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[ STUDENT > # Solve desertfox' example problem
[ STUDENT > # (Convert to metric)
[ STUDENT > # Find angle to determine if bearing reaction moment is
[           significant
[ STUDENT >
[ STUDENT > # Assume bearings and pulley have zero width
[ STUDENT >
[ STUDENT > # two sections: m = motor (between bearings); e =
[           extension
[ STUDENT > # Lm = distance between motor bearings
[ STUDENT > # Le = distance from inboard motor bearing to Pulley
[           center.
[ STUDENT > # x = distance coordinate.
[ STUDENT > # x=0 at OB brg. x=Lm at IB brg. x = Lm + Le at pulley
[ STUDENT >
[ STUDENT > # V = shear
[ STUDENT > # M = moment
[ STUDENT > # T = slope (T for theta)
[ STUDENT > # Y = displacement
[ STUDENT >
[ STUDENT > # Dirac = unit impulse (for concentrated load)
[ STUDENT > # Heaviside = unit step function
[ STUDENT >
[ STUDENT > restart; # initialization
[ STUDENT >
[ STUDENT > # Convert Inputs to metric
[ STUDENT > Dm:=evalf(4/40); # Diam of motor section of shaft (betw
[           brgs)
[
[            $Dm := .1000000000$ 
[ STUDENT > De:=evalf(2/40); # Diam of extension section of shaft
[           (betw brgs)
[
[            $De := .05000000000$ 
[ STUDENT > Lm:=evalf(10/40); # Diam of motor section of shaft (betw
[           brgs)
[
[            $Lm := .2500000000$ 
[ STUDENT > Le:=evalf(5/40); # Length of extension section of shaft
[           (betw brgs)
[
[            $Le := .1250000000$ 
[ STUDENT > E:=200E9; # Young's Modulus in Pascals
[
[            $E := .200 \cdot 10^{12}$ 
[ STUDENT > # Area Moment Of Inertia
[ STUDENT > I_m:=evalf(Pi*Dm^4/64); # motor section

