

N, V, M loading at surface: Stresses in Cartesian system of coordinates



Setup for Units is the default to SI.

Initialization

ORIGIN \equiv 1 Count with fingers TOL := 0.01 CTOL := 0.01

ton := 1000·kgf ksi := 70.307· $\frac{\text{kgf}}{\text{cm}^2}$ psi := $\frac{\text{ksi}}{1000}$ kip := 453.592·kgf MPa := 10.197· $\frac{\text{kgf}}{\text{cm}^2}$ day := 86400·sec
month := 30·day
year := 365·day

AND2(a,b) := $\left\{ \begin{array}{l} \text{if } a = 1 \\ \quad \left\{ \begin{array}{l} 1 \text{ if } b = 1 \\ 0 \text{ otherwise} \end{array} \right. \\ 0 \text{ otherwise} \end{array} \right.$ OR2(a,b) := $\left\{ \begin{array}{l} 1 \text{ if } a = 1 \\ \text{otherwise} \\ \quad \left\{ \begin{array}{l} 1 \text{ if } b = 1 \\ 0 \text{ otherwise} \end{array} \right. \end{array} \right.$



N := 200· $\frac{\text{ton}}{\text{m}}$ compression perpendicular and towards the ground, along y
of slice depth

V := 20· $\frac{\text{ton}}{\text{m}}$ horizontal (shear) force applied at the same point, along x service level, line loadings through slices' depth
of slice depth

M := 50· $\frac{\text{m}\cdot\text{ton}}{\text{m}}$ moment applied at the same point
of slice depth

x := 3·m distance from point of loads

y := 5·m depth into the ground



We have used as reference *Elasticidad, 4th ed.*, Eduardo Torroja, Dossat, Madrid 1967, p- 245

$r(x,y) := \sqrt{x^2 + y^2}$ $\theta(x,y) := \text{atan}\left(\frac{x}{y}\right)$

$r(x,y) = 5.83 \text{ m}$ $\theta(x,y) = 30.96 \text{ deg}$

we use these as data and then get polar stresses

$$\sigma_r(x,y) := \frac{2 \cdot N \cdot \cos(\theta(x,y))}{\pi \cdot r(x,y)} + \frac{2 \cdot V \cdot \sin(\theta(x,y))}{\pi \cdot r(x,y)} - \frac{2 \cdot M \cdot \sin(2 \cdot \theta(x,y))}{\pi \cdot r(x,y)^2}$$

$$\sigma_\theta(x,y) := 0 \cdot \text{MPa}$$

$$\tau_{r\theta}(x,y) := \frac{M \cdot (\cos(2 \cdot \theta(x,y)) + 1)}{\pi \cdot r(x,y)^2}$$

$$\sigma_r(x,y) = 0.19 \text{ MPa}$$

$$\sigma_\theta(x,y) = 0 \text{ MPa}$$

$$\tau_{r\theta}(x,y) = 0.01 \text{ MPa}$$

which are polar results that we want to convert to cartesian.
 This we do by setting the transformation equations as a system of equations with the stresses in xy as unknowns, that we solve inverting the matrix.

$$AA(x,y) := \begin{bmatrix} \cos(\theta(x,y))^2 & \sin(\theta(x,y))^2 & 2 \cdot \sin(\theta(x,y)) \cdot \cos(\theta(x,y)) \\ \sin(\theta(x,y))^2 & \cos(\theta(x,y))^2 & -2 \cdot \sin(\theta(x,y)) \cdot \cos(\theta(x,y)) \\ -(\sin(\theta(x,y)) \cdot \cos(\theta(x,y))) & \sin(\theta(x,y)) \cdot \cos(\theta(x,y)) & \cos(\theta(x,y))^2 - \sin(\theta(x,y))^2 \end{bmatrix}$$

$$\sigma(x,y) := AA(x,y)^{-1} \cdot \begin{pmatrix} 0 \cdot \text{MPa} \\ \sigma_r(x,y) \\ \tau_{r\theta}(x,y) \end{pmatrix} \qquad \sigma(x,y) = \begin{pmatrix} 0.04 \\ 0.14 \\ -0.08 \end{pmatrix} \text{MPa}$$

$$\sigma_y(x,y) := \sigma(x,y)_2 \qquad \sigma_x(x,y) := \sigma(x,y)_1 \qquad \tau_{xy}(x,y) := \sigma(x,y)_3$$



$$\sigma_y(x,y) = 0.14 \text{ MPa}$$

$$\sigma_x(x,y) = 0.04 \text{ MPa}$$

$$\tau_{xy}(x,y) = -0.08 \text{ MPa}$$

to guess the shear effect imagine a horizontal element and the outwards acting load being resisted at bottom

Chart the solution, uncollapse area



- Please note in the following charts that if shear load and moment are input the chart can not be symmetrical.

