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On the Strength of Cast Iron Columns

Research Report No R829

By

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Kim JR Rasmussen MScEng PhD

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Abstract:

The aim of the report is to show that modern methods of verification of steel structures can be used for the structural appraisal of old cast-iron columns. The proposed method of verification takes account of the geometrical imperfections which are typical for cast-iron columns and of the particular behaviour of the material which is distinct from modern structural steel in that the tension strength is significant lower than the compression strength, the stress-strain curve is non-linear rather than bi-linear and the Young's modulus is about 2/5 the value for modern steels. The report proposes equations for calculating the strength of cast iron columns by checking failure by yielding in compression and fracture in tension. The proposed strength equations are compared with experimental results obtained by Hodgkinson (1840) and Tetmayer (1901).

Keywords:

Cast iron, columns, buckling, historical buildings, design, imperfections, nonlinear material

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1 Introduction

Cast iron beams and columns were used for textile mills and warehouses from about 1800, and from about 1820 for other buildings. Cast iron columns (mainly hollow) continued to be used until 1900, and even later, but from the 1880s they were progressively superseded by wrought steel columns (Bussell 1997).

The current tendency of public authorities to preserve as much of our architectural heritage as possible requires the assessment of structural members before refurbishment. Many of the historical buildings currently being refurbished were constructed between 1840 and 1940. In many cases, they therefore feature cast iron columns as part of the load-carrying structure. As part of the refurbishment process, it is usually required to assess the load-carrying capacity of the structure, notably that of the columns. For cast iron columns, this assessment is difficult because our knowledge about the material and the fabrication techniques is rather fragmental, although the material properties are known to differ significantly from those of modern structural steels. In addition, the behaviour of old cast iron columns is sensitive to geometrical and structural imperfections. Several articles (Frey and Kapplein 1993; Blanchard et al. 1997; Kapplein 1997) have recently been published on the appraisal of historical buildings featuring cast iron, and methods of estimating the strengths of columns and beams have been presented.

The strength of cast iron columns cannot be estimated on the basis of current design guidance for steel structures, such as Eurocode3, Part 1.1 (1992). The main reason is that the stress-strain curve is nonlinear which leads to a reduced buckling strength when the material gradually looses stiffness. This phenomenon is well known from the design of aluminium and stainless steel structures and is accounted for in structural standards for these materials. However, it is possible to obtain a direct relationship between the degree of material softening and the column strength (Rasmussen and Rondal 1997). In obtaining this relationship, the stress-strain curve is assumed to be approximated by a Ramberg-Osgood curve (Ramberg and Osgood 1943),

$$\varepsilon = \frac{\sigma}{E_0} + 0.002\left(\frac{\sigma}{\sigma_{0.2}}\right)^n \tag{1}$$

and hence, the Ramberg-Osgood parameters (E_0, $\sigma_{0.2}$, n) are assumed to be known. Based on a Perry curve, the nondimensional column strength can then be expressed in terms of n and $e=\sigma_{0.2}/E_0$ (Rasmussen and Rondal 1997). This approach is used in the present report to establish design strength equations for cast iron steel columns. An additional equation is included to check for failure by tension fracture.

2 Material behaviour

Structural cast iron as produced in the 19th century basically consisted of iron and carbon with the carbon content varying between 2% and 5%. As a result of the high carbon content, high compressive strengths were achieved. However, the high carbon content also produced graphite which acted as voids or cracks within the iron matrix and led to a much reduced tensile strength.

The cast iron used in the 19th century was primary "grey cast iron" characterised by a high carbon content and essentially no other alloying elements. Several other types of cast iron exist (*eg* see Angus (1976)) and today's cast iron types contain several alloying elements, notably silicon and magnesium, to enhance their ductility and tensile properties.

The stress-strain diagram of cast iron as produced in the 19th century shows a continuous hardening and a very different response in compression and tension. In fact, cast iron can be considered as ductile in compression but brittle in tension. Figure 1 gives two examples of the initial parts of the stress-strain curves for cast iron under tension and compression, as obtained from Blanchard et al. (1997).

The design procedures prevailing between 1840 and 1930 determined the column strength in terms of the ultimate compressive strength of the material (f_{uc}) and the column slenderness (L/r), where the ultimate strength was obtained as the crushing strength of a small block of material. The concept of a yield stress did not exist in the design of cast iron steel columns. Accordingly, values are readily available for the ultimate compressive strength and the initial elastic modulus (E_0), as summarised in Table 1. The values shown in Table 1 were arrived at on the basis of many tests performed by various authors. However, actual stress-strain diagrams do not appear to be readily available. For instance, Salmon's text summarising research on columns up to the 1920s (Salmon 1921), contains no stress-strain curves. The diagrams shown in recent papers (Blanchard et al. 1997; Kapplein 1997) appear to have been obtained from recent tests of coupons cut from old columns.

Current design standards for aluminium and stainless steel columns operate with an equivalent yield stress, chosen as the 0.2% proof stress ($\sigma_{0.2}$). In order to apply modern design procedures to cast iron steel columns, it is therefore necessary to determine a nominal value of the 0.2% proof stress. Kapplein (1997) suggested that at the elastic strain ($\varepsilon_{euc}=\sigma_{uc}/E_0$) corresponding to the ultimate compressive strength (σ_{uc}), the stress is approximately half the ultimate compressive strength,

$$\sigma(\varepsilon_{euc}) = \frac{\sigma_{uc}}{2} \tag{2}$$

as shown in Fig. 2. Combining eqns (1,2), the proof stress can be achieved as,

$$\sigma_{0.2} = \frac{\sigma_{uc}}{2\left[\dfrac{\sigma_{uc}}{2E_0}/0.002\right]^{\frac{1}{n}}}. \tag{3}$$

For typical values of σ_{uc}, E_0 and n for cast iron, the term $\left[\dfrac{\sigma_{uc}}{2E_0}/0.002\right]^{\frac{1}{n}}$ is approximately equal to 1.1. Given the approximate nature of eqn. (2), it appears reasonable to determine the 0.2% proof stress simply as,

$$\sigma_{0.2} = \frac{\sigma_{uc}}{2}. \tag{4}$$

On the basis of the values of σ_{uc} shown in Table 1, a representative value of the 0.2% proof stress of cast iron is then $\sigma_{0.2}$ = 750/2 MPa = 375 MPa. Similarly, the initial elastic modulus may be taken as E_0 = 88,000 MPa, which is about 40% of the value for modern structural steels.