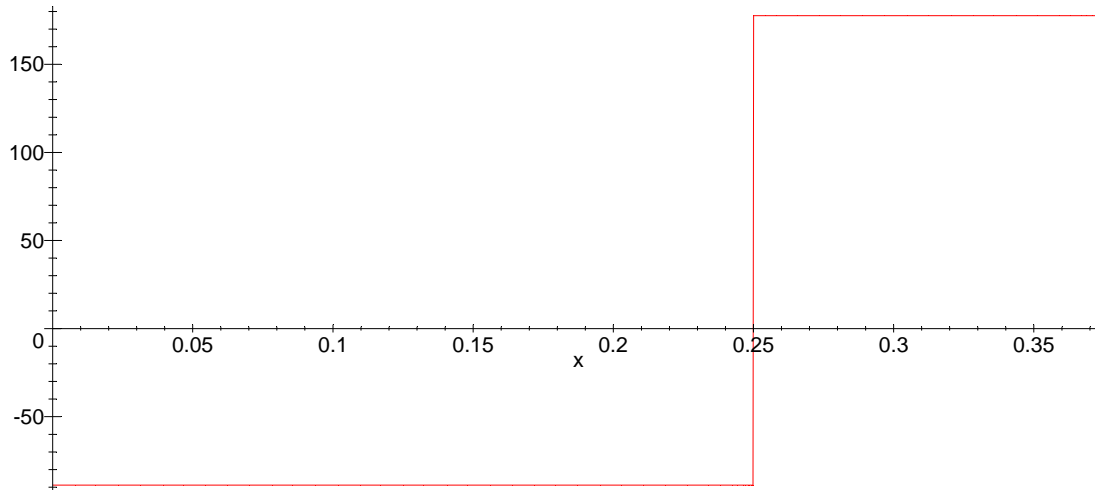
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[                                      $I_m := .4908738522 \cdot 10^{-5}$ 
STUDENT > I_e:=evalf(Pi*De^4/64); # extension section
[                                      $I_e := .3067961576 \cdot 10^{-6}$ 
STUDENT > Fp:=-40*4.44; # Pulley Force
[                                      $F_p := -177.60$ 
STUDENT > # Solve Reaction Forces from Force & Moment Balance
STUDENT > ForceBalanceEqn:=Fp+Rib+Rob=0;
[                                      $ForceBalanceEqn := -177.60 + Rib + Rob = 0$ 
STUDENT > MomentBalanceEqn:=Rib*L_e+Rob*(L_e+L_m)=0;
[                                      $MomentBalanceEqn := .1250000000 Rib + .3750000000 Rob = 0$ 
STUDENT > Rsolution:=solve({ForceBalanceEqn,MomentBalanceEqn},{Rib,Rob});
[                                      $Rsolution := \{ Rib = 266.4000000, Rob = -88.80000000 \}$ 
[
STUDENT > Rob:=subs(Rsolution,Rob); # Reaction force @ ob brg
[                                      $Rob := -88.80000000$ 
STUDENT > Rib:=subs(Rsolution,Rib); # Reaction force @ ib brg
[                                      $Rib := 266.4000000$ 
STUDENT > # Force profile is sum of applied force and reactions
STUDENT > Force(x):=Rob*Dirac(x-0)+Rib*Dirac(x-L_m)+Fp*Dirac(x-L_m-L_e)
[                                     ;
[                                      $Force(x) := -88.80000000 Dirac(x) + 266.4000000 Dirac(x - .2500000000)$ 
[                                      $- 177.60 Dirac(x - .3750000000)$ 
STUDENT > # note dirac is unit impulse
[
STUDENT > # Solve shear as integral of force (no integration
[                                     constant since forces sum to 0)
STUDENT > V(x):=int(Force(x),x);
[                                      $V(x) := -88.80000000 Heaviside(x) + 266.4000000 Heaviside(x - .2500000000)$ 
[                                      $- 177.6000000 Heaviside(x - .3750000000)$ 
STUDENT >
STUDENT > plot(V(x),x=0..L_m+L_e,title='ShearNewtons');

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ShearNewtons



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STUDENT > # Solve Moment as integral of shear
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STUDENT > M(x):=simplify(int(V(x),x))+CM;
```

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M(x) := -88.80000000 Heaviside(x) x + 266.4000000 Heaviside(x - .2500000000) x
        - 66.60000000 Heaviside(x - .2500000000) - 177.6000000 Heaviside(x - .3750000000) x
        + 66.60000000 Heaviside(x - .3750000000) + CM
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STUDENT >
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STUDENT > # Evaluate integration constant assuming free end moment
           is 0
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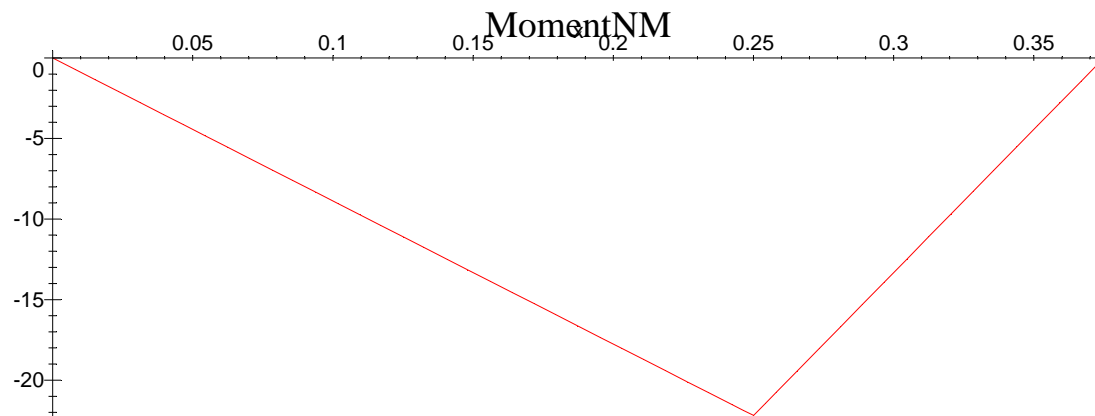
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STUDENT > CMsolution:=solve(subs(x=Le+Lm,M(x))=0,CM);
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CMsolution := 0
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STUDENT > M(x):=simplify(subs(CM=CMsolution,M(x)));
```

