$$H_D = H_A - h_f^{A-D} = 60 - 13.26 \Rightarrow H_D = 46.74\ m$$

that would be equivalent to the earlier result if Example 5.1 was carried to 2 decimal places.

5.1.2 Pipes in Parallel

When one or more pipes connect the same locations (junctions), the hydraulics are much more interesting. The relationships in these small networks lead to the fundamental relationships for full network modeling. Locations A and B in Figure 5-2 are described as nodes or junctions of several pipes. As in Example 2.1, conservation of mass must be preserved at these locations. That is, in steady state the known incoming flow at node A must balance with the outgoing flows in pipes 1, 2, and 3. Similarly, the incoming flows to node B in the incoming pipes 1, 2, and 3 must equal the known withdrawal at node B.

$$q_A = q_B = Q_1 + Q_2 + Q_3 \tag{5-3}$$

where Q_l and q_j define the flow rate in pipe l and the nodal withdrawal/supply at node j, respectively. The mass balance for node B provides the same information as the above and is redundant.

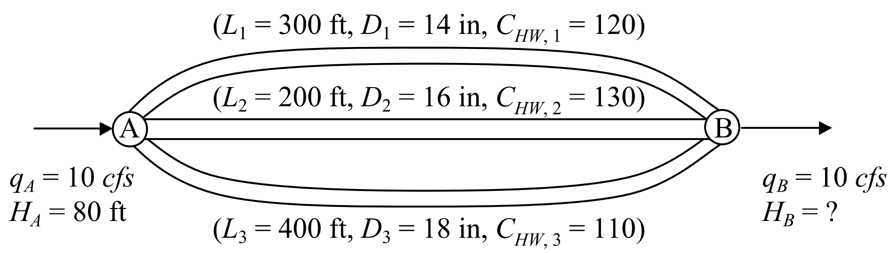

Figure 5-2: Pipes in parallel with data for Example 5.3.

The second relationship that must hold is that the head loss in pipes 1, 2, and 3 must be the same. Since all begin at a single node (A) and all end at a single node (B) and the difference in head between those two nodes is unique, regardless of the pipe characteristics the head loss in the pipes is the same or:

$$H_A - H_B = h_{L,1} = h_{L,2} = h_{L,3} = h_L \tag{5-4}$$

where H_A and H_B are the total heads at nodes A and B, respectively, $h_{L,l}$ is the head loss in pipe l, and h_L is the single value of head loss between nodes A and

B. Eq. 5-4 is a statement of conservation of energy for a pipe and is used in several formulations for solving for flows and heads in a general network.

In other network solution methods, we write conservation of energy for closed loops. A closed loop is a path of pipes that begins and ends at the same node. Pipes 1 and 2 form a closed loop beginning and ending at node A. Starting at node A around the path, energy, $h_{L,1}$, is lost as water flows from A to B. As we follow the path back to node A to close the loop, we gain energy, $h_{L,2}$, since we are moving in the direction opposite to the flow. We can write the path equation around the loop and manipulate it to show:

$$H_A - h_{L,1} + h_{L,2} = H_A \Rightarrow -h_{L,1} + h_{L,2} = 0 \Rightarrow h_{L,1} = h_{L,2} \qquad (5\text{-}5)$$

Now using Eq. 5-3 and either Eqs. 5-4 or 5-5, we can determine the head loss and flow for each pipe and an equivalent pipe coefficient, K_{eq}^{p}. In any pipe network system the nodal inflows and outflows (q_A and q_B) and at least one nodal head's total energy (H_A in this case) must be known to provide a datum for the pressure head. For steady flow conditions in the network in Figure 5-2, we have a total of seven unknowns, node B's total energy (H_B), three pipe flows (Q_1, Q_2, and Q_3) and three head losses ($h_{L,1}$, $h_{L,2}$, and $h_{L,3}$).

Eq. 5-4 provides two independent equations relating the head losses ($h_{L,1}$ = $h_{L,2}$, and $h_{L,1}$, = $h_{L,3}$). The third equation is that the head loss in any pipe equals the difference in head between nodes A and B (the first part of Eq. 5-4). Conservation of mass at node A (Eq. 5-3) is the fourth relationship. The final three equations are the head loss versus discharge equations:

$$h_{L,l} = K_l\, Q_l^n \text{ or } Q_l = \left(\frac{h_{L,l}}{K_l}\right)^{1/n} \qquad (5\text{-}6)$$

We can substitute Eq. 5-6 in the mass balance equations (Eq. 5-3) with h_L equal to each pipe's head loss or:

$$\left(\frac{h_L}{K_1}\right)^{1/n} + \left(\frac{h_L}{K_2}\right)^{1/n} + \left(\frac{h_L}{K_3}\right)^{1/n} = q_A \qquad (5\text{-}7)$$

In this equation, all terms except for h_L are known. After solving for h_L, the unknown pipe flows can be computed by Eq. 5-6 and H_B can be determined in Eq. 5-4.

