4 Symmetrical Components: A Review

4.1 INTRODUCTION AND BACKGROUND

The method of symmetrical components provides a practical technology for understanding and analyzing the operation of a system during power unbalanced conditions, such as those caused by faults between phases and ground, open phases, unbalanced impedances, and so on. In addition, many protective relays operate from symmetrical component quantities. Thus, a good understanding of this subject is of great value and another very important tool in protection.

In a sense, symmetrical components can be called the language of the relay engineer or technician. Its value is both in thinking or visualizing unbalances, and it is a means of detailed analysis of them from the system parameters. In this, it is like a language in that it requires experience and practice for each access and application. Faults and unbalances occur infrequently and many do not require detailed analysis, so it becomes difficult to practice the language. This has increased with the ready availability of fault studies by computers. These provide rapid access to voluminous data, often with little understanding of the background or method that provides the data. Hence, this review of the method is intended to provide the fundamentals, basic circuits and calculations, and an overview directed at clear understanding and visualization.

The method of symmetrical components was discovered by Charles L. Fortescue, who was mathematically investigating the operation of induction motors under unbalanced conditions, late in 1913. At the 34th Annual Convention of the AIEE—on June 28, 1918, in Atlantic City—he presented an 89-page paper entitled "Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks." The six discussants, including Charles Proteus Steinmetz, added 25 pages. Practical application for system fault analysis was developed by C.F. Wagner and R.D. Evans in the later part of 1920s and early 1930s, with W.A. Lewis adding valuable simplifications in 1933. Tables of fault and unbalance connections were provided by E.L. Harder in 1937. At the same time Edith Clarke was also developing notes and lecturing in this area, but formal publication of her work did not occur until 1943. Additional material and many examples for further study are found in Blackburn (1993).

Only symmetrical components for three-phase systems are reviewed in this chapter. For these systems there are three distinct sets of components: positive, negative, and zero for both current and voltage. Throughout this discussion, the sequence quantities are always line-to-neutral or line-to-ground and appropriate to the situation. This is an exception for voltage connections, whereas while in the power system line-to-line voltages are commonly indicated, in symmetrical components they are always given as line-to-neutral (or possibly line-to-ground).

4.2 POSITIVE-SEQUENCE SET

The positive-sequence set consists of balanced three-phase currents and line-to-neutral voltages supplied by the system generators. Thus, they are always equal in magnitude and are phase-displaced by 120°C. Figure 4.1 shows a positive-sequence set of phase currents, with the power system phase sequence in the order of *a*, *b*, *c*. A voltage set is similar, except for line-to-neutral voltage of the three phases, with equal magnitude and which displaces at 120°C. These are phasors that rotate in the counterclockwise direction at the system frequency.

To document the angle displacement, it is convenient to use a unit phasor with an angle displacement of 120°. This is designated as *a* so that

$$\begin{aligned} a &= 1\angle 120^\circ = -0.5 + j0.866 \\ a^2 &= 1\angle 240^\circ = -0.5 - j0.866 \\ a^3 &= 1\angle 360^\circ = 1\angle 0^\circ = 1.0 + j0. \end{aligned} \tag{4.1}$$

Therefore, the positive-sequence set can be designated as

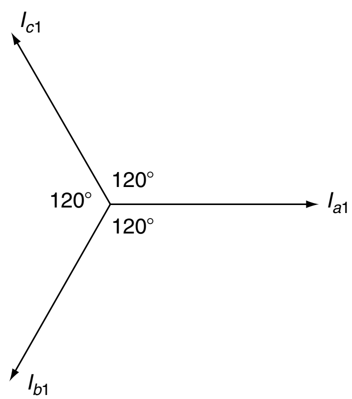

FIGURE 4.1 Positive-sequence current phasors. Phasor rotation is counterclockwise.

$$I_{a1} = I_1 \quad V_{a1} = V_1,$$
$$I_{b1} = a^2 I_{a1} = a^2 I_1 = I_1 \angle 240^\circ \quad V_{b1} = a^2 V = V_1 \angle 240^\circ,$$
$$I_{c1} = a I_{a1} = a I_1 = I_1 \angle 120^\circ \quad V_{c1} = a V_1 = V_1 \angle 120^\circ. \tag{4.2}$$

It is most important to emphasize that the set of sequence currents or sequence voltages always exists as defined. The phasors I_{a1} or I_{b1} or I_{c1} can never exist alone or in pairs, but always as a set of three. Thus, it is necessary to define only one of the phasors (any one) from which the other two will be as documented in Equation 4.2.

4.3 NOMENCLATURE CONVENIENCE

It will be noted that the designation subscript for phase *a* was dropped in the second expression for the currents and voltages in Equation 4.2 (and also in the following equations). This is a common shorthand notation used for convenience. When the phase subscript is not mentioned, it can be assumed that the reference is to phase *a*. If phase *b* or phase *c* quantities are intended, the phase subscript must be correctly designated; otherwise, it is assumed as phase *a*. This shortcut will be used throughout the book and is common in practice.

4.4 NEGATIVE-SEQUENCE SET

The negative-sequence set is also balanced with three equal magnitude quantities at 120° separately, but only when the phase rotation or sequence is reversed as illustrated in Figure 4.2. Thus, if positive sequence is *a*, *b*, *c*; negative will be *a*, *c*, *b*. When positive sequence is *a*, *c*, *b*, as in some power systems; negative sequence is *a*, *b*, *c*.

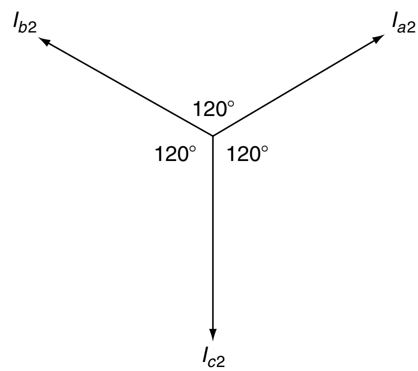

FIGURE 4.2 Negative-sequence current phasors. Phasor rotation is counterclockwise.

The negative-sequence set can be designated as

$$I_{a2} = I_2 \; V_{a2} = V_2,$$
$$I_{b2} = aI_{a2}aI_2 = I_2 \angle 120^\circ \;\; V_{b2} = aV_2 = V_2 \angle 120^\circ,$$
$$I_{c2} = a^2I_{a2}a^2I_2 = I_2 \angle 240^\circ \; V_{c2} = a^2V_2 = V_2 \angle 240^\circ. \tag{4.3}$$

Again, negative sequence always exists as a set of current or voltage as defined in the foregoing or as shown in Figure 4.2: I_{a2} or I_{b2} or I_{c2} can never exist alone. When one current or voltage phasor is known, the other two of the set can be defined as mentioned earlier.

4.5 ZERO-SEQUENCE SET

The members of this set of rotating phasors are always equal in magnitude and exist in phase (Figure 4.3).

$$I_{a0} = I_{b0} = I_{c0} = I_0 V_{a0} = V_{b0} = V_{c0} = V_0. \tag{4.4}$$

Similarly, I_0 or V_0 exists equally in all three phases, but never alone in a phase.

4.6 GENERAL EQUATIONS

Any unbalanced current or voltage can be determined from the sequence components given in the following fundamental equations:

$$I_a = I_1 + I_2 + I_0, \quad V_a = V_1 + V_2 + V_0, \tag{4.5}$$

$$I_b = a^2I_1 + aI_2 + I_0, \quad V_b = a^2V_1 + aV_2 + V_0, \tag{4.6}$$

$$I_c = aI_1 + a^2I_2 + I_0, \quad V_c = aV_1 + a^2V_2 + V_0, \tag{4.7}$$

where I_a, I_b, and I_c or V_a, V_b, and V_c are general unbalanced line-to-neutral phasors.

From these, equations defining the sequence quantities from a three-phase unbalanced set can be determined:

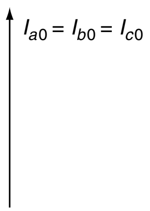

FIGURE 4.3 Zero-sequence current phasors. Phasor rotation is counterclockwise.

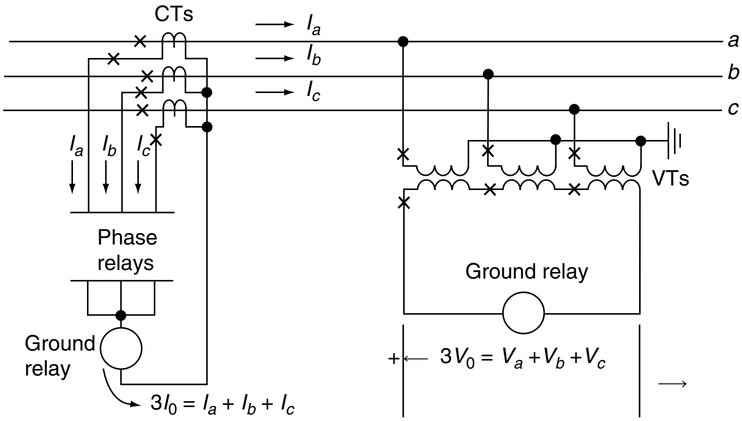

FIGURE 4.4 Zero-sequence current and voltage networks used for ground-fault protection. See Figure 3.9 and Figure 3.10 for typical fault operations.

$$I_0 = \tfrac{1}{3}(I_a + I_b + I_c), \quad V_0 = \tfrac{1}{3}(V_a + V_b + V_c), \tag{4.8}$$

$$I_1 = \tfrac{1}{3}(I_a + aI_b + a^2I_c), \quad V_1 = \tfrac{1}{3}(V_a + aV_b + a^2V_c), \tag{4.9}$$

$$I_2 = \tfrac{1}{3}(I_a + a^2I_b + aI_c), \quad V_2 = \tfrac{1}{3}(V_a + a^2V_b + aV_c). \tag{4.10}$$

These three fundamental equations are the basis for determining if the sequence quantities exist in any given set of unbalanced three-phase currents or voltages. They are used for protective-relaying operations from the sequence quantities. For example, Figure 4.4 shows the physical application of current transformers (CTs) and voltage transformers (VTs) to measure zero sequence as required in Equation 4.8 and as used in ground-fault relaying.

Networks operating from CTs or VTs are used to provide an output proportional to I_2 or V_2 and are based on physical solutions (Equation 4.10). This can be accomplished with resistors, transformers, or reactors, by digital solutions of Equation 4.8 through Equation 4.10.

4.7 SEQUENCE INDEPENDENCE

The factor that makes the concept of dividing the unbalanced three-phase quantities into the sequence components practical is the independence of the components in a balanced system network. For all practical purposes, electric power systems are balanced or symmetrical from the generators to the point of single-phase loading, except in an area of a fault or unbalance, such as an open conductor. In this effectively balanced area, the following conditions exist:

1. Positive-sequence currents flowing in the symmetrical or balanced network produce only positive-sequence voltage drops, no negative- or zero-sequence drops.
2. Negative-sequence currents flowing in the balanced network produce only negative-sequence voltage drops, no positive- or zero-sequence voltage drops.
3. Zero-sequence currents flowing in the balanced network produce only zero-sequence voltage drops, no positive- or negative-sequence voltage drops.

This is not true for any unbalanced or nonsymmetrical point or area, such as an unsymmetrical fault, open phase, and so on.

4. Positive-sequence current flowing in an unbalanced system produces positive-, negative-, and possibly zero-sequence voltage drops.
5. Negative-sequence currents flowing in an unbalanced system produces positive-, negative-, and possibly zero-sequence voltage drops.
6. Zero-sequence current flowing in an unbalanced system produces all three: positive-, negative-, and zero-sequence voltage drops.

This important fundamental condition permits setting up three independent networks, one for each of the three sequences, which can be interconnected only at the point or area of unbalance. Before continuing with the sequence networks, a review of the source of fault current is useful.

4.8 POSITIVE-SEQUENCE SOURCES

A single-line diagram of the power system or area under study is the starting point for setting up the sequence networks. A typical diagram for a section of a power system is shown in Figure 4.5. In these diagrams, circles are used to designate the positive-sequence sources, which are the rotating machines in the system; generators, synchronous motors, synchronous condensers, and probably induction motors. The symmetrical current supplied by these to the power-system faults decreases exponentially with time from a relatively high initial value to a low steady-state value. During this transient period three reactance values are possible for use in the positive-sequence network and for the calculation of fault currents. These are the direct-axis subtransient reactance X''_d, the direct-axis transient reactance X'_d, and the unsaturated direct-axis synchronous reactance X_d.

