```
STUDENT > #  File CT_ResponseR2.mws
STUDENT >
STUDENT > # =-================  OBJECTIVE ===============
STUDENT > #  Objective: to analyse effects of CT (or clamp-on)
          effects
STUDENT > #    when measuring motor starting current
STUDENT >
STUDENT >
STUDENT > #  ========== OVERVIEW / CONCLUSIONS =========
STUDENT > # Overview/Conclusions:
STUDENT > #  A simple model for CT (and clamp probe) dynamic
          response
          #        is derived (under assumptions)
STUDENT > #  The model predicts step response has a step followed by
STUDENT > #        an exponential decay decay
STUDENT > #  *  This form of step response was confirmed by
          measurement
STUDENT > #         of clamp-on response to actual DC step current
          input
STUDENT > #  ** Note that any LTI system can be characterized
          completely
          #        by either it's impulse response or it's step
          response.
STUDENT > #    =>  Even if the model assumptions are not perfect,
          the
STUDENT > #         conclusions are correct if step response
          matches
          #            and system is LTI
STUDENT > #  Under this model, CT (and clamp probe) behavior can be

          #                characterized by time constant Tau_meas
STUDENT > #  As long as Tau_meas >> Tau_motor, output is ~ accurate
STUDENT > #  where Tau_motor represents motor and system
          characteristics
STUDENT >
STUDENT > #  When Tau_meas gets down close to Tau_mot,
STUDENT > #    then we expect to see oscillations in INDICATED
          current
STUDENT > #  These oscillations match qualitatively what has been
          observed
          #            in practice
STUDENT >
STUDENT > # ====================  MODEL ================
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STUDENT > # CT or clamp-on is modeled at 1:1 ct
STUDENT > # Input current is i1(t)
STUDENT > # Output current is i2(t)
STUDENT > # Output connected impedance is represented as reistance R
          *
STUDENT > #    (ASSUME purely resistive burden)
STUDENT > #  (*Note for clamp-on there may be output conversion
          circuits
STUDENT > #    and this R does not necessarily represent an actual
          measuring resitance
STUDENT > #      external to the clamp-on probe
STUDENT > # Non-ideality of CT is represented by a magnetizing
          branch with inductance L
STUDENT > # Current in magnetizing branch is im(t)
STUDENT > #  KCL:  i1(t) = i2(t) + im(2)
STUDENT >
STUDENT > # Tau_meas = L/R
STUDENT >
STUDENT > #=============== APPROACH =================
STUDENT > #  Develop Laplace transform transfer function H(s) =
          I2(s)/I1(s)
STUDENT > #  Use that to calculate response to worst case offset
          motor starting waveform
STUDENT >
STUDENT > # ================ Parameters / Values  ================
STUDENT > #  Tau_motor will be assumed 20 msec.  This is value
          estimated from a motor starting trace
STUDENT > #   It corresponds to a T1/2 = 14 msec and p.f. = 0.13
STUDENT > #   Note Tau_motor may incorporate some effects of the
          power supply impedance
STUDENT > #     although power supply does not play as important a
          role in motor starting time constant
STUDENT > #        as it does in fault analysis time constant
STUDENT >
STUDENT > #  Tau_meas will be varied over a range
STUDENT > #     Based on measurement of dc step input to an ac
          clamp-on probe,...
STUDENT > #        actual T_meas is believed to be 110 msec (T1/2 =
          80 msec)
STUDENT > # === Initialization =======
STUDENT > restart;
STUDENT > with(inttrans);
```

[*addtable*, *fourier*, *fouriercos*, *fouriersin*, *hankel*, *hilbert*, *invfourier*, *invhilbert*, *invlaplace*,

