# Shell 4. Concrete Shells' Buckling

reference is *Láminas de Hormigón*, Haas, p. 383...

## 0. Important Notes

- It is typical from shells of sound design if something to fail by local buckling failure under a buckling studies here won't prevent local buckling coming from such concentrated local loa wanted per design.
- So inaccessibility of shells susceptible to such buckling damage must be established by d useful life. Maintenance will take care of not disturbing the membranal status to such degr
- All holes made in the membrane by design will be properly rounded by rings which effects c as to prevent buckling failure originating in them.
- Our recommendation is: make a FEM model and subject it to the load case more likely to natural deflections of the load case) likely or worst case imperfections. Check you don't ha effects taken into account.

$\gamma_c := 2400 \cdot \frac{kgf}{m^3}$ concrete specific weight

$f_c := 35 \cdot MPa$ specified strength

$$E_c := 33 \cdot \left( \frac{\gamma_c}{\frac{lbf}{ft^3}} \right)^{1.5} \cdot \left( \frac{f_c}{psi} \right)^{0.5} \cdot psi$$

$$E_{cLT} := \frac{E_c}{5}$$

- You will see we will be using one long term degraded modulus of elasticity to take into accour
- This may be necessary for cases where the stresses are those common in beams and colum compressed circular piles, etc), but might not be needed for all cases where the stresses are time-dependent effects may not be significant.
- Note however that lantern inclusions and other irregularities may cause the referred high stres
- Whenever you have disruptions to the basic geometry of the shell, I would recommend to use

## 1. Folded plates

Assuming long term loading and simply supported conditions at all sides (which is conservati can be assumed to remain constant...

$b := 3 \cdot m$ width of an element component of the folded plate section, assumed uniformly loaded

$t := 12 \cdot cm$ its thickness

$$\frac{b}{t} = 25$$

$$\sigma_{cr} := \frac{\pi^2 \cdot E_{cLT} \cdot t^2}{3 \cdot \left(1 - \nu^2\right) \cdot b^2}$$

$\sigma_{cr} = 32.6\,MPa$ Remain under this value

- Most folded plates won't exceed these dimensions nor materials will be worse or real coercion folded plates have serious buckling problems. We are in the 1/25 slenderness range.
- Note however that box piles are some kind of folded plates and for them no more slenderness
- We could roughly say that current recommendations for both steel and concrete are b/t=30 fo

## 2. Cylindrical shells / Haas

From the practical approach in p. 391...

$r := 15 \cdot m$ radius of the cylindrical shell

$t := 12 \cdot cm$ thickness of the shell

$b := 3 \cdot m$ arc length (measured on it) (within transversal section, uniformly loaded)

$$K := \frac{b^2}{r \cdot t} \qquad K = 5 \qquad \frac{4 \cdot \pi^2}{\sqrt{12 \cdot \left(1 - \nu^2\right)}} = 11.63$$

$$K \geq \frac{4 \cdot \pi^2}{\sqrt{12 \cdot \left(1 - \nu^2\right)}} = 0$$

must be 1 for OK, and then...

$$\sigma_{cr\_Haas} := \frac{E_{cLT} \cdot t^2}{b^2} \cdot \frac{K}{\sqrt{3\left(1 - \nu^2\right)}}$$

$\sigma_{cr\_Haas} = 28.03\,MPa$

having curvature enhances very much the buckling strength.

We can make better and automatize the search for the lesser buckling strength of a uniformly compressed pipe by the above formulations. Will accept t, r dimensions can remain constant

$$b := 3 \cdot m$$

$$\sigma_{cr}(b) := \frac{E_{cLT} \cdot t^2}{b^2} \cdot \frac{\frac{b^2}{r \cdot t}}{\sqrt{3\left(1 - \nu^2\right)}}$$

Given

$b \geq 0 \cdot m \qquad b \leq \pi \cdot r$

$b := \text{Minimize}\left(\sigma_{cr}, b\right) \qquad b = 300\,cm$

$\sigma_{cr}(b) = 28.03\,MPa$

not surprisingly equal to the already obtained result since the formulation for critical stress is independent of b, as can be seen once K is expressed explicitly

So change the blue input to guess acceptable proportions of compressed pipes or cylindrical shells from the buckling standpoint

See below other evaluations

## 3. Spherical domes

$t := 12 \cdot cm$ thickness

$r := 15 \cdot m$ radius

$r := 15 \cdot m$ radius

$\alpha := 0.15$ buckling parameter, about 0.58/4 of p. 395 or select from p. 399. It seems 0.