- Note also that in my no-patch Mathcad 8 Pro release the numbering in the contour lines must be enabled within the format special tab each time you load the sheet

Chart the effects of line load perpendicular to slice in halfspace

$$N_{\text{partsx}} := 40 \quad N_{\text{partsy}} := 20 \quad B_0 := 10 \cdot \text{m} \quad \text{halfwidth} \quad H_0 := 9 \cdot \text{m}$$

region of width and depth to be charted

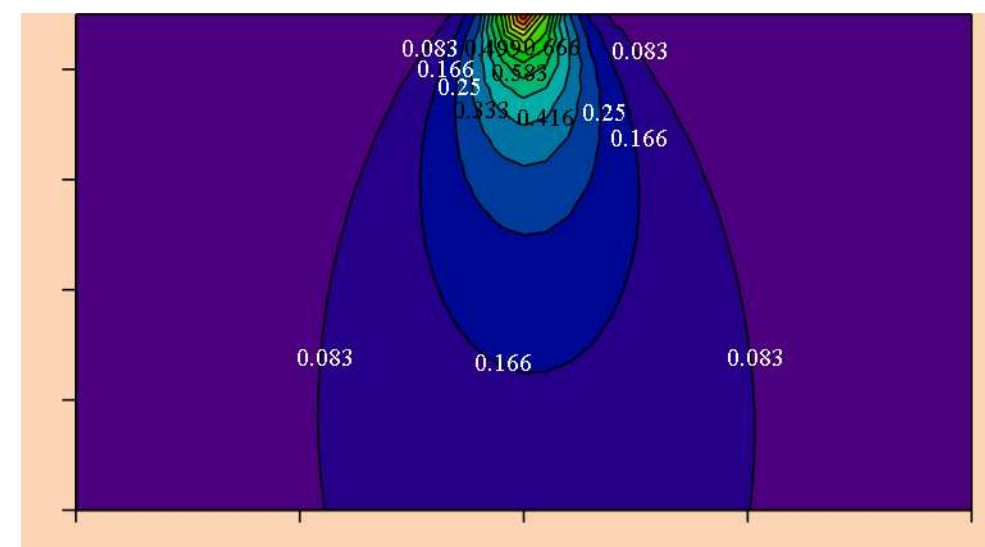
$$i := 1..N_{\text{partsx}} + 1 \quad j := 1..N_{\text{partsy}} + 1$$

$$x_{i,j} := -B_0 + \frac{2 \cdot B_0}{N_{\text{partsx}}} \cdot (i - 1) \quad y_{i,j} := 1 \cdot \text{m} + \frac{H_0}{N_{\text{partsy}}} \cdot (j - 1)$$

$$z1_{i,j} := \sigma_y(x_{i,j}, y_{i,j}) \quad z2_{i,j} := \sigma_x(x_{i,j}, y_{i,j}) \quad z3_{i,j} := \tau_{xy}(x_{i,j}, y_{i,j})$$