$laplace, mellin]$

```
STUDENT >
STUDENT > #  ====== Calculations =========
STUDENT > #  Ohms law applied at CT output terminal:
STUDENT > eq1:=v(t)=R*i2(t);
```

$$eq1 := \mathrm{v}(t) = R\,\mathrm{i2}(t)$$

```
STUDENT >
STUDENT > #  Terminal Relations applied to magneitizing branch
STUDENT > eq2:=v(t)=L*diff(im(t),t);
```

$$eq2 := \mathrm{v}(t) = L\left(\frac{\partial}{\partial t}\mathrm{im}(t)\right)$$

```
STUDENT >
STUDENT > # Simplify by eliminating im(t) using KCL:
STUDENT > eq2:=subs(im(t)=i1(t)-i2(t),eq2);
```

$$eq2 := \mathrm{v}(t) = L\left(\frac{\partial}{\partial t}(\mathrm{i1}(t) - \mathrm{i2}(t))\right)$$

```
STUDENT > # Combine the two equations:
STUDENT > eq2:=subs(eq1,eq2);
```

$$eq2 := R\,\mathrm{i2}(t) = L\left(\left(\frac{\partial}{\partial t}\mathrm{i1}(t)\right) - \left(\frac{\partial}{\partial t}\mathrm{i2}(t)\right)\right)$$

```
STUDENT > # Take the Laplace Transform:
STUDENT > eq2s:=laplace(eq2,t,s);
```

$$eq2s := R\,\mathrm{laplace}(\mathrm{i2}(t), t, s) = L\,(s\,\mathrm{laplace}(\mathrm{i1}(t), t, s) - \mathrm{i1}(0) - s\,\mathrm{laplace}(\mathrm{i2}(t), t, s) + \mathrm{i2}(0))$$

```
STUDENT > I2solution(s):=solve(eq2s,laplace(i2(t),t,s));
```

$$\mathrm{I2solution}(s) := \frac{L\,(s\,\mathrm{laplace}(\mathrm{i1}(t), t, s) - \mathrm{i1}(0) + \mathrm{i2}(0))}{R + L\,s}$$

```
STUDENT > # Assume currents are at 0 initially for any system to be
          analysed:
STUDENT > I2solution(s):=subs({i1(0)=0,i2(0)=0},I2solution(s));
```

$$\mathrm{I2solution}(s) := \frac{L\,s\,\mathrm{laplace}(\mathrm{i1}(t), t, s)}{R + L\,s}$$

```
STUDENT > H(s):=I2solution(s)/laplace(i1(t),t,s);
```

$$\mathrm{H}(s) := \frac{L\,s}{R + L\,s}$$

```
STUDENT > # Transfer fucnttio Has a zero at s=0 and a pole at s =
          -R/L = -1/Tau_meas
STUDENT >
STUDENT > # Rewrite by substituting Tau_meas = L/R:
STUDENT > H(s):=s/(1/Tau_meas+s);
```

$$H(s) := \frac{s}{\frac{1}{Tau\_meas} + s}$$

STUDENT > # Let StepInput(t) be DC step input

STUDENT > StepInput(t):=Heaviside(t);

$$StepInput(t) := Heaviside(t)$$

STUDENT > StepInput(s):=laplace(StepInput(t),t,s);

$$StepInput(s) := \frac{1}{s}$$

STUDENT > StepOutput(t):=invlaplace(StepInput(s)*H(s),s,t)*Heaviside(t);