0.06 minimum, 0.16 typical, and never more than 0.32

$$\sigma_{cr} := \frac{\alpha}{2} \cdot E_{cLT} \cdot \frac{t}{r} \qquad \sigma_{cr} = 3.57\,\text{MPa}$$

Remain under this value at the fact

Note this formulation of the buckling risk seems not consistent with that of cylindrical shells, w
even the differences of loading (radial pressure on the shell for the dome and longitudinal comp

The above formulation in terms of supported loas is

$$q_{cr} := \alpha \cdot E_{cLT} \cdot \frac{t^2}{r^2} \qquad q_{cr} = 5820.63 \frac{\text{kgf}}{\text{m}^2}$$

Remain under this value at the fact

# 4. Double curvature shells

I think referred to both principal stresses compressive, hence mostly for sinclastic shells.
Otherwise the tension if kept in place would be increasing buckling strength.

$t := 12 \cdot cm$ thickness

$r_1 := 15 \cdot m$ radius 1

$r_2 := 5 \cdot m$ radius 2

$\alpha := 0.15$ buckling parameter, about 0.58/4 of p. 395 or select from p. 399. It seems (

0.06 minimum, 0.16 typical, and never more than 0.32

In this case the formulation (p. 397) is given in critical load terms...

$q_{cr} := \alpha \cdot E_{cLT} \cdot \frac{}{r_1 \cdot r_2}$ $q_{cr} = 17461.9 \frac{}{m^2}$ Remain under this value at the fa

This formulation is subject to the same critic than the spherical dome above; furthermore, it p radiuses is bigger than the other, which seems contrary to experience; one shell of one radius before the one having the bigger radius in place, which seems unrealistic.

## 2.b Cylindrical shells' buckling strength / Spampinato

reference is *Teoría y Cálculo de las Bóvedas Cáscaras Cilíndricas*, Agripino R. Spampinato p.

These refer to half cylinder roofs of eliptical section acting as beams, and the compression is a Spampinato values seem more reasonable than those of Haas for these shells in reason of the

for a elliptical section of a cylindrical shell of half horizontal breadth of ellipse a an half vertical

$a := 10 \cdot m$ $b := 6.5 \cdot m$ $r := \frac{a^2}{b}$ $r = 15.38\,m$

$t := 12 \cdot cm$

## Long cylindrical shells

$$\sigma_{xk} := \frac{80 \cdot \frac{kgf}{cm^2}}{1 + \frac{r}{200 \cdot t}}$$

$\sigma_{xk} = 4.78\,MPa$ this is the allowable (service level

$\frac{240}{80} \cdot \sigma_{xk} \cdot \sqrt{\frac{f_c}{24 \cdot MPa}} = 17.32\,MPa$ maybe the true bucklin

Remain under this value at the factored lo

## Short cylindrical shells

$L := 20 \cdot m$ rest as above

$$\sigma_{xk} := \frac{80 \cdot \frac{kgf}{cm^2}}{}$$

$\sigma_{xk} = 2.89\,MPa$ this is the allowable (service lev

$$1 + \frac{L}{1100 \cdot t} \cdot \sqrt{\frac{r}{t}}$$

$\sigma_{xk} = 2.89\,MPa$ this is the allowable (service lev

$$\frac{240}{80} \cdot \sigma_{xk} \cdot \sqrt{\frac{f_c}{24 \cdot MPa}} = 10.47\,MPa$$

maybe the true buckl

Remain under this value at the factore

## 2.c Cylindrical shells' buckling strength / Roark

reference is *Roark's Formulas for Stress and Strain*, Warren. C. Young p. 689.

Assuming a fully compressed cylinder to represent the case...

$r := 15 \cdot m$

$t := 12 \cdot cm$

$$\sigma_{cr\_Roark} := 0.3 \cdot E_{cLT} \cdot \frac{t}{r}$$

$\sigma_{cr\_Roark} = 14.27\,MPa$

Remain under t

## 2.d Cylindrical shells' buckling strength / Ugural

reference is *Stresses in Plates and Shells*, Ansel C.Ugural p. 467 and p. 471

Assuming a fully compressed cylinder to represent the case...

