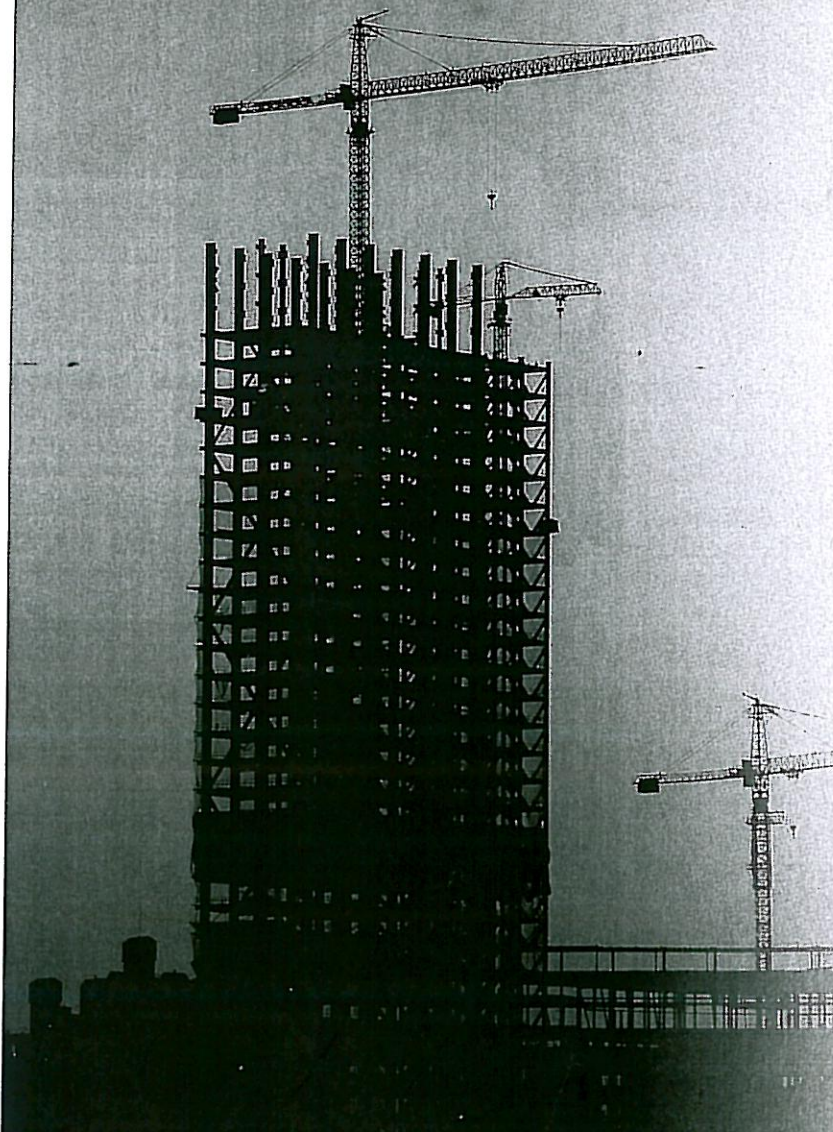


Stable improvements

Direct analysis method
for stability design
of steel-framed buildings

By Gregory Deierlein, Ph.D., P.E.



When published next year, the 2005 edition of the American Institute of Steel Construction's (AISC) Specification for Structural Steel Buildings will introduce a new alternative for the stability design of steel structures, called the *direct analysis method*. These new provisions will eliminate the requirement to calculate effective buckling length factors (*K*-factors) for column design. Instead, the provisions emphasize the fundamental tenet of stability design — that the structure must have sufficient strength and stiffness to sustain internal forces generated by the applied loads acting upon the deformed structural geometry. To do this, the direct analysis provisions use second-order (geometrically nonlinear) elastic analysis and include supplemental provisions to account for the destabilizing effects of geometric imperfections, material yielding, and other factors not explicitly modeled in analysis. The new provisions also provide a more consistent treatment of stability effects for moment frames, braced frames, and combined systems which will facilitate the design of innovative structural forms.

This article will briefly discuss the history of stability design provisions, describe the direct analysis method, contrast the method to the effective length method that is currently used, illustrate the new method with a design example, and touch upon a few practical applications.

