

# **COMBINED STRESSES IN GUSSET PLATES**

**W. A. Thornton, Ph.D., P.E.\***

## **GENERAL**

It has been the traditional approach in the design of connections in the United States, to consider only simple stresses, i.e., shear, normal, and flexure. This was perhaps due to the unsophisticated nature of the times, i.e., 70-80 years ago, when such things as von-Mises stresses were not known in the construction industry, but also this approach of using only simple uncombined stresses is perceived to be adequate because of the very crude nature of the stress calculations. The formulas generally used to determine stresses in connections are those that apply correctly to long slender members. With connections, the members are not slender, plane sections do not remain plane, and the usual formulas give only a very rough estimate of the actual stresses. It makes no sense to then impose a very precise calculation of principal stresses and the von-Mises criterion for combined stresses on top of the original very imprecise stresses.

In more recent times, much work has been done in the U.S. on design of bracing connections with gusset plates. Richard (1986) showed that the maximum gusset stress occurs in the gusset at the inboard end of the brace to gusset connection. This location of high effective stress is one of two limit states that control the gusset thickness. The other is gusset buckling. Both of these limit states are used in conjunction with “Whitmore’s” section (Whitmore-1952), to determine the required gusset thickness. On Whitmore’s section, the only stress is a normal stress so this is in effect the von-Mises stress.

Six full-scale connection tests on gusset plates by Gross (1990) and Bjorhovde/

\*President, Cives Engineering Corporation, Roswell, Georgia  
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Chakrabarti (1985) were used to validate the currently approved U.S. gusset plate design method, the Uniform Force Method (Thornton-1995). The calculations used to compare the predictions of the Uniform Force Method (UFM) with these six tests were performed in exactly the same way as the calculations presented by the American Institute of Steel Construction (AISC) Manual, Vol. II, Connections, which was published in 1994. With one exception, no combined stresses were used and excellent agreement between theory and test was achieved in this way. For Bjorhovde's tests, the predictions were within about 5-10% of the tests on the safe side. For Gross' tests, the predictions gave 60% of the test failure load, i.e. on the conservative side. Using combined stresses can only widen the distance between the theory and tests and are therefore found not to be necessary.

The exception alluded to above involves block shear rupture. In Fig. 16 of Thornton (1995), the Bjorhovde/Chakrabarti test specimen had a gusset to column connection with connection angles bolted to the gusset (Connection C of Fig. 16). This connection is subjected to both normal and shear forces. In this case, either a straight line or an elliptical interaction equation can be used (see Thornton-Kane (1999) pp. 163-164). For the capacities given in Table 3 of Thornton (1995) for Connection C, i.e.  $216^k$  for the  $30^\circ$  test,  $164^k$  for the  $45^\circ$  test, and  $155^k$  for the  $60^\circ$  test, a conservative straight line interaction equation was used, but the brace to gusset connection (Whitmore's section) controlled anyway. It will be noted from Table 3 that the brace to gusset connection controlled in five out of six tests, and in this one test where it didn't (Gross/Cheek No. 1), prying action and not gusset stresses controlled the capacity.

The von-Mises criterion can be written as:

$$\sigma_e = \left( \sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2 \right)^{\frac{1}{2}} \leq \sigma_{yp}$$

and this requires three stresses at any one point. If we know  $\tau_{xy}$  and  $\sigma_x$  say, on any one cut section, we will not know  $\sigma_y$  on the perpendicular cut section. Therefore, the unknown stress needs to be assumed at some value.  $\sigma_y$  could be zero or it could be equal to  $\sigma_x$  or it could be of opposite sign. The von-Mises effective stress  $\sigma_e$  will vary significantly depending on what is assumed. As mentioned above, the stress calculations are very crude. Any assumption for  $\sigma_y$  just adds a further degree of crudeness to the calculations. So, what looks like a sophisticated approach to the stress calculations really does not add any dependable accuracy to the numbers.

### EXAMPLE

Consider the example in Thornton-Kane (1999) that starts on page 39. Look at Figure 2.4 on page 51. The resultant forces are shown as  $V = 291^k$ ,  $P$  (or  $N$ )  $= 314^k$ , and  $M = 7279$  k-in. The stresses for these forces are:

Shear:  $f_v = 9.24$  ksi

Normal:  $f_n = 9.97$  ksi

Bending:  $f_b = 33$  ksi.

If these stresses are fed into the von-Mises formula with  $\sigma_x = 9.97+33$ ,  $\sigma_y = 0$  (assumed) and,  $\tau_{xy} = 9.24$ ,  $\sigma_e = ((9.97+33)^2 + 3 \times 9.24^2)^{1/2} = 45.9$  ksi, which is greater than  $\phi F_y = .9 \times 36 = 32.4$  ksi, and the connection fails. But the discussion on page 53

points out that this is not a failure because most of the cut section is still elastic. When the stresses are arranged to give full plasticity, so that  $\sigma_x = 25.9$ ,  $\sigma_y = 0$ ,  $\tau_{xy} = 9.24$ ,  $\sigma_e = (25.9^2 + 3 \times 9.24^2)^{1/2} = 30.4 \text{ ksi} < 32.4 \text{ ksi}$ , the connection is okay. This calculation gives an excess capacity of  $((32.4 - 30.4)/32.4 \times 100 = 6.2\%$ . It will be noted that the calculation on page 55 using uncombined stress gave  $((32.4 - 25.9)/32.4 \times 100 = 20\%$  excess capacity. But, pages 56 and 57 show that the block shear excess capacity is 9.9% and the Whitmore Section tension strength gives 5% excess capacity. Thus, the Whitmore capacity controls and combined stresses are not necessary.

