing branches, wich in turn leads to significant decrease of the u.m.p. acting on the rotor of the machine.

Some change in the losses can be expected due to the change in currents and field distribution.

5 - Experimentals

The radial forces were measured on a setup designed and constructed at our laboratory. The setup consists of two induction motors with a common shaft, an external panel permitting a variety of connection patterns for the stator coils, and a measuring device for the forces. Readings were taken in two mutually perpendicular directions with the aid of strain gauges serving as branches in differential bridges. A personal computer was used for acquisition and processing of the data.

The experimental findings confirmed the theoretical results; the measured u.m.p. was 25 times smaller than in the absence of the equalizing connections.

The influence of the connection mode on machine power losses was examined on a three-phase induction motor with three pairs of end-shields with different degrees of eccentricity. No significant increase in power losses was found.

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