The *n*-parameter of the Ramberg-Osgood equation can be obtained from two points on the stress-strain curve, usually chosen as the 0.01% and 0.2% proof stresses. However, the available stress-strain curves are not sufficiently precise to allow determination of the 0.01% proof stress, and in stead the *n*-parameter will here be based on the 0.1% and 0.2% proof stress. Figure 1 shows the 0.1% and 0.2% proof stresses for the two stress-strain curves shown. Utilising that $n = \ln(2)/\ln(\sigma_{0.2}/\sigma_{0.1})$, the *n*-values of the two stress-strain curves are 5.3 and 7.1. A similar calculation using the four stress-strain curves presented in Kapplein (1997), results in *n*-values between 5 and 6. In view of these results, a representative value of *n* emerges as *n*=6. The Ramberg-Osgood parameters thus obtained are summarised in Table 2. The table also shows the nondimensional proof stress (*e*),

$$e = \frac{\sigma_{0.2}}{E_0}. \tag{5}$$

As shown in Table 1, the ultimate tensile strength (σ_{ut}) is significantly lower than the ultimate compressive strength. Introducing the ratio,

$$f = \frac{\sigma_{ut}}{\sigma_{uc}} \tag{6}$$

the ultimate strength values shown in Table 1 suggests that the ratio (*f*) typically lies in the range between 0.1 and 0.2, *ie* the ultimate tensile strength is merely 10% to 20% of the compressive strength.

3 Geometric imperfections

3.1. Member imperfection

Like other metallic columns, cast iron columns exhibit geometrical imperfections such as initial crookedness. Salmon (1921) collected data for wrought iron columns and concluded that a reasonable upper bound to the initial crookedness (δ_0) (taken as the maximum deviation of the column axis from a straight line connecting the ends) can be obtained as,

$$\delta_{0,\max} = \frac{L}{750} \tag{7}$$

as shown in Fig. 3. The mean crookedness is approximately,

$$\delta_{0,mean} = \frac{L}{1500} \tag{8}$$

While these measurements were obtained for wrought iron columns, they are indicative of the fabrication tolerances met in the 19th century. It is noted that the mean crookedness (*L*/1500) matches that suggested by Bjorhovde (1972) and is also the value on which the generalised column curve formulation (Rasmussen and Rondal 1997) is based.

3.2. *Cross-section imperfections*

Hollow cast-iron sections are peculiar in that the hole is frequently eccentric to the outer perimeter, as shown in Fig. 4, see also Kapplein (1997) and Bussell (1997). The irregular wall thickness is the result of lifting forces, dislocations and/or deflections of the casting core used for producing the hole of the member during casting in the horizontal position. The eccentricity of the hole leads to an eccentricity (g) of the load with reference to the centroid of the cross-section, as shown in Fig. 4, which, for given values of external diameter (d_e), internal diameter (d_i), and minimum thickness (t_{min}), can be obtained from,

$$g = j\frac{d_i^2}{d_e^2 - d_i^2} \tag{9}$$

where

$$j = \frac{d_e - d_i}{2} - t_{\min} \tag{10}$$

The distances from the centroid (G) to the extreme fibres are,

$$v' = \frac{d_e}{2} - g \tag{11}$$

$$v = \frac{d_e}{2} + g. \tag{12}$$

Salmon (1921) mentions that the size of the eccentricity (g) is difficult to evaluate but can be estimated by,

$$\frac{g}{d_e} \approx \frac{1}{30} \text{ to } \frac{1}{40} \tag{13}$$

where d_e is the external diameter.

4 Design equations

4.1. *Compression failure*

According to the generalised column curve formulation (Rasmussen and Rondal 1997), the ultimate stress ($\sigma_{ult,c}$) of a column made from a metal with given Ramberg-Osgood parameters (E_0, $\sigma_{0.2}$, n) can be obtained as,

$$\sigma_{ult,c} = \chi_c \sigma_{0.2,c} \tag{14}$$

where $\sigma_{0.2,c}$ is the 0.2% proof stress for compression and the slenderness reduction factor (χ_c) is given by,

$$\chi_c = \frac{1}{\varphi_c + \sqrt{\varphi_c^2 - \lambda_c^2}} \tag{15}$$

$$\varphi_c = \frac{1}{2}(1 + \eta_c + \lambda_c^2) \tag{16}$$

$$\lambda_c = \sqrt{\frac{\sigma_{0.2,c}}{\sigma_{E_0}}} \tag{17}$$

$$\sigma_{E_0} = \frac{\pi^2 E_0}{(L/r)^2}. \tag{18}$$

In eqn. (16), the imperfection parameter (η_c) is given by,

$$\eta_c = \alpha\left((\lambda_c - \lambda_1)^\beta - \lambda_0\right) \tag{19}$$

where the parameters (α, β, λ_0, λ_1) are functions of the material parameters n and $e_c=\sigma_{0.2,c}/E_0$. By substituting the values of (n, e_c) into the expressions for (α, β, λ_0, λ_1) given in Rasmussen and Rondal (1997), the following values are obtained,

$$\eta_c = \alpha = 0.85 \qquad \beta = 0.095 \qquad \lambda_0 = 0.70 \qquad \lambda_1 = 0.55 \tag{20}$$

The column strengths ($\sigma_{ult,c}$) obtained from eqns (14-20) are compared with tests reported by Hodgkinson (1840) and Tetmayer (1901) in Figs 5 and 6 respectively. The test data is contained in Appendix II of this report. It follows from Fig. 5 that the strength curve produces a reasonable mean to the test strengths presented by Hodgkinson, although several test strengths lie below the strength curve in the intermediate slenderness range. The strength curve provides essentially a lower bound to the test strengths presented by Tetmayer, as shown in Fig. 6.