The values of these reactances vary with the designs of the machines and the specific values are supplied by the manufacturer. In their absence, typical

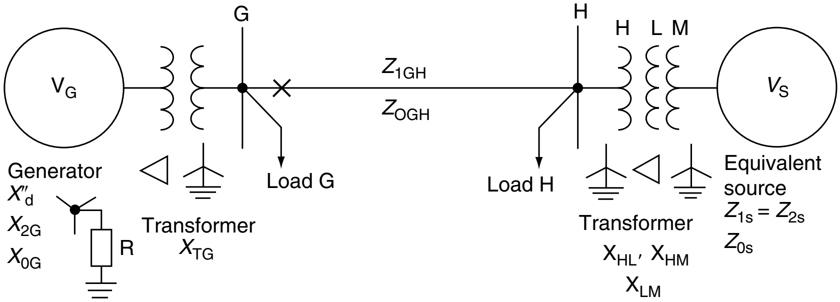

FIGURE 4.5 Single-line diagram of a section of a power system.

values are shown in Blackburn (1993, p. 279) and in many other references. Generally, typical values at the machines rated MVA (kVA) and kV are: $X''_d = 0.1$ to 0.3 pu, with time constants of about 0.35 sec; $X'_d = 1.2 - 2.0$ times X''_d, with time constants in the order of 0.6–1.5 sec; X_d for faults is the unsaturated value that can range from 6 to 14 times X''_d.

For system-protection fault studies, the almost universal practice is to use the subtransient (X''_d) for the rotating machines in the positive-sequence networks. This provides a maximum value of fault current that is useful for high-speed relaying. Although slower-speed protection may operate after the subtransient reactance has decayed into the transient reactance period, the general practice is to use X'_d, except possibly for special cases where X'_d would be used. There are special programs to account for the decremental decay in fault current with time in setting the slower-speed protective relays, but these tend to be difficult and tedious, and may not provide any substantial advantages. A guide to aid in the understanding of the need for special considerations is outlined in Figure 4.6. The criteria are very general and approximate.

Cases A and B (see Figure 4.6) are the most common situations, so that the use of X''_d has a negligible effect on the protection. Here the higher system Z_s tends to negate the source decrement effects.

Case C (see Figure 4.6) can affect the overall operation time of a slower-speed protection, but generally the decrease in fault current level with time will not cause coordination problems unless the time–current characteristics of various devices that are used are significantly different. When Z_M predominates, the fault levels tend to be high and well above the maximum-load current. The practice of setting the protection as sensitive as possible, but not operating on maximum load (phase devices) should provide good protection sensitivity in the transient reactance period. If protection-operating times are very long, such that the current decays into the synchronous reactance period, special phase relays are required, as discussed in Chapter 8.

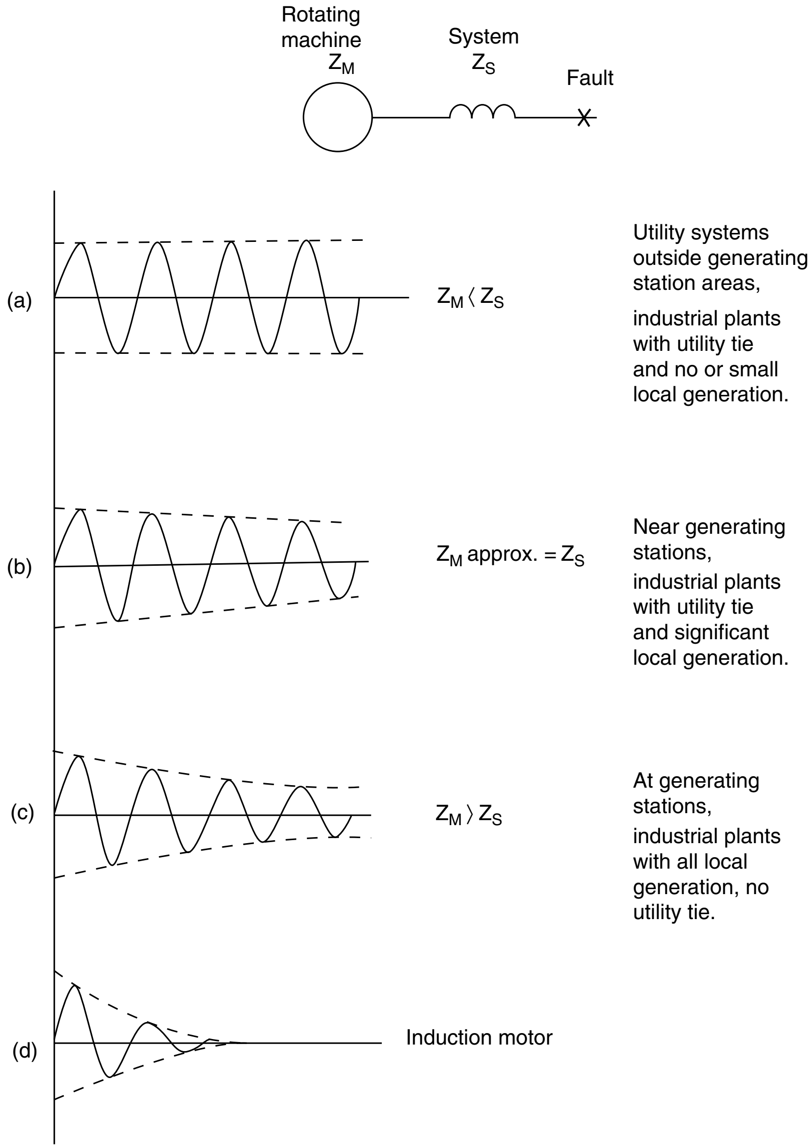

FIGURE 4.6 Guide illustrating the effects of rotating machine decrements on the symmetrical fault current.

Usually, induction motors are not considered as sources of fault current for protection purposes (see Figure 4.6, case D). However, it must be emphasized that these motors must be considered in circuit breakers' applications under the ANSI/IEEE standards. Without a field source, the voltage that is developed by induction motors decays rapidly, within a few cycles; thus, they generally have a negligible effect on the protection. The DC offset that can result from sudden changes in current in the ac networks is neglected in symmetrical components. It is an important consideration in all protection.

An equivalent source, such as that shown in Figure 4.5, represents the equivalent of all the systems that are not shown up to the point of connection to that part of the system under study. This includes one or many rotating machines that may be interconnected together with any network of transformers, lines, and so on. In general, a network system can be reduced to two equivalent sources at each end of an area to be studied, with an equivalent interconnecting tie between these two equivalent sources. When the equivalent tie is large or infinite, indicating that little or no power is exchanged between the two source systems, it is convenient to express the equivalent source system up to a specified bus or point in short-circuit MVA (or kVA). Appendix 4.1 outlines this and the conversion to the impedance or the reactance values. In Figure 4.5, the network to the right has reduced to a single equivalent impedance to represent it up to the M terminal of the three-winding transformer bank.

4.9 SEQUENCE NETWORKS

The sequence networks represent one of the three-phase-to-neutral or to-ground circuits of the balanced three-phase power system and document how their sequence currents will flow if they can exist. These networks are best explained by an example: let us now consider the section of a power system in Figure 4.5.

Reactance values have been indicated only for the generator and the transformers. Theoretically, impedance values should be used, but the resistances of these units are small and negligible for fault studies. However, if loads are included, impedance values should be used unless their values are small in relation to the reactances.

It is important that all values should be specified with a base [voltage if ohms are used, or MVA (kVA) and kV if per-unit or percent impedances are used]. Before applying these to the sequence networks, all values must be changed to one common base. Usually, per-unit (percent) values are used, and a common base in practice is 100 MVA at the particular system kV.

4.9.1 Positive-Sequence Network

This is the usual line-to-neutral system diagram for one of the three symmetrical phases modified for fault conditions. The positive-sequence networks for the system in Figure 4.5 are shown in Figure 4.7. The voltages V_G and V_S are the system line-to-neutral voltages. V_G is the voltage behind the generator subtransient direct-axis reactance X'_d, and V_S is the voltage behind the system equivalent impedance Z_{1S}.

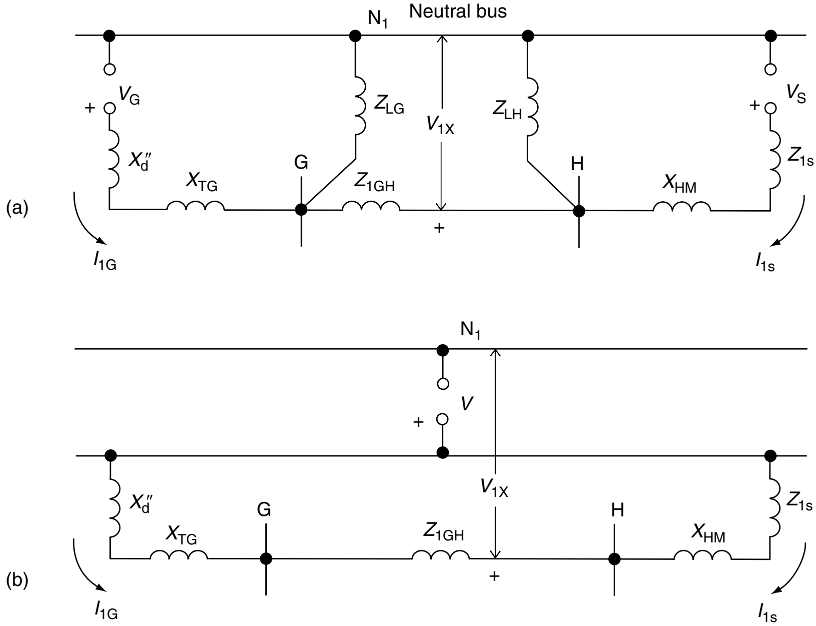

FIGURE 4.7 Positive-sequence networks for the system in Figure 4.5: (a) network including loads; (b) simplified network with no load—all system voltages equal and in phase.

X_{TG} is the transformer leakage impedance for the bank bus G, and X_{HM} is the leakage impedance for the bank at H between the H and M windings. More details on these are given in Appendix 4.2. The delta-winding L of this three-winding bank is not involved in the positive-sequence network unless a generator or synchronous motor is connected to it or unless a fault is to be considered in the L delta system. The connection would be as in Figure A4.2-3.

For the line between buses G and H, Z_{1GH} is the line-to-neutral impedance of this three-phase circuit. For open-wire transmission lines, an approximate estimating value is 0.8 Ω/mi for single conductor and 0.6 Ω/mi for bundled conductors. Typical values for shunt capacitance of these lines are 0.2 MΩ/mi for single conductor and 0.14 MΩ/mi for bundled conductors. Normally, this capacitance is neglected, as it is very high in relation to all other impedances that are involved in fault calculations. These values should be used either for estimating or in the absence of specific line constants. The impedances of cables vary considerably, so specific data are necessary for these.

The impedance angle of lines can vary quite widely, depending on the voltage and type of cable or open wire that is used. In computer fault programs, the angles are considered and included, but for hand calculation, it is often practical and convenient to simplify calculations by assuming that all the equipment involved in the fault calculation is at 90°. Otherwise, it is better to use reactance values only. Sometimes it may be preferred to use the line impedance values and treat them as reactances. Unless the network consists of a large proportion of low-angle circuits, the error of using all values as 90° will not be too significant.

Load is shown to be connected at buses G and H. Normally, this would be specified as kVA or MVA and can be converted into impedance.

$$I_{\text{load}} = \frac{1000\ \text{MVA}_{\text{load}}}{\sqrt{3}\text{kV}} \quad \text{and} \quad V_{\text{LN}} = \frac{1000\ \text{kV}}{\sqrt{3}}$$

$$Z_{\text{load}} = \frac{V_{\text{LN}}}{I_{\text{load}}} = \frac{\text{kV}^2}{\text{MVA}_{\text{load}}} = \text{ohms at kV.} \tag{4.11}$$

Equation 4.11 is a line-to-neutral value and could be used for Z_{LG} and Z_{LH}, representing the loads at G and H as shown in Figure 4.7a. If load is represented, the voltages V_{G} and V_{S} will be different in magnitude and angle, varying according to the system load.