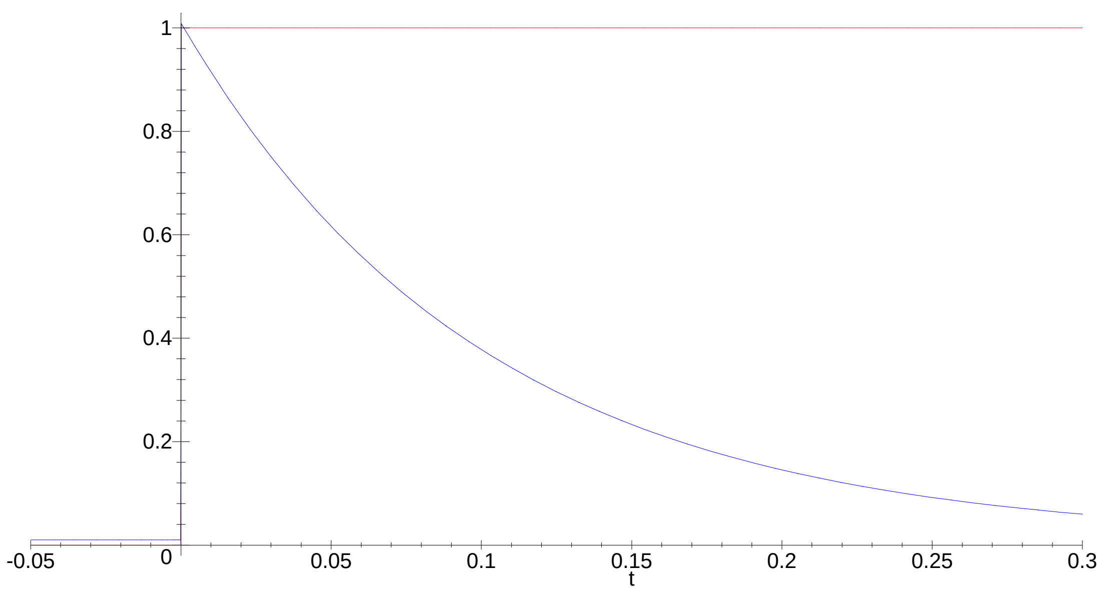
$$StepOutput(t) := \mathbf{e}^{\left(-\frac{t}{Tau\_meas}\right)} Heaviside(t)$$

STUDENT > plotassistmag := 0.01; # offset plot slightly for ease of viewing

$$plotassistmag := .01$$

STUDENT > plot([StepInput(t),subs({Tau_meas=0.1},StepOutput(t))+plotassistmag],t=-0.05..0.3,thickness=[4,4],color=[red,blue],title='StepInputRedStepOutputBlue');