In both Figs 5 and 6, many test strengths lie above the Euler curve. The relatively small variation in initial elastic modulus (E_0) shown in Table 1 does not explain why strength higher than the Euler stress were achieved, and it is apparent that some degree of end restraint was present in many tests, as also pointed out by Salmon (1921).

While the imperfection parameter given by eqn. (19) accounts for the effect of material softening, it does not consider the effect of an additional eccentricity arising from the asymmetric position of the hole in hollow sections. To take this effect into account, the imperfection parameter may be augmented to (Maquoi and Rondal 1983),

$$\eta_c^* = \eta_c + gA\frac{v}{I} \tag{21}$$

where A and I are the cross-section area and second moment of area respectively, which for a thin tube can be approximated by $\pi t d_e$ and $\pi t d_e^3/8$ respectively. Assuming g=d_e/30 by eqn. (13), and using eqn. (12) for v, the second term of eqn. (21) reduces to 32/225, *ie*

$$\eta_c^* = \eta_c + \frac{32}{225}. \tag{22}$$

The strength curves obtained using eqn. (22) for the imperfection parameter are also shown in Figs 5 and 6. The added eccentricity has greatest effect at small and intermediate slenderness values, and, while it improves the agreement with Hodgkinson's tests at intermediate slenderness values (Fig. 5), it appears to produce too low a strength curve in the comparison with Tetmayer's tests (Fig. 6). This may be partly explained by Hodgkinson's comment that the displacement of the core "does not produce so great a diminution in strength as might be expected, for the thinner part of a casting is much harder than the thicker, and this usually becomes the compressed side". On the basis of these results it is considered appropriate to maintain the imperfection parameter in the form of eqn. (19).

Figures 5 and 6 also include the Rankine-Gordon formula (Salmon 1921),

$$\sigma_{ult,c} = \frac{552\ \text{MPa}}{1 + \frac{1}{1600}\left(\frac{L}{r}\right)^2} \tag{23}$$

which was commonly used for the design of cast iron columns in the 19th century. It fits the Tetmayer tests very well as a lower bound but several of the Hodgkinson tests lie below the curve, possibly as a result of tension failure.

4.2. Tension failure

As mentioned under Material Behaviour, cast iron is relatively weak in tension and it is therefore possible that column failure may be initiated by fracture as tension develops on one side on the column during overall bending. However, while the literature implies that this failure mode exists, it will only occur in relatively imperfect columns, since according to Shanley (1947) the compressive strains increase during the initial overall buckling of a perfect column.

A design check on tension failure can easily be obtained by applying the Perry-Robertson criterion for column failure to the tension side,

$$\sigma_{t,\max} = \sigma_{0.2,t} \tag{24}$$

where

$$\sigma_{t,\max} = \delta_0 A \frac{v'}{I} \frac{\sigma}{1 - \sigma / \sigma_{E_0}} - \sigma. \tag{25}$$

In eqn. (25), the first term of the right-hand side is the tension stress due to bending while the second term is the compression stress resulting from the applied load. Combining eqns (24,25), the stress causing tension failure ($\sigma_{ult,t}$) is to be obtained from,

$$\eta_t \sigma_{ult,t} \sigma_{E_0} = (\sigma_{0.2,t} + \sigma_{ult,t})(\sigma_{E_0} - \sigma_{ult,t}) \tag{26}$$

where

$$\eta_t = \delta_0 A \frac{v'}{I} \tag{27}$$

The imperfection parameter η_t is the same as that obtained in expressing compression failure (Maquoi and Rondal 1983) except that it features v' rather than v. Ignoring differences between v' and v, noting Hodgkinson's comment suggesting that hole eccentricity may affect the tension failure strength, and using the generalised formulation to account for the effect of material nonlinearity, the imperfection parameter for tension failure is taken as,

$$\eta_t = \eta_c + \frac{32}{225} \tag{28}$$

where η_c is given by eqn. (19). Solving eqn. (26), the tension failure strength is obtained as,

$$\sigma_{ult,t} = \chi_t f \sigma_{0.2,c} \tag{29}$$

where, in view of Kapplein's suggestion, the strength ratio (*f*) has been assumed to be the same for the tensile and compressive 0.2% proof stresses as for the ultimate strengths (see eqn. (6)), *ie*

$$\sigma_{0.2,t} = f\sigma_{0.2,c} \tag{30}$$

The slenderness reduction factor (χ_t) in eqn. (29) is given by,

$$\chi_t = \frac{1}{\varphi_t + \sqrt{\varphi_t^2 + f\lambda_c^2}} \tag{31}$$

$$\varphi_t = \frac{1}{2}(-1 + \eta_t + f\lambda_c^2) \tag{32}$$

$$\eta_t = \alpha\left(\left(\lambda_c - \lambda_1\right)^{\beta} - \lambda_0\right) + \frac{32}{225} \tag{33}$$

The column strength obtained using eqns (29-33) are shown in Figs 5 and 6 for the representative value of *f* of 0.2. It follows from the figures that tension failure becomes critical at a slenderness ratio (*L/r*) of about 56, and that the tension failure strength curve provides an accurate lower bound to the tests strengths for *L/r*>56. An accurate lower bound to the test strengths can now be achieved by combining the strength curves for compression failure (using eqn. (19) for the imperfection parameter) and tension failure (using eqn. (33) for the imperfection parameter).