The value of load impedance is usually quite large compared with the system impedances, such that the load has a negligible effect on the faulted-phase current. Thus, it becomes practical and simplifies the calculations to neglect load for shunt faults. With no load, Z_{LG} and Z_{LH} are infinite. V_{G} and V_{S} are equal and in phase, and so they are replaced by a common voltage V as in Figure 4.7b. Normally, V is considered as 1 pu, the system-rated line-to-neutral voltages.

Conventional current flow is assumed to be from the neutral bus N_1 to the area or point of unbalance. With this the voltage drop V_{1x} at any point in the network is always

$$V_{1x} = V - \sum I_1 Z_1, \tag{4.12}$$

where V is the source voltage (V_{G} or V_{S} in Figure 4.7a) and $\Sigma I_1 Z_1$ is the sum of the drops along any path from the N_1 neutral bus to the point of measurement.

4.9.2 Negative-Sequence Network

The negative-sequence network defines the flow of negative-sequence currents when they exist. The system generators do not generate negative sequence, but negative-sequence current can flow through their windings. Thus, these generators and sources are represented by an impedance without voltage, as shown in Figure 4.8. In transformers, lines, and so on, the phase

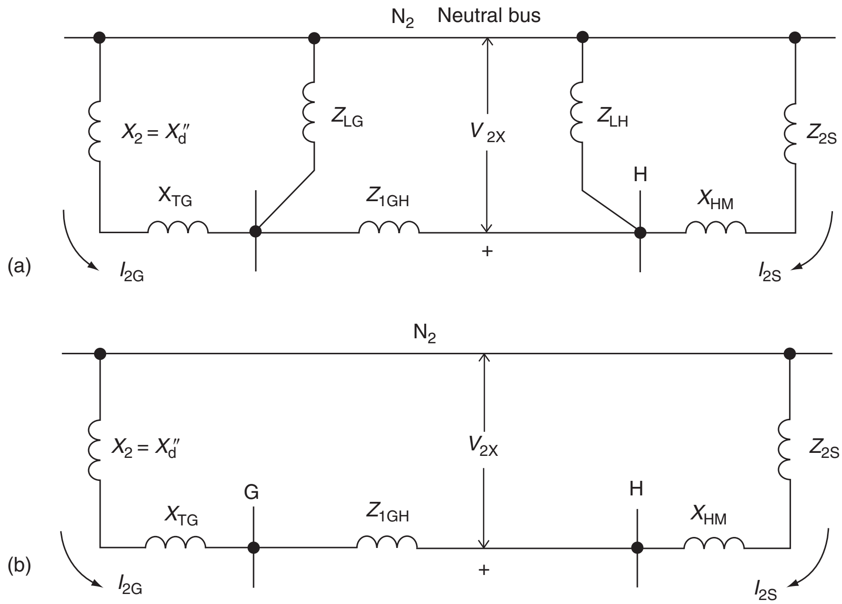

FIGURE 4.8 Negative-sequence networks for the system in Figure 4.5: (a) network including loads; (b) network neglecting loads.

sequence of the current does not change the impedance encountered; hence, the same values as in the positive-sequence network are used.

A rotating machine can be visualized as a transformer with one stationary and one rotating winding. Thus, DC in the field produces positive sequence in the stator. Similarly, the DC offset in the stator ac current produces an ac component in field. In this relative-motion model, with the single winding rotating at synchronous speed, negative sequence in the stator results in a double-frequency component in the field. Thus, the negative-sequence flux component in the air gap alternates between and under the poles at this double frequency. One common expression for the negative-sequence impedance of a synchronous machine is

$$X_2 = \frac{1}{2}\left(X_d'' + X_q''\right) \qquad (4.13)$$

or the average of the direct and substransient reactance of quadrature axes. For a round-rotor machine, $X_d'' = X_q''$, so that $X_2 = X_d''$. For salient-pole machines, X_2 will be different, but this is frequently neglected unless calculating a fault very near the machine terminals. Where normally $X_2 = X_d''$, the negative-sequence network is equivalent to the positive-sequence network except for the omission of voltages.

Loads can be shown, as in Figure 4.8a, and will be the same impedance as that for positive sequence, provided they are static loads. Rotating loads, such as those of induction motors, have quite a different positive- and negative-sequence impedances when in operation. This is discussed further in Chapter 11.

Similarly, when the load is normally neglected, the network is as shown in Figure 4.8b and is the same as the positive-sequence network (see Figure 4.7b), except that there is no voltage.

Conventional current flow is assumed to be from the neutral bus N_2 to the area or point of unbalance. With this the voltage drop V_{2x} at any point in the network is always

$$V_{2x} = 0 - \sum I_2 Z_2, \tag{4.14}$$

where $\sum I_2 Z_2$ is the sum of the drops along any path from the N_2 neutral bus to the point of measurement.

4.9.3 Zero-Sequence Network

The zero-sequence network is always different. It must satisfy the flow of equal and in-phase currents in the three phases. If the connections for this network are not apparent, or in doubt, these can be resolved by drawing the three-phase system to see how the equal in-phase, zero-sequence currents can flow. For the example in Figure 4.5, a three-phase diagram is shown in Figure 4.9. The convention is that the current always flows to the unbalance. Therefore, assuming an unbalance between buses G and H, the top left diagram shows I_{0G} flowing from the transformer at bus G. Zero sequence

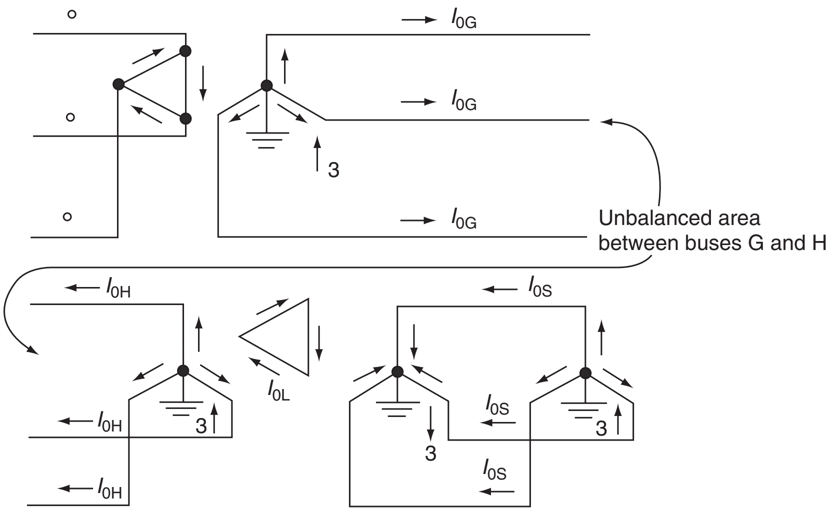

FIGURE 4.9 Diagrams illustrating the flow of zero-sequence current as an aid in drawing the zero-sequence network. Arrows indicate current directions only, not relative magnitudes.

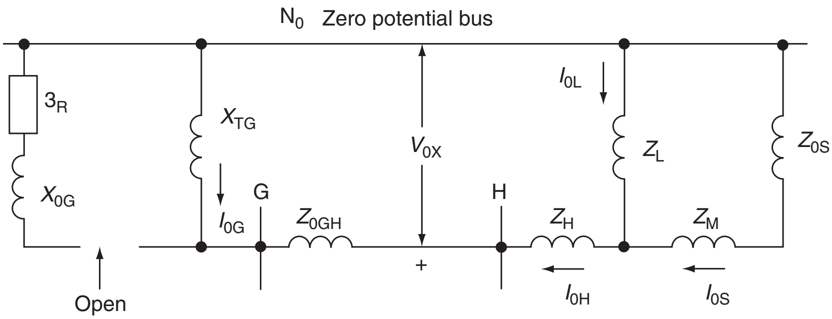

FIGURE 4.10 Zero-sequence network for the system of Figure 4.5.

can flow in the grounded wye and to the fault because there is a path for it to flow in the delta. Thus X_{TG} is connected between the zero potential bus and bus G as shown in Figure 4.10. This connection for the grounded-wye–delta transformer bank is also shown in Figure A4.2-1.

Zero-sequence impedance for transformer banks is equal to the positive and negative sequences and is the transformer leakage impedance. The exception to this is three-phase core-type transformers, for which the construction does not provide an iron flux path for zero sequence. For these the zero-sequence flux must pass from the core to the tank and return. Hence, for these types X_0 usually is 0.85–0.9 X_1 and, when known, the specific value should be used.

The lower right-hand diagram in Figure 4.9 is for the system connected to bus H (see Figure 4.5). Currents out of the three-winding transformer will flow as shown in the L and M windings. The three currents can flow in the M-grounded wye because the equivalent source is shown grounded with Z_{0S} given. Thus, the three-winding equivalent circuit is connected in the zero-sequence network (see Figure 4.10) as shown, which follows the connections documented in Figure A4.2-3b.

Note that in the right-hand part of Figure 4.9, if any of the wye connections were not grounded, the connections would be different. If the equivalent system or the M winding were ungrounded, the network would be open between Z_M and Z_{0S}, since zero-sequence currents could not flow as shown. Loads, if desired, would be shown in the zero-sequence network only if they were wye grounded; delta loads would not pass zero sequence.

Zero-sequence line impedance is always different, as it is a loop impedance: the impedance of the line plus a return path either in the earth or in a parallel combination of the earth and ground wire, cable sheath, and so on. The positive-sequence impedance is a one-way impedance from one end to the other end. As a result, zero sequence varies from two to six times X_1 for lines. For estimating open wire lines, a value of $X_0 = 3$ or 3.5 X_1 is commonly used.

The zero-sequence impedance of generators is low and variable, depending on the winding design. Except for very low-voltage units, generators are never solidly grounded. This is discussed in Chapter 7. In Figure 4.5, the generator G is shown grounded through a resistor R. Faults on bus G and in the system to the right do not involve the generator as far as zero sequence is concerned because the transformer delta blocks the flow of zero-sequence current, as shown.

Conventional current flow is assumed to be from the zero-potential bus N_0 to the area or point of unbalance. Thus, the voltage drop V_{0x} at any point in the network is always

$$V_{0x} = 0 - \sum I_0 Z_0, \tag{4.15}$$

where $\Sigma I_0 Z_0$ is the sum of the drops along the path from the N_0 bus to the point of measurement.

4.9.4 Sequence Network Reduction

For shunt fault calculations, the sequence networks can be reduced to a single equivalent impedance commonly designated as Z_1 or X_1, Z_2 or X_2, and Z_0 or X_0 from the neutral or zero-potential bus to the fault location. This is the Thevenin theorem equivalent impedance, and in the positive-sequence network, it is termed as the Thevenin voltage. These values are different for each fault location. Short-circuit studies with computers use various techniques to reduce complex power systems and to determine fault currents and voltages.

For the positive-sequence network in Figure 4.7b consider faults at bus H. Then by paralleling the impedances on either side, Z_1 becomes

$$Z_1 = \frac{(X''_{\mathrm{d}} + X_{\mathrm{TG}} + Z_{1\mathrm{GH}})(Z_{1\mathrm{S}} + X_{\mathrm{HM}})}{X''_{\mathrm{d}} + X_{\mathrm{TG}} + Z_{1\mathrm{GH}} + Z_{1\mathrm{S}} + X_{\mathrm{HM}}}.$$

Each term in parentheses in the numerator, divided by the denominator, provides a per-unit value to define the portion of current flowing in the two parts of the network. These are known as distribution factors and are necessary to determine the fault currents in various parts of the system. Thus, the per-unit current distribution through bus G is

$$I_{1\mathrm{G}} = \frac{Z_{1\mathrm{S}} + X_{\mathrm{HM}}}{X''_{\mathrm{d}} + X_{\mathrm{TG}} + Z_{1\mathrm{GH}} + Z_{1\mathrm{S}} + X_{\mathrm{HM}}}\,\mathrm{pu} \tag{4.16}$$

and the current distribution through bus H is

$$I_{1\mathrm{S}} = \frac{X''_{\mathrm{d}} + X_{\mathrm{TG}} + Z_{1\mathrm{GH}}}{X''_{\mathrm{d}} + X_{\mathrm{TG}} + Z_{1\mathrm{GH}} + Z_{1\mathrm{S}} + X_{\mathrm{HM}}}\,\mathrm{pu}. \tag{4.17}$$

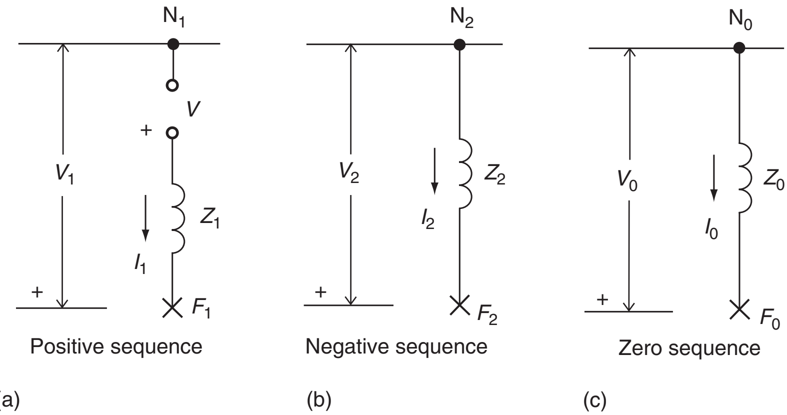

FIGURE 4.11 Reduced sequence networks where Z_1, Z_2, and Z_0 are the equivalent impedances of the networks to the fault point.