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M(x) := -88.80000000 Heaviside(x) x + 266.4000000 Heaviside(x - .2500000000) x
        - 66.60000000 Heaviside(x - .2500000000) - 177.6000000 Heaviside(x - .3750000000) x
        + 66.60000000 Heaviside(x - .3750000000)
```

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STUDENT > plot(M(x),x=0..Lm+Le,title='MomentNM');
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[ STUDENT > # Solve theta as 1/(E*I) * integral of Moment
[ STUDENT > # Divide into regions m (motor) and e (extension) because
[ I differs
[ STUDENT > Tm(x):=1/(E*I_m)*int(M(x),x)+CTm;
Tm(x) := -.00004522546864 Heaviside(x) x^2 + .0001356764059 Heaviside(x - .2500000000) x^2
+ .8479775370 10^-5 Heaviside(x - .2500000000)
- .00006783820296 Heaviside(x - .2500000000) x
- .00009045093728 Heaviside(x - .3750000000) x^2
- .00001271966305 Heaviside(x - .3750000000)
+ .00006783820296 Heaviside(x - .3750000000) x + CTm
[ STUDENT > Te(x):=1/(E*I_e)*int(M(x),x)+CTe;
Te(x) := -.0007236074979 Heaviside(x) x^2 + .002170822494 Heaviside(x - .2500000000) x^2
+ .0001356764059 Heaviside(x - .2500000000)
- .001085411247 Heaviside(x - .2500000000) x
- .001447214996 Heaviside(x - .3750000000) x^2
- .0002035146088 Heaviside(x - .3750000000)
+ .001085411247 Heaviside(x - .3750000000) x + CTe
[ STUDENT >
[ STUDENT >
[ STUDENT > # Solve displacement as integral of slop
[ STUDENT > # Again solve in two regions
[ STUDENT > Ym(x):=int(Tm(x),x)+CYm;
Ym(x) := -.00001507515621 Heaviside(x) x^3 + .00004522546863 %2 x^3 - .7066479474 10^-6 %2
+ .8479775370 10^-5 %2 x - .00003391910148 %2 x^2 - .00003015031243 %1 x^3
+ .1589957880 10^-5 %1 - .00001271966305 %1 x + .00003391910148 %1 x^2 + CTm x + CYm
%1 := Heaviside(x - .3750000000)
%2 := Heaviside(x - .2500000000)
[ STUDENT > Ye(x):=int(Te(x),x)+CYe;
Ye(x) := -.001507515621 Heaviside(x) x^3 + .004522546863 %2 x^3 - .00007066479474 %2
+ .0008479775370 %2 x - .003391910148 %2 x^2 - .003015031243 %1 x^3 + .0001589957880 %1
- .001271966305 %1 x + .003391910148 %1 x^2 + CTm x + CYe
%1 := Heaviside(x - .3750000000)
%2 := Heaviside(x - .2500000000)
Ye(x) := -.0002412024993 Heaviside(x) x^3 + .0007236074980 %2 x^3 - .00001130636716 %2
+ .0001356764059 %2 x - .0005427056235 %2 x^2 - .0004824049987 %1 x^3
+ .00002543932610 %1 - .0002035146088 %1 x + .0005427056235 %1 x^2 + CTe x + CYe
