

APPENDIX A

Broms method for analysis of single piles under lateral loading

The method was presented in three papers published in 1964 and 1965 (Broms 1964a, 1964b, 1965). As shown in the following paragraphs, a pile can be designed to sustain a lateral load by solving some simple equations or by referring to charts and graphs.

A.1 PILES IN COHESIVE SOIL

A.1.1 *Ultimate lateral load for piles in cohesive soil*

Broms adopted a distribution of soil resistance, as shown in Figure A.1, that allows the ultimate lateral load to be computed by equations of static equilibrium. The elimination of soil resistance for the top 1.5 diameters of the pile is a result of lower resistance in that zone because a wedge of soil can move up and out when the pile is deflected. The selection of nine times the undrained shear strength times the pile diameter as the ultimate soil resistance, regardless of depth, is based on calculations with movement of soil from the front toward the back of the pile.

A.1.1.1 *Short, free-head piles in cohesive soil*

For short piles that are unrestrained against rotation, the patterns that were selected for behavior are shown in Figure A.2. The following equation results from the integration of the upper part of the shear diagram to the point of zero shear (the point of maximum moment)

$$M_{\max}^{\text{pos}} = P_t(e + 1.5b + f) - \frac{9c_u b f^2}{2} \quad (\text{A.1})$$

But the point where shear is zero is

$$f = \frac{P_t}{9c_u b} \quad (\text{A.2})$$

Therefore,

$$M_{\max}^{\text{pos}} = P_t(e + 1.5b + 0.5f) \quad (\text{A.3})$$

Integration of the lower portion of the shear diagram yields

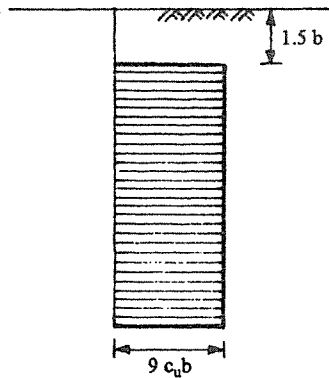


Figure A.1. Assumed distribution of soil resistance for cohesive soil.

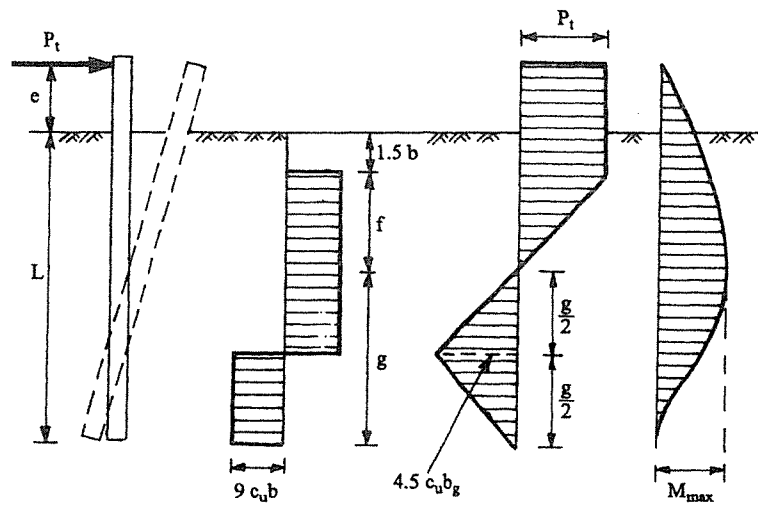


Figure A.2. Diagrams of deflection, soil resistance, shear, and moment for short pile in cohesive soil, unrestrained against rotation.

$$M_{\max}^{\text{pos}} = 2.25c_u b g^2 \quad (\text{A.4})$$

It may be seen that

$$L = (1.5b + f + g) \quad (\text{A.5})$$

Equations A.2 through A.5 may be solved for the load P_{ult} that will produce a soil failure. After obtaining a value of P_{ult} , the maximum moment can be computed and compared with the moment capacity of the pile. An appropriate factor of safety should be employed.