The reduction of the positive-sequence network with load (see Figure 4.7a) requires determining the load current flow throughout the network before a fault, determining the open-circuit voltage (Thevenin voltage) at the fault point, and then the equivalent impedance looking into the network from the fault point with all calculations. The total currents in the network are the sum of the prefault load and the fault currents.

The negative- and zero-sequence networks can be reduced in a manner similar to a single impedance to a fault point and with appropriate distribution factors. These three independent equivalent networks are shown in Figure 4.11 with I_1, I_2, and I_0 representing the respective sequence currents in the fault and V_1, V_2, and V_0 representing the respective sequence voltages at the fault.

As indicated earlier, the sequence networks, such as those shown in Figure 4.11, are completely independent of each other. Next, we discuss interconnections to represent faults and unbalances.

4.10 SHUNT UNBALANCE SEQUENCE NETWORK INTERCONNECTIONS

The principal shunt unbalances are faults: three-phase, two-phase-to-phase, two-phase-to-ground, and one-phase-to-ground. Two fault studies are normally made: (1) three-phase faults for applying and setting phase relays and (2) one-phase-to-ground faults for applying and setting ground relays. The other two faults (phase-to-phase and two-phase-to-ground) are rarely calculated for relay applications. With $Z_1 = Z_2$, as is common, then, a solid phase-to-phase fault is 0.866 of the three-phase fault.

The phase currents for solid two-phase-to-ground faults will vary depending on the zero-sequence impedances, but generally tend to be near the phase-to-phase or three-phase fault values (see Section 4.16.1).

4.10.1 Fault Impedance

Faults are seldom solid, but involve varying amounts of resistance. However, it is generally assumed in protective relaying and most fault studies that the connection or contact with the ground involves very low and in general negligible impedance. For the higher voltages of transmission and subtransmission, this is essentially true. In distribution systems (34.5 kV and lower) a very large to basically infinite impedance can exist. This is true, particularly at the lower voltages. Many faults are tree contacts, which can be high impedance, intermittent, and variable. Conductors lying on the ground may or may not result in significant fault current and yet can be highly variable. Many tests have been conducted over the years on wet soil, dry soil, rocks, asphalt, concrete, and so on, with quite a different variable and sometimes unpredictable result. Thus, in most fault studies, the practice is to assume zero ground mat and fault impedances for maximum fault values. Protective relays are set as sensitively as possible, however, to respond properly to these maximum values.

Consequently, although arcs are quite variable, a commonly accepted value for currents between 70 and 20,000 A has been an arc drop of 440 V per phase, basically independent of current magnitude. Therefore,

$$Z_{\text{arc}} = \frac{440l}{I}\ \Omega \tag{4.18}$$

where l is the arc length in feet and I the current in amperes: 1/kV at 34.5 kV and higher is approximately 0.1–0.05. The arc is essentially resistance, but can appear to protective relays as an impedance, with a significant reactive component resulting from out-of-phase contributions from remote sources. This is discussed in more detail in Chapter 12. In low-voltage (480 V) switchboard-type enclosures, typical arc voltages of about 150 V can be experienced. This is relatively of current magnitude.

It appears that because arcs are variable, their resistances tend to start at a low value and continue at the same value for an appreciable time, then build up exponentially. On reaching a high value, an arc breaks over to shorten its path and resistance.

4.10.2 Substation and Tower-Footing Impedance

Another highly variable factor, which is difficult both to calculate and measure, is the resistance between a station ground mat, line pole, or tower, and the ground. In recent years several technical papers have been written, and computer programs have been developed in this area but there are still many variables and assumptions. All this is beyond the scope of this book. The general practice is to neglect these in most fault studies and relay applications and settings.

4.10.3 Sequence Interconnections for Three-Phase Faults

Three-phase faults are assumed to be symmetrical; hence, no analysis is necessary for their calculation. The positive-sequence network, which is the normal balanced diagram for a symmetrical system, can be used, and the connection is shown in Figure 4.12. For a solid fault the fault point F_1 is connected back to the neutral bus (see Figure 4.12a); with fault impedance the connection includes this impedance, as shown in Figure 4.12b. From these,

$$I_1 = I_{aF} = \frac{V}{Z_1} \quad \text{or} \quad I_1 = I_{aF} = \frac{V}{Z_1 + Z_F} \tag{4.19}$$

and $I_{bF} = a^2 I_1$, $I_{cF} = aI_1$, according to Equation 4.2. There is no difference between a three-phase fault and a three-phase-to-ground fault.

4.10.4 Sequence Interconnections for Single-Phase-to-Ground Faults

A phase-a-to-ground fault is represented by connecting the three sequence networks together as shown in Figure 4.13, with diagram 4.13a for solid faults and 4.13b for faults with impedance. From these:

$$I_1 = I_2 = I_0 = \frac{V}{Z_1 + Z_2 + Z_0} \quad \text{or}$$

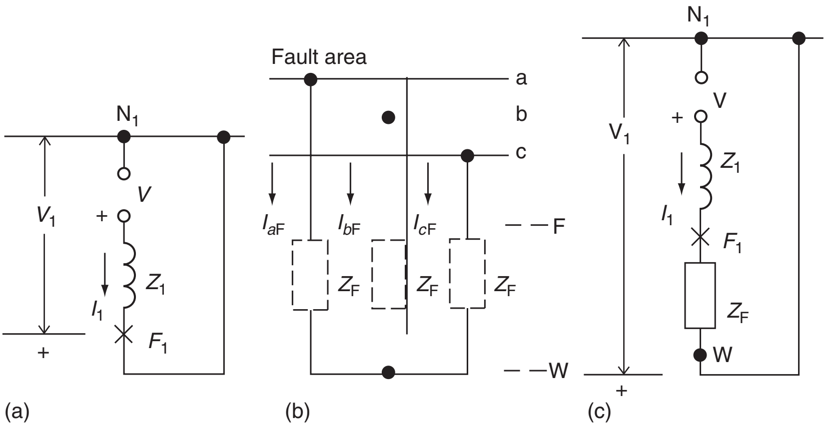

FIGURE 4.12 Three-phase fault and its sequence network interconnections: (a) solid fault; (b) system fault; (c) with fault impedance.

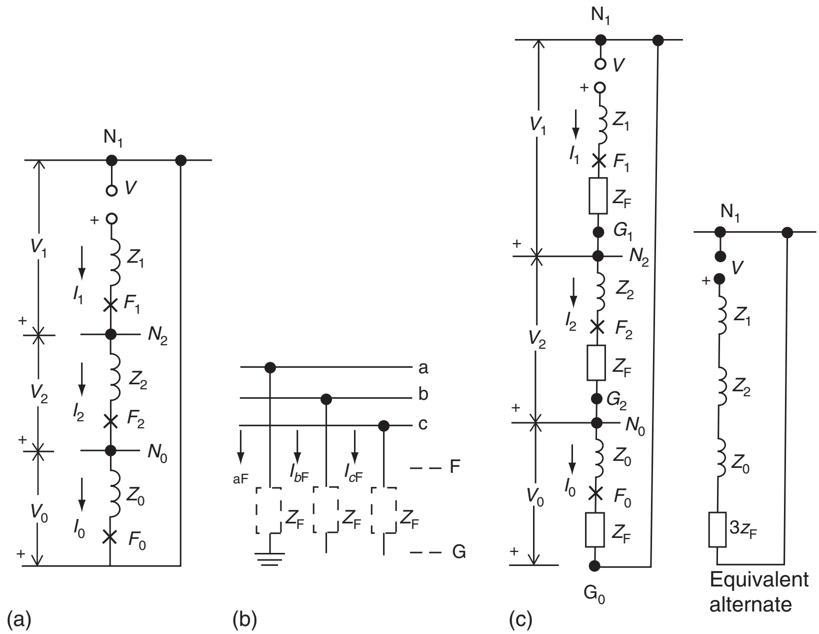

FIGURE 4.13 Single-phase-to-ground fault and its sequence network interconnections: (a) solid fault; (b) system fault; (c) with fault impedance.

$$I_1 = I_2 = I_0 = \frac{V}{Z_1 + Z_2 + Z_0 + 3Z_F}, \tag{4.20}$$

$$I_{aF} = I_1 + I_2 + I_0 = 3I_1 = 3I_2 = 3I_0. \tag{4.21}$$

From Equation 4.6 and Equation 4.7, it can be seen that $I_{bF} = I_{cF} = 0$, which is corrected in the fault. In addition, $V_{aF} = 0$, which is supported by the sequence connections because $V_1 + V_2 + V_0 = 0$.

4.10.5 Sequence Interconnections for Phase-to-Phase Faults

For this type of fault, it is convenient to show that the fault is between phases b and c. Then, the sequence connections are as shown in Figure 4.14. From these,

$$I_1 = -I_2 = \frac{V}{Z_1 + Z_2} \quad \text{or} \quad I_1 = -I_2 = \frac{V}{Z_1 + Z_2 + Z_F}. \tag{4.22}$$

From the fundamental Equation 4.5 through Equation 4.7,

$$I_{aF} = I_1 - I_2 = 0,$$

as it should be in the fault

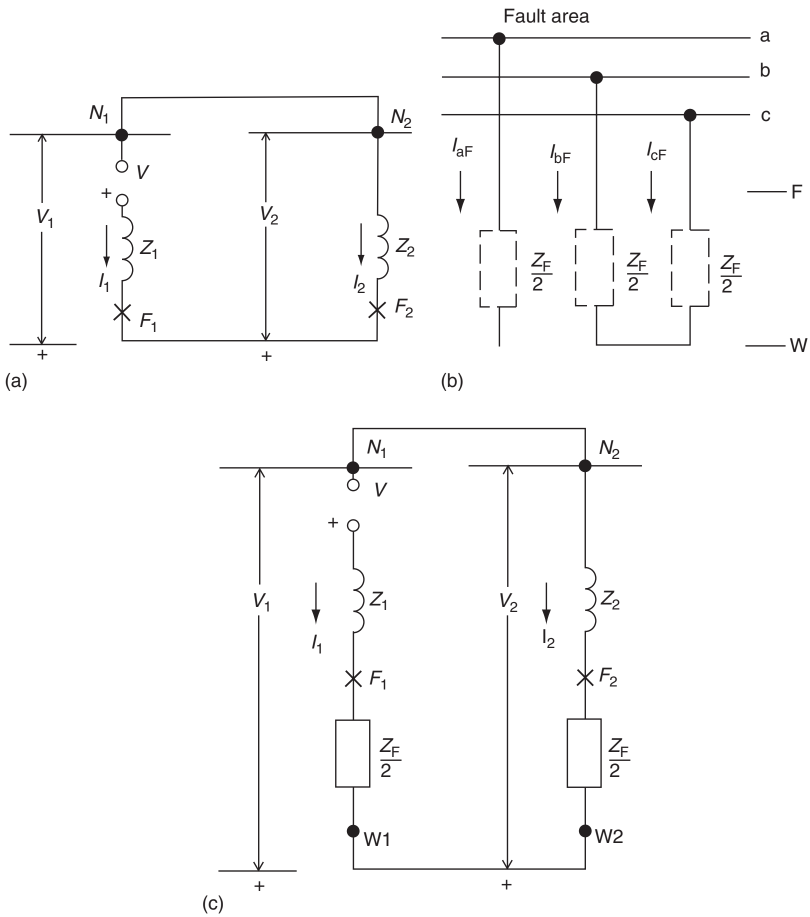

FIGURE 4.14 Phase-to-phase fault and its sequence network interconnections: (a) solid fault; (b) system fault; (c) with fault impedance.

$$I_{bF} = a^2 I_1 + aI_2 = (a^2 - a)I_1 = -j\sqrt{3}I_1, \tag{4.23}$$

$$I_{cF} = aI_1 + a^2 I_2 = (a - a^2)I_1 = +j\sqrt{3}I_1. \tag{4.24}$$

As is common, $Z_1 = Z_2$; then $I_1 = V/2Z_1$; disregarding $\pm j$ and considering only the magnitude yields

$$I_{\phi\phi} = \sqrt{\frac{3V}{2Z_1}} = 0.866, \quad \frac{V}{Z_1} = 0.866 I_{3\phi}. \tag{4.25}$$

Thus, the solid phase-to-phase fault is 86.6% of the solid three-phase fault when $Z_1 = Z_2$.