If a horizontal section in the gusset at the beam is checked for combined stresses or a vertical section at the column is checked, both will be found to be satisfactory. As stated earlier, the Whitmore section or the block shear limit states of the brace to gusset connections will almost always control, even if combined stresses are used. Similar results will be found for the example of Figure 2.8.

If, in spite of the above discussion, combined stresses are going to be used, a better way to check for them is to use an interaction equation from plasticity theory. For a rectangular cross-section subjected shear  $V$ , normal force  $N$ , and moment  $M$ ,

$$\left[ \frac{M}{\phi M_p} \right] + \left( \frac{N}{\phi N_y} \right)^2 + \left( \frac{V}{\phi V_y} \right)^4 \leq 1$$

$$\text{where } M_p = \frac{1}{4} \sigma_{yp} t h^3$$

$$N_y = t h \sigma_{yp}$$

$$V_y = \frac{1}{\sqrt{3}} t h \sigma_{yp}$$

where  $t$  is the gusset thickness and  $h$  is the length of the cut section. This interaction equation is from Neal (1977). It does not require any assumption about the perpendicular normal stress  $\sigma_y$ . Using the above interaction equation, the previously discussed example will give

$$\frac{7279}{.9 \times .25 \times 36 \times .75 \times 42^2} + \left( \frac{341}{.9 \times 36 \times .75 \times 42} \right)^2 + \left( \frac{291}{.9 \times .6 \times 36 \times .75 \times 42} \right)^4 = .679 + .112 + .051 = .842 < 1 \text{ OK}$$

Using this approach, the excess capacity is  $\left( \frac{1 - .842}{1} \right) \times 100 = 15.8\%$  which is close to the 20% excess predicted by the uncombined stress method, but much more than Whitmore (5%) or block shear (9.9%) predict. Remember that the latter two checks will usually control the design. It can be seen from the above that checking combined stresses will not change the final design, and is therefore not necessary.

## ADDITIONAL MATTERS

The shear stress is  $f_v = V/A$ , not  $1.5V/A$ . As noted on pages 51 and 52, the 1.5 factor only applies to long slender beam sections, and the gusset obviously does not fall into this category. Gusset buckling is checked, see page 42. Gross (1990) determined the  $k$  factor of 0.5. There is no need to check free edges or “ $b/t$ ” ratios. The physical tests do not require it.

The gusset buckling approach just discussed will also be found in the AISC Manual Vol. II (1994). Recent work by Cheng (1999) has confirmed that this approach, which was originally proposed by Thornton (1984), is conservative, even for cyclic loadings.

It is noted above that gusset stress determinations are relatively crude. Section 2.2.1.5 on page 68 dealing with “frame action” shows one reason why this is true. Frame action gives rise to what are called “secondary stresses” in, for instance, Blodgett (1966). Frame action also explains why gusset buckling is not as big a problem as might be expected. When the brace is in compression and the gusset might be expected to buckle, the 90° angle between the beam and column increases and creates a “tension field” in the gusset (Figure 2.12) which inhibits buckling. This explains why the k factor of 0.5 is satisfactory.

## CONCLUSION

The use of the von-Mises criterion, which is technically correct, is not justified in the design of gusset plates. Blodgett (1966) uses principal stresses, but not the von-Mises combined or effective stress. Blodgett’s approach is typically the method used in machine design where fatigue is a consideration. It is not typical in U.S. standard practice in gusset design. If it is nevertheless decided to use combined stresses, the interaction equation from Neal (1977) [also Astaneh 1998] using M, N, and V is the most reasonable approach, but as stated above, it will probably never control the thickness of a gusset plate.

## REFERENCES

1. Astaneh-Asl, A. (1998), “Seismic Behavior and Design of Gusset Plates,” **Steel Tips**, Structural Steel Education Council, Moraga, CA.

2. Bjorhovde, R., and S.K. Chakrabarti (1985), “Tests of Full Size Gusset Plate Connections,” **ASCE Journal of Structural Engineering**, Vol. 111, No. 3, March, pp. 667-684.
3. Blodgett, O.W. (1966), **Design of Welded Structures**, The James F. Lincoln Arc Welding Foundation, Cleveland, OH.
4. Cheng, R.J.J. (1999), “Recent Developments in the Behavior of Cyclically Loaded Gusset Plates,” **Proceedings, AISC North American Steel Construction Conference**, Toronto, pp. 8-1 through 8-22, AISC, Chicago, IL.
5. Gross, J.L. (1990), “Experimental Study of Gusseted Connections,” **AISC Engineering Journal**, Vol. 27, No. 3, 3<sup>rd</sup> Quarter, pp. 89-97.
6. Neal, B.G. (1977), **The Plastic Methods of Structural Analysis**, Halsted Press, Wiley, New York.
7. Richard, R.M. (1986), “Analysis of Large Bracing Connections Designs for Heavy Construction,” **Proceedings, AISC National Engineering Conference**, Nashville, pp. 31-1 through 31-24, AISC, Chicago, IL.
8. Thornton, W.A. (1984), “Bracing Connections for Heavy Construction,” **AISC Engineering Journal**, Vol. 21, No. 3, 3<sup>rd</sup> Quarter, pp. 139-148.
9. Thornton, W.A. (1995), “Connections—Art, Science and Information in the Quest for Economy and Safety,” **AISC Engineering Journal**, Vol. 32, No. 4, 4<sup>th</sup> Quarter, pp. 132-144.

10. Thornton, W.A. and T. Kane (1999), “Designs of Connections for Axial, Moment, and Shear Forces,” Chapter 2 of **Handbook of Structural Steel Connection Design and Details**, Edited by A.R. Tamboli, McGraw-Hill, New York.
11. Whitmore, R.E. (1952), “Experimental Investigation of Stresses in Gusset Plates,” **University of Tennessee Experiment Station Bulletin No. 16**, University of Tennessee, Knoxville.