4.3. *Limiting slenderness*

A design equation for checking against tension failure did not exists in the 19th century. Rather, limits were imposed on the column slenderness to guard against this type of failure. For instance, the London County Council (1909) Act imposed the limit,

$$\frac{L}{r} \le 80 \tag{34}$$

while Goodman (1926) ("to avoid the buckling stress causing a residual tension at the surface") suggested,

$$\frac{L}{d_e} \le 15 \tag{35}$$

which for circular hollow sections leads to,

$$\frac{L}{r} \le 50 \tag{36}$$

To investigate how the limits given by eqns (34,36) compare with the design checks for compression and tension failure presented above, the switch-over point defined by $\sigma_{ult,t} = \chi_t f \sigma_{0.2,c} = \chi_c \sigma_{0.2,c} = \sigma_{ult,c}$ has been solved for λ_c, producing a lengthy expression which upon linearisation in *f* can be expressed as,

$$\lambda_c \approx 1 + 0.85 f \tag{37}$$

The coefficients "1" and "0.85" have been evaluated for η_c=0.25 which is representative for η_c in the *L*/*r*-range of interest from 50 to 80. In terms of *L*/*r*, the switch-over point is,

$$\frac{L}{r} \approx \sqrt{\frac{\pi^2 E_0}{\sigma_{0.2,c}}}\left(1 + 0.85 f\right) \tag{38}$$

or

$$\frac{L}{r} \approx 48\left(1 + 0.85 f\right) \tag{39}$$

for $\sigma_{0.2,c}$=375 MPa and E_0=88,000 MPa. For *f*=0.2, the switch-over point is at *L*/*r*=56, as also shown in Figs 5 and 6. This value is in line with that (*L*/*r*=50) suggested by Goodman (1926).

5 Recommendations

It is recommended that the ultimate strength of cast iron columns be calculated as the minimum of the compression and tension failure strengths, *ie*

$$\sigma_{ult} = \min\{\sigma_{ult,c}, \sigma_{ult,t}\} \tag{40}$$

where the compression failure strength ($\sigma_{ult,c}$) shall be calculated using eqns (14-20) and the tension failure strength ($\sigma_{ult,t}$) shall be calculated using eqns (29-33). The representative value of $f = \sigma_{0.2,t}/\sigma_{0.2,c}$=0.2 may be used in calculating the tension failure strength.

The recommended strength equations have been shown to provide an accurate lower bound to tests on cast iron columns. In a limit state design situation, it would be required to apply a resistance factor, (safety factor in an allowable stress design), to the recommended nominal strength. A resistance factor could conceivably be determined using a modern reliability analysis, *eg* Eurocode3, Annex Z (1992) or the LRFD framework (Ravindra and Galambos 1978), in which case the strength data contained in Appendix II would be required. However, in needs to be borne in mind that many test strengths were artificially enhanced by end restraints, as previously mentioned, and hence the strength data may not be suitable for a statistical analysis. Alternatively, some guidance on an appropriate safety factor can be found in Blanchard et al. (1997) and Kapplein (1997).

If a resistance factor was to be derived, subject to a screening of the test data contained in Appendix II, the target reliability index should possibly be chosen higher for tension failure because of the perceived more brittle failure associated with tension failure compared to compression failure. Similarly a higher safety factor should possibly be applied to the tension failure strength.

6 Conclusions

Design equations have been presented for determining the strength of cast iron columns. The equations account for the material characteristics of cast iron which include a nonlinear stress-strain curve, significantly lower tensile strength than compressive strength and an elastic modulus of approximately 2/5 of that of modern structural steels. The design equations for determining the tension failure strength were derived using the Perry-Robertson approach with respect to the edge of the cross-section undergoing tension during overall bending. The generalised column curve formulation (Rasmussen and Rondal 1997) was used to amend the imperfection parameter to account for the effect of gradual yielding.

The design equations have been shown to produce an accurate lower bound to some 300 tests performed in the 19th century. The design equations indicate that tension failure may occur at column slenderness values (L/r) greater than about 56. The trend of the test strengths supports this result, which is also in agreement with contemporary recommendations (Goodman 1926). However, tension failure is not likely to occur for columns with small initial imperfections, irrespective of their slenderness value, because the initial buckling of such columns occurs under increasing compressive strains.

The presented design method is based on the modern approach to determining the strength of metallic columns which embraces the Perry curve. It differs from the methods used more than century a century ago in that firstly, the 0.2% proof stress is used as the upper bound, rather than the ultimate compressive strength, and secondly, explicit design equations are used to determine the tension failure strength rather than imposing a limit on the column slenderness.

The presented design equations are useful for assessing the strength of cast iron steel columns as part of the refurbishing of historical buildings.

Tables

Table 1: Ultimate strength and initial modulus of elasticity of cast iron

Tests	σ_u (MPa)	E_o (MPa)
Compression		
English grey iron (Hodgkinson 1840)	750	88,000
English iron (Twelvetrees 1900)	590- 780	85,000-90,000
Tension		
English iron (Morin 1862)	110	-
English grey iron (Hodgkinson 1840)	75-160 (mean: 124)	91,000
English iron (Twelvetrees 1900)	124	66,000-93,000

Table 2: Representative values of the Ramberg-Osgood parameters for cast iron, (values for compression)

E_0 (MPa)	$\sigma_{0.2,c}$ (MPa)	n	e_c
88,000	375	6	0.0043

Figures

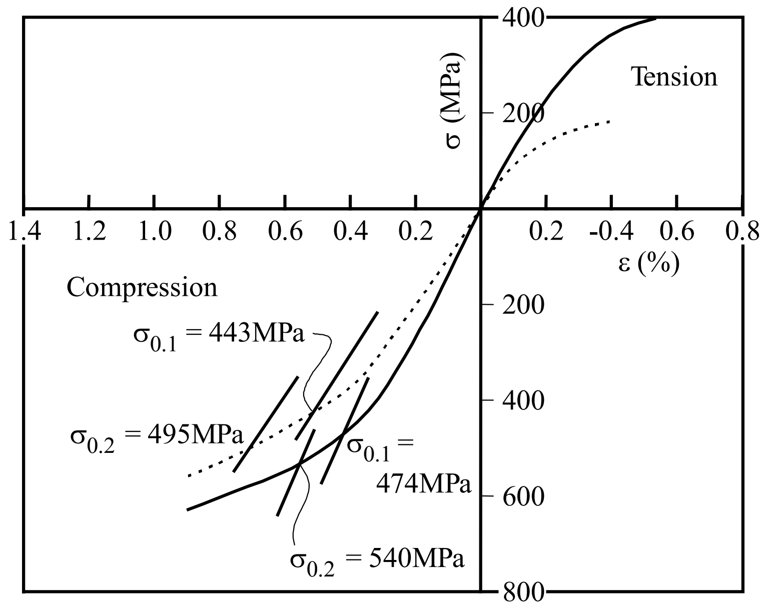

Figure 1: Stress-strain curves for typical structural cast iron (Blanchard et al. 1997)