As an example of the use of the equations, assume the following:

$b = 305 \text{ mm}$ (assume 305-mm O.D. steel pipe by 19 mm wall),

$I_p = 1.75 \times 10^{-4} \text{ m}^4$, $e = 0.61 \text{ m}$, $L = 2.44 \text{ m}$, and

$$c_u = 47.9 \text{ kPa.}$$

Equations A.2 through A.5 are solved simultaneously and the following quadratic equation is obtained.

$$P_t^2 + 1,083P_t - 67,900 = 0$$

$$P_t = 59.4 \text{ kN}$$

Substituting into Equation A.3 yields the maximum moment

$$M_{\max} = 59.4 \left[0.61 + 1.5(0.305) + \frac{(0.5)(59.4)}{(9)(47.9)(0.305)} \right]$$

$$= 77 \text{ kN-m.}$$

Assuming no axial load, the maximum stress is

$$f_b = \frac{(77)(0.1525)}{1.75 \times 10^{-4}} = 67,000 \text{ kPa}$$

The computed maximum stress is tolerable for a steel pipe, especially when a factor of safety is applied to P_{ult} . The computations, then, show that the short pile would fail due to a soil failure.

Broms presented a convenient set of curves for solving the problem of the short-pile (see Fig. A.3). Entering the curves with L/b of 8 and e/b of 2, one obtains a value of P_{ult} of 60 kN, which agrees with the results computed above.

A.1.1.2 Long, free-head piles in cohesive soil

As the pile in cohesive soil with the unrestrained head becomes longer, failure will occur with the formation of a plastic hinge at a depth of $1.50b + f$. Equation A.3 can then be used directly to solve for the ultimate lateral load that can be applied. The shape of the pile under load will be different than that shown in Figure A.2 but the equations of mechanics for the upper portion of the pile remain unchanged.

A plastic hinge will develop when the yield stress of the steel is attained over the entire cross-section. For the pile that is used in the example, the yield moment is 430 m-kN if the yield strength of the steel is selected as 276 MPa.

Substituting into Equation A.3

$$430 = P_{ult} \left(0.61 + 0.4575 + \frac{P_{ult}}{2.53} \right)$$

$$P_{ult} = 224 \text{ kN}$$

Broms presented a set of curves for solving the problem of the long pile (see Fig. A.4). Entering the curves with a value of $M_y/c_u b^3$ of 316.4, one obtains a value of P_{ult} of about 220 kN.

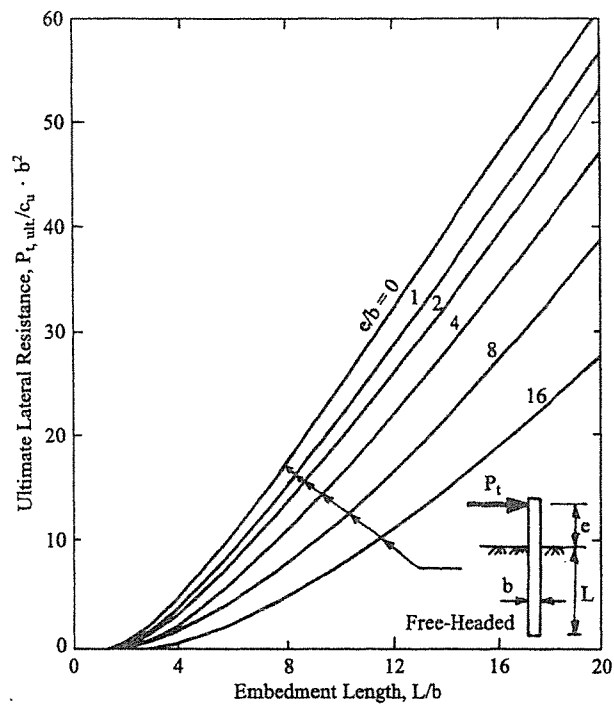


Figure A.3. Curves for design of short piles under lateral load in cohesive soil.

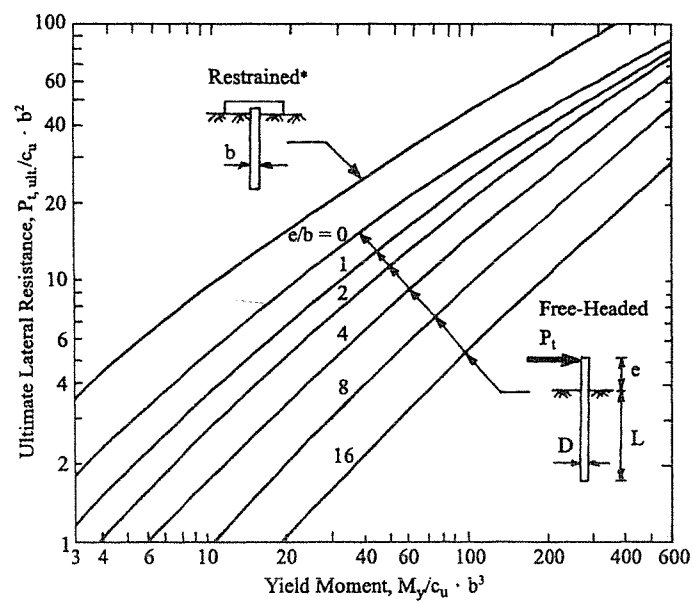


Figure A.4. Curves for design of long piles under lateral load in cohesive soil.

