```
STUDENT > # Solve desertfox' example problem
STUDENT > # (Convert to metric)
STUDENT > # Find angle to determine if bearing reaction moment is
          significant
STUDENT >
STUDENT > # Assume bearings and pulley have zero width
STUDENT >
STUDENT > # two sections: m = motor (between bearings); e =
          extension
STUDENT > # Lm = distance between motor bearings
STUDENT > # Le = distance from inboard motor bearing to Pulley
          center.
STUDENT > # x = distance coordinate.
STUDENT > # x=0 at OB brg. x=Lm at IB brg. x = Lm + Le at pulley
STUDENT >
STUDENT > # V = shear
STUDENT > # M = moment
STUDENT > # T = slope (T for theta)
STUDENT > # Y = displacement
STUDENT >
STUDENT > # Dirac = unit impulse (for concentrated load)
STUDENT > # Heaviside = unit step function
STUDENT >
STUDENT > restart;  # initialization
STUDENT >
STUDENT > # Convert Inputs to metric
STUDENT > Dm:=evalf(4/40); # Diam of motor section of shaft (betw
          brgs)
```

$$Dm := .1000000000$$

```
STUDENT > De:=evalf(2/40); # Diam of extension section of shaft
          (betw brgs)
```

$$De := .05000000000$$

```
STUDENT > Lm:=evalf(10/40); # Diam of motor section of shaft (betw
          brgs)
```

$$Lm := .2500000000$$

```
STUDENT > Le:=evalf(5/40); # Length of extension section of shaft
          (betw brgs)
```

$$Le := .1250000000$$

```
STUDENT > E:=200E9;  # Young's Modulus in Pascals
```

$$E := .200\ 10^{12}$$

```
STUDENT > # Area Moment Of Inertia
STUDENT > I_m:=evalf(Pi*Dm^4/64); # motor section
```

$$I\_m := .4908738522\ 10^{-5}$$

```
STUDENT > I_e:=evalf(Pi*De^4/64); # extension section
```

$$I\_e := .3067961576\ 10^{-6}$$

```
STUDENT > Fp:=-40*4.44;  # Pulley Force
```

$$Fp := -177.60$$

```
STUDENT > # Solve Reaction Forces from Force & Moment Balance
STUDENT > ForceBalanceEqn:=Fp+Rib+Rob=0;
```

$$ForceBalanceEqn := -177.60 + Rib + Rob = 0$$

```
STUDENT > MomentBalanceEqn:=Rib*Le+Rob*(Le+Lm)=0;
```

$$MomentBalanceEqn := .1250000000\ Rib + .3750000000\ Rob = 0$$

```
STUDENT > Rsolution:=solve({ForceBalanceEqn,MomentBalanceEqn},{Rib,R
          ob});
```

$$Rsolution := \{ Rib = 266.4000000, Rob = -88.80000000 \}$$

```
STUDENT > Rob:=subs(Rsolution,Rob); # Reaction force @ ob brg
```

$$Rob := -88.80000000$$

```
STUDENT > Rib:=subs(Rsolution,Rib); # Reaction force @ ib brg
```

$$Rib := 266.4000000$$

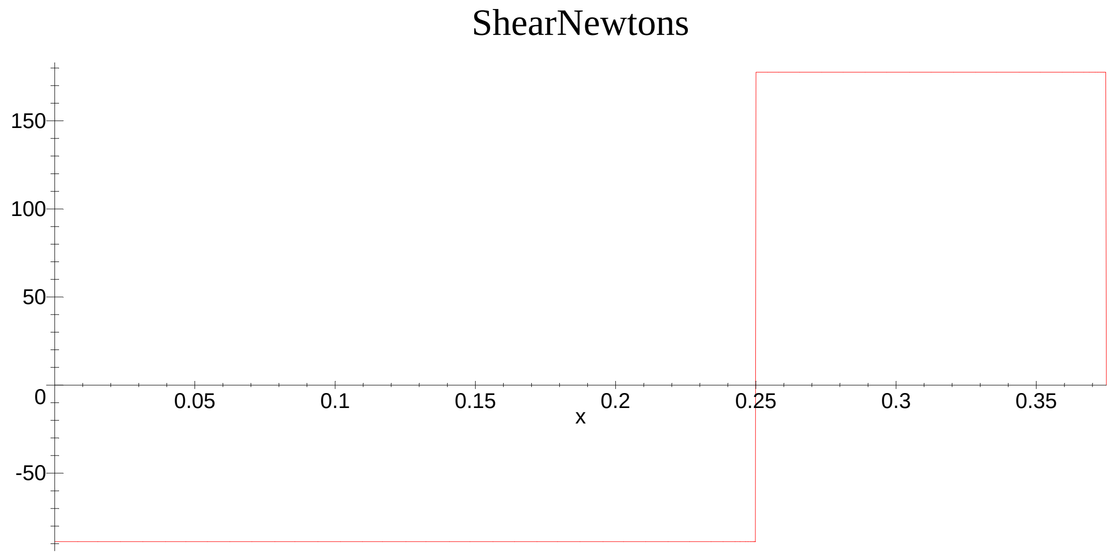
```
STUDENT > # Force profile is sum of applied force and reactions
STUDENT > Force(x):=Rob*Dirac(x-0)+Rib*Dirac(x-Lm)+Fp*Dirac(x-Lm-Le)
          ;
```

