

Figure 11-15 Procedures for location of point of application of P_a for (a) irregular backfill; (b) concentrated or line load inside failure zone; (c) concentrated or line load outside failure zone (but inside zone ABC).

The author suggests the best solution for total wall force and point of application when there are backfill loads of any type and location is to use one of the following:

1. If the backfill load is inside the Rankine zone, convert it to an equivalent surcharge over the Rankine zone, then obtain the wall pressure and resultant using either the Coulomb or Rankine equations.
2. If the backfill load is either inside or outside the Rankine zone, use the Coulomb or Rankine equations for the soil wedge with no backfill load. Next use the Theory of Elasticity equations given in Sec. 11-13 to find the wall forces from the backfill loads. Then to find total force and point of application use $\sum P = R = \sum P_i$ and $R\bar{y} = \sum P_i y_i$.

11-13 LATERAL PRESSURES BY THEORY OF ELASTICITY

The Boussinesq Equation

The trial wedge method seems to be too conservative in estimating the lateral force against a wall when there are surcharges (or loads) on the backfill—particularly outside the Rankine zone. For this reason, at present this procedure does not seem to be used much. The Theory of Elasticity method can be used to compute the lateral pressure profile against the wall from the surface surcharge (point, line, strip) loading. The Boussinesq equation—or some variation of it—is commonly used. The equation form usually credited to Boussinesq is

$$\sigma_r = \frac{P}{2\pi z^2} \left[3 \sin^2 \theta \cos^3 \theta - \frac{(1 - 2\mu) \cos^2 \theta}{1 + \cos \theta} \right] \quad (11-20)$$

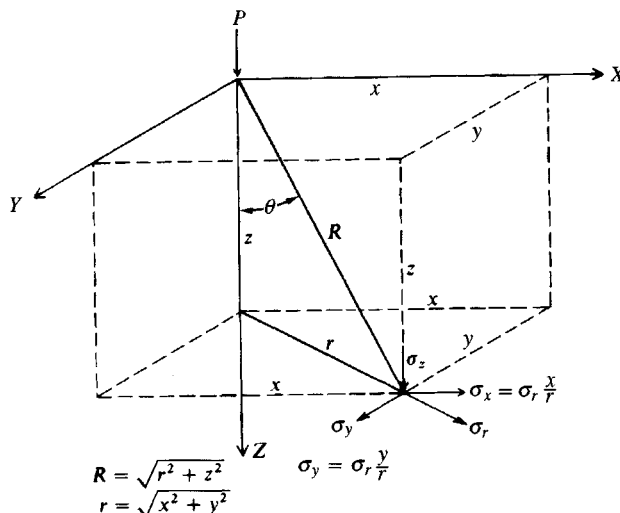


Figure 11-16 Identification of terms used in the Boussinesq equation [Eq. (11-20)] for lateral pressure.

Equation (11-20) can also be written as

$$\sigma_r = \frac{P}{2\pi} \left[\frac{3r^2z}{R^5} - \frac{1-2\mu}{R(R+z)} \right] \quad (11-20a)$$

where the several terms including θ , z , r , and R are identified on Fig. 11-16. This form of the equation is particularly suitable for programming on a small calculator, since the point load P is usually fixed with given x , y coordinates and we want to vary z to obtain the wall pressure profile.

The computer programming of this equation allows one to solve any of the given backfill surcharge loads of Fig. 11-17 defined as follows:

1. Point load. Use the equation in the given form.
2. Line load. Treat as either one load or a series of concentrated loads along a line of unit width acting on unit areas.
3. Strip load. Treat as a series of parallel line loads acting on strips of some unit width.
4. Loaded area. Treat as a series of parallel line loads acting on strips of finite length.

We can easily analyze a constant uniformly loaded area (say, the interior part of an embankment) or one with a linear varying load (say, the embankment side slopes). In either of these cases the loaded area is divided into strips with some load intensity q and some small "unit" width B , on the order of 0.25 to 0.5 m. These strips are then subdivided into "unit" areas of some length L also on the order of 0.25 to 0.50 m. These "unit" areas are treated as a series of point loads of $Q = qBL$ acting at the center of each of the unit areas. The several "unit" area contributions making up the total loaded area are then summed to obtain the total lateral pressure acting at some point at the depth of interest (either in the soil or on a wall). This is the procedure used in program SMBLP1 (B-8) on your program diskette.

The general validity of using a form of Eq. (11-20) for surcharges was established in several publications, including the work of Spangler (1936), Spangler and Mickle (1956), Rehnman and Broms (1972), and others.