4.10.6 SEQUENCE INTERCONNECTIONS FOR DOUBLE-PHASE-TO-GROUND FAULTS

The connections for this type are similar to those for the phase-to-phase fault, but only with the addition of the zero-sequence network connected in parallel as shown in Figure 4.15. From these,

$$I = \frac{V}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}},$$

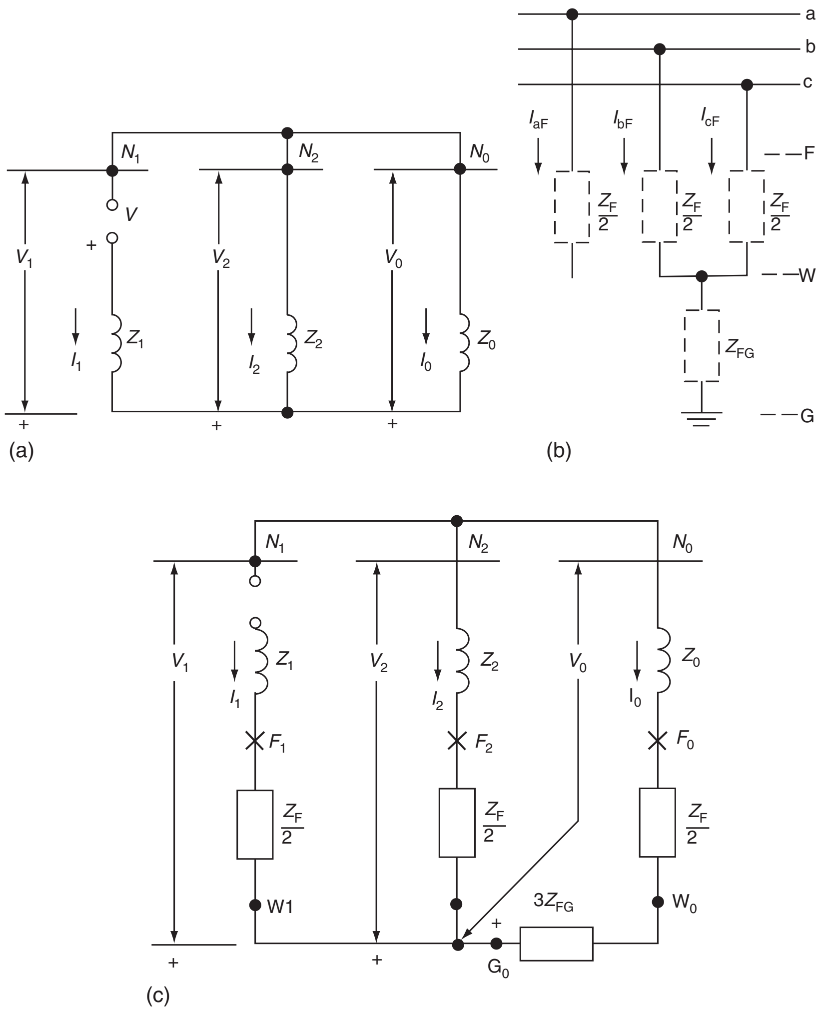

FIGURE 4.15 The double-phase-to-ground fault and its sequence network interconnections: (a) solid fault; (b) system fault; (c) with fault impedance.

or

$$I_1 = \frac{V}{Z_1 + \frac{Z_F}{2} + \frac{(Z_2 + (Z_F/2))(Z_0 + (Z_F/2) + 3Z_{FG})}{Z_2 + Z_0 + Z_F + 3Z_{FG}}}$$

$$I_2 = -I_1 \frac{Z_0}{Z_2 + Z_0} \quad \text{and} \quad I_0 = -I_1 \frac{Z_2}{Z_2 + Z_0}, \tag{4.26}$$

or

$$I_2 = -I_1 \frac{Z_0 + (Z_F/2) + 3Z_{FG}}{Z_2 + Z_0 + Z_F + 3Z_{FG}},$$

and

$$I_0 = -I_1 \frac{Z_2 + (Z_F/2)}{Z_2 + Z_0 + Z_F + 3Z_{FG}}. \tag{4.27}$$

Equation 4.5 through Equation 4.7 provide $I_{aF} = 0$ and fault magnitudes for I_{bF} and I_{cF}.

4.10.7 Other Sequence Interconnections for Shunt System Conditions

The impedances at the fault point in Figure 4.12 through Figure 4.15 were considered to result from the fault arc. However, they can also be considered as a shunt load, shunt reactor, shunt capacitor, and so on, connected at a given point to the system. Various types and their sequence interconnections are covered in Blackburn (1993).

4.11 EXAMPLE: FAULT CALCULATIONS ON A TYPICAL SYSTEM SHOWN IN FIGURE 4.16

The system of Figure 4.16 is the same as that shown in Figure 4.5, but with typical constants for the various parts. These are on the bases indicated, so the first step is to transfer them to a common base, as discussed in Chapter 2. The positive- and negative-sequence networks (negative is the same as positive, except for the omission of the voltage) are shown in Figure 4.17. The conversion to a common base of 100 MVA is shown as necessary.

For a fault at bus G, the right-hand impedances ($j0.18147 + j0.03667 + j0.03 = j0.5481$) are paralleled with the left-hand impedances ($j0.20 + j0.1375 = j0.3375$). Reactance values, rather than impedance values, are used, as is typical when the resistance is relatively small.

$$X_1 = X_2 = \frac{\overset{(0.5763)}{0.3375} \times \overset{(0.4237)}{0.2481}}{0.5856} = j0.1430 \text{ pu}. \tag{4.28}$$

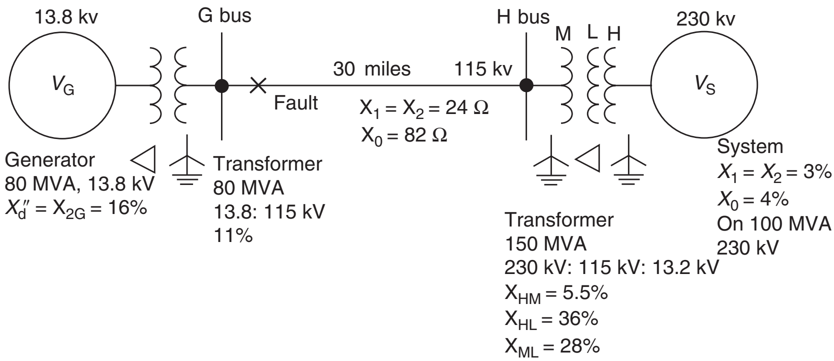

FIGURE 4.16 Power system example for fault calculations.

The division of $0.3375/0.5856 = 0.5763$ and $0.2481/0.5856 = 0.4237$, as shown, provides a partial check, for $0.5763 + 0.4237$ must equal 1.0, which are the distribution factors indicating the per-unit current flow on either side of the fault. These values are added to the network diagram. Thus, for faults at bus G, $X_1 = X_2 = j0.1430$ pu on a 100 MVA base.

The zero-sequence network for Figure 4.16 is shown in Figure 4.18. Again the reactance values are converted to a common 100 MVA base.

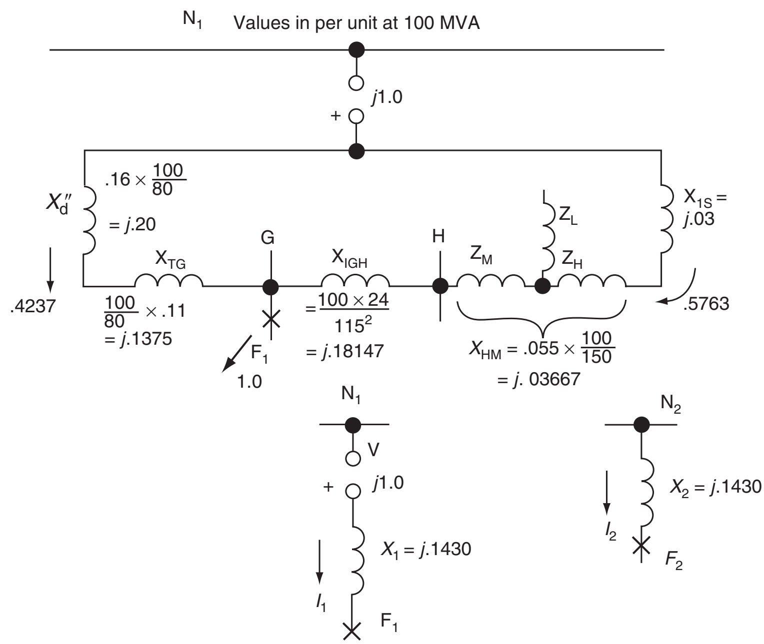

FIGURE 4.17 Positive- and negative-sequence networks and their reduction to a single impedance for a fault at bus G in the power system in Figure 4.16.

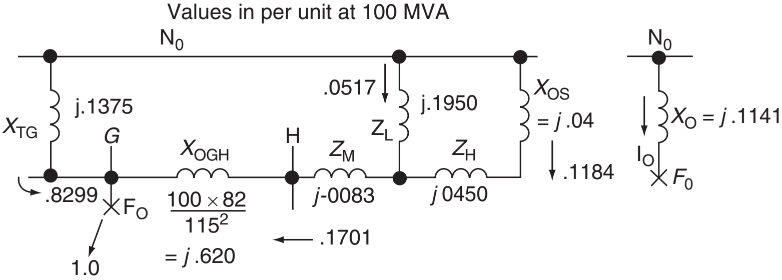

FIGURE 4.18 Zero-sequence network and its reduction to a single impedance for a fault at bus G in the power system in Figure 4.16.

The three-winding bank connections are as indicated in Figure A4.2-3b with $Z_{NH} = Z_{NM} = 0$ because the neutrals are shown as solidly grounded.

The conversions to a common 100 MVA base are shown, except for the three-winding transformer. For this bank,

$$X_{HM} = 0.055 \times \frac{100}{150} = 0.03667 \text{ pu},$$
$$X_{HL} = 0.360 \times \frac{100}{150} = 0.2400 \text{ pu},$$
$$X_{ML} = 0.280 \times \frac{100}{150} = 0.18667 \text{ pu},$$

and from Equation A4.2-13 through Equation A4.2-15,

$$\begin{aligned} X_H &= \tfrac{1}{2}(0.03667 + 0.2400 - 0.18667) = 0.0450 \text{ pu}, \\ X_M &= \tfrac{1}{2}(0.03667 + 0.18667 - 0.240) = -0.00833 \text{ pu}, \\ X_L &= \tfrac{1}{2}(0.2400 + 0.18667 - 0.3667) = 0.1950 \text{ pu}. \end{aligned} \tag{4.29}$$

These are shown in Figure 4.18.

This network is reduced for a fault at bus G by the first paralleling $X_{0S} + Z_H$ with Z_L and then adding Z_M and X_{0GH};

$$\frac{\overset{(0.6964)}{0.1950} \times \overset{(0.3036)}{0.0850}}{0.280} = j0.0592$$

$$\begin{aligned} &-j0.0083 \; (Z_M) \\ &\underline{j0.620} \;\; (X_{0GH}). \\ &j0.6709 \end{aligned}$$

This is the right-hand branch paralleling with the left-hand branch,

$$X_0 = \frac{\overset{(0.8299)}{0.6709} \times \overset{(0.1701)}{0.1375}}{0.8084} = j0.1141 \text{ pu at 100 MVA.} \tag{4.30}$$

The values (0.8299) and (0.1701) add to 1.0 as a check and provide the current distribution on either side of the bus G fault, as shown on the zero-sequence network. The distribution factor 0.1701 for the right side is further divided by $0.6964 \times 0.1701 = 0.1184$ pu in the 230 kV system neutral and $0.3036 \times 0.1701 = 0.0517$ pu in the three-winding transformer H neutral winding. These are shown on the zero-sequence network.