A.1.1.3 *Influence of pile length, free-head piles in cohesive soil*

Consideration may need to be given to the pile length at which the pile ceases to be a short pile. The value of the yield moment may be computed from the pile geometry and material properties and used with Equations A.2 through A.5 to solve for a critical length. Longer piles will fail by yielding. Or a particular solution may start with use of the short-pile equations; if the resulting moment is larger than the yield moment, the long-pile equations may be used.

For the example problem, the length at which the short-pile equations cease to be valid may be found by substituting a value of P_{ult} of 224 kN into Equation A.2 and solving for f and substituting a value of M_{max} of 430 m-kN into Equation A.4 and solving for g . Equation A.5 can then be solved for L . The value of L was found to be 5.8m. Thus, for the example problem the value of P_{ult} increases from zero to 224 kN as the length of the pile increases from 0.46m to 5.8m, and above a length of 5.8m the value of P_{ult} remains constant at 224 kN.

A.1.1.4 *Short, fixed-head piles in cohesive soil*

For a pile that is fixed against rotation at its top, the mode of failure depends on the length of the pile. For a short pile, failure consists of a horizontal movement of the pile through the soil with the full soil resistance developing over the length of the pile except for the top one and one-half pile diameters, where it is expressly eliminated. A simple equation can be written for this mode of failure, based on force equilibrium.

$$P_{ult} = 9c_u b(L - 1.5b) \quad (A.6)$$

A.1.1.5 *Intermediate length, fixed-head piles in cohesive soil*

As the pile becomes longer, an intermediate length is reached such that a plastic hinge develops at the top of the pile. Rotation at the top of the pile will occur and a point of zero deflection will exist somewhere along the length of the pile. Figure A.5 presents the diagrams of mechanics for the case of the restrained pile of intermediate length.

The equation for moment equilibrium for the point where the shear is zero (where the positive moment is maximum) is:

$$M_{pos}^{max} = P_i(1.5b + f) - f(9c_u b)\left(\frac{f}{2}\right) - M_y$$

Substituting a value of f ,

$$M_{pos}^{max} = P_i(1.5b + 0.5f) - M_y \quad (A.7)$$

Employing the shear diagram for the lower portion of the pile,

$$M_{pos}^{max} = 2.25c_u bg^2 \quad (A.8)$$

The other equations that are needed to solve for P_{ult} are:

$$L = 1.5b + f + g \quad (A.9)$$

and

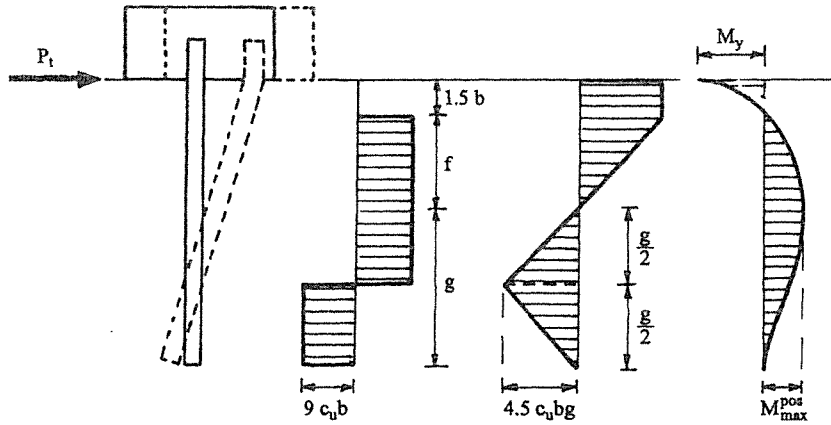


Figure A.5. Diagrams of deflection, soil resistance, shear, and moment for intermediate-length pile in cohesive soil, fixed against rotation.

$$f = \frac{P_t}{9} c_u b \quad (\text{A.10})$$

Equations A.7 through A.10 can be solved for the behavior of the restrained pile of intermediate length.