$$\mathrm{Force}(x) := -88.80000000\ \mathrm{Dirac}(x) + 266.4000000\ \mathrm{Dirac}(x - .2500000000) - 177.60\ \mathrm{Dirac}(x - .3750000000)$$

```
STUDENT > # note dirac is unit impulse
STUDENT >
STUDENT > # Solve shear as integral of force (no integration
          constant since forces sum to 0)
STUDENT > V(x):=int(Force(x),x);
```

$$\mathrm{V}(x) := -88.80000000\ \mathrm{Heaviside}(x) + 266.4000000\ \mathrm{Heaviside}(x - .2500000000) - 177.6000000\ \mathrm{Heaviside}(x - .3750000000)$$

```
STUDENT >
STUDENT > plot(V(x),x=0..Lm+Le,title='ShearNewtons');
```




```
STUDENT > # Solve Moment as integral of shear
STUDENT > M(x):=int(V(x),x)+CM;
```

$$M(x) := -88.80000000\,\text{Heaviside}(x)\,x + 266.4000000\,\text{Heaviside}(x - .2500000000)\,x - 66.60000000\,\text{Heaviside}(x - .2500000000) - 177.6000000\,\text{Heaviside}(x - .3750000000)\,x + 66.60000000\,\text{Heaviside}(x - .3750000000) + CM$$

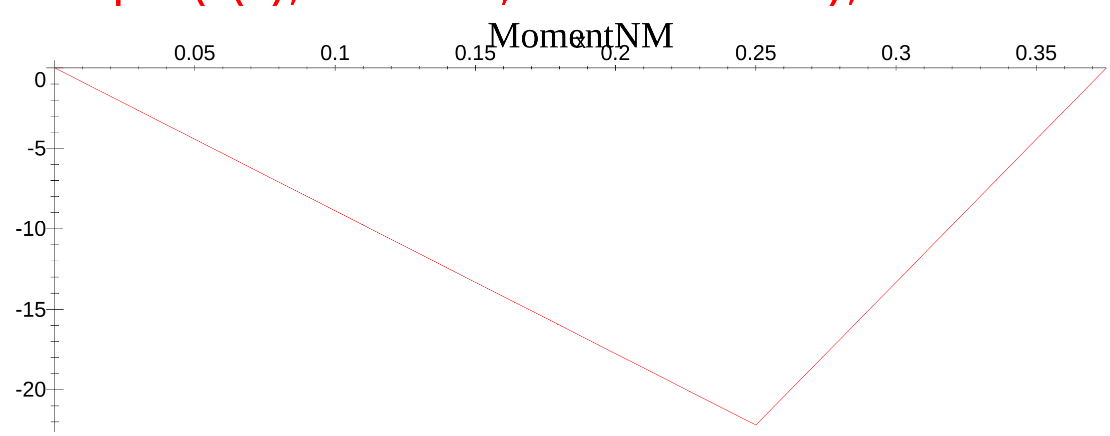
```
STUDENT >
STUDENT > # Evaluate integration constant assuming free end moment
          is 0
STUDENT > CMsolution:=solve(subs(x=Le+Lm,M(x))=0,CM);
```