StepInputRedStepOutputBlue



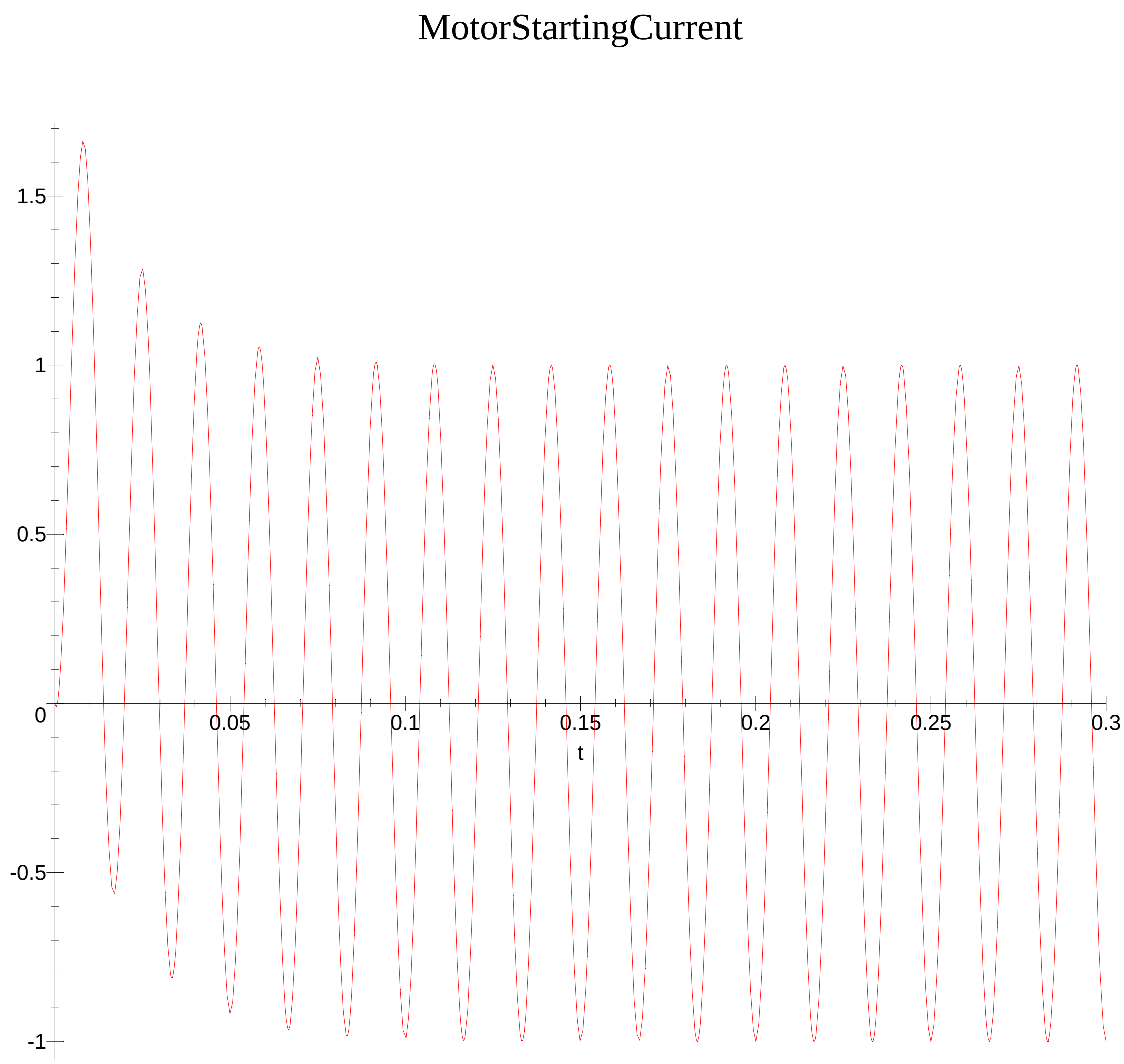
STUDENT >

STUDENT > # Developo an expressio for MotorStarting waveform assuming close at worst voltage

STUDENT > MotorStart(t):=(exp(-t/Tau_mot)-cos(w*t))*Heaviside(t);

$$MotorStart(t) := \left(\mathbf{e}^{\left(-\frac{t}{Tau\_mot}\right)} - \cos(w\,t)\right) Heaviside(t)$$

```
STUDENT > plot(subs({Tau_mot=0.02,w=377},MotorStart(t)),t=0..0.3,title='MotorStartingCurrent');
```




```
STUDENT > MotorStart(s):=laplace(MotorStart(t),t,s);
```

$$\mathrm{MotorStart}(s) := \frac{1}{s + \frac{1}{Tau\_mot}} - \frac{s}{s^2 + w^2}$$

```
STUDENT > MotorStartOut(t):=invlaplace(MotorStart(s)*H(s),s,t)*Heaviside(t);
```

$$\mathrm{MotorStartOut}(t) := \left(- \frac{Tau\_meas\ \mathbf{e}^{\left(-\frac{t}{Tau\_mot}\right)}}{Tau\_mot - Tau\_meas} - \frac{w^2\ Tau\_meas^2 \cos(w\,t)}{1 + w^2\ Tau\_meas^2} + \frac{w\ Tau\_meas \sin(w\,t)}{1 + w^2\ Tau\_meas^2} \right.$$

$$+\frac{Tau\_meas\,\mathbf{e}^{\left(-\frac{t}{Tau\_meas}\right)}}{(1+w^2\,Tau\_meas^2)\,(Tau\_mot-Tau\_meas)}+\frac{Tau\_meas^2\,\mathbf{e}^{\left(-\frac{t}{Tau\_meas}\right)}\,w^2\,Tau\_mot}{(1+w^2\,Tau\_meas^2)\,(Tau\_mot-Tau\_meas)}\Bigg)$$

Heaviside(t)