A.1.1.6 Long, fixed-head piles in cohesive soil

The mechanics for a long pile that is restrained at its top is similar to that shown in Figure A.5 except that a plastic hinge develops at the point of the maximum positive moment. Thus, the $M_y^{pos, max}$ in Equation A.7 becomes M_y and the following equation results

$$P_t = \frac{2M_y}{(1.5b + 0.5f)} \quad (\text{A.11})$$

Equations A.10 and A.11 can be solved to obtain P_{ult} for the long pile.

A.1.1.7 Influence of pile length, fixed-head piles in cohesive soil

The example problem will be solved for the pile lengths where the pile goes from one mode of behavior to another. Starting with the short pile, an equation can be written for moment equilibrium for the case where the yield moment has developed at the top of the pile and where the moment at its bottom is zero. Referring to Figure A.5, but with the soil resistance only on the right-hand side of the pile, taking moments about the bottom of the pile yields the following equation.

$$P_t L - 9c_u b \left(\frac{L - 1.5b}{2} \right) - M_y = 0$$

Summing forces in the horizontal direction yield the next equation.

$$P_{ult} = 9c_u b(L - 1.5b) = 0 \quad (\text{same as Eq. A.6})$$

The simultaneous solution of the two equations yields the desired expression.

$$P_{ult} = \frac{M_y}{(0.5L + 0.75b)} \quad (\text{A.12})$$

Equations A.6 and A.12 can be solved simultaneously for P_{ult} and for L , as follows

$$\text{from Equation A.6, } P_{ult} = (9)(47.9)(0.305)(L - 0.4575),$$

$$\text{from Equation A.12, } P_{ult} = 430/(0.5L + 0.229),$$

$$\text{then } L = 2.6 \text{ m and } P_{ult} = 281 \text{ kN.}$$

For the determination of the length where the behavior changes from that of the pile of intermediate length to that of a long pile, Equations A.7 through A.10 can be used with M_{max} set equal to M_y , as follows:

$$\text{from Equation A.7, } P_{ult} = \frac{(2)(430)}{(1.5)(0.305) + 0.5f}$$

$$\text{from Equation A.8, } g = \left[\frac{439}{(2.25)(47.9)(0.305)} \right]^{0.5} = 3.62 \text{ m}$$

$$\text{from Equation A.9, } L = (1.5)(0.305) + f + g$$

$$\text{from Equation A.10, } f = \frac{P_{ult}}{9} (47.9)(0.305)$$

$$\text{then } L = 7.27 \text{ m and } P_{ult} = 419 \text{ kN.}$$

In summary, for the example problem the value of P_{ult} increases from zero to 281 kN as the length of the pile increases from 0.46 m to 2.6 m, increases from 281 kN to 419 kN as the length increases from 2.6 m to 7.3 m, and above a length of 7.3 m the value of P_{ult} remains constant at 419 kN.

In his presentation, Broms showed a curve in Figure A.3 for the short pile that was restrained against rotation at its top. That curve is omitted here because the computation can be made so readily with Equation A.6. Broms' curve for the long pile that is fixed against rotation at its top is retained in Figure A.4 but a note is added to ensure proper use of the curve. For the example problem, a value of 415 kN was obtained for P_{ult} , which agrees well with the computed value. No curves are presented for the pile of intermediate length.

A.1.2 Deflection of piles in cohesive soil

Broms suggested that for cohesive soils the assumption of a coefficient of subgrade reaction that is constant with depth can be used with good results for predicting the lateral deflection at the groundline. He further suggests that the coefficient of subgrade reaction α should be taken as the average over a depth of $0.8\beta L$, where

$$\beta = \left(\frac{\alpha}{4E_p I_p} \right)^{0.25} \quad (\text{A.11})$$

where α soil reaction modulus and; $E_p I_p$ = pile stiffness.

Broms presented equations and curves for computing the deflection at the groundline. His presentation follows the procedures presented elsewhere in this text.

With regard to values of the reaction modulus, Broms used work of himself and Vesic (1961a, 1961b) for selection of values, depending on the unconfined compressive strength of the soil. The work of Terzaghi (1955) and other with respect to the reaction modulus have been discussed fully in the text.

