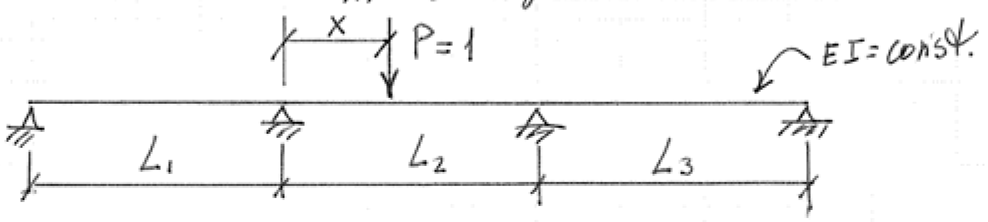
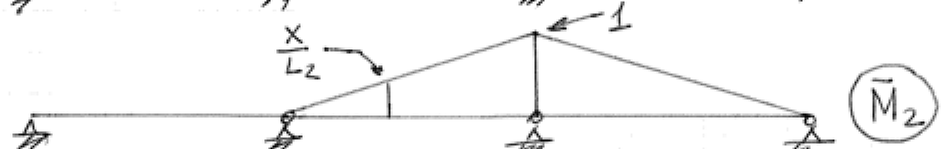
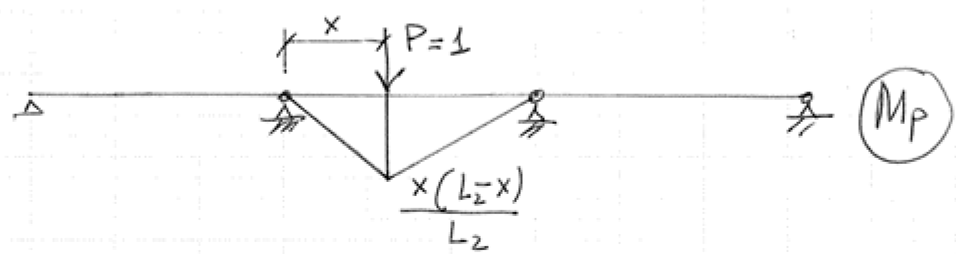
Method of forces as applied to Construction of influence lines in Continious beams.

Example: Find bending moment and shear force at the point of application of unit concentrated force in 3-span continious beam







$$\delta_{11} = \int \frac{\bar{M}_1^2}{EI} dx = \frac{L_1 + L_2}{3EI}$$

$$\delta_{22} = \int \frac{\bar{M}_2^2}{EI} dx = \frac{L_2 + L_3}{3EI}$$

$$\delta_{12} = \int \frac{\bar{M}_1 \bar{M}_2}{EI} dx = \frac{L_2}{6EI} = \delta_{21}$$

$$\Delta_{1p} = \int \frac{\bar{M}_1 M_p}{EI} dx = \frac{-x}{6EI}\left(0 + 2\frac{(L_2 - x)}{L_2} \cdot \frac{x(L_2-x)}{L_2} + \frac{x(L_2-x)}{L_2} + 0\right) = -\frac{x^2(3L_2 - x)(L_2 - x)}{6EI \cdot L^2}$$

$$\Delta_{2p} = \int \frac{\bar{M}_2 M_p}{EI} dx = \frac{-(L_2 - x)}{6EI}\left(\frac{2x}{L_2} \cdot \frac{x(L_2-x)}{L_2} + 0 + 0 + \frac{x(L_2-x)}{L_2}\right) = -\frac{(L_2-x)^2(2x+L_2)x}{6EI \cdot L^2}$$

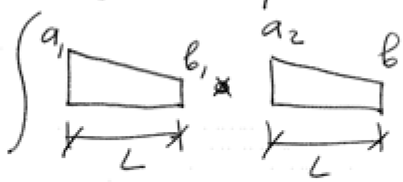
$$\begin{cases} \delta_{11} X_1 + \delta_{12} X_2 + \Delta_{1p} = 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \Delta_{2p} = 0 \end{cases} \quad \text{OR} \quad X = -\delta^{-1} \times \Delta p$$

after X_1 & X_2 found

$$M = \bar{M}_1 X_1 + \bar{M}_2 X_2 + M_p = -\left(\frac{L_2 - x}{L_2}\right) X_1 - \left(\frac{x}{L_2}\right) X_2 + \frac{x(L_2 - x)}{L_2}$$

$$V = \bar{V}_1 X_1 + \bar{V}_2 X_2 + V_p$$

integration of two linear areas



$$dx = \frac{L}{6}\left(2a_1 a_2 + 2b_1 b_2 + a_1 b_2 + b_1 a_2\right)$$