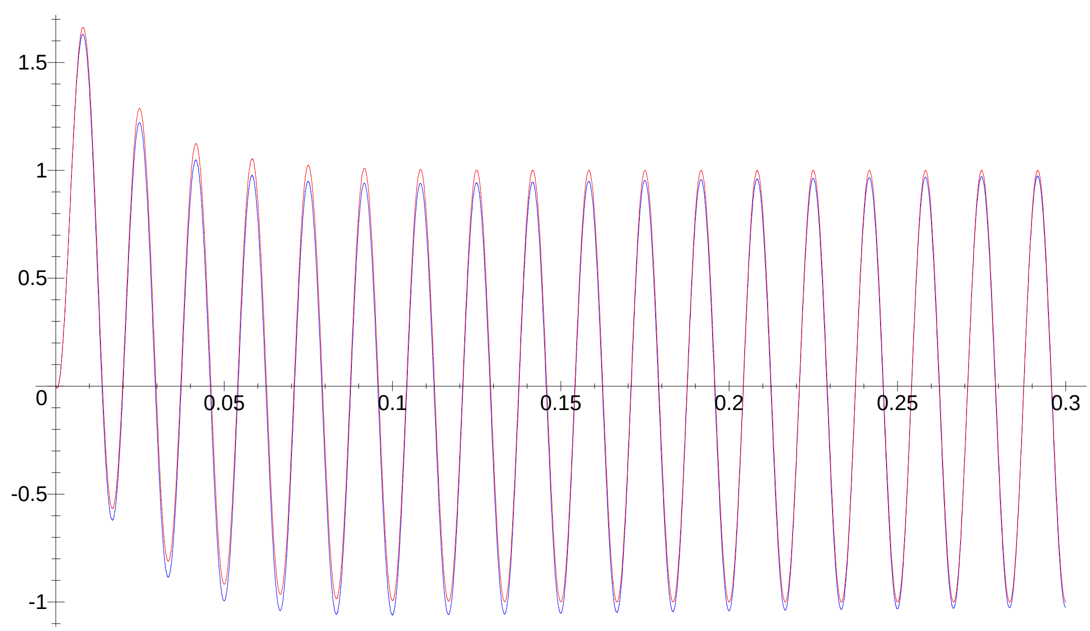
```
STUDENT > MotorStartOut(t):=simplify(MotorStartOut(t));
```

$$\mathrm{MotorStartOut}(t) := -\,Tau\_meas\left(\mathbf{e}^{\left(-\frac{t}{Tau\_mot}\right)}+\mathbf{e}^{\left(-\frac{t}{Tau\_mot}\right)}\,w^2\,Tau\_meas^2\right.$$
$$+\,w^2\,Tau\_meas\cos(w\,t)\,Tau\_mot-w^2\,Tau\_meas^2\cos(w\,t)-w\sin(w\,t)\,Tau\_mot$$
$$\left.+\,w\,Tau\_meas\sin(w\,t)-\mathbf{e}^{\left(-\frac{t}{Tau\_meas}\right)}-Tau\_meas\,\mathbf{e}^{\left(-\frac{t}{Tau\_meas}\right)}\,w^2\,Tau\_mot\right)\mathrm{Heaviside}(t)\Big/\big($$
$$Tau\_mot-Tau\_meas+w^2\,Tau\_meas^2\,Tau\_mot-w^2\,Tau\_meas^3\big)$$

```
STUDENT >
STUDENT > # ========= Plot Actual and Indicated Current for Range of
          Tau_meas =========
STUDENT > # Start with measurement time constant 10 times motor time
          constant
STUDENT > system1:=[Tau_meas=0.2,Tau_mot=0.02,w=377];
STUDENT > plot([subs(system1,MotorStartOut(t)),subs(system1,MotorSta
          rt(t))],t=0..0.3,numpoints=2000,color=[blue,red],title='Re
          dActualBlueIndicated');          system1;
```