Broms suggested that the use of a constant for the reaction modulus is valid only for a load of one-half to one-third of the ultimate lateral capacity of a pile.

A.1.3 *Effects of nature of loading on piles in cohesive soil*

The values of reaction modulus presented by Terzaghi are apparently for short-term loading. Terzaghi did not discuss dynamic loading or the effects of repeated loading. Also, because Terzaghi's coefficients were for overconsolidated clays only, the effects of sustained loading would probably be minimal. Because the nature of the loading is so important in regard to pile response, some of Broms' remarks are presented here.

Broms suggested that the increase in the deflection of a pile under lateral loading due to consolidation can be assumed to be the same as would take place with time for spread footings and rafts founded on the ground surface or at some distance below the ground surface. Broms suggested that test data for footings on stiff clay indicate that the coefficient of subgrade reaction to be used for long-time lateral deflections should be taken as 1/2 to 1/4 of the initial reaction modulus. The value of the coefficient of subgrade reaction for normally consolidated clay should be 1/4 to 1/6 of the initial value.

Broms suggested that repetitive loads cause a gradual decrease in the shear strength of the soil located in the immediate vicinity of a pile. He stated that unpublished data indicate that repetitive loading can decrease the ultimate lateral resistance of the soil to about one-half its initial value.

A.2 PILES IN COHESIONLESS SOILS

A.2.1 *Ultimate lateral load for piles in cohesionless soil*

As for the case of cohesive soil, two failure modes were considered; a soil failure and a failure of the pile by the formation of a plastic hinge. With regard to a soil failure in cohesionless soil, Broms assumed that the ultimate lateral resistance is equal to three times the Rankine passive pressure. Thus, at a depth z below the ground surface the soil resistance per unit of length P_z can be obtained from the following equations.

$$P_z = 3b\gamma z K_p \quad (\text{A.12})$$

$$K_p = \tan^2 \left(45 + \frac{\phi}{2} \right) \quad (\text{A.13})$$

where γ = unit weight of soil; K_p = Rankine coefficient of passive pressure; and ϕ = friction angle of soil.

A.2.1.1 Short, free-head piles in cohesionless soil

For short piles that are unrestrained against rotation, a soil failure will occur. The curve showing soil reaction as a function of depth is shaped approximately as shown in Figure A.6. The use of M_a as an applied moment at the top of the pile follows the procedure adopted by Broms. If both P_t and M_a are acting, the result would be merely to increase the magnitude of e . It is unlikely in practice that M_t alone would be applied.

The patterns that were selected for behavior are shown in Figure A.7. Failure takes place when the pile rotates such that the ultimate soil resistance develops from the ground surface to the center of rotation. The high values of soil resistance that develop at the toe of the pile are replaced by a concentrated load as shown in Figure A.7.

The following equation results after taking moments about the bottom of the pile.

$$P_t(e + L) + M_t = (3\gamma b L K_p) \left(\frac{L}{2} \right) \left(\frac{L}{3} \right) \quad (\text{A.14})$$

Solving for P_t when M_t is equal to zero,

$$P_t = \frac{\gamma b L^3 K_p}{2(e + L)} \quad (\text{A.15})$$

And solving for M_t when P_t is equal to zero,

$$M_t = 0.5\gamma b L^3 K_p \quad (\text{A.16})$$

Equations A.14 through A.16 can be solved for the load or moment, or a combination of the two, that will cause a soil failure. The maximum moment will then be found, at the depth f below the ground surface, and compared with the moment capacity of the pile. An appropriate factor of safety should be used. The distance f can be computed by solving for the point where the shear is equal to zero.

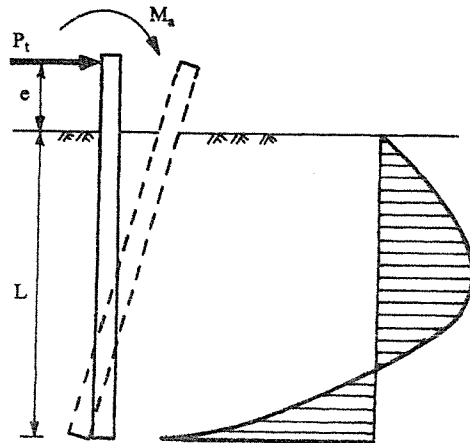


Figure A.6. Assumed distribution of soil resistance for cohesionless soil for a short pile, unrestrained against rotation.