4.11.1 Three-Phase Fault at Bus G

For this fault,

$$I_1 = I_{a\mathrm{F}} = \frac{j1.0}{j0.143} = 6.993 \text{ pu}$$

$$= 6.993 \frac{100{,}000}{\sqrt{3} \times 115} = 3510.8 \text{ A at 115 kV.} \tag{4.31}$$

The divisions of current from the left ($I_{a\mathrm{G}}$) and right ($I_{a\mathrm{H}}$) are:

$$I_{a\mathrm{G}} = 0.4237 \times 6.993 = 2.963 \text{ pu,} \tag{4.32}$$

$$I_{a\mathrm{H}} = 0.5763 \times 6.993 = 4.030 \text{ pu.} \tag{4.33}$$

4.11.2 Single-Phase-to-Ground Fault at Bus G

For this fault,

$$I_1 = I_2 = I_0 = \frac{j1.0}{j(0.143 + 0.143 + 0.1141)} = 2.50 \text{ pu,} \tag{4.34}$$

$$I_{a\mathrm{F}} = 3 \times 2.5 = 7.5 \text{ pu at 100 MVA,}$$

$$= 7.5 \times \frac{100{,}000}{\sqrt{3} \times 115} = 3764.4 \text{ A at 115 kV.} \tag{4.35}$$

Normally, the $3I_0$ currents are documented in the system, for these are used to operate the ground relays. As an aid to understanding these are illustrated in Figure 4.19 along with the phase currents. Equation 4.5 through Equation 4.7 provide the three-phase currents. Because $X_1 = X_2$, such that $I_1 = I_2$, these reduce to $I_b = I_c = -I_1 + I_0$ for the phase b and c currents, since $a + a^2 = -1$. The currents shown are determined by adding $I_1 + I_2 + I_0$ for I_a, $-1_1 + I_0$ for I_b, and I_c and $3I_0$ for the neutral currents.

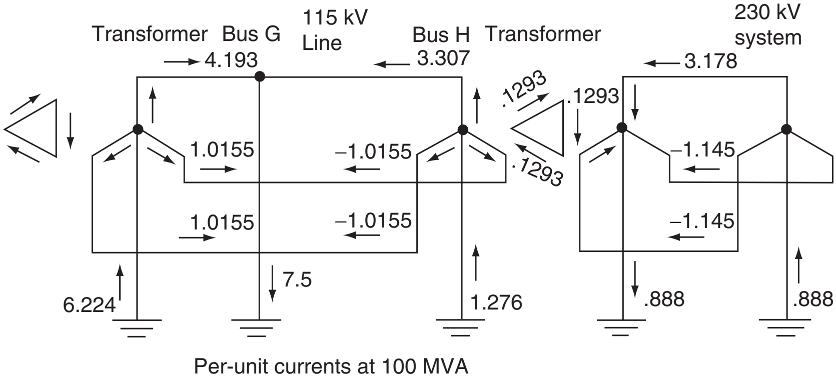

FIGURE 4.19 Phase and $3I_0$ current distribution for a single-phase-to-ground fault at bus G in Figure 4.16.

In the 115 kV system the sum of the two neutral currents is equal and opposite to the current in the fault. In the 230 kV system the current up the neutral equals the current down the other neutral.

The calculations assumed no load; accordingly, prefault, all currents in the system were zero. With the fault involving only phase *a*, it will be observed that current flows in the *b* and *c* phases. This is because the distribution factors in the zero-sequence network are different from the positive- and negative-sequence distribution factors. On a radial system where positive-, negative-, and zero-sequence currents flow only from one source and in the same direction, the distribution factors in all three networks will be 1.0, in spite of the zero-sequence impedances. Then $I_b = I_c = -I_1 + I_0$ becomes zero, and fault current flows only in the faulted phase. In this type $I_a = 3I_0$ throughout the system for a single-phase-to-ground fault.

4.12 EXAMPLE: FAULT CALCULATION FOR AUTOTRANSFORMERS

Autotransformers have become quite common in recent years. They provide some different and interesting problems. Consider a typical autotransformer in a system, as shown in Figure 4.20, and assume that a single-phase-to-ground fault occurs at the H or 345 kV system values that are given based on this consideration. For the autotransformer, the equivalent network is as in Figure A4.2-3d, with values as follows: On a 100 MVA base,

$$X_{\mathrm{HM}} = 8 \times \frac{100}{150} = 5.333\% = 0.05333 \text{ pu},$$

$$X_{\mathrm{HL}} = 34 \times \frac{100}{150} = 68\% = 0.68 \text{ pu},$$

$$X_{\mathrm{ML}} = 21.6 \times \frac{100}{40} = 54\% = 0.54 \text{ pu}, \tag{4.36}$$

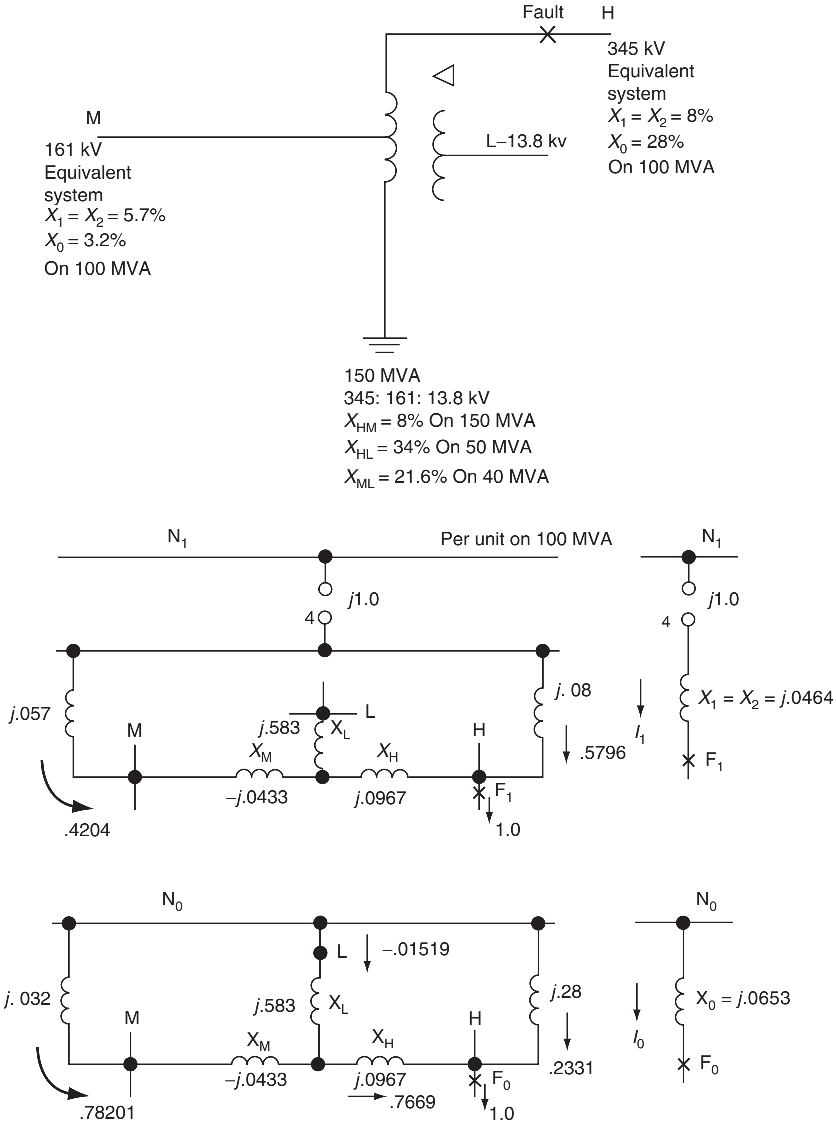

FIGURE 4.20 Example of fault calculation for an autotransformer.

and from Equation A4.2-13 through Equation A4.2-15,

$$
\begin{aligned}
X_H &= \tfrac{1}{2}(0.0533 + 0.68 - 0.54) = 0.09667 \text{ pu},\\
X_M &= \tfrac{1}{2}(0.0533 + 0.54 - 0.68) = -0.04334 \text{ pu},\\
X_L &= \tfrac{1}{2}(0.68 + 0.54 - 0.0533) = 0.58334 \text{ pu}.
\end{aligned} \tag{4.37}
$$

These values are shown in the sequence diagrams in Figure 4.20. In the positive- (and negative)-sequence networks for a fault at H,

$$X_1 = X_2 \frac{\overset{(0.5796)\quad(0.4204)}{(0.0533 + 0.057)(0.08)}}{0.1903} = j0.04637 \text{ pu.} \tag{4.38}$$

The zero-sequence network reduces as follows: first paralleling the left side,

$$\begin{aligned} &= \frac{(0.032 - 0.0433)(0.5833)}{0.032 - 0.0433 + 0.5833} = \frac{\overset{(-0.0198)(1.0198)}{(-0.0113)(0.5833)}}{0.5720} \\ &= -0.01156 \text{ pu} \end{aligned}$$

$$\begin{aligned} X_0 &= \frac{(-0.01156 + 0.09667)(0.28)}{0.08511 + 0.28} = \frac{\overset{(0.2331)(0.7669)}{(0.08511)(0.28)}}{0.36511} \\ &= j0.06527 \text{ pu.} \end{aligned} \tag{4.39}$$

The current distribution factor through the X_M path is $0.7669 \times 1.0198 = 0.78207$, and through the X_L path is $0.7669 \times -0.0198 = -0.01519$. These current distributions are shown on the network.

4.12.1 Single-Phase-to-Ground Fault at H Calculation

$$\begin{aligned} I_1 = I_2 = I_0 &= \frac{j1.0}{j(0.0464 + 0.0464 + 0.0653)} = \frac{1.0}{0.1580} \\ &= 6.3287 \text{ pu} \\ &= 6.3287 \times \frac{100{,}000}{\sqrt{3} \times 345} = 1059.1 \text{ A at 345 kV} \end{aligned} \tag{4.40}$$

$$\begin{aligned} I_{aF} &= 3I_0 = 3 \times 6.3287 = 18.986 \text{ pu} \\ &= 3 \times 1059.1 = 3177.29 \text{ A at 345 kV.} \end{aligned} \tag{4.41}$$

It is recommended that amperes, rather than per unit, may be used for fault current distribution, particularly in the neutral and common windings. The autotransformer is unique in that it is both a transformer and a direct electrical connection. Thus, amperes at the medium-voltage base I_M are combined directly with amperes at the high-voltage base I_H for the common winding current I or for the high-side fault,

$$I = I_H(\text{in amperes at kV}_H) - I_M(\text{in amperes at kV}_M). \tag{4.42}$$

For the current in the grounded neutral,

$$3I_0 = 3I_{0H} \text{ (in amperes at kV}_H) - 3I_{0M} \text{ (in amperes at kV}_M). \qquad (4.43)$$

Both of the foregoing currents are assumed to flow up the neutral and to the M junction point.

Correspondingly, for a fault on the M or medium-voltage system, the current flowing up the grounded neutral is

$$3I_0 = 3I_{0M}\text{(in amperes at kV}_M) - 3I_{0H}\text{(in amperes at kV}_H). \qquad (4.44)$$

Thus, these currents in the common winding and neutral are a mixture of high- and medium-voltage currents; therefore, there is no base to which they can be referred. This makes per unit difficult, as it must have a base. When, or if, per unit must be used, a fictional base can be devised based on the ratios of the transformer parts. This is quite complex. Because it is the fundamental base, amperes are easy to handle and they will be used in the following.

The sequence, phase a, and neutral currents are documented in Figure 4.21 for the example in Figure 4.20. There will be current flowing in phases b and c because the current distribution factors are different in the positive- and zero-sequence networks. These are not shown, for they are of little importance in protection.