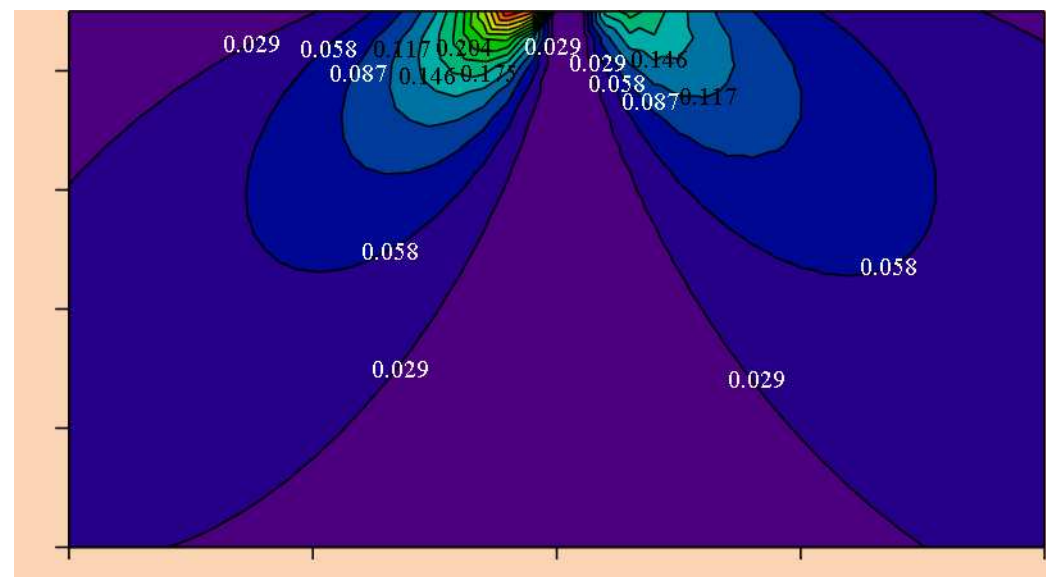
$$z4_{i,j} := \sqrt{(z1_{i,j})^2 + (z3_{i,j})^2} \quad z5_{i,j} := \sqrt{(z2_{i,j})^2 + (z3_{i,j})^2}$$

σ_y vertical stress at a depth (MPa)



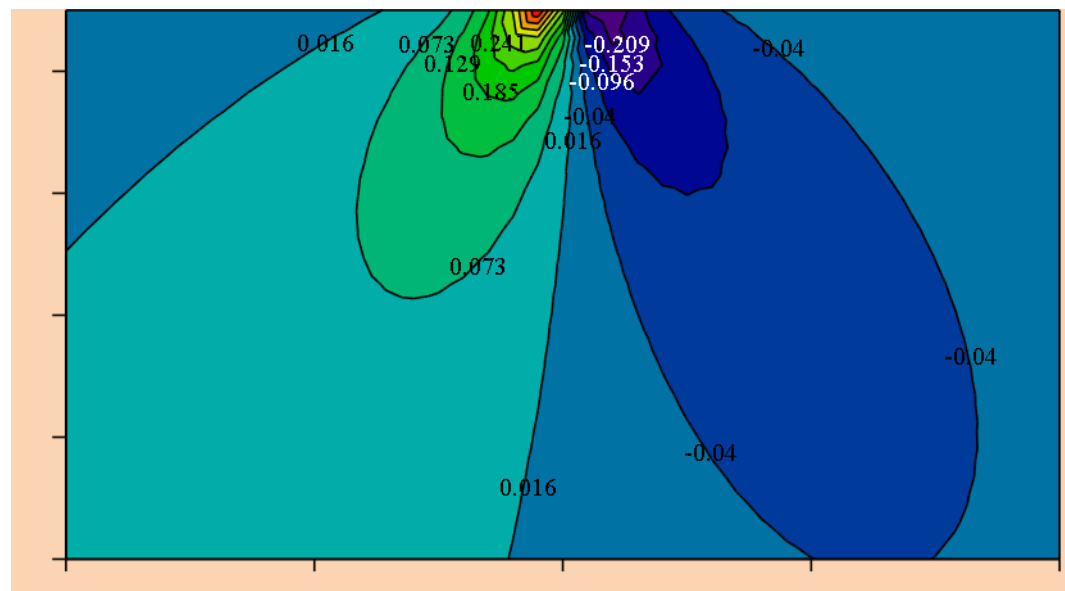
$$\left(\frac{x}{\text{m}}, -\frac{y}{\text{m}}, \frac{z1}{\text{MPa}} \right)$$

σ_x horizontal stress at a depth (MPa)