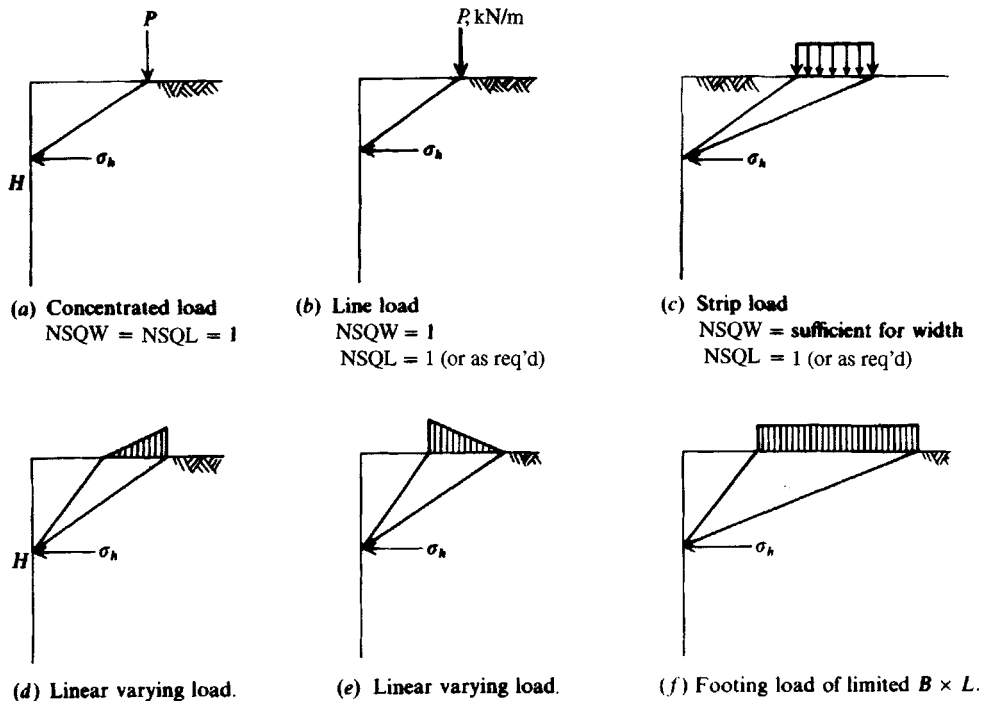


Figure 11-17 Surcharge loads that can be used with the computer program SMBLP1 (B-8) on your program diskette. NSQW, NSQL = number of unit elements away from wall and parallel to wall, respectively, as used in the computer program.

The early work of Spangler and Spangler and Mickle introduced an error into the general application of the equations; however, that can be avoided by direct use of Eq. (11-20) and an appropriate value for Poisson's ratio μ .

When the work of Spangler was first published, he used $\mu = 0.5$ [and later in Spangler and Mickle (1956)], which substantially simplifies Eq. (11-20)—but may not be correct. Spangler's work consisted of trying to measure the lateral pressure against a 1.829 m (2.134 m total height) high \times 4.572 m wide retaining wall with a constant stem thickness of 0.150 m. He used metal ribbons (since earth-pressure cells were not readily available in the early 1930s) and simply dumped a granular ($w_L = 17.5$, $w_P = 13.2\%$) backfill behind the wall with no compaction at all to produce an extremely loose state. After a time, he had a truck backed onto the loose backfill so that the rear wheels could simulate two concentrated loads. To simulate a line load he laid a railroad crosstie parallel to the wall, onto which the rear wheels (a single axle with dual tires) of a loaded truck were backed. Since the wall was only 4.572 m long and a railroad crosstie is about 3 m long, a strip model was not very likely to have been produced.

From these efforts Spangler (both references) found that the measured lateral pressure was about twice that predicted by Eq. (11-20) with $\mu = 0.5$. From the reported results, Mindlin (1936a), in discussing Spangler's (1936) work suggested that the factor of 2 could be explained by a rigid wall producing the effect of a mirror load placed symmetrically in front of the wall (Fig. 11-18a). The author began looking at this problem more closely and decided that the mirror load is not an explanation. As shown in Fig. 11-18b, a mirror load on a rigid