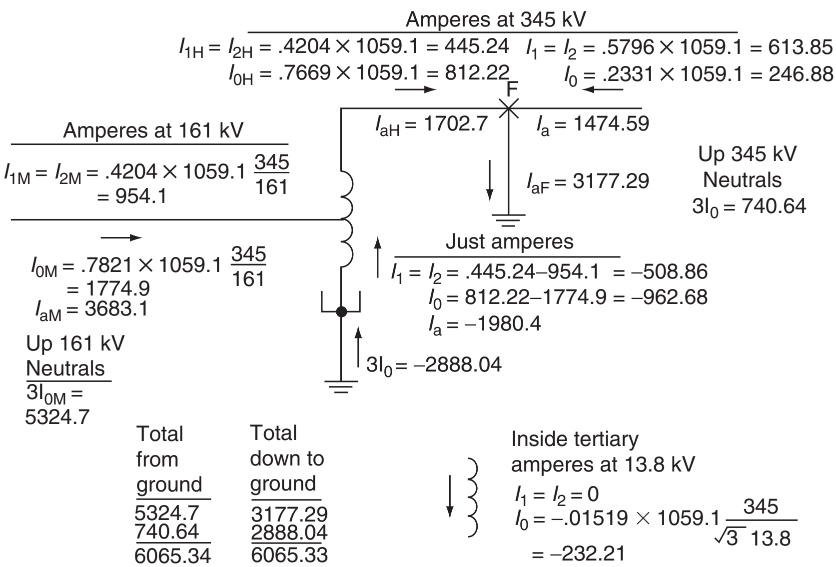

FIGURE 4.21 Fault current distribution for the autotransformer in Figure 4.20.

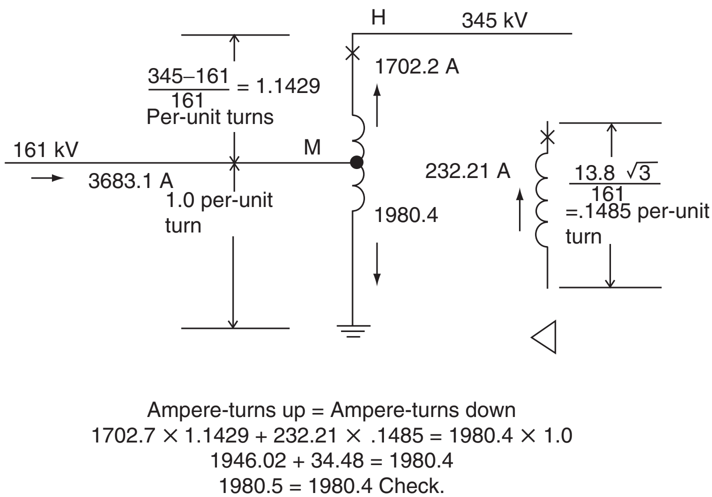

FIGURE 4.22 Ampere-turn check to confirm or establish the direction of current flow in the tertiary.

The example indicates that current flows in the downward direction of the neutral sequence of the autotransformer instead of the upward direction, as might be expected. In addition, in this example, the current in the delta has reversed, because the negative branch of the transformer-equivalent circuit is larger than the very solidly grounded 161 kV connected system. Both these effects influence the protection. This is discussed further in Chapter 12.

There can be a question about the direction of current in the tertiary. This can be checked by ampere turns, as shown in Figure 4.22. Arbitrarily, one per-unit turn was assumed for the 161 kV winding and the others were derived. Any convenient winding or group could be used for the base.

4.13 EXAMPLE: OPEN-PHASE CONDUCTOR

A blown fuse or broken conductor that opens one of the three phases represents an unbalance series that is dealt more in detail in Blackburn (1993). As an example, consider phase *a* open on the 34.5 kV line at bus H that is given in Figure 4.23. All constants are in per unit on a 30 MVA base.

The three sequence networks are shown in Figure 4.24. With no load, opening any phase makes no difference in the current flow because it is already zero. Constantly, in these series unbalances, it is necessary to consider load; therefore, the 30 MVA, 90%, is as shown. With induction motor loads, the negative-sequence load impedance is less than the positive-sequence impedance. This is covered in Chapter 11.

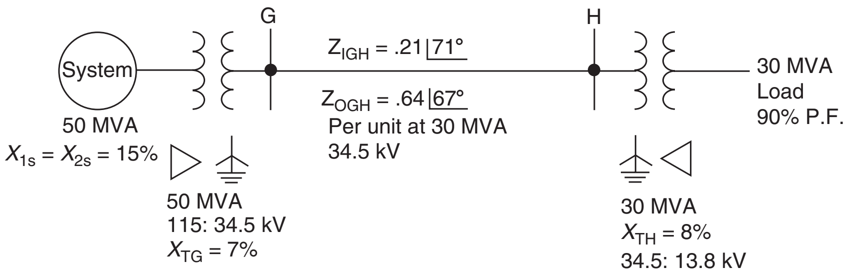

FIGURE 4.23 Example for series unbalance calculations.

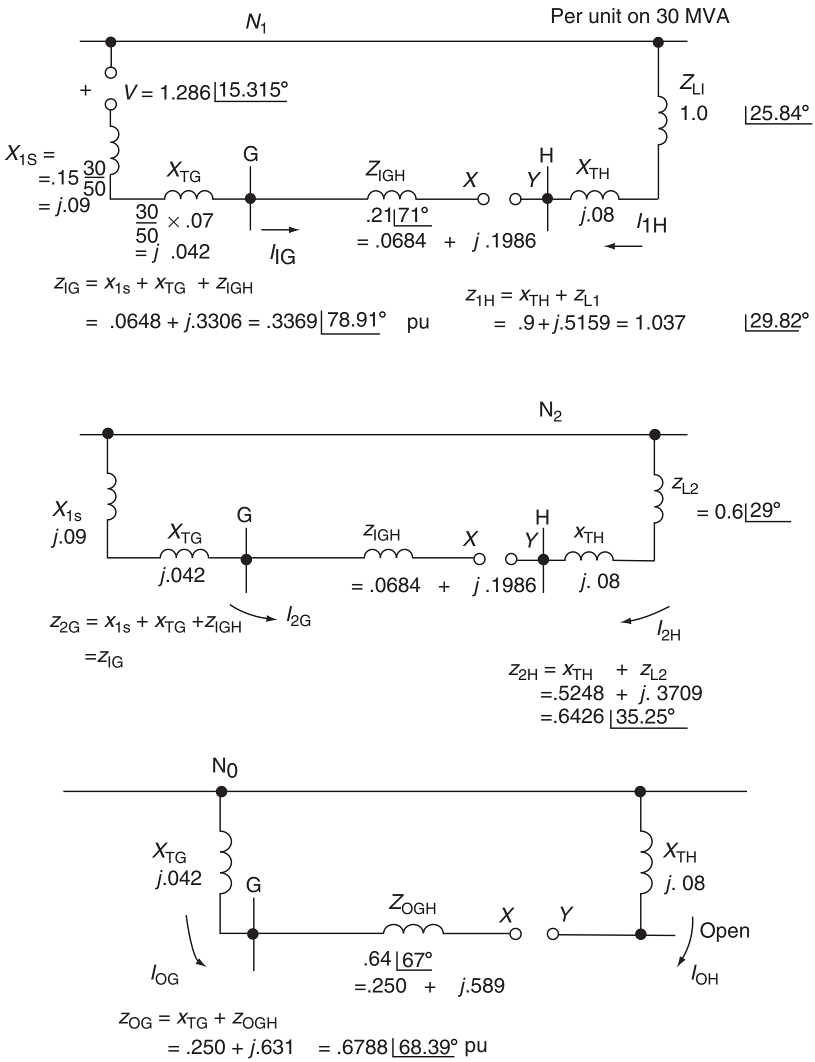

FIGURE 4.24 The three sequence networks for the system in Figure 4.23.

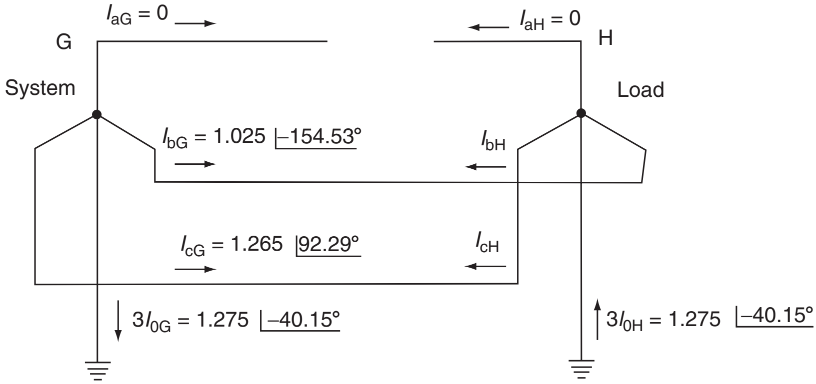

FIGURE 4.25 Per-unit current distribution for an open conductor in the power system in Figure 4.23 (per unit at 30 MVA where 1 pu = 502 A at 34.5 kV).

If we assume the voltage at the load to be 1.00° pu, then the voltage at the generator will be 1.286 $\angle 15.315°$. Phase *a* open is represented by connecting the three-sequence *X* points together and the three-sequence *Y* points together. This connects the total zero-sequence impedances in parallel with the total negative-sequence impedances across the open *X*–*Y* of the positive-sequence network. From these data, I_1, I_2, and I_0 can be easily calculated.

The resulting currents flowing in the system are shown in Figure 4.25 and are in the order of normal load currents. Thus, it is difficult to locate and provide protection for these faults.

4.14 EXAMPLE: OPEN PHASE FALLING TO GROUND ON ONE SIDE

In the system shown in Figure 4.23, the phase *a* conductor on the line at bus H opens and falls to ground value on the bus H side. The sequence networks are the same as those indicated in Figure 4.24, but are interconnected, as can be noted in Figure 4.26. These are simultaneous faults: a series open-phase fault and a phase-to-ground fault. Thus, three ideal or perfect transformers are used for isolation of the open-phase *X*–*Y* interconnection from three networks in series for the ground fault. Because these transformers have no leakage or exciting impedances, the voltage drop across them cannot be expressed by the current in their windings. The currents I_1, I_2, and I_0 can be determined by solving various voltage drop equations around the networks. The resulting fault currents are shown in Figure 4.27. In this instance, it is possible to obtain currents by neglecting the load. These are shown in parentheses in Figure 4.27.

The other possibility is that the open conductor falls to ground value on the line side. Here, the three ideal transformers are moved to the left or *X* side of the

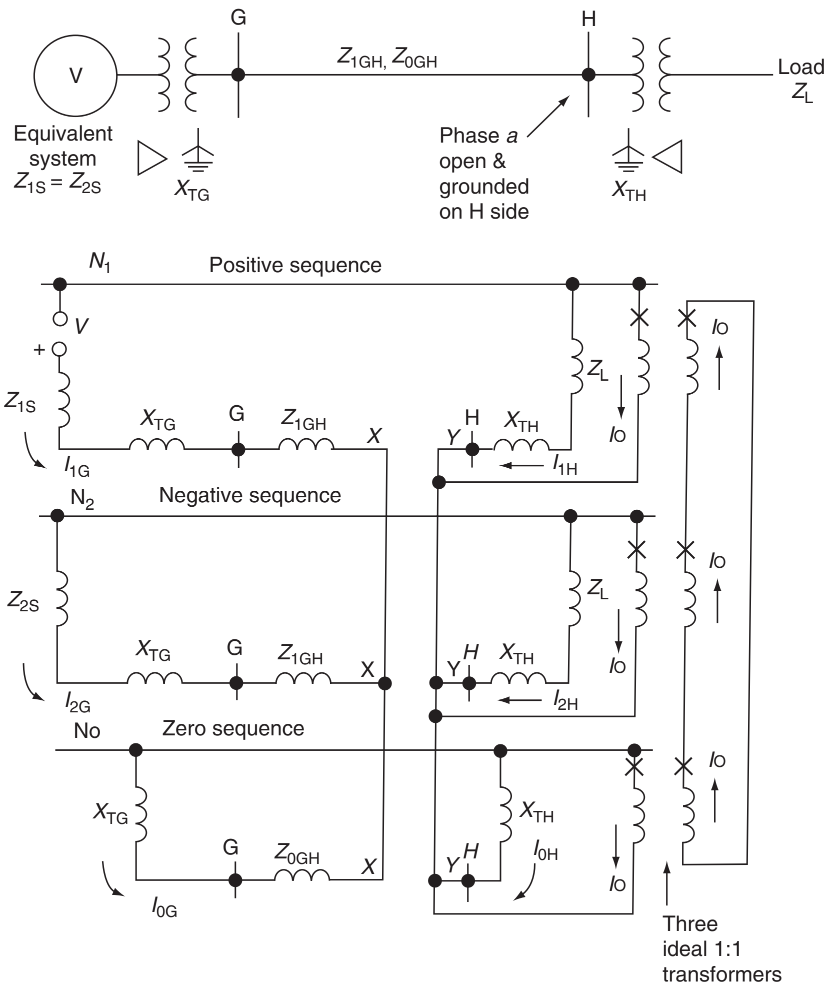

FIGURE 4.26 Example of the sequence network interconnections for phase *a* open and grounded at bus H (broken conductor has fallen to ground value).

three-sequence networks: there is now no option—load must be considered. The fault currents are shown in Figure 4.28. Note that, as in the open phase (see Figure 4.25), the currents are still quite low, not higher than load currents.