$$CMsolution := 0$$

```
STUDENT > M(x):=subs(CM=CMsolution,M(x));
```

$$M(x) := -88.80000000\,\text{Heaviside}(x)\,x + 266.4000000\,\text{Heaviside}(x - .2500000000)\,x - 66.60000000\,\text{Heaviside}(x - .2500000000) - 177.6000000\,\text{Heaviside}(x - .3750000000)\,x + 66.60000000\,\text{Heaviside}(x - .3750000000)$$

```
STUDENT > plot(M(x),x=0..Lm+Le,title='MomentNM');
```



```
STUDENT > # Solve theta as 1/(E*I) * integral of Moment
STUDENT > # Divide into regions m (motor) and e (extension) because
          I differs
STUDENT > Tm(x):=1/(E*I_m)*int(M(x),x)+CTm;
```

$$Tm(x) := -.00004522546864 \text{ Heaviside}(x)\, x^2 + .0001356764059 \text{ Heaviside}(x - .2500000000)\, x^2 + .8479775370\ 10^{-5} \text{ Heaviside}(x - .2500000000) - .00006783820296 \text{ Heaviside}(x - .2500000000)\, x - .00009045093728 \text{ Heaviside}(x - .3750000000)\, x^2 - .00001271966305 \text{ Heaviside}(x - .3750000000) + .00006783820296 \text{ Heaviside}(x - .3750000000)\, x + CTm$$

```
STUDENT > Te(x):=1/(E*I_e)*int(M(x),x)+CTe;
```

$$Te(x) := -.0007236074979 \text{ Heaviside}(x)\, x^2 + .002170822494 \text{ Heaviside}(x - .2500000000)\, x^2 + .0001356764059 \text{ Heaviside}(x - .2500000000) - .001085411247 \text{ Heaviside}(x - .2500000000)\, x - .001447214996 \text{ Heaviside}(x - .3750000000)\, x^2 - .0002035146088 \text{ Heaviside}(x - .3750000000) + .001085411247 \text{ Heaviside}(x - .3750000000)\, x + CTe$$

```
STUDENT >
STUDENT >
STUDENT > # Solve displacement as integral of slop
STUDENT > # Again solve in two regions
STUDENT > Ym(x):=int(Tm(x),x)+CYm;
```

$$Ym(x) := -.00001507515621 \text{ Heaviside}(x)\, x^3 + .00004522546863\ \%2\ x^3 - .7066479474\ 10^{-6}\ \%2 + .8479775370\ 10^{-5}\ \%2\ x - .00003391910148\ \%2\ x^2 - .00003015031243\ \%1\ x^3 + .1589957880\ 10^{-5}\ \%1 - .00001271966305\ \%1\ x + .00003391910148\ \%1\ x^2 + CTm\, x + CYm$$

$\%1 := \text{Heaviside}(x - .3750000000)$

$\%2 := \text{Heaviside}(x - .2500000000)$

```
STUDENT > Ye(x):=int(Tm(x),x)+CYe;
```

$$Ye(x) := -.001507515621 \text{ Heaviside}(x)\, x^3 + .004522546863\ \%2\ x^3 - .00007066479474\ \%2 + .0008479775370\ \%2\ x - .003391910148\ \%2\ x^2 - .003015031243\ \%1\ x^3 + .0001589957880\ \%1 - .001271966305\ \%1\ x + .003391910148\ \%1\ x^2 + CTm\, x + CYe$$

$\%1 := \text{Heaviside}(x - .3750000000)$

$\%2 := \text{Heaviside}(x - .2500000000)$

$$Ye(x) := -.00001507515621 \text{ Heaviside}(x)\, x^3 + .00004522546863\ \%2\ x^3 - .7066479474\ 10^{-6}\ \%2 + .8479775370\ 10^{-5}\ \%2\ x - .00003391910148\ \%2\ x^2 - .00003015031243\ \%1\ x^3 + .1589957880\ 10^{-5}\ \%1 - .00001271966305\ \%1\ x + .00003391910148\ \%1\ x^2 + CTm\, x + CYe$$