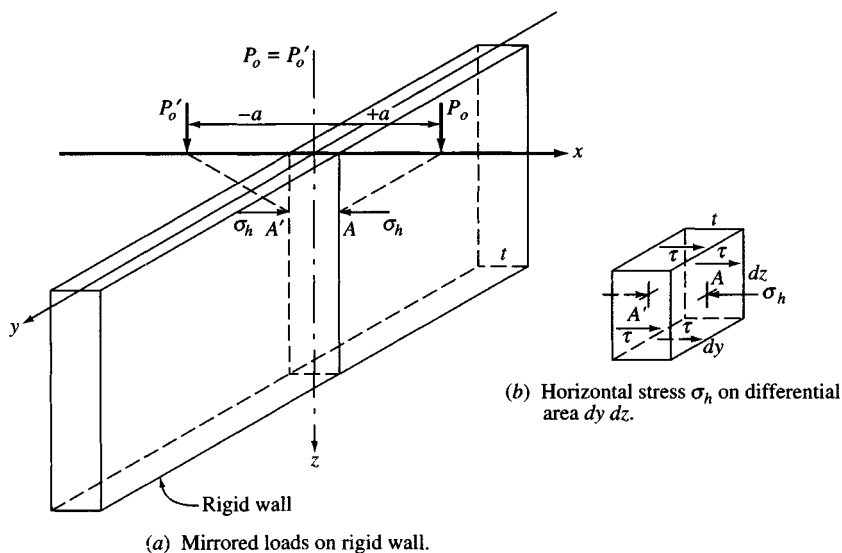


Figure 11-18 The case for lateral pressure on a rigid wall.

wall would simply cancel the lateral shear stresses in the wall and certainly not double the horizontal pressure. A flexible wall could possibly double the lateral pressure but would have to be extremely flexible (and have the loads applied sequentially). Referring to Fig. 11-18b, we see that the horizontal pressure σ_h produced by P_o (applied first) acting on a rigid area of $dy dz$ would develop shear resistance τ such that $\tau t(2dz + 2dy) = \sigma_h$. When mirror load P'_o is applied, a second shear stress τ would develop on the element but in the opposite direction, so the shear stress would cancel and we would simply have on each side of the wall a horizontal stress σ_h (not $2\sigma_h$). If the wall is flexible and little shear stress develops, the element would (if the loads were applied sequentially) displace laterally toward the $-a$ direction to produce a resisting soil stress on the $-a$ side of $\sigma'_h = \sigma_h - \tau(2dz + 2dy)$. Since this would become locked in when P'_o is applied, the stress on the left would become $\sigma_h + \sigma'_h$, and since σ'_h is transmitted through the wall, the right side would also have the existing σ_h + transmitted σ'_h value from load P'_o . If the loads were applied simultaneously the stresses would simply be σ_h on each side (and not $2\sigma_h$).

Mindlin got around this complication by inserting a statement that the wall was rigid but could not carry shear. There is no such wall type known to the author.

Because the Spangler work was done in the early 1930s, it is difficult to speculate on the cause of the high stresses except to note that the wall had rather finite dimensions. The surcharge load was caused by a truck backing onto the backfill. When it stopped at the desired position it would have produced an inertial force that was amplified because the fill was not well compacted. The backfill probably was of limited extent, so that it is also possible some type of arching occurred that increased the lateral pressure.

More recently Rehnman and Broms (1972) showed (using modern earth-pressure cells) that when the soil behind the wall was dense the lateral pressure from point loads was much less than when the soil was loose. They also found that gravelly backfill produced larger lateral pressures than finer-grained materials. This observation indicates that both soil state and Poisson's ratio are significant parameters.

Theory of elasticity gives the limiting range of Poisson's ratio as

$$-1 \leq \mu \leq 0.5$$

Also note that, strictly, there is a sign with μ , so that (+) means an elongation strain with lateral contraction, as for a tensile steel test that gives $\mu = 0.3$, and a compression strain with lateral expansion, as for a concrete test cylinder giving $\mu \approx 0.15$. No engineering material is known that might give a (-) ratio where there is lateral expansion with elongation or lateral contraction with compression.

In Chap. 2 it was stated that for soils, values of μ can be greater than 0.5 with values of 0.6 and 0.7 fairly common because soil is only pseudoelastic.

Equation (11-20) and similar expressions do not compute reliable wall pressures for surcharge loads (point, strip, or line) unless the loads are located beyond some critical distance from the wall. Laba and Kennedy (1986) suggested this distance might be $0.4H$. Terzaghi (1954) had also suggested $0.4H$ might be the critical distance and provided two equations, based on Eq. (2-20) but with $\mu = 0.5$: one for the surcharge distance $< 0.4H$ from the wall and a second for the surcharge distance $> 0.4H$.

With this background, it is clear that approximations to using Eq. (11-20) should be used cautiously. There are a number of closed-form solutions for select cases of backfill loads; however, the author has found substantial differences, particularly for variable intensity loading. For these several reasons, and because Eq. (11-20) can be easily programmed *for all the cases*, it is the only method recommended by the author. Comparison with closed-form solutions indicates almost no error from using discrete methods for continuous loadings.