%1 := Heaviside(x - .3750000000)
%2 := Heaviside(x - .2500000000)

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[ STUDENT >
[ STUDENT > # 4 Boundary conditions to solve the integration constants
[ STUDENT > BC1:=simplify(subs(x=0,Ym(x))=0); # Displacment # ob is 0
                                     BC1 := CYm = 0
[ STUDENT > BC2:=simplify(subs(x=Lm,Ym(x))=0); # Displacment # ib is
0
                                     BC2 := -.2355493158 10-6 + .2500000000 CTm + CYm = 0
[ STUDENT > BC3:=simplify(subs(x=Lm,Ye(x))=0); # Displacment # ib is 0
applies to both displacement solutions
                                     BC3 := -.3768789052 10-5 + .2500000000 CTe + CYe = 0
[ STUDENT > BC4:=simplify(subs(x=Lm,Tm(x))=subs(x=Lm,Te(x))); #
continuity of slope @ ib
                                     BC4 := -.2826591790 10-5 + CTm = -.00004522546862 + CTe
[ STUDENT > Csolution:=solve({BC1,BC2,BC3,BC4},{CTm,CTe,CYm,CYe});
Csolution :=
{ CYm = 0, CYe = -.7066479471 10-5, CTe = .00004334107409, CTm = .9421972632 10-6 }
[ STUDENT >
[ STUDENT > # Plug solution constants back into T and Y
[ STUDENT > Tm(x):=simplify(subs(Csolution,Tm(x)));
Tm(x) := -.00004522546864 Heaviside(x) x2 + .0001356764059 Heaviside(x - .2500000000) x2
+ .8479775370 10-5 Heaviside(x - .2500000000)
- .00006783820296 Heaviside(x - .2500000000) x
- .00009045093728 Heaviside(x - .3750000000) x2
- .00001271966305 Heaviside(x - .3750000000)
+ .00006783820296 Heaviside(x - .3750000000) x + .9421972632 10-6
[ STUDENT > Te(x):=simplify(subs(Csolution,Te(x)));
Te(x) := -.0007236074979 Heaviside(x) x2 + .002170822494 Heaviside(x - .2500000000) x2
+ .0001356764059 Heaviside(x - .2500000000)
- .001085411247 Heaviside(x - .2500000000) x
- .001447214996 Heaviside(x - .3750000000) x2
- .0002035146088 Heaviside(x - .3750000000)
+ .001085411247 Heaviside(x - .3750000000) x + .00004334107409
[ STUDENT > Ym(x):=subs(Csolution,Ym(x));
Ym(x) := -.00001507515621 Heaviside(x) x3 + .00004522546863 %2 x3 - .7066479474 10-6 %2
+ .8479775370 10-5 %2 x - .00003391910148 %2 x2 - .00003015031243 %1 x3
+ .1589957880 10-5 %1 - .00001271966305 %1 x + .00003391910148 %1 x2
+ .9421972632 10-6 x

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%1 := Heaviside($x - .3750000000$)

%2 := Heaviside($x - .2500000000$)

STUDENT > Ye(x):=subs(Csolution,Ye(x));