$$system1 := [\,Tau\_meas = .2,\ Tau\_mot = .02,\ w = 377\,]$$

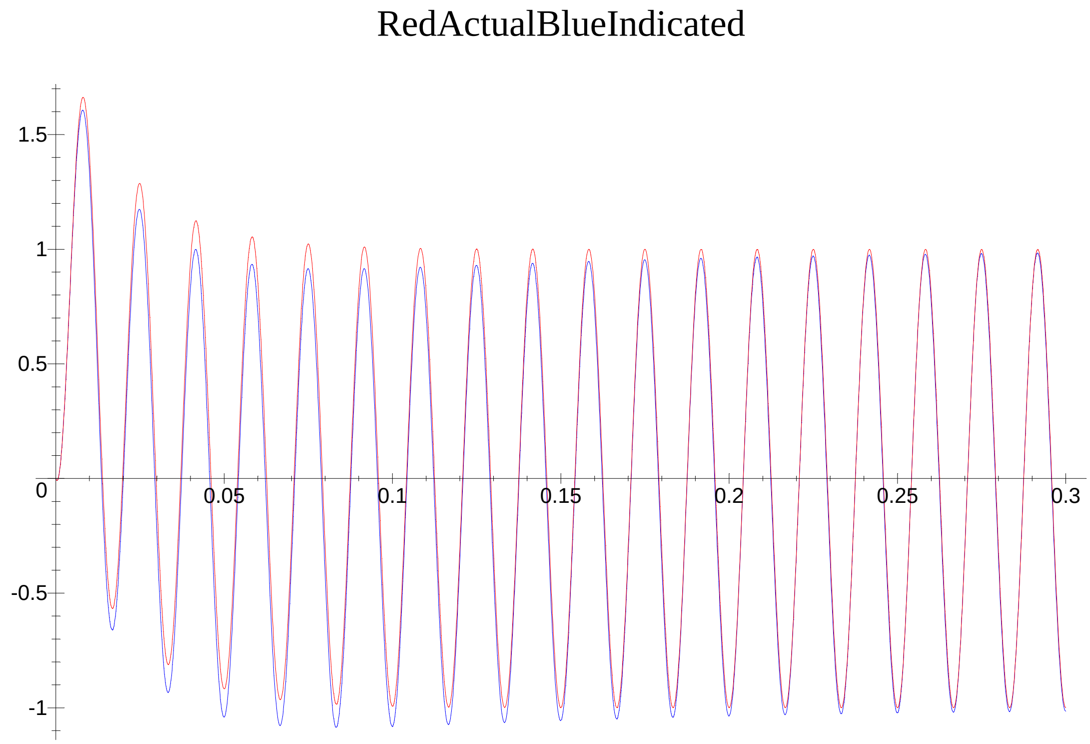
RedActualBlueIndicated



$$[\,Tau\_meas = .2,\ Tau\_mot = .02,\ w = 377\,]$$

```
STUDENT > # Try best guess measurement constant based on observed
          response of small probe to dc step
STUDENT > system1:=[Tau_meas=0.11,Tau_mot=0.02,w=377];
STUDENT > plot([subs(system1,MotorStartOut(t)),subs(system1,MotorSta
          rt(t))],t=0..0.3,numpoints=2000,color=[blue,red],title='Re
          dActualBlueIndicated');        system1;
STUDENT >
```