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Historical perspective on stability design provisions

Provisions for designing columns based upon their effective buckling lengths, rather than their actual unbraced lengths, were introduced into the 1961 edition of the AISC Specification in recognition of the interaction of member and system stability effects. Derived from elastic buckling theory, the effective length provisions provided an approximate technique for equating the critical load of a column with indeterminate boundary conditions to an equivalent column (or so-called Euler column) with pin-ended boundary conditions. Graphical alignment charts and other techniques, such as critical load analyses, have been used to solve for effective buckling lengths, but all require simplifying assumptions that are rarely met in construction. Over the past 40 years, many modifications to the original effective length procedures have been proposed to overcome these simplifying assumptions. Many of these developments are summarized in a comprehensive report by the American Society of Civil Engineer's (ASCE) Task Committee on Effective Length titled, "Effective Length and Notional Load Approaches for Assessing Frame Stability," which was published in 1997.

Effective length procedures were introduced at a time when approximate methods or moment-distribution were the norm for calculating load effects — both member and connection forces — in structures. Computerized linear-elastic (first-order elastic) methods have increased the speed of analysis, but all of these methods share the common trait of solving for equilibrium on the undeformed geometry of the structure. With the advent of computerized second-order analysis in the 1970s, design approaches began to place greater reliance on modeling stability effects directly in the calculation of required member strengths. Second-order analysis was formally incorporated as the basis for design in the first edition of the AISC Load and Resistance Factor Design Specification for Structural Steel Buildings published in 1986; and the American Concrete Institute's (ACI-318) Building Code Requirements for Structural Concrete introduced second-order analysis design provisions in 1995. Through the use of second-order analysis, design provisions have become less reliant on effective column buckling length calculations. The direct analysis method extends these developments to fully eliminate the use of effective length requirements for frame design. Key attributes of the direct analysis and effective length methods are contrasted in Table 1.

Direct analysis method for stability design

The direct analysis method begins with the basic requirement that internal member and connection forces are determined using an accurate second-order elastic analysis. The second-order analysis can be accomplished using a computer program with second-order capabilities (one that considers both frame effects and member effects) or by amplifying the results of a first-order analysis by using the B1-B2 amplifier method in the AISC Specification, for example. Since second-order effects are nonlinear, the second-order analyses must be conducted under factored load and resistance factor design (LRFD) load combinations or allowable stress design load combinations amplified by the factor of safety.

To account for the destabilizing effects of initial geometric imperfections and other physical variations between the actual structure and the idealized model, the direct analysis provisions require the application of a notional load, equal to 0.2 percent of the gravity load, in conjunction with the other applied loads. The notional load magnitude roughly corresponds to a frame out-of-plumb ratio of 1/500, which is the maximum value permitted in the AISC Code of Standard Practice.

To further account for system reliability and inelastic effects under the factored loads, the effective elastic stiffness of members that comprise the lateral load resisting system are reduced from their nominal values for the second-order analysis. As summarized in Table 1, the effective flexural stiffness, EI , and axial stiffness, EA , are reduced to $0.8\tau EI$ (where τ is the stiffness reduction factor defined in Table 1) and $0.8EA$, respectively. The reduced stiffness coefficients are justified by considering structures at limiting ends of the slenderness spectrum. For frames with slender members, where the governing limit state is elastic instability, the

VARIABLES

EI	— nominal elastic flexural member stiffness
EA	— nominal elastic axial member stiffness
K	— effective column buckling length factor
L	— unbraced member length
M_2/M_1	— ratio of second-order to first-order moments
M_u	— LRFD factored moment
M_n	— nominal member moment strength
P_n	— nominal column compressive strength
$P_{n,L}$	— nominal column compressive strength for unbraced length L
$P_{n,KL}$	— nominal column compressive strength for unbraced length KL
P_u	— LRFD factored column compressive load
P_y	— nominal member yield strength
ϕ	— LRFD resistance factor
τ	— inelastic stiffness reduction factor

0.8 stiffness adjustment provides roughly an equivalent margin of safety for frames as implied by current design provisions for slender columns. For frames with intermediate or stocky columns, the 0.8τ factor reduces the stiffness to account for inelastic softening prior to the members reaching their design strength. The τ factor is similar to the inelastic stiffness reduction factor implied in the AISC column curve to account for loss of stiffness under high-compression loads, such that $P_u > 0.5P_y$, and the 0.8 factor accounts for additional softening under combined axial compression and bending. It is a fortuitous coincidence that the reduction coefficients for the slender and stocky columns are close enough, such that the 20-percent stiffness reduction is reasonable over the full range of frame slenderness.

Axial forces and moments determined by the second-order analysis are applied in

Table 1: Attributes of the effective length method versus the direct analysis method.