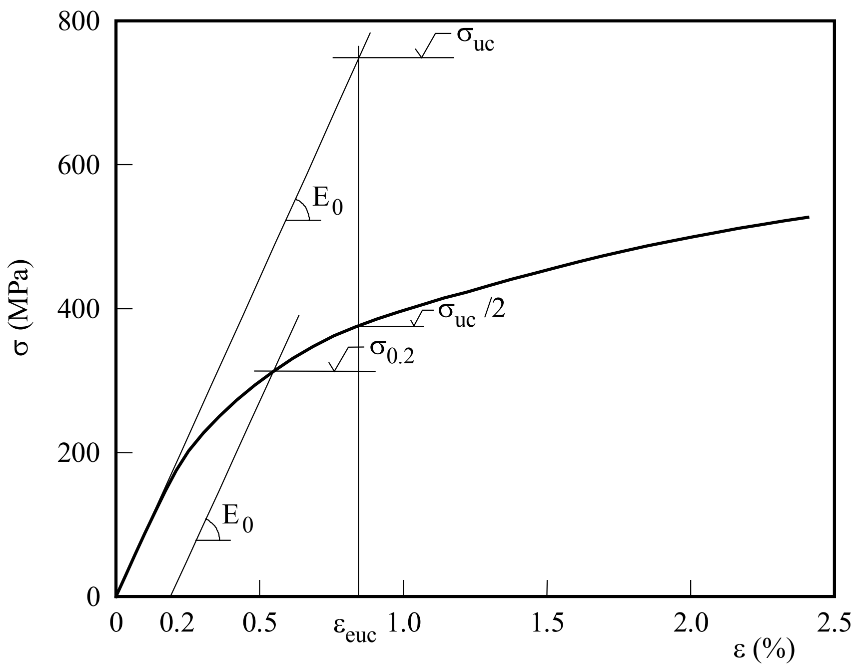

Figure 2: Kapplein’s proposal (Kapplein 1997) and 0.2% proof stress

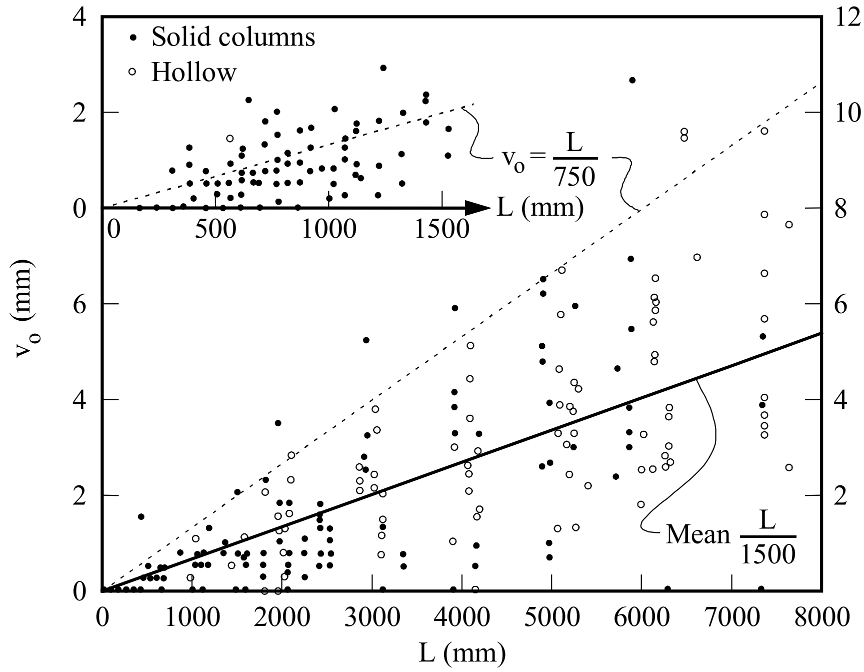

Figure 3: Member imperfection measurements of wrought iron columns (Salmon 1921)