There is some misuse of Eq. (11-20) or its equivalent caused by setting $\mu = 0.5$ so the μ term disappears. For example, line and strip loads of infinite length should be treated as plane strain problems. That is, take a unit length parallel to the wall similar to the procedure for surcharges.

For both line and strip surcharges, Terzaghi (1943) performed integrations on the modified Eq. (11-20) using $\pm\infty$ for the limits—but stated that these loadings were plane strain problems. In using Eq. (11-20) one should be using a plane strain μ' [see Eq. (2-65)] instead of the triaxial value (or a value estimated at around 0.3 to 0.4). Note the following short table for the plane strain μ' versus triaxial μ :

$\mu =$	0.3	0.33	0.35	0.40	0.45	0.50	0.60
$\mu' =$	0.42	0.50	0.54	0.67	0.82	1.00	1.50

Table 11-7 illustrates the case of a small retaining wall with a concentrated load at varying distance, using a range of Poisson's ratio. This wall also includes the trial wedge solution for the several load positions and the Rankine lateral pressure computed for no surcharge. From this table several conclusions can be drawn:

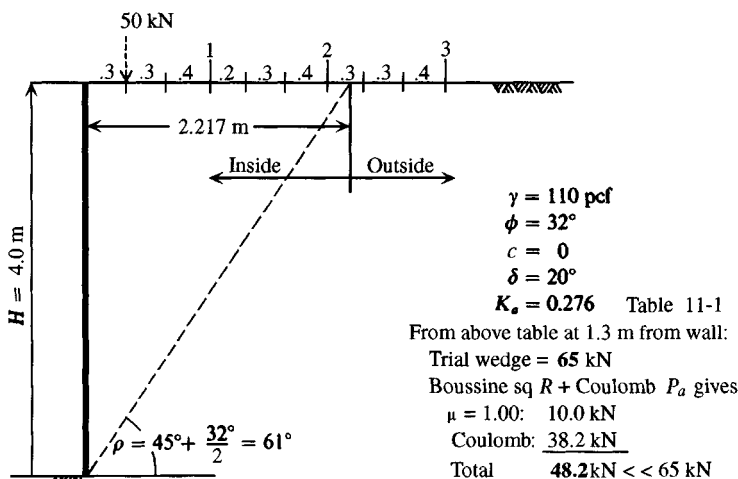
1. The trial wedge method gives the Coulomb (or Rankine if $\delta = 0$) wall force for a horizontal backfill and correctly locates the failure surface using angle ρ measured from the horizontal. For concentrated surcharges on the backfill, much larger wall forces are obtained than by any other method.
2. Poisson's ratio $\mu = 1$ gives a substantial increase in wall pressure versus $\mu = 0.3$ to 0.5. A plane strain $\mu' = 1.00$ may be possible for soil in a very loose state.

TABLE 11-7

Comparison of trial wedge and Boussinesq wall forces computed using Eq. (11-20).

Also shown is the Coulomb active pressure force.

2.217 m									
Load position from wall, m	0	0.3	0.6	1.0	1.3	1.6	2.0	2.3	2.6 3.0 m
Trial wedge, kN		65.0	65.0	65.0	65.0	64.8	63.4	61.6	59.3 55.5
Coulomb $P_a = \frac{1}{2}(17.30)(4)^2(0.276) = 38.2 \text{ kN (vs. 38.1 of trial wedge)}$									
Boussinesq	Inside							Outside	
$\mu = 0.3$	—	8.8	8.3	5.3	4.1	3.3	2.5	2.1	1.8 1.4
0.5	—	13.1	11.5	7.5	5.8	4.6	3.6	3.0	2.5 2.1
0.7	—	18.0	14.8	9.7	7.5	6.0	4.6	3.9	3.3 2.7
1.0	—	25.3	19.9	13.0	10.0	8.1	6.2	5.2	4.5 3.7



- Concentrated loads well outside the Rankine zone contribute to P_a in the trial wedge case, leading one to the opinion that the trial wedge is not correct, but conservative.
- Since Eq. (11-20) gives small lateral pressures when the load is very close to the wall, this result may mean either that the surcharge load is being carried downward by vertical wall friction rather than by lateral pressure or that Eq. (11-20) is not valid for a load close to such a massive discontinuity in the elastic half-space.
- Wall pressures computed by Eq. (11-20) are rather small once the wall-to-load distance is greater than the Rankine zone.

Computer Program for Lateral Pressure

Computer program SMBLP1 (B-8) on your program diskette can be used for all the lateral pressure problems shown on Fig. 11-17. By superposition, almost any conceivable surcharge load can be analyzed, quite rapidly in most cases.