Like pipes in series, an equivalent pipe coefficient can be computed for parallel pipes. In Eq. 5-7, h_L can be pulled from each term on the left hand side or for a general discharge and three parallel pipes:

$$\left[\left(\frac{1}{K_1}\right)^{1/n}+\left(\frac{1}{K_2}\right)^{1/n}+\left(\frac{1}{K_3}\right)^{1/n}\right]h_L^{1/n}=q \tag{5-8}$$

The equivalent coefficient is then:

$$\left(\frac{1}{K_1}\right)^{1/n}+\left(\frac{1}{K_2}\right)^{1/n}+\left(\frac{1}{K_3}\right)^{1/n}=\left(\frac{1}{K_{eq}^{p}}\right)^{1/n}=\sum_{l=1}^{lp}\left(\frac{1}{K_l}\right)^{1/n} \tag{5-9}$$

where K_{eq}^{p} is the equivalent pipe coefficient for parallel pipes. As shown in the last term, Eq. 5-9 can be generalized for *lp* parallel pipes.

The head loss between the two end nodes is:

$$h_L = K_{eq}^{p}\,(Q_{total})^{n} \tag{5-10}$$

Example 5.3

Problem: Given the data for the three parallel pipes in Figure 5-2, compute (1) the equivalent parallel pipe coefficient, (2) the head loss between nodes A and B, (3) the flow rates in each pipe, and (4) the total head at node B.

Solution: (1) The equivalent parallel pipe coefficient allows us to determine the head loss that can then be used to disaggregate the flow between pipes. The loss coefficient for the Hazen-Williams equation for pipe 1 with English units is:

$$K_1=\frac{4.73\,L_1}{C_1^{1.85}\;D_1^{4.87}}=\frac{4.73\,(300)}{120^{1.85}\;(14/12)^{4.87}}=0.0954$$

Similarly, K_2 and K_3 equal 0.0286 and 0.0439, respectively.

The equivalent loss coefficient is:

$$\left(\frac{1}{K_1}\right)^{1/n}+\left(\frac{1}{K_2}\right)^{1/n}+\left(\frac{1}{K_3}\right)^{1/n}=\left(\frac{1}{0.0954}\right)^{0.54}+\left(\frac{1}{0.0286}\right)^{0.54}+\left(\frac{1}{0.0439}\right)^{0.54}=$$

$$=3.557+6.817+5.408=\left(\frac{1}{K_{eq}^{p}}\right)^{0.54}\;\Rightarrow K_{eq}^{p}=0.00607$$

(2 and 4) The head loss between nodes A and B is then:

$$h_L = K_{eq}^{p} (Q_{total})^n = 0.00607\,(10)^{1.85} = 0.43\ ft$$

(4) So the head at node B, H_B, is:

$$H_A - H_B = h_L = 80 - H_B = 0.43\,ft \;\Rightarrow\; H_B = 79.57\,ft$$

(3) The flow in each pipe can be computed from the individual pipe head loss equations since the head loss is known for each pipe ($h_L = 0.43\,ft$).

$$Q_1 = \left(\frac{1}{K_1}\right)^{1/n} h_L^{1/n} = \left(\frac{1}{0.0954}\right)^{0.54} (0.43)^{0.54} = 2.26\ cfs$$

The flows in pipes 2 and 3 can be computed by the same equation and are 4.32 and 3.43 cfs, respectively. The sum of the three pipe flows equals 10 cfs, which is same as inflow to node A.

5.2 SYSTEM OF EQUATIONS FOR STEADY FLOW

Conservation of mass at a junction node (Eq. 5-3) and conservation of energy (Eq. 5-1) can be extended from parallel pipes to general networks for steady state hydraulic conditions. The resulting set of simultaneous quasi-linear equations can be solved for the pipe flows and nodal heads for steady state and step-wise (quasi) dynamic (known as extended period simulation or EPS) analyses. EPS analysis requires an additional relationship describing changes in tank levels due to inflow/outflows and is discussed in later sections. Only steady state hydraulics is considered in Sections 5.2 and 5.3.

5.2.1 Conservation of Mass

As defined earlier, a junction node is a connection of two or more pipes. Although demands are distributed along pipes, these demands are lumped at junctions and defined as q_{node}. Conservation of mass at a node was presented in Section 2.1.2.1. For a junction node i, conservation of mass can be written as:

$$\sum_{l \in J_{in}} Q_l - \sum_{l \in J_{out}} Q_l = q_i \qquad (5\text{-}11)$$

where q_i is the external demand (withdrawal), $J_{in,i}$ and $J_{out,i}$ are the set of pipes supplying and carrying flow from node i, respectively, and $l \in J_{in}$ denotes that l is in the set of pipes in J_{in}. This equation can be written for every junction node in the system.