$$system1 := [Tau\_meas = .11, Tau\_mot = .02, w = 377]$$



$$[Tau\_meas = .11, Tau\_mot = .02, w = 377]$$

```
STUDENT >
STUDENT >
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STUDENT >
STUDENT > # Try just above motor time constant
STUDENT >
STUDENT > system1:=[Tau_meas=0.025,Tau_mot=0.02,w=377];
STUDENT > plot([subs(system1,MotorStartOut(t)),subs(system1,MotorSta
          rt(t))],t=0..0.3,numpoints=2000,color=[blue,red],title='Re
          dActualBlueIndicated');        system1;
```
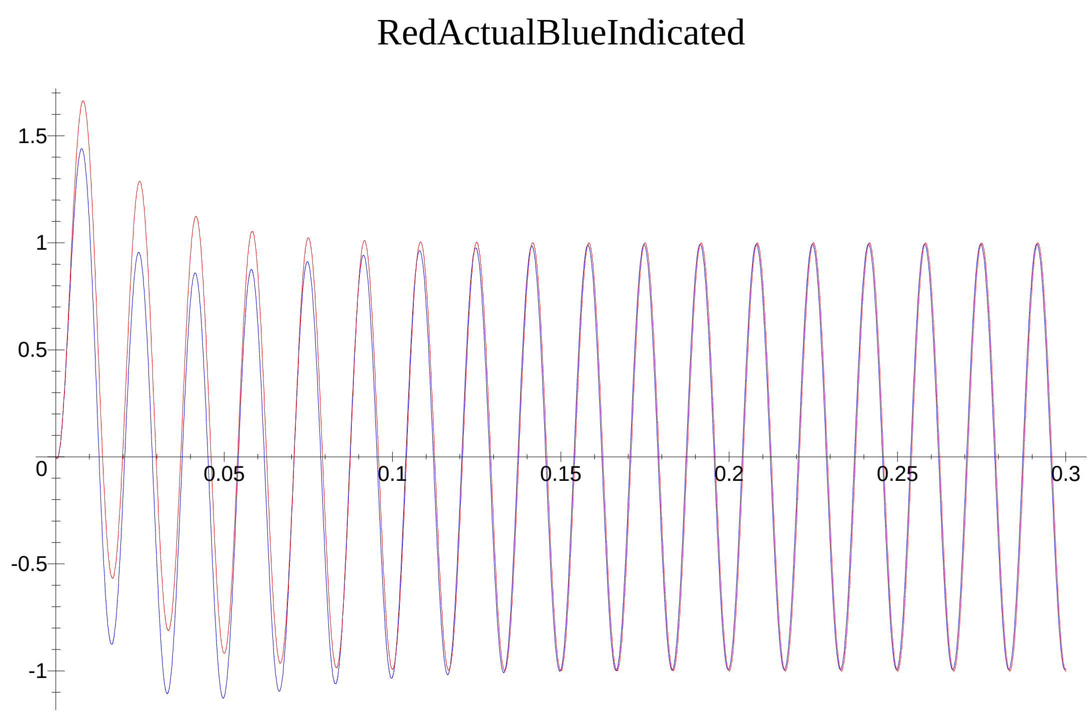
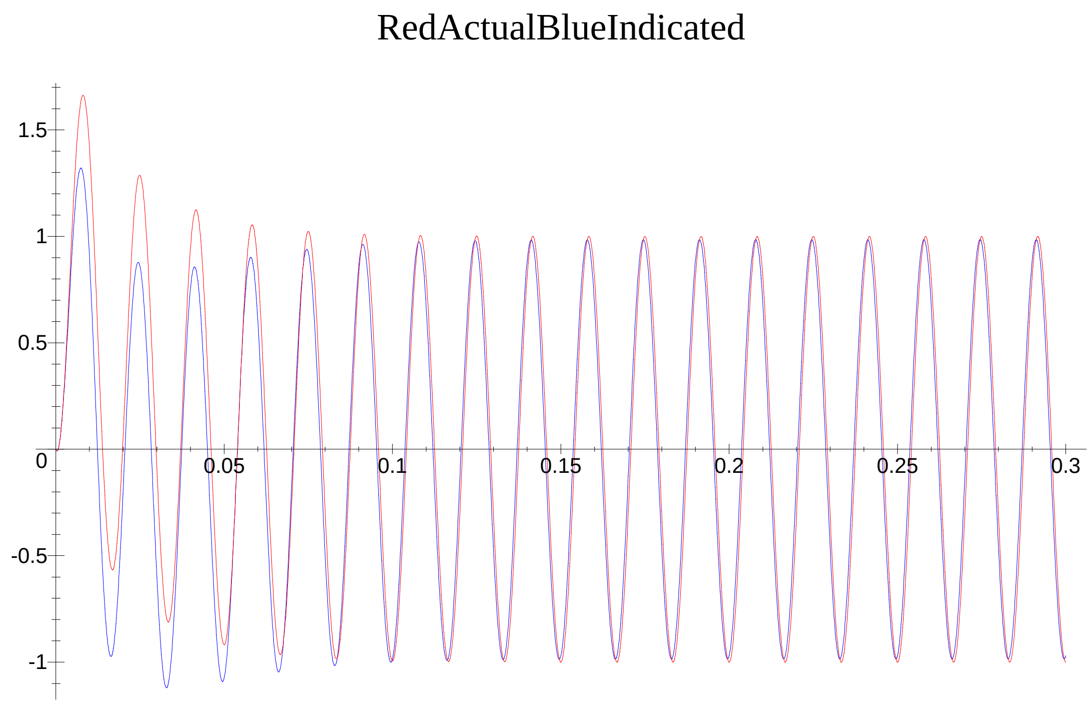