$$\begin{aligned} \text{Ye}(x) := & -.0002412024993 \text{Heaviside}(x) x^3 + .0007236074980 \%2 x^3 - .00001130636716 \%2 \\ & + .0001356764059 \%2 x - .0005427056235 \%2 x^2 - .0004824049987 \%1 x^3 \\ & + .00002543932610 \%1 - .0002035146088 \%1 x + .0005427056235 \%1 x^2 + .00004334107409 x \\ & - .7066479471 10^{-5} \end{aligned}$$

%1 := Heaviside($x - .3750000000$)

%2 := Heaviside($x - .2500000000$)

STUDENT > T(x):=piecewise(x>Lm,Te(x),Tm(x));

$$\begin{aligned} \text{T}(x) := & \{ -.0007236074979 \text{Heaviside}(x) x^2 + .002170822494 \%2 x^2 + .0001356764059 \%2 \\ & - .001085411247 \%2 x - .001447214996 \%1 x^2 - .0002035146088 \%1 + .001085411247 \%1 x \\ & + .00004334107409, .2500000000 < x \\ & -.00004522546864 \text{Heaviside}(x) x^2 + .0001356764059 \%2 x^2 + .8479775370 10^{-5} \%2 \\ & - .00006783820296 \%2 x - .00009045093728 \%1 x^2 - .00001271966305 \%1 \\ & + .00006783820296 \%1 x + .9421972632 10^{-6}, \text{otherwise} \end{aligned}$$

%1 := Heaviside($x - .3750000000$)

%2 := Heaviside($x - .2500000000$)

STUDENT > Y(x):=piecewise(x>Lm,Ye(x),Ym(x));

$$\begin{aligned} \text{Y}(x) := & \{ -.0002412024993 \text{Heaviside}(x) x^3 + .0007236074980 \%2 x^3 - .00001130636716 \%2 \\ & + .0001356764059 \%2 x - .0005427056235 \%2 x^2 - .0004824049987 \%1 x^3 \\ & + .00002543932610 \%1 - .0002035146088 \%1 x + .0005427056235 \%1 x^2 + .00004334107409 x \\ & - .7066479471 10^{-5}, .2500000000 < x \\ & -.00001507515621 \text{Heaviside}(x) x^3 + .00004522546863 \%2 x^3 - .7066479474 10^{-6} \%2 \\ & + .8479775370 10^{-5} \%2 x - .00003391910148 \%2 x^2 - .00003015031243 \%1 x^3 \\ & + .1589957880 10^{-5} \%1 - .00001271966305 \%1 x + .00003391910148 \%1 x^2 \\ & + .9421972632 10^{-6} x, \text{otherwise} \end{aligned}$$

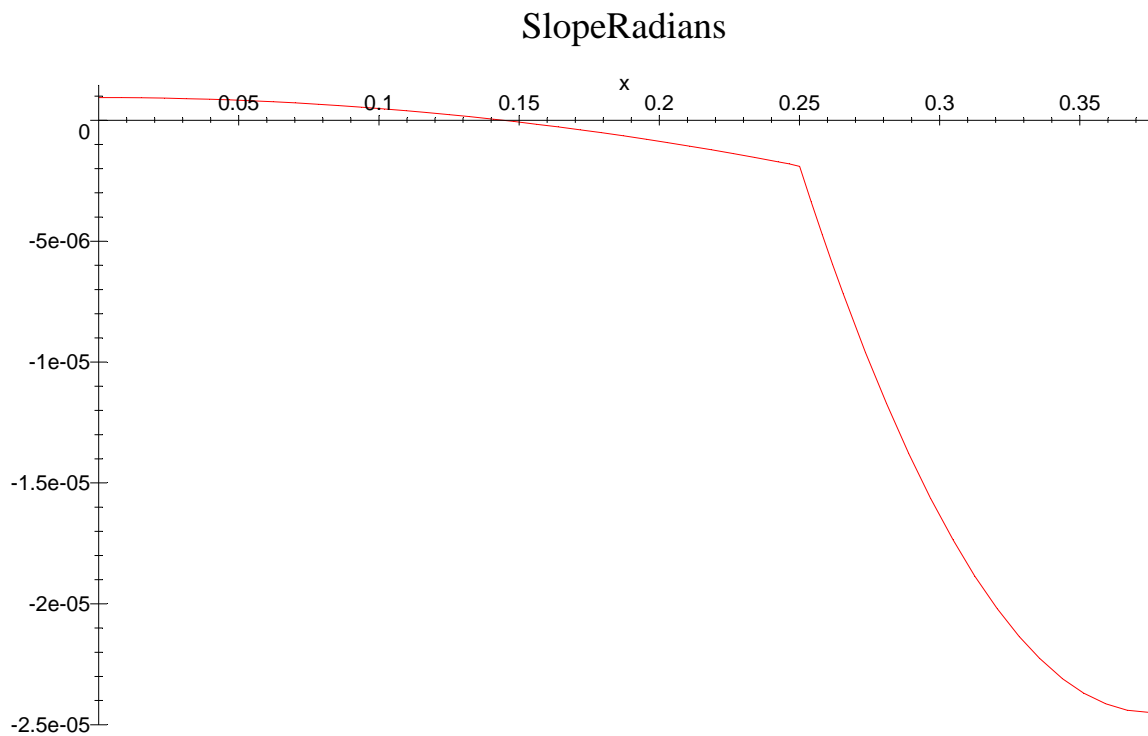
%1 := Heaviside($x - .3750000000$)

%2 := Heaviside($x - .2500000000$)

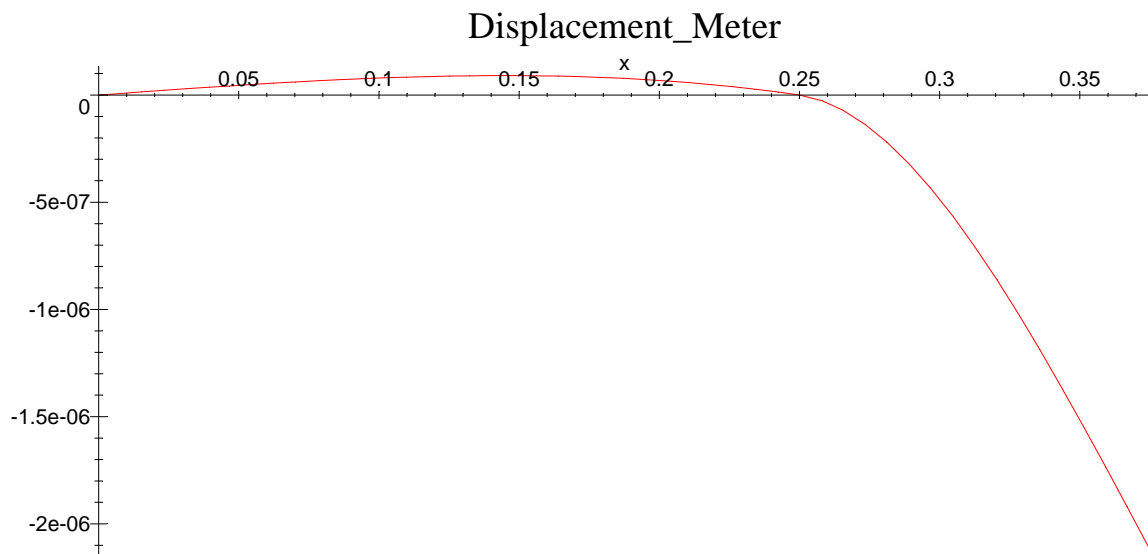
STUDENT >

STUDENT > # Plot results

STUDENT > plot(T(x),x=0..Lm+Le,title='SlopeRadians');



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STUDENT > plot(Y(x),x=0..Lm+Le,title='Displacement_Meter');
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```
STUDENT > # Find the angle at motor inboard bearing (T at x=Lm)
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STUDENT > AngleRadians:=evalf(subs(x=Lm,Te(x)));
```