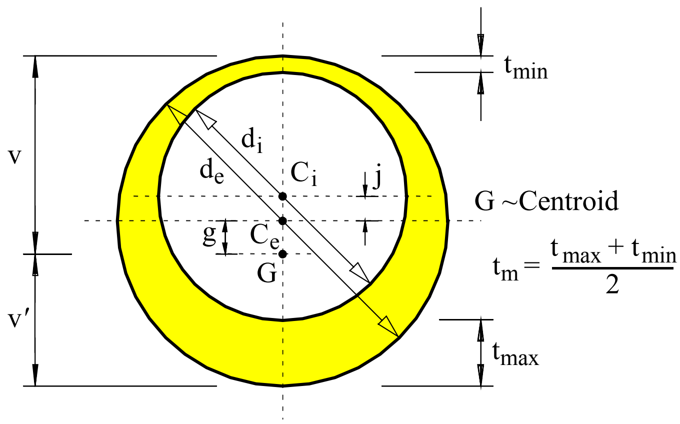

Figure 4: Cross-section imperfection in hollow cast iron columns

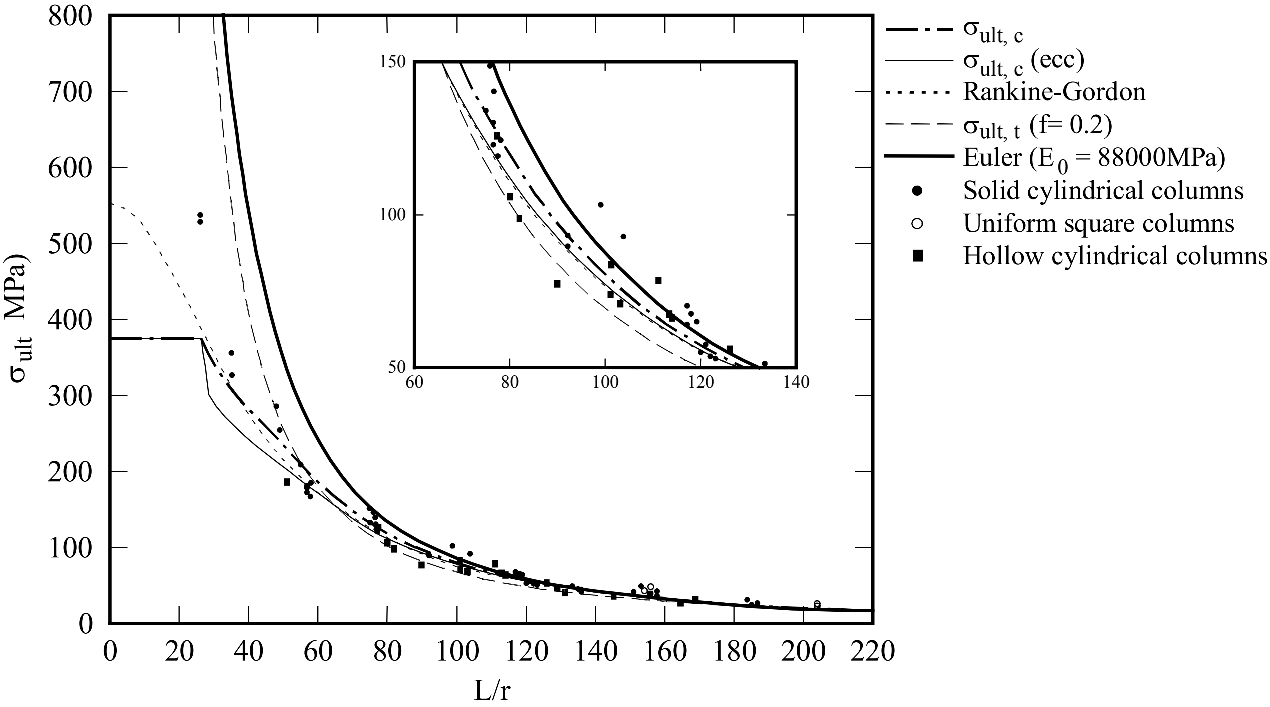

Figure 5: Comparison of design strengths with Hodgkinson's tests (Hodgkinson 1840)

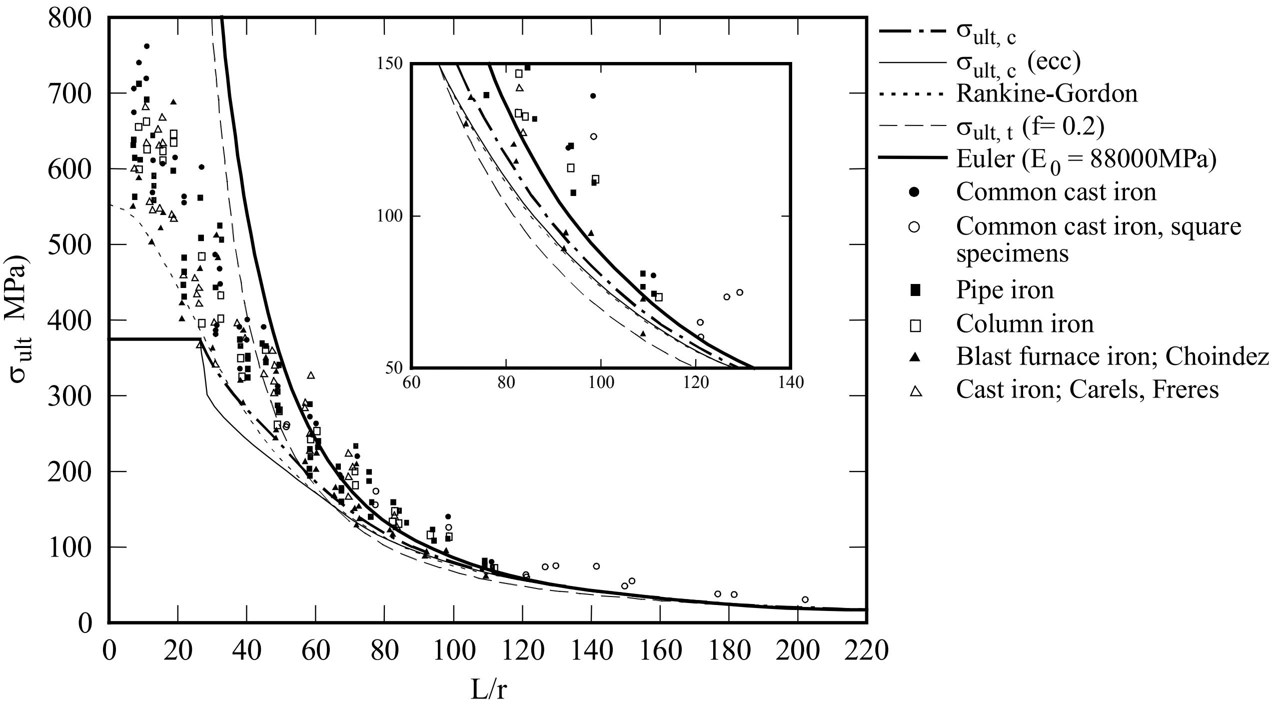

Figure 6: Comparison of design strengths with Tetmayer's tests (Tetmayer 1901)

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Appendix II: Column strength data

II.1 General

This appendix lists values of column slenderness values (*L/r*) and column strengths (σ_{ult}) (Hodgkinson 1840; Tetmayer 1901) as presented in Figs 72 and 74 of Salmon (1921). The values were read off the printed figures and thus may contain slight errors compared with the actual test data.