$$system1 := [Tau\_meas = .025, Tau\_mot = .02, w = 377]$$



$[Tau\_meas = .025, Tau\_mot = .02, w = 377]$

```
STUDENT >
STUDENT > # try just below motor time constant
STUDENT > system1:=[Tau_meas=0.015,Tau_mot=0.02,w=377];
STUDENT > plot([subs(system1,MotorStartOut(t)),subs(system1,MotorSta
          rt(t))],t=0..0.3,numpoints=2000,color=[blue,red],title='Re
          dActualBlueIndicated');          system1;
```

$system1 := [Tau\_meas = .015, Tau\_mot = .02, w = 377]$

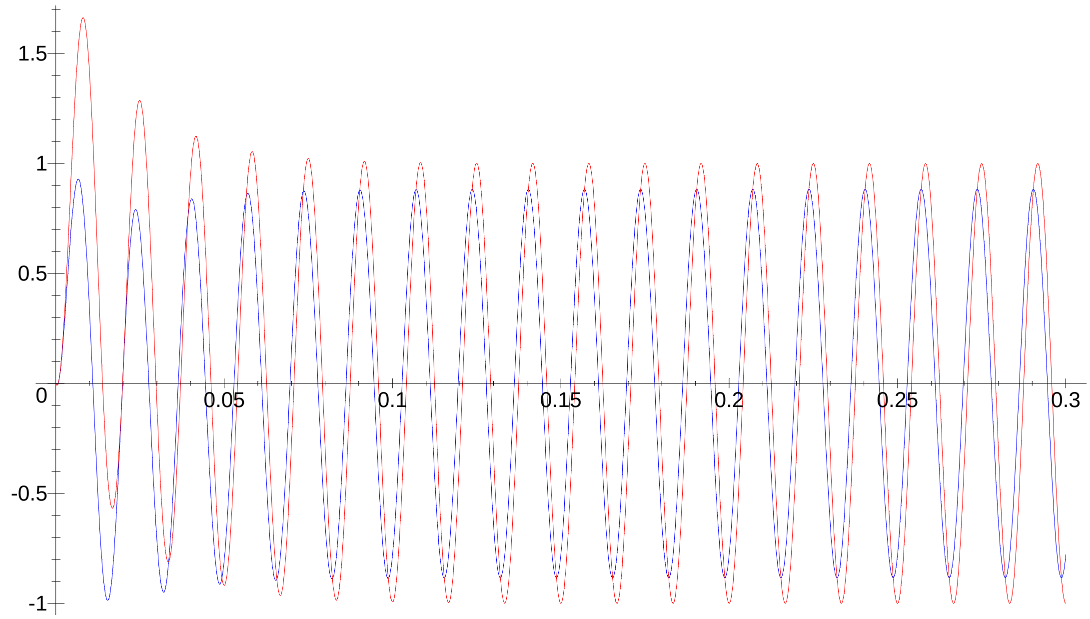


$[Tau\_meas = .015, Tau\_mot = .02, w = 377]$

```
STUDENT > # try 4x lower (slower) than motor time constant
STUDENT > system1:=[Tau_meas=0.005,Tau_mot=0.02,w=377];
STUDENT > plot([subs(system1,MotorStartOut(t)),subs(system1,MotorStart(t))],t=0..0.3,numpoints=2000,color=[blue,red],title='RedActualBlueIndicated');          system1;
```

$system1 := [Tau\_meas = .005, Tau\_mot = .02, w = 377]$

RedActualBlueIndicated



[$Tau\_meas = .005$, $Tau\_mot = .02$, $w = 377$]

STUDENT >

STUDENT > # We approach ideal response at very high Tau_meas (slow decay)

STUDENT > # At the very low Tau_meas = 5 msec, the probe dynamic behavior starts to affect the SS amplitude

STUDENT > # => minimum frequency range listed on probe spec sheet may provide some indirect indication of the Tau_meas

STUDENT > # Lower sinusoidal frequencies demand higher tau (slower decay) for accurate amplitude

STUDENT >

STUDENT >