$\%1 := \text{Heaviside}(x - .3750000000)$

$\%2 := \text{Heaviside}(x - .2500000000)$

```
STUDENT >
STUDENT > # 4 Boundary conditions to solve the integration constants
STUDENT > BC1:=simplify(subs(x=0,Ym(x))=0); # Displacment # ob is 0
```

$$BC1 := CYm = 0$$

```
STUDENT > BC2:=simplify(subs(x=Lm,Ym(x))=0);  # Displacment # ib is
          0
```

$$BC2 := -.2355493158\ 10^{-6} + .2500000000\ CTm + CYm = 0$$

```
STUDENT > BC3:=simplify(subs(x=Lm,Ye(x))=0); # Displacment # ib is 0
          applies to both displacement solutions
```

$$BC3 := -.2355493158\ 10^{-6} + .2500000000\ CTm + CYe = 0$$

```
STUDENT > BC4:=simplify(subs(x=Lm,Tm(x))=subs(x=Lm,Te(x))); #
          continuity of slope @ ib
```

$$BC4 := -.2826591790\ 10^{-5} + CTm = -.00004522546862 + CTe$$

```
STUDENT > Csolution:=solve({BC1,BC2,BC3,BC4},{CTm,CTe,CYm,CYe});
```

$$Csolution := \{ CYe = 0,\ CYm = 0,\ CTm = .9421972632\ 10^{-6},\ CTe = .00004334107409 \}$$

```
STUDENT >
STUDENT > # Plug solution constants back into T and Y
STUDENT > Tm(x):=subs(Csolution,Tm(x));
```

$$\begin{aligned}\mathrm{Tm}(x) := &-.00004522546864\ \mathrm{Heaviside}(x)\ x^2 + .0001356764059\ \mathrm{Heaviside}(x - .2500000000)\ x^2\\ &+ .8479775370\ 10^{-5}\ \mathrm{Heaviside}(x - .2500000000)\\ &- .00006783820296\ \mathrm{Heaviside}(x - .2500000000)\ x\\ &- .00009045093728\ \mathrm{Heaviside}(x - .3750000000)\ x^2\\ &- .00001271966305\ \mathrm{Heaviside}(x - .3750000000)\\ &+ .00006783820296\ \mathrm{Heaviside}(x - .3750000000)\ x + .9421972632\ 10^{-6}\end{aligned}$$

```
STUDENT > Te(x):=subs(Csolution,Te(x));
```

$$\begin{aligned}\mathrm{Te}(x) := &-.0007236074979\ \mathrm{Heaviside}(x)\ x^2 + .002170822494\ \mathrm{Heaviside}(x - .2500000000)\ x^2\\ &+ .0001356764059\ \mathrm{Heaviside}(x - .2500000000)\\ &- .001085411247\ \mathrm{Heaviside}(x - .2500000000)\ x\\ &- .001447214996\ \mathrm{Heaviside}(x - .3750000000)\ x^2\\ &- .0002035146088\ \mathrm{Heaviside}(x - .3750000000)\\ &+ .001085411247\ \mathrm{Heaviside}(x - .3750000000)\ x + .00004334107409\end{aligned}$$

```
STUDENT > Ym(x):=subs(Csolution,Ym(x));
```

$$\begin{aligned}\mathrm{Ym}(x) := &-.00001507515621\ \mathrm{Heaviside}(x)\ x^3 + .00004522546863\ \%2\ x^3 - .7066479474\ 10^{-6}\ \%2\\ &+ .8479775370\ 10^{-5}\ \%2\ x - .00003391910148\ \%2\ x^2 - .00003015031243\ \%1\ x^3\\ &+ .1589957880\ 10^{-5}\ \%1 - .00001271966305\ \%1\ x + .00003391910148\ \%1\ x^2\\ &+ .9421972632\ 10^{-6}\ x\\ \%1 := &\ \mathrm{Heaviside}(x - .3750000000)\end{aligned}$$

$\%2 := \text{Heaviside}(x - .2500000000)$

```
STUDENT > Ye(x):=subs(Csolution,Ye(x));
```

$$Ye(x) := -.00001507515621\,\text{Heaviside}(x)\,x^3 + .00004522546863\,\%2\,x^3 - .7066479474\;10^{-6}\,\%2 + .8479775370\;10^{-5}\,\%2\,x - .00003391910148\,\%2\,x^2 - .00003015031243\,\%1\,x^3 + .1589957880\;10^{-5}\,\%1 - .00001271966305\,\%1\,x + .00003391910148\,\%1\,x^2 + .9421972632\;10^{-6}\,x$$