Attribute	Effective length method	Direct analysis method
Analysis type	second-order elastic	second-order elastic
Notional load	None*	$0.002P_u$
Effective stiffness	Nominal	$EI_{eff} = 0.8\tau EI$ and $EA_{eff} = 0.8EA$ where, $\tau = 1$ for $P_u \leq 0.5P_y$, or $\tau = 4[P_u/P_y(1 - P_u/P_y)]$ for $P_u > 0.5P_y$
Axial strength, P_n	$P_{n,KL}$ based on KL	$P_{n,L}$ based on L

* The 2005 AISC Specification will introduce a minimum notional load for structures for cases where the lateral load requirements are small or non-existent.

The direct analysis method eliminates the need for effective length buckling calculations.



conventional member force interaction equations to check the in-plane flexural buckling and out-of-plane torsional-flexural buckling limit states in beam-columns. A key distinction between the direct analysis and the effective length methods is in how the nominal column strength term, P_n , is computed. For example, the flexural, in-plane limit state for beam-columns is given by the following LRFD equations:

(Eq. 1a)

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left(\frac{M_u}{\phi_b M_n} \right) \leq 1.0 \quad \text{for} \quad \frac{P_u}{\phi_c P_n} \geq 0.2$$

(Eq. 1b)

$$\frac{P_u}{2\phi_c P_n} + \frac{M_u}{\phi_b M_n} \leq 1.0 \quad \text{for} \quad \frac{P_u}{\phi_c P_n} < 0.2$$

where P_u and M_u are the required strengths, calculated from second-order analysis under the design loads; P_n and M_n are the nominal compression and bending strengths, calculated in the plane of the frame; and ϕ_c and ϕ_b are the resistance factors (which are both equal to 0.9 in the 2005 AISC Specification). For the effective length method, P_n is determined using the effective buckling length KL in the plane of bending, whereas in the direct analysis method, P_n is calculated using $K=1$, therefore $KL=L$, in the plane of bending. For compact shapes that are braced out-of-plane, M_n is equal to the plastic moment M_p in both methods.

Contrasting direct analysis and effective length methods

Differences between the effective length and the direct analysis approaches lie mainly in the in-plane interaction check. Figure 1a shows a plot of the in-plane interaction equation for the effective length approach, where the anchor point on the vertical axis, P_{nKL} , is determined using an effective buckling length factor.

The two nonlinear plots represent the change in axial force and internal second-order moment in the column under increasing applied load. The response plot labeled "actual response" is determined from a second-order, spread-of-plasticity analysis, whereas the second plot is from a second-order elastic analysis. The "actual response" curve reveals larger moments than the second-order elastic curve due to the combined effects of partial yielding and geometric imperfections, which are not included in the elastic second-order analysis. The plateau in the "actual response" curve corresponds to the maximum axial force, P_u , that the member can sustain prior to the onset of instability. The design strength is defined by the intersection of the force point trace of the elastic second-order analysis curve with the P_{nKL} interaction envelope. The effective length provisions have been calibrated such that the resultant design axial strength, P_u , at the point of intersection with the interaction equations coincides with the actual strength.

The reduced stiffness and notional load provisions of the direct analysis method are calibrated to simulate the actual response using an elastic second-order analysis response. This is shown in Figure 1b, where the force point trace of internal axial load and moment determined

by the elastic second-order analysis (conducted with the reduced stiffness and notional load) is close to the actual response plot. The calibration is done to achieve parity between the actual compression strength (determined by the plateau in the axial response plot) with the design strength. By accounting for more of the destabilizing effects in the analysis, the direct analysis strength check is made using an interaction curve based on P_{nL} , for example based on the actual unbraced member length L . In this method, the direct analysis provisions for the second-order analysis take account of system stability effects, while the interaction check accounts for member stability effects — including, for example, member out-of-straightness and torsional-flexural response, which are not simulated in the analysis.

Illustrative example

To illustrate an application of the two stability design methods, both methods are applied to the cantilever beam-column shown in Figure 2. The cantilever is subjected to the vertical and proportional horizontal load shown. Bending is about the major axis, and there is out-of-plane restraint provided along the full member length. The column slenderness, L/r , is equal to 40. Using LRFD with the in-plane interaction

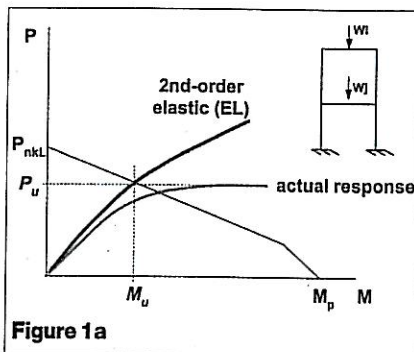


Figure 1a

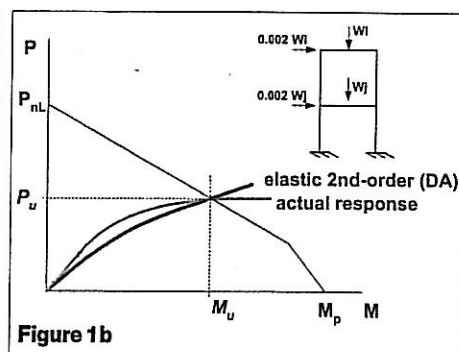


Figure 1b

Figure 1: These graphs represent a comparison of beam-column interaction checks for (1a) the effective length approach and (1b) the direct analysis approach.