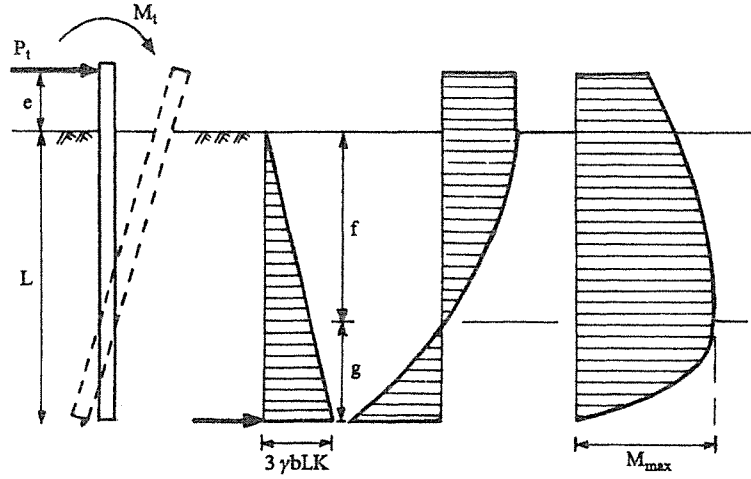


Figure A.7. Diagrams of deflection, soil resistance, shear, and moment for short pile in cohesionless soil, unrestrained against rotation.

$$P_t - (3\gamma bfK_p) \left(\frac{f}{2} \right) = 0 \quad (\text{A.17})$$

Solving Equation A.17 for an expression for f

$$f = 0.816 \left(\frac{P_t}{\gamma b K_p} \right)^{0.5} \quad (\text{A.18})$$

The maximum positive bending moment can then be computed by referring to Figure A.7.

$$M_{\max}^{\text{pos}} = P_t(e + f) - \frac{K_p \gamma b f^3}{2} + M_t$$

Or, by substituting expression for Equation A.17 into the above equation, the following expression is obtained for the maximum moment.

$$M_{\max}^{\text{pos}} = P_t(e + f) - \frac{P_t f}{3} + M_t \quad (\text{A.19})$$

As an example of the use of the equations, the pile used previously is considered. The friction angle of the sand is assumed to be 34 degrees and the unit weight is assumed to be 8.64 kN/m³ (the water table is assumed to be above the ground surface). Assume M_t is equal to zero. Equations A.13 and A.15 yield the following:

$$K_p = \tan^2 \left(45 + \frac{\phi}{2} \right) = 3.54,$$

$$P_{ult} = \frac{(8.64)(0.305)(2.44)^3(3.54)}{(0.61 + 2.44)} = 22.2 \text{ kN}$$

The distance f can be computed by solving Equation A.18.

$$f = 0.816 \left(\frac{22.2}{(8.64)(0.305)(3.54)} \right)^{0.5} = 1.259 \text{ m}$$

The maximum positive bending moment can be found using Equation A.19.

$$M_{max} = (22.2)(0.61 + 1.259) - \frac{(22.2)(1.259)}{3} = 32.2 \text{ m-kN}$$

Assuming no axial load, the maximum bending stress is

$$f_b = \frac{(32.2)(0.1525)}{1.75 \times 10^{-4}} = 28,000 \text{ kPa}$$

The computed maximum stress is undoubtedly tolerable, especially when a factor of safety is used to reduce P_{ult} . Broms presented curves for the solution of the case where a short, unrestrained pile undergoes a soil failure; however, Equations A.14 and A.17 are so elementary that such curves are unnecessary.

A.2.1.2 Long, free-head piles in cohesionless soil

As the pile in cohesionless soil with the unrestrained head becomes longer, failure will occur with the formation of a plastic hinge in the pile at the depth f below the ground surface. It is assumed that the ultimate soil resistance develops from the ground surface to the point of the plastic hinge. Also, the shear is zero at the point of maximum moment. The value of f can be obtained from Equation A.18 shown above. The maximum positive moment can then be computed and Equation A.19 is obtained as before. Assuming that M_t is equal to zero, an expression can be developed for P_{ult} as follows:

$$P_{ult} = \frac{M_y}{e + 0.544 \left[\frac{P_{ult}}{(\gamma b K_p)} \right]^{0.5}} \quad (\text{A.20})$$

For the example problem, Equation A.20 can be solved, as follows:

$$P_{ult} = \frac{430}{0.61 + 0.544 \left[\frac{P_{ult}}{(8.64)(0.305)(3.54)} \right]^{0.5}} = 153 \text{ kN}$$

Broms presented a set of curves for solving the problem of the long pile in cohesionless soils (see Fig. A.8). Entering the curves with a value of $M_y/b^4\gamma K_p$ of 1926, one obtains a value of P_{ult} of about 160 kN. The logarithmic scales are somewhat difficult to read and it may be desirable to make a solution using Equation A.20. Equations A.19 and A.20 must be used in any case if a moment is applied at the top of the pile.