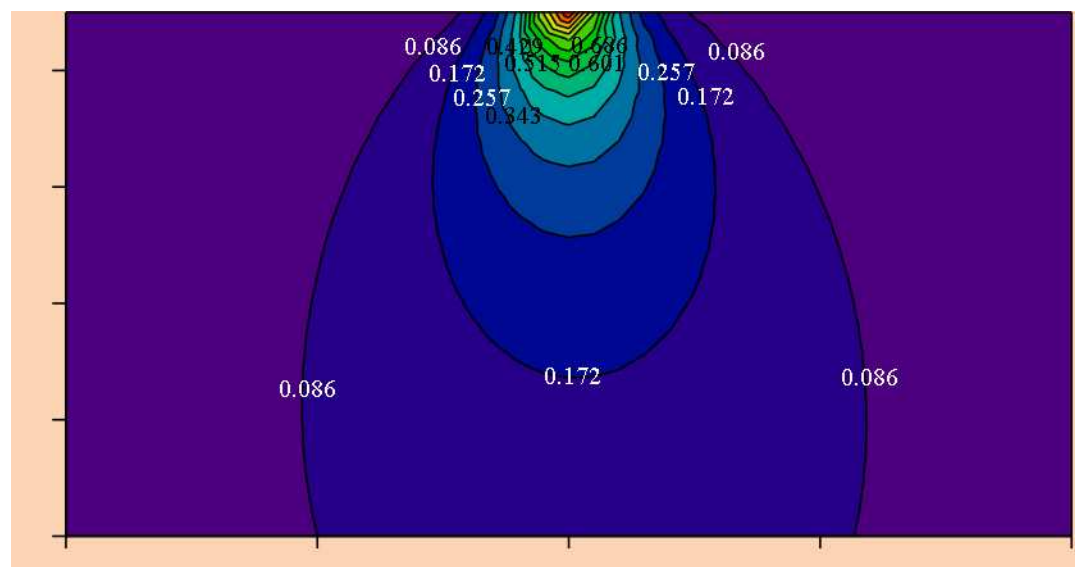
$$\left(\frac{x}{m}, -\frac{y}{m}, \frac{z2}{\text{MPa}} \right)$$

τ_{xy} shear stress at a depth (MPa)



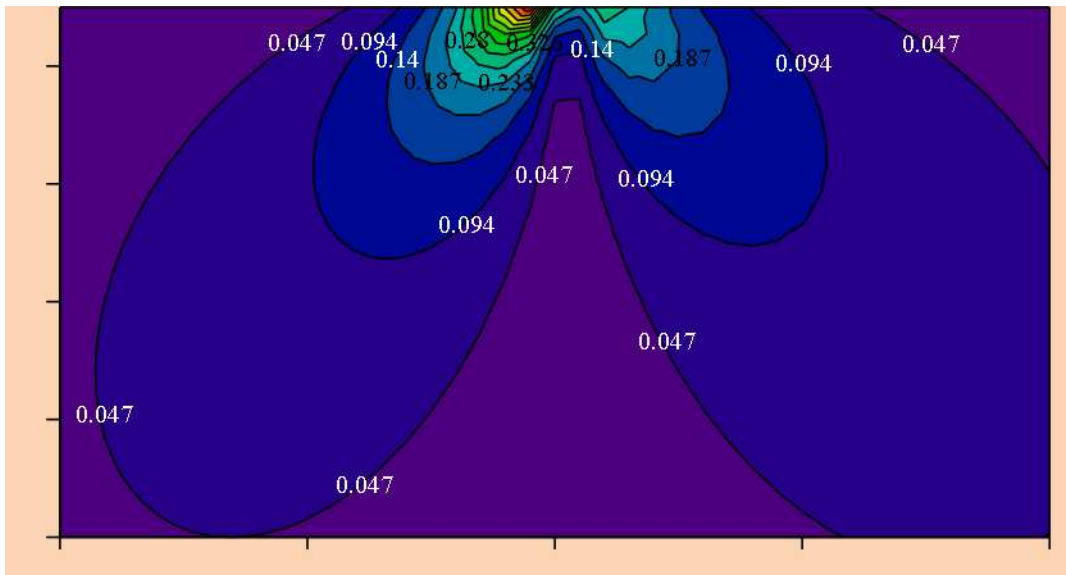
$$\left(\frac{x}{m}, -\frac{y}{m}, \frac{z_3}{\text{MPa}} \right)$$

Total stress on a horizontal plane at a depth (MPa)



$$\left(\frac{x}{m}, -\frac{y}{m}, \frac{z_4}{\text{MPa}} \right)$$

Total stress on a vertical plane at a depth (MPa)



$$\left(\frac{x}{m}, -\frac{y}{m}, \frac{z5}{\text{MPa}}\right)$$