II.2 Tests presented by Hodgkinson

Hodgkinson (1840) presented tests on solid cylindrical columns, uniform square columns and hollow cylindrical columns, as set out in the following three tables.

Table AII.1: Solid cylindrical columns

Spec no.	*L/r*	σ_{ult} (lb/in^2)	σ_{ult} (MPa)
1	26.0	77798	536.8
2	26.0	76330	526.7
3	35.0	51489	355.3
4	35.0	47481	327.6
5	48.0	41327	285.2
6	48.6	36867	254.4
7	54.8	30318	209.2
8	57.8	26704	184.3
9	56.5	26253	181.1
10	56.6	25801	178.0
11	56.7	24954	172.2
12	57.8	24333	167.9
13	74.7	21849	150.8
14	75.9	21454	148.0
15	76.5	20212	139.5
16	74.8	19308	133.2
17	76.5	18800	129.7
18	77.6	17953	123.9
19	76.5	17728	122.3
20	77.5	17219	118.8
21	91.8	13437	92.7
22	91.8	12929	89.2
23	98.7	14905	102.8
24	103.5	13380	92.3
25	116.8	10106	69.7
26	116.8	9259	63.9
27	117.6	9711	67.0
28	118.7	9372	64.7
29	120.7	8299	57.3
30	119.7	7904	54.5
31	121.7	7678	53.0
32	122.7	7622	52.6
33	133.0	7339	50.6
34	134.8	6718	46.4
35	135.8	6718	46.4
36	135.8	6041	41.7
37	150.9	6154	42.5
38	152.8	7114	49.1
39	155.9	5872	40.5

40	155.6	5533	38.2
41	157.8	6154	42.5
42	157.8	5307	36.6
43	184.0	4629	31.9
44	184.1	4291	29.6
45	185.8	3839	26.5
46	186.9	4065	28.0
47	240.2	2936	20.3
48	240.2	2653	18.3
49	242.4	2879	19.9
50	242.4	2145	14.8

Table AII.2: Uniform square columns

Spec no.	*L/r*	σ_{ult} (lb/in^2)	σ_{ult} (MPa)
1	153.8	6323	43.6
2	155.9	7114	49.1
3	204.0	3952	27.3
4	204.1	3500	24.2

Table AII.3: Hollow cylindrical columns

Spec no.	*L/r*	σ_{ult} (lb/in^2)	σ_{ult} (MPa)
1	50.8	26930	185.8
2	77.1	18179	125.4
3	79.8	15356	106.0
4	81.7	14227	98.2
5	89.6	11179	77.1
6	100.8	12082	83.4
7	100.8	10614	73.2
8	102.9	10219	70.5
9	110.7	11348	78.3
10	112.8	9711	67.0
11	113.6	9485	65.4
12	125.7	8073	55.7
13	128.6	7057	48.7
14	130.9	5815	40.1
15	144.9	5194	35.8
16	155.1	5872	40.5
17	164.9	4121	28.4
18	168.7	4629	31.9

III.2 Tests presented by Tetmayer

Tetmayer (1901) presented tests on six types of cast iron columns, as presented in the following tables.

Table AII.4: Common cast iron

Spec no.	*L/r*	σ_{ult} (t/cm^2)	σ_{ult} (MPa)
1	7.8	7.20	706.2
2	7.8	6.89	676.3
3	9.4	7.57	742.2
4	11.5	7.78	763.2
5	11.5	7.35	720.6
6	13.2	6.24	612.1
7	13.2	5.81	570.1
8	16.1	6.21	608.8
9	19.4	6.29	617.1
10	19.4	5.44	533.5
11	22.2	5.75	564.5
12	22.2	5.68	556.8
13	27.0	6.17	605.5
14	27.0	5.17	507.0
15	31.4	4.97	487.6
16	32.6	4.78	469.3
17	32.6	4.58	448.9
18	31.4	4.02	394.1
19	31.4	3.95	387.4
20	31.4	3.90	383.0
21	38.2	4.00	392.4
22	38.2	3.42	335.9
23	40.3	4.11	402.9
24	40.3	3.83	375.8
25	45.1	4.00	392.4
26	48.9	3.20	314.4
27	48.9	2.89	283.9
28	58.3	2.78	272.3
29	60.1	2.70	264.6
30	58.8	2.56	250.7
31	60.3	2.49	244.6
32	58.2	2.27	222.5
33	66.8	1.99	194.8
34	66.9	1.96	192.6
35	71.0	2.34	229.7
36	71.6	2.25	220.3
37	71.6	2.10	205.9
38	71.5	1.86	182.1
39	82.2	1.58	155.0
40	92.7	1.24	121.8
41	98.0	1.42	139.5
42	110.4	0.82	80.3

Table AII.5: Common cast iron, square specimens

Spec no.	*L/r*	σ_{ult} (t/cm^2)	σ_{ult} (MPa)
1	51.4	2.65	259.6
2	51.6	2.67	261.8
3	77.5	1.76	172.7
4	77.3	1.59	156.1
5	98.0	1.29	126.2
6	120.4	0.66	64.8
7	120.5	0.61	59.8
8	125.9	0.74	73.1
9	128.8	0.76	74.7
10	140.8	0.76	74.2
11	141.0	0.76	74.2
12	148.7	0.49	47.6
13	150.8	0.55	54.2
14	175.5	0.38	37.1
15	180.2	0.38	37.1
16	201.0	0.30	29.3

Table AII.6: Column iron

Spec no.	*L/r*	σ_{ult} (lb/in^2)	σ_{ult} (MPa)
1	9.3	6.71	658.6
2	9.3	6.12	599.9
3	11.6	6.39	627.1
4	16.2	6.39	626.5
5	16.2	6.26	614.3
6	19.2	6.49	637.0
7	27.1	4.95	485.9
8	27.1	4.06	398.5
9	32.6	4.43	434.5
10	32.6	4.11	402.9
11	38.2	3.57	349.8
12	38.6	3.33	326.5
13	45.2	3.70	363.1
14	49.4	2.83	277.8
15	49.0	2.66	261.2
16	58.3	2.49	244.1
17	60.4	2.59	254.0
18	71.3	2.03	199.2
19	71.3	1.87	183.7
20	82.5	1.50	146.7
21	82.4	1.36	133.4
22	83.9	1.35	132.3
23	93.2	1.18	115.7
24	98.3	1.15	112.4
25	111.5	0.74	73.1