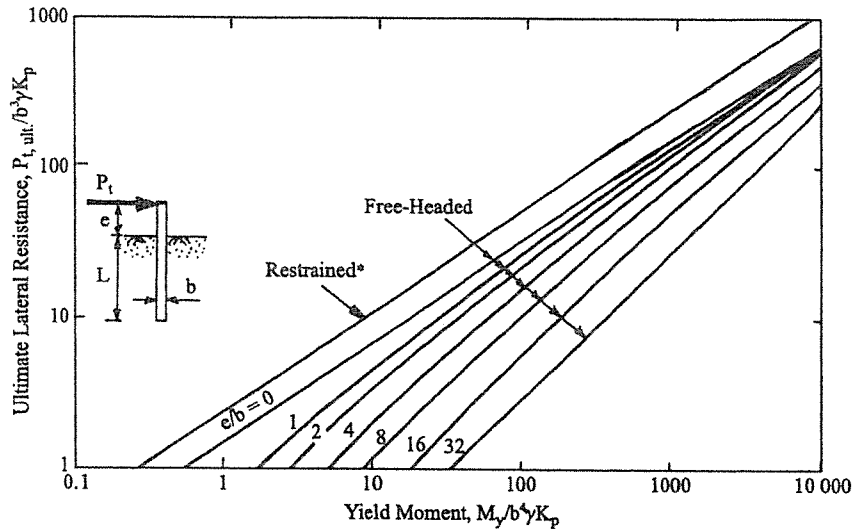


Figure A.8. Curves for design of long piles under lateral load in cohesionless soil.

A.2.1.3 Influence of pile length, free-head piles in cohesionless soil

There may be a need to solve for the pile length where there is a change in behavior from the short-pile case to the long-pile case. As for the case of the pile in cohesive soils, the yield moment may be used with Equations A.14 through A.16 to solve for the critical length of the pile. Alternatively, the short-pile equations would then be compared with the yield moment. If the yield moment is less, the long-pile equations must be used.

For the example problem, the value of P_{ult} of 153 kN is substituted into Equation A.16 and a value of L of 6.0 m is computed. Thus, for the pile that is unrestrained against rotation the value of P_{ult} increases from zero when L is zero to a value of 153 kN when L is 6.0 m. For larger values of L , the value of P_{ult} remains constant at 153 kN.

A.2.1.4 Short, fixed-head piles in cohesionless soil

For a pile that is fixed against rotation at its top, as for cohesive soils, the mode of failure for a pile in cohesionless soil depends on the length of the pile. For a short pile, the mode of failure will be a horizontal movement of the pile through the soil, with the ultimate soil resistance developing over the full length of the pile. The equation for static equilibrium in the horizontal direction leads to a simple expression.

$$P_{ult} = 1.5\gamma L^2 b K_p \quad (A.21)$$

A.2.1.5 Intermediate length, fixed-head piles in cohesionless soil

As the pile becomes longer, an intermediate length is reached such that a plastic hinge develops at the top of the pile. Rotation at the top of the pile will occur, and a point of zero deflection will exist somewhere along the length of the pile. The assumed soil resistance will be the same as shown in Figure A.7. Taking moments about the toe of the pile leads to the following equation for the ultimate load.

$$P_{ult} = \frac{M_y}{L} + 0.5\gamma bL^2 K_p \quad (\text{A.22})$$

Equation A.22 can be solved to obtain P_{ult} for the pile of intermediate length.