$\%1 := \text{Heaviside}(x - .3750000000)$

$\%2 := \text{Heaviside}(x - .2500000000)$

```
STUDENT > T(x):=piecewise(x>Lm,Te(x),Tm(x));
```

$$T(x) := \begin{cases} -.0007236074979\,\text{Heaviside}(x)\,x^2 + .002170822494\,\%2\,x^2 + .0001356764059\,\%2 - .001085411247\,\%2\,x - .001447214996\,\%1\,x^2 - .0002035146088\,\%1 + .001085411247\,\%1\,x + .00004334107409, & .2500000000 < x \\ -.00004522546864\,\text{Heaviside}(x)\,x^2 + .0001356764059\,\%2\,x^2 + .8479775370\;10^{-5}\,\%2 - .00006783820296\,\%2\,x - .00009045093728\,\%1\,x^2 - .00001271966305\,\%1 + .00006783820296\,\%1\,x + .9421972632\;10^{-6}, & otherwise \end{cases}$$

$\%1 := \text{Heaviside}(x - .3750000000)$

$\%2 := \text{Heaviside}(x - .2500000000)$

```
STUDENT > Y(x):=piecewise(x>Lm,Ye(x),Ym(x));
```

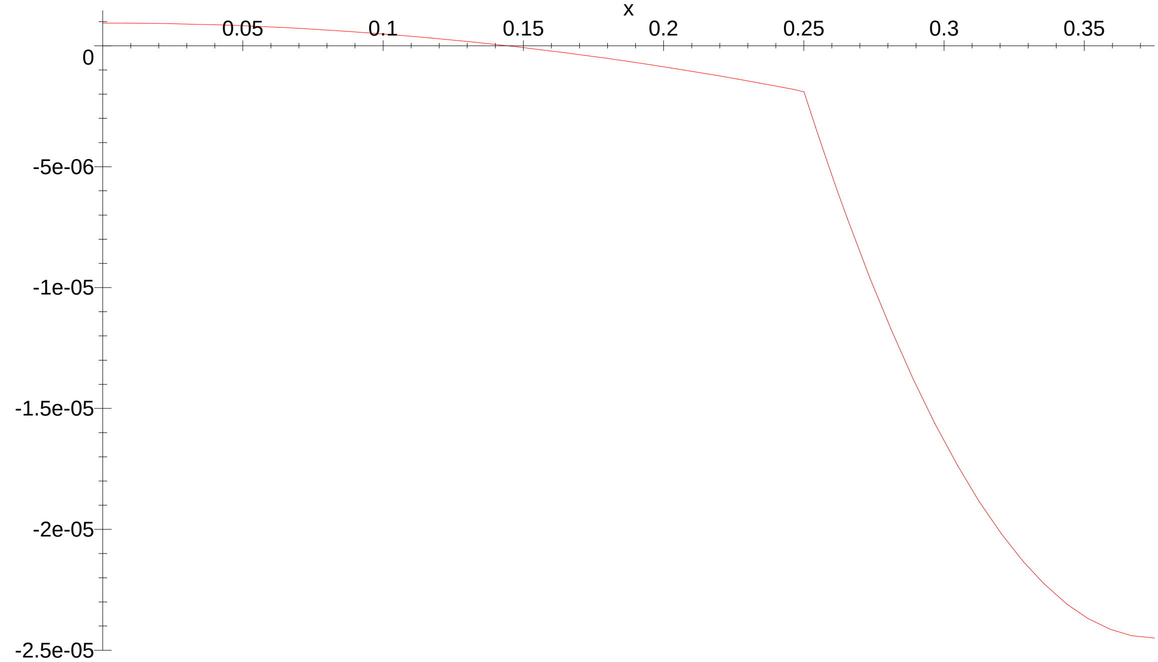
$$Y(x) := \begin{cases} -.00001507515621\,\text{Heaviside}(x)\,x^3 + .00004522546863\,\%2\,x^3 - .7066479474\;10^{-6}\,\%2 + .8479775370\;10^{-5}\,\%2\,x - .00003391910148\,\%2\,x^2 - .00003015031243\,\%1\,x^3 + .1589957880\;10^{-5}\,\%1 - .00001271966305\,\%1\,x + .00003391910148\,\%1\,x^2 + .9421972632\;10^{-6}\,x, & .2500000000 < x \\ -.00001507515621\,\text{Heaviside}(x)\,x^3 + .00004522546863\,\%2\,x^3 - .7066479474\;10^{-6}\,\%2 + .8479775370\;10^{-5}\,\%2\,x - .00003391910148\,\%2\,x^2 - .00003015031243\,\%1\,x^3 + .1589957880\;10^{-5}\,\%1 - .00001271966305\,\%1\,x + .00003391910148\,\%1\,x^2 + .9421972632\;10^{-6}\,x, & otherwise \end{cases}$$