Table AII.7: Pipe iron

Spec no.	*L/r*	σ_{ult} (t/cm^2)	σ_{ult} (MPa)
1	7.8	6.53	640.9
2	7.8	6.44	632.0
3	7.8	6.27	614.9
4	7.8	5.77	565.6
5	9.5	7.28	714.0
6	9.5	6.26	614.3
7	11.4	7.06	692.9
8	11.4	6.77	664.1
9	13.4	6.59	646.4
10	13.4	6.05	593.3
11	13.4	5.91	580.0
12	13.4	5.70	559.5
13	16.3	6.48	635.4
14	19.2	6.61	648.6
15	19.2	6.11	599.4
16	22.1	4.94	484.8
17	22.1	4.75	466.0
18	22.1	4.56	447.2
19	22.1	4.41	432.8
20	27.0	5.73	562.3
21	27.0	5.21	510.8
22	32.6	5.37	526.9
23	32.6	5.18	508.6
24	31.3	4.53	444.4
25	38.3	3.83	375.2
26	38.3	3.72	364.7
27	40.3	3.60	353.1
28	40.3	3.57	350.3
29	40.3	3.42	335.4
30	40.3	3.32	325.4
31	44.3	3.79	371.9
32	45.2	3.76	368.6
33	45.4	3.51	344.2
34	49.1	3.49	342.0
35	48.9	3.13	307.2
36	48.9	2.94	288.4
37	49.2	2.87	281.2
38	58.1	2.94	288.9
39	58.1	2.49	244.1
40	58.1	2.35	230.2
41	58.1	2.23	219.2
42	58.1	2.07	203.1
43	58.1	2.02	198.1
44	60.5	2.46	241.3
45	60.5	2.38	233.6
46	66.1	2.10	205.9
47	67.1	1.82	178.8
48	67.2	1.79	175.4
49	67.1	1.63	159.9
50	71.3	2.39	234.7
51	75.3	2.04	199.8
52	75.3	1.91	187.1
53	76.0	1.62	159.4
54	75.7	1.42	139.5

55	82.2	1.62	159.4
56	84.1	1.51	148.3
57	85.8	1.34	131.7
58	93.6	1.25	122.9
59	93.9	1.09	107.4
60	98.0	1.13	110.7
61	108.4	0.82	80.8
62	108.4	0.78	76.4
63	110.5	0.76	74.2

Table AII.8: Blast furnace iron; Choindez

Spec no.	*L/r*	σ_{ult} (t/cm^2)	σ_{ult} (MPa)
1	7.5	5.64	553.5
2	11.5	6.79	666.4
3	11.5	6.39	627.1
4	9.3	6.03	591.1
5	13.1	5.60	549.0
6	13.1	5.16	506.4
7	16.2	5.54	543.5
8	19.2	7.03	689.6
9	19.3	6.64	651.4
10	21.4	4.34	425.6
11	21.7	4.10	402.4
12	27.0	4.81	471.5
13	27.0	4.52	443.9
14	31.5	5.26	515.8
15	32.2	4.94	484.8
16	30.5	3.72	365.3
17	30.5	3.56	349.2
18	38.2	3.80	372.5
19	38.2	3.27	321.0
20	39.2	2.98	292.2
21	45.4	3.57	350.3
22	45.1	3.40	333.2
23	48.7	3.40	333.7
24	48.1	2.61	255.7
25	48.1	2.49	244.6
26	56.8	2.18	213.6
27	60.1	2.30	225.3
28	60.1	2.08	203.7
29	65.2	1.84	180.4
30	65.0	1.74	170.5
31	71.2	2.16	212.0
32	71.5	1.82	178.8
33	72.4	1.59	155.5
34	71.2	1.56	152.8
35	72.5	1.42	138.9
36	71.6	1.33	130.6
37	81.3	1.26	123.4
38	82.0	1.20	117.9
39	92.1	0.96	94.6
40	91.7	0.91	89.1
41	97.4	0.96	94.6
42	108.2	0.74	73.1
43	108.5	0.63	61.4

Table AII.9: Common cast iron

Spec no.	*L/r*	σ_{ult} (t/cm^2)	σ_{ult} (MPa)
1	7.8	6.16	604.4
2	11.3	6.98	684.6
3	15.1	6.48	635.4
4	11.4	6.50	637.6
5	12.3	5.69	558.4
6	13.2	5.59	547.9
7	16.2	6.84	671.3
8	16.2	6.50	638.1
9	15.6	5.61	550.7
10	15.6	5.34	524.1
11	18.8	5.53	542.9
12	19.2	5.49	538.5
13	22.0	4.72	463.2
14	26.6	4.55	446.6
15	26.6	4.33	425.1
16	26.6	3.74	366.9
17	25.5	4.68	459.4
18	25.9	4.45	436.1
19	30.6	4.08	400.1
20	31.5	3.50	343.7
21	37.5	4.07	399.6
22	39.3	3.96	388.5
23	39.5	3.86	379.1
24	45.2	3.37	330.4
25	47.8	3.68	360.9
26	48.0	3.48	341.5
27	48.0	3.28	321.6
28	48.0	3.11	305.0
29	59.0	3.34	327.6
30	56.9	2.98	292.2
31	57.1	2.89	283.4
32	58.3	2.58	253.5
33	58.3	2.34	229.7
34	69.3	2.29	224.7
35	70.0	2.10	205.9
36	69.3	1.98	194.3
37	69.3	1.70	167.1
38	82.7	1.45	142.2
39	83.3	1.30	127.3
40	15.1	6.68	655.3