A.2.1.6 Long, fixed-head piles in cohesionless soil

As the length of the pile increases more, the mode of behavior will be that of a long pile. A plastic hinge will form at the top of the pile where there is a negative bending moment and at some depth f where there is a positive bending moment. The shear at depth f is zero and the ultimate soil resistance is as shown in Figure A.7. The value of f may be determined from Equation A.18 but that equation is re-numbered and presented here for convenience.

$$f = 0.816 \left(\frac{P_t}{\gamma b K_p} \right)^{0.5} \quad (\text{A.23})$$

Taking moments at point f leads to the following equation for the ultimate lateral load on a long pile that is fixed against rotation at its top.

$$P_{ult} = \frac{M_y^+ + M_y^-}{e + 0.544 \left(\frac{P_{ult}}{\gamma b K_p} \right)^{0.5}} \quad (\text{A.24})$$

Equations A.23 and A.24 can be solved to obtain P_{ult} for the long pile.

A.2.1.7 Influence of pile length, fixed-head piles in cohesionless soil

The example problem will be solved for the pile lengths where the pile goes from one mode of behavior to another. An equation can be written for the case where the yield moment has developed at the top of the short pile. The equation is:

$$P_{ult} = \frac{M_y}{L} + 0.5\gamma bL^2 K_p \quad (\text{A.25})$$

Equations A.22 and A.25 are, of course, identical but the repetition is for clarity. Equations A.21 and A.25 can be solved for P_{ult} and for L , as follows:

$$\text{from Equation A.21, } P_{ult} = 14.0L^2$$

$$\text{from Equation A.25, } P_{ult} = \frac{430}{L} + 15.3L^2$$

$$\text{then } L = 3.59 \text{ m and } P_{ult} = 180 \text{ kN.}$$

For the determination of the length where the behavior changes from that of a pile of intermediate length to that of a long pile, the value of P_{ult} from Equation A.22 may be set equal to that in Equation A.24. It is assumed that the pile has the same yield moment over its entire length in this example.

$$\text{from Equation A.22, } P_{\text{ult}} = \frac{430}{L} + 4.664L^2$$

$$\text{from Equation A.24, } P_{\text{ult}} = \frac{2(430)}{0.61 + 0.544 \left[\frac{P_{\text{ult}}}{(8.64)(0.305)(3.54)} \right]^{0.5}}$$

$$\text{then } L = 6.25 \text{ m, } P_{\text{ult}} = 251 \text{ kN}$$

In summary, for the example problem the value of P_{ult} increases from zero to 180 kN as the length of the pile increases from zero to 3.59 m, 180 kN to 251 kN as the length increased from 3.59 m to 6.25 m, and above 6.25 m the value of P_{ult} remains constant at 251 kN.

In his presentation, Broms showed curves for short piles that were restrained against rotation at their top. Those curves are omitted because the equations for those cases are so easy to solve. Broms' curve for the long pile that is fixed against rotation at its top is retained in Figure A.8 but a note is added to ensure proper use of the curve. For the example problem, a value of 300 kN was obtained for P_{ult} , which agrees poorly with the computed value. The difficulty probably lies in the inability to read the logarithmic scales accurately. No curves are presented for the pile of intermediate length with fixed head.

A.2.2 *Deflection of piles in cohesionless soil*

Broms noted that Terzaghi (1955) has shown that the reaction modulus for a cohesionless soil can be assumed to increase approximately linearly with depth. As noted earlier, and using the formulations of this work, Terzaghi recommends the following equation for the soil modulus.

$$E_{py} = k_{py} z \quad (\text{A.26})$$

Broms suggested that Terzaghi's values can be used only for computing deflections up to the working load and that the measured deflections are usually larger than the computed ones except for piles that are placed with the aid of jetting.

Broms presented equations and curves for use in computing the lateral deflection of a pile; however, the methods presented herein are considered to be appropriate.

A.2.3 *Effects of nature of loading on piles in cohesionless soil*

Broms noted that piles installed in cohesionless soil will experience the majority of the lateral deflection under the initial application of the load. There will be only a small amount of creep under sustained loads.

Repetitive loading and vibration, on the other hand, can cause significant additional deflection, especially if the relative density of the cohesionless soil is low. Broms noted that work of Prakash (1962) shows that the lateral deflection of a pile group in sand increased to twice the initial deflection after 40 cycles of load. The increase in deflection corresponds to a decrease in the soil modulus to one-third its initial value.

For piles subjected to repeated loading, Broms recommended for cohesionless soils of low relative density that the reaction modulus be decreased to $1/4$ its initial value and that the value of the reaction modulus be decreased to $1/2$ its initial value for soils of high relative density. He suggested that these recommendations be used with caution because of the scarcity of experimental data.