$r := 15 \cdot m$

$t := 12 \cdot cm$

## Symmetrical Buckling

$$\sigma_{cr} := 0.605 \cdot E_{cLT} \cdot \frac{t}{r}$$

$\sigma_{cr} = 28.78\,MPa$

## Unsymmetrical Buckling

more likely

$$\psi := \frac{1}{16}\cdot\sqrt{\frac{r}{t}} \qquad K := \left[1 - 0.901\cdot\left(1 - e^{-\psi}\right)\right] \qquad K = 0.55$$

$$\sigma_{cr\_Ugural} := 0.605\cdot K\cdot E_{cLT}\cdot\frac{t}{r} \qquad \sigma_{cr\_Ugural} = 15.74\,MPa$$

Remain under tl

## 2e. Cylinder shells' buckling / Pilkey

in p. 262

$L := 20\cdot m$ $a := 6\cdot m$ $b := 3\cdot m$

$$Z := \frac{L^2}{r\cdot t}\cdot\sqrt{1 - \nu^2} \qquad Z = 217.73$$

must be bigger than 50, so OK

$$\eta := \frac{r}{t} \qquad K_c := 0.22195 + \frac{29.7611}{\eta} - \frac{2322.08667}{\eta^2} + \frac{65832.1484}{\eta^3}$$

$$\sigma_{cr\_Pilkey} := K_c\cdot E_{cLT}\cdot\frac{t}{r} \qquad \sigma_{cr\_Pilkey} = 16.42\,MPa$$

Remain under this value

## 2f. Long cylinder shells' buckling. Discussion

Mean Buckling strength:

$$\sigma_{cr\_cyl} := \frac{\sigma_{cr\_Spampinato} + \sigma_{cr\_Ugural} + \sigma_{cr\_Roark}}{4}$$

$$\sigma_{cr\_cyl} = 15.85\,MPa$$

Remain under this value at

- The preceding evaluation of critical pseudoelastic buckling strength have detected (Ugural) The lower critical strength corresponds to the unsymmetrical mode, and the bigger to the s about twice the unsymmetrical strength, and seems to be the value reported by Haas. Sinc symmetrical failure remains possible) it seems the Haas/symmetrical buckling strength mu
- The Spampinato values are based in concretes of 24 MPa strength, so the buckling streng
- The predicted value more accurate per the equal weight vote is that of Ugural, which is als

ı concentrated load and maybe time dependent effects. The
ıding, causing excess of flexure where a membranal status is

esign and warranted by owner's agents inspection during
ee as to cause local or general buckling failure.
on the shell will have been taken into account (long term) and

produce buckling. Include in it concomitant (augmenting the
ve a inestable model by running the model with second order

$\nu := 0.20$ coeff. of Poisson of concrete

Long Term modulus of deformation

ıt time dependent rheological effects.
s (shells like folded plates, flexural cylindrical beams,
so low (many domes and shells) that the additional

ısses and ignite the buckling failure, long term.
e the long term degraded modulus of deformation.

ve) and considering some (short) length at which stresses

: at the factored loads level

ıs more exacting than considered, so it is unlikely that

; than b/t=15 is currently being recommended.
r static loads and b/t=15 or lower for dynamic.

30 could also be accepted.

:ored loads level

/hich portrait as unsecure when compared to spherical domes,
>ression for the cylinder) are accounted for.

:ored loads level

).30 could also be accepted.

ctored loads level

outs the critical (but buckling) load higher when one of the
s equal to the lower in that of two radiuses would be tearing

258...

along the length of the cylinder.
e given critical stresses values for spherical domes

breadth b is

) buckling strength for the LONG cylindrical shell

g load

$$\sigma_{cr\_Spampinato} := \frac{240}{80} \cdot \sigma_{xk} \cdot \sqrt{\frac{f_c}{25 \cdot MPa}}$$

ads level

al) buckling strength for the SHORT cylindrical shell el) buckling strength for the SHORT cylindrical shell

ing load

ed loads level

his value at the factored loads level

his value at the factored loads level

e at the factored loads level

$+ \sigma_{cr\_Pilkey}$

: the factored loads level

) a symmetrical and an unsymmetrical buckling failure modes.
ymmetrical mode. The symmetrical failure mode happens at
:e the unsymmetrical value must control design (even if a
ıst be discarded.
ȷth is corrected on modulus of deformation for consistency.
o the more modern text.