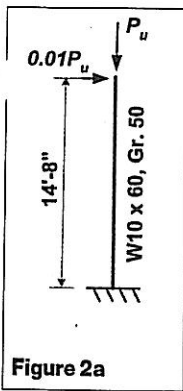


Figure 2a

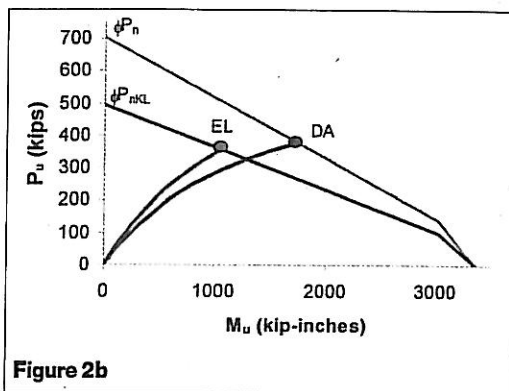


Figure 2b

Table 2: This compares the calculated strengths of a cantilever column designed with both analysis methods.

Analysis method	P_u (kips)	M_n (kip-inch)	M_2/M_1	$P_u/\phi P_n$
Effective length	357	1,061	1.69	0.72
Direct analysis	380	1,753	2.18	0.54

Figure 2 (left): For the cantilevered column example shown, the graph compares the axial load versus moment at the column base for the effective length (EL) and the direct analysis (DA) method.

check of Eq. 1, the maximum design strength is determined based on the critical combination of axial load and bending at the column base.

Shown in Figure 2 are plots of the axial load versus moment at the column base, determined using the direct analysis (DA) and effective length (EL) methods. Notice that the internal moments M_u increase faster with P_u for the direct analysis method. This is because of the reduced stiffness of the member ($0.8\tau EI$) and the notional lateral load of $0.002P_u$ that is added to the external load of $0.01P_u$. Overlaid on these force-point traces are the beam-column strength interaction diagrams, where the ϕP_n anchor point for the effective length method is $\phi P_{n, KL} = 496$ kips is based upon $\lambda L = 2L$. For the direct analysis method, $\phi P_{n, L} = 704$ kips is based upon L .

The calculated strengths, as determined by the maximum combination of P_u and M_u that satisfy the in-plane interaction check, are summarized in Table 2. The ratios of maximum base moments calculated by second-order versus first-order analysis (shown as M_2/M_1 in Table 2) provide an indication of the geometric nonlinearity. Comparing the axial load strengths, which are representative of the total applied load, the strength determined by direct analysis is approximately 6 percent larger than the strength calculated by the effective length method. The difference in the calculated moments is much larger. Referring back to the comparison in Figure 1, this difference is expected since the effective length procedure compensates for underestimation of the actual moment by reducing the design axial strength, whereas the direct analysis method imposes requirements on the analysis to calculate the moments more accurately. This is further evident in the difference in column axial strength ratios (shown as $P_u/\phi P_n$ in Table 2) that contribute to the interaction checks.

Subject to the assumed geometric imperfections, such as out-of-plumbness and residual stresses, the moments calculated by the direct analysis procedure are generally conservative and



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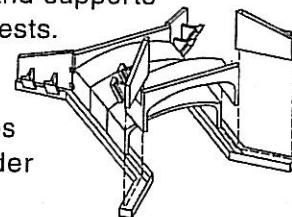
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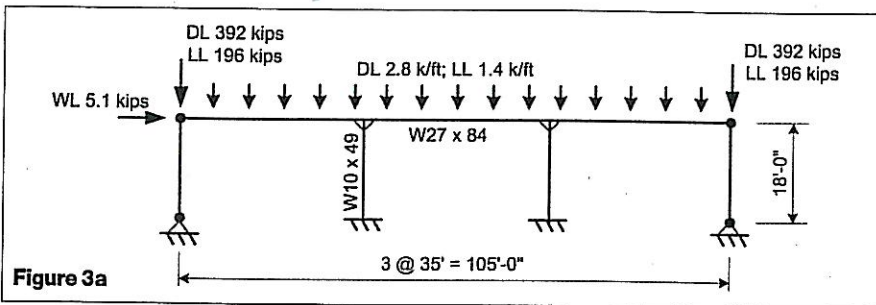