$\%1 := \text{Heaviside}(x - .3750000000)$

$\%2 := \text{Heaviside}(x - .2500000000)$

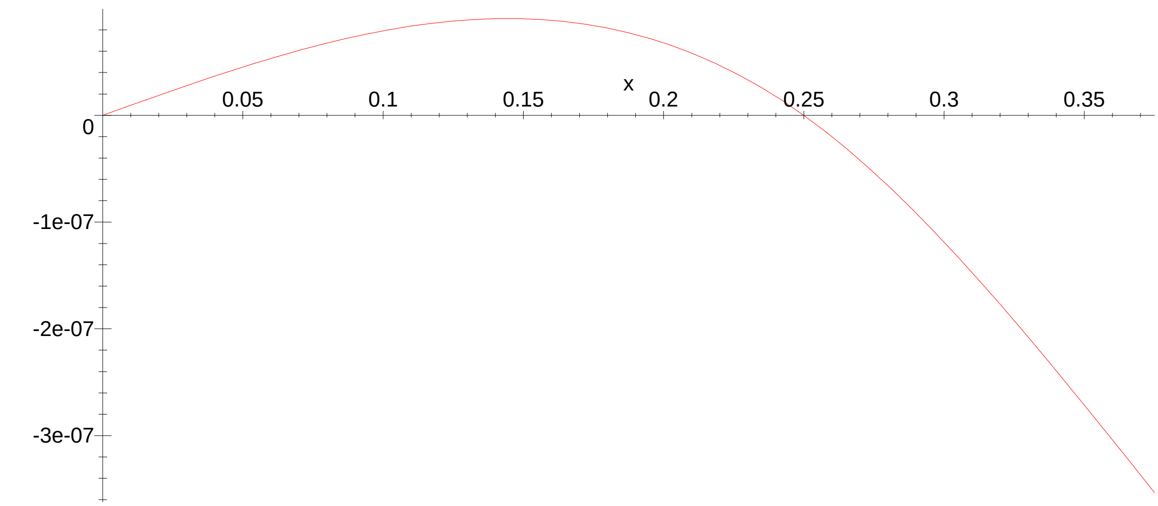
```
STUDENT >
STUDENT > # Plot results
STUDENT > plot(T(x),x=0..Lm+Le,title='SlopeRadians');
```

SlopeRadians




**STUDENT >** `plot(Y(x),x=0..Lm+Le,title='Displacement_Meter');`

Displacement_Meter




**STUDENT >** `# Find the angle at motor inboard bearing (T at x=Lm)`

**STUDENT >** `AngleRadians:=evalf(subs(x=Lm,Te(x)));`

$$AngleRadians := -.188439453\ 10^{-5}$$

**STUDENT >** `AngleDegrees:=AngleRadians*evalf(180/Pi);`

$$AngleDegrees := -.0001079678535$$

**STUDENT >** `AngleMinutes:=AngleDegrees*60;`

*AngleMinute* 006478071210

```
STUDENT > #  Moment is negligible at this low angle per NSK figure 4
          page 136
STUDENT > #
          http://www.nsk.com/services/basicknowledge/technicalreport
          /05distribution.html
STUDENT >
STUDENT > # *** Conclusion - bearing reaction moment is not
          significant for this case, and probably not for any
          electric motor since the shaft is stout.  It may become a
          factor only for very long thin shafts under high load
STUDENT >
STUDENT > # Compute Bending stress
STUDENT > M_NM:=evalf(subs(x=Lm,M(x)));  # moment at ib brg in
          Newtons
```

$$M\_NM := -22.20000000$$

```
STUDENT > Sbending_PA:=M_NM*(De/2)/I_e; # Bending stress in PA
```

$$Sbending\_PA := -.1809018745\ 10^{7}$$

```
STUDENT > Sbending_PSI:=Sbending_PA/6895; # convert PA to PSI
```

$$Sbending\_PSI := -262.3667505$$