AngleRadians := -.188439453 10⁻⁵

```
STUDENT > AngleDegrees:=AngleRadians*evalf(180/Pi);
```

AngleDegrees := -.0001079678535

```
STUDENT > AngleMinutes:=AngleDegrees*60;
```

AngleMinute Page 7 006478071210

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[ STUDENT > # Moment is negligible at this low angle per NSK figure 4
page 136
[ STUDENT > #
http://www.nsk.com/services/basicknowledge/technicalreport
/05distribution.html
[ STUDENT >
[ STUDENT > # *** Conclusion - bearing reaction moment is not
significant for this case, and probably not for any
electric motor since the shaft is stout. It may become a
factor only for very long thin shafts under high load
[ STUDENT >
[ STUDENT > # Compute Bending stress
[ STUDENT >  $M_{NM} := \text{evalf}(\text{subs}(x=Lm, M(x)))$ ; # moment at ib brg in
Newtons

 $M_{NM} := -22.20000000$ 
[ STUDENT >  $Sbending\_PA := M_{NM} * (De/2) / I_e$ ; # Bending stress in PA

 $Sbending\_PA := -.1809018745 \cdot 10^7$ 
[ STUDENT >  $Sbending\_PSI := Sbending\_PA / 6895$ ; # convert PA to PSI

 $Sbending\_PSI := -262.3667505$ 
[ STUDENT >

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