Figure 3a

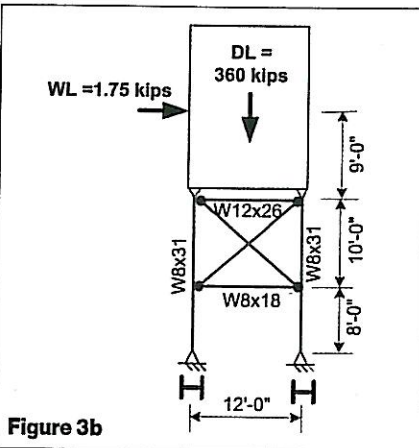


Figure 3b

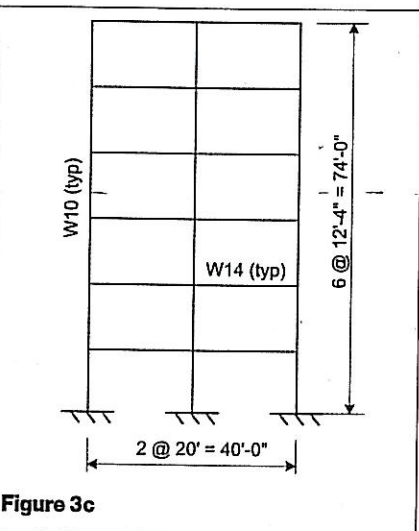


Figure 3c

Figure 3: Design applications for the direct analysis test methods included (a) low-rise frame, (b) a support frame, and (c) a multi-story frame.

closer to the true values. The large difference in the calculated internal moments highlights a significant benefit of the direct analysis method, since the more accurate calculation of the internal moments facilitates straightforward design of restraining members and their connections. For instance, in the cantilever column example, the base moments from the direct analysis procedure take into account initial out-of-plumb and inelastic second-order

effects, which are not captured in the effective length procedure. In this sense, the direct analysis procedure provides a direct measure of required strengths of all members, whereas the effective length approach necessitates supplementary requirements to design restraining members for adequate bracing forces.

Design applications

Shown in Figure 3 are three design applications that the development team examined as part of the development process of the direct analysis method. The frame in Figure 3a represents a framing bent from a large floor plan, single-story industrial building, such as an automobile plant. With heavy material handling equipment hung from the roof, a small wind exposure, and a high fraction of leaning columns, such structures are dominated by gravity loads with large second-order effects. The second example, shown in Figure 3b, is a support rack for a grain storage bin. This is a case where calculation of the column effective lengths is not obvious and the direct analysis method is much more straightforward to apply. The third example, shown in Figure 3c, is a multi-story frame that has been used in various stability benchmark studies and is representative of modern building construction. All three structures were designed to meet strength criteria outlined above and stiffness criteria to limit service load drift ratios to 1/400. The high gravity loading and configurations of the first two examples resulted in large second-order amplification factors, on the order of 1.7 to 2.5, whereas the amplifier in the third example was approximately 1.1. This difference highlights the fact that low-rise structures with high gravity-to-lateral load ratios are often more stability sensitive than multi-story structures and warrant careful assessment of second-order effects.

In general, the design applications revealed similar trends to those in the cantilever column example. The direct analysis method was more straightforward to apply, provided more

realistic values of the internal member forces, and resulted in more efficient designs. The differences were particularly apparent in the first two examples of Figure 3, where significant second-order effects dominated the response and resulting design checks.

Conclusion

By taking a more explicit account of geometric nonlinearities, initial geometric imperfections, and inelastic effects in analysis, the direct analysis method provides a transparent approach to stability design that is applicable to a broad range of structural types. This, together with elimination of the need for effective length buckling calculations, makes the procedure straightforward to apply in design. Perhaps most importantly, it provides a new framework for stability design that is more amenable to the use of inelastic analysis for applications in performance-based design of structures for earthquakes, fires, and other extreme loads where post-elastic response must be considered. ■

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Acknowledgments

The developments summarized in this report are the result of contributions by many individuals over the course of deliberations of a joint AISC Structural Stability Research Council Ad Hoc Committee on Frame Stability and AISC Task Committee 10 on Stability. Portions of this paper are excerpted from an internal report by Gregory Deierlein, Ph.D., P.E., to AISC Task Committee 10, "Background and Illustrative Examples on Proposed Direct Analysis Method for Stability Design of Moment Frames," published in July 2003. Further background and information on the direct analysis method will be available in the commentary to the 2005 AISC Specification and other supporting documents currently under preparation.