4.15 SERIES AND SIMULTANEOUS UNBALANCES

Series and simultaneous unbalances certainly occur in power systems. One type is a blown fuse or open (broken) phase conductor. The broken conductor can contact the ground making a simultaneous unbalance. Several instances of these are covered with examples in Blackburn (1993).

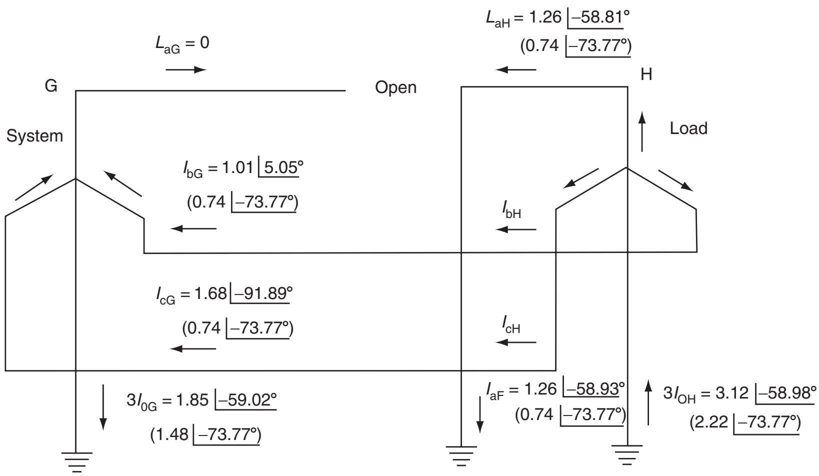

FIGURE 4.27 Per-unit current distribution for a broken conductor at bus H that falls to ground value on the bus H side in the power system in Figure 4.23. Top values are seen with a 30 MVA load. Values in parentheses are with load neglected (no load). Per unit is at 30 MVA where 1 pu = 502 A at 34.5 kV.

4.16 OVERVIEW

Faults and the sequence quantities can be visualized and perhaps be better understood by an overall view in contrast to the specific representations and calculations. Accordingly, several overviews are presented next.

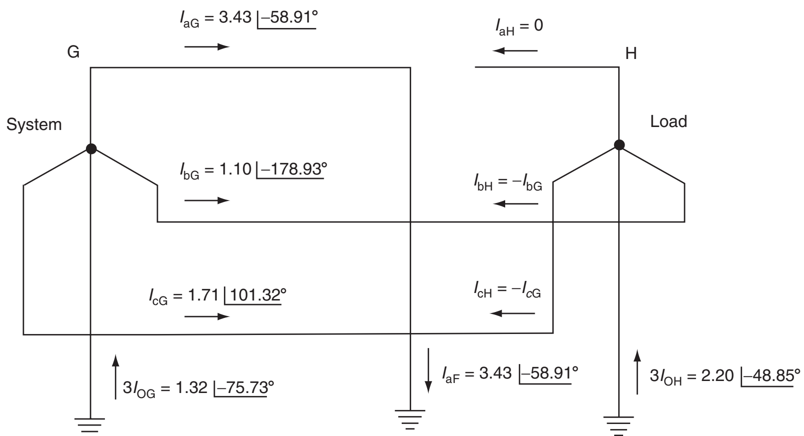

FIGURE 4.28 Per-unit current distribution for a broken conductor at bus H that falls to ground value on the line side in the power system in Figure 4.29. Per unit is at 30 MVA when 1 pu = 502 A at 34.5 kV.

4.16.1 Voltage and Current Phasors for Shunt Faults

The first overview is a review of shunt faults, which are the common types experienced on a power system. These are illustrated in Figure 4.29, with Figure 4.29a showing the normal balanced voltage and load current phasors. Load is slightly lagging, normally from unit power factor to about a 30° lag. With capacitors at light load, the currents may slightly be direct.

When faults occur, the internal voltage of the generators does not change; that is true unless the fault is left unattended for long, and the voltage regulators attempt to increase the terminal voltage of the fault-reduced machine.

A three-phase fault (see Figure 4.29a) reduces all three voltages and causes a large increase and higher lagging by the system, and usually varies from about a 30° to 45° lag, and sometimes nearly a 90° lag (see Figure 4.29b).

The single-phase-to-ground fault (see Figure 4.29c) is the most common one. The faulted-phase voltage collapses, and its current increases, as shown. Load current is neglected, for it is usually relatively small, and $I_b = I_c = 0$. As has been

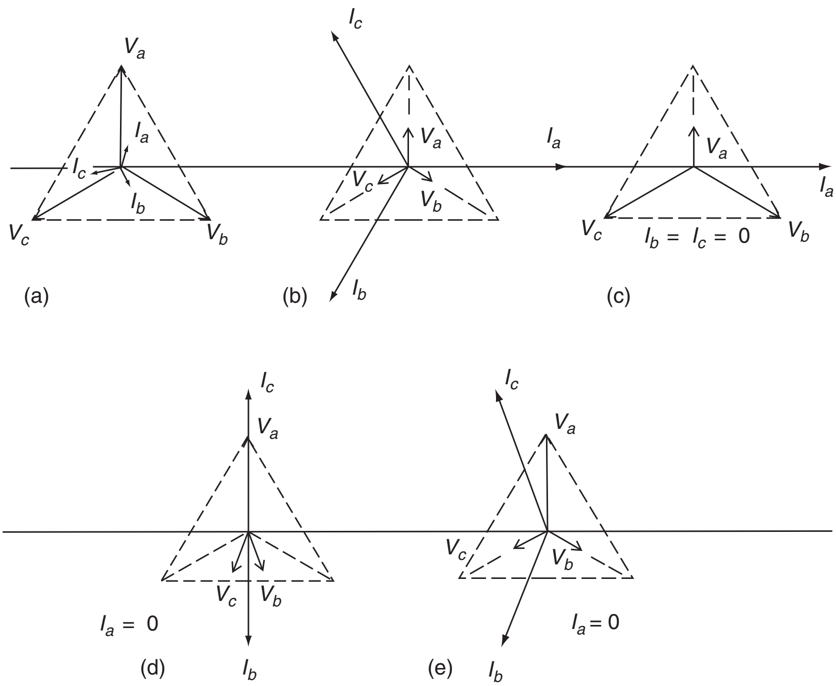

FIGURE 4.29 Typical current and voltage phasors for common shunt faults: The fault currents are shown at 90° lagging or for a power system where $Z = X$. During faults the load is neglected. (a) Normal balanced system; (b) three-phase faults; (c) phase-to-ground *a*-Gnd faults; (d) phase-to-phase *bc* faults; (e) two-phase-to-ground *bc*-Gnd faults.

indicated earlier, fault current flows in the unfaulted phases on loop systems in which the distribution factors for the three sequence networks are different. Once again the fault current lags normally. It is shown at 90° in Figure 4.29c.

The phase-to-phase fault is seen in Figure 4.29d. Neglecting the load, for a b-to-c phase fault, V_a is normal, $I_a = 0$. V_b, and V_c collapse from their normal positions to vertical phasors at a solid fault point where $V_{bc} = 0$. I_b and I_c are normally equal and opposite and lagging is 90°, as shown in Figure 4.29c.

The two-phase-to-ground fault (see Figure 4.29c) results in the faulted phase voltages collapsing along their normal position until, for a solid fault, they are zero. Thus, at the fault, $V_b = V_c = 0$, which is not true for the phase-to-phase fault (see Figure 4.29d). I_b and I_c will be in the general area, as shown. An increasing amount of zero-sequence current will cause I_b and I_c to swing closer to each other; contrarily, a low zero-sequence current component will result in the phasors approaching the phase-to-phase fault in Figure 4.29d. This can be seen from the sequence network connections of Figure 4.15. For a phase-to-phase fault, if Z_0 becomes infinite (essentially ungrounded system), the interconnection becomes as indicated in Figure 4.14. On the other hand, for a very solidly grounded system where Z_0 approached zero relatively, the negative network becomes shorted, and this fault becomes similar to the three-phase fault as shown in Figure 4.12.

Total fault current in per unit

Based on $V = j1$ pu; $Z_1 = Z_2 = j1$ pu; and $Z_0 = jX_1$ pu

$I_{30} = 1.0$

$I_{00} = 0.866^a$

Fault	**X_0 pu: 0.1**	**0.5**	**1.0**	**2.0**	**10.0**
10 Gnd	1.43	1.2	1.0	0.75	0.25
00 Gnd	1.52	1.15	1.0	0.92	0.87
$3I_{00\ \mathrm{Gnd}}$	−2.5	−1.5	−1.0	−0.6	−0.143
Angle of $I_{b00\ \mathrm{Gnd}}$	−145.29°	−130.89°	−120°	−109.11°	94.69°
Angle of $I_{c00\ \mathrm{Gnd}}$	145.29°	130.89°	120°	109.11°	94.69°

For the phase-to-phase fault:

$$I_1 = -I_2 = 1/1 + 1 = 0.5.\ I_b = a^2 I_1 + aI_2 = (a^2 - a)I_1 = -j\sqrt{3}(0.5) = -j0.866.$$

In some parts of a loop network it is possible for the zero-sequence current to flow opposite the positive- and negative-sequence currents. In this area I_c may lag the V_c phasor, rather than lead it as shown, correspondingly with I_b leading the position, as shown.

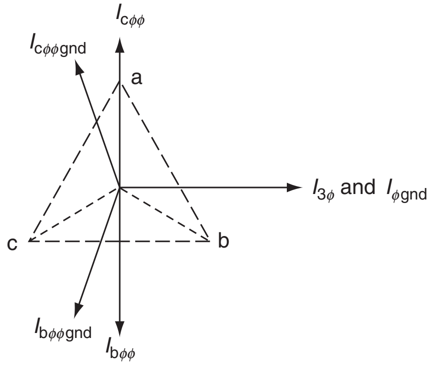

FIGURE 4.30 Comparison of solid shunt-type faults.

These trends are further amplified by Figure 4.30, which compares the various solid shunt faults. The effect of the zero sequence for ground faults is illustrated by various values of X_0 reactances relative to $X_1 = X_2$. As has been indicated, the zero-sequence network is always different from the positive- and negative-sequence networks. However, X_0 can be approximately equal to X_1, X_2 for secondary bus faults on distribution feeders connected to large power systems. In these cases, the systems X_1, X_2 are very small relative to the primary delta-secondary wye-grounded distribution transformer. Thus, the case of $X_1 = X_2 = X_0$ is quite practicable.

4.16.2 System Voltage Profiles during Faults

The trends of the sequence voltages for the various faults in Figure 4.29 are illustrated in Figure 4.31. Only the phase *a* sequence voltages are shown for an ideal case where $Z_1 = Z_2 = Z_0$. This makes the presentation less complex and does not affect the trends shown.

With the common assumption of no load, the system voltage is equal throughout the system, as indicated by the dashed lines. When a solid three-phase fault occurs, the voltage at the fault point becomes zero, but as indicated earlier, does not change in the source until the regulators act to change the generator fields. Meanwhile, the fault should have been cleared by protective relays. Thus, the voltage profile is as shown in Figure 4.31a.

For phase-to-phase faults (see Figure 4.31b), the positive-sequence voltage drops to half value ($Z_1 = Z_2$). This unbalance fault is the source of negative sequence and the V_2 drops, which are zero in the generators, are as shown.

For two-phase-to-ground faults (see Figure 4.31c) with $Z_1 = Z_2 = Z_0$, the positive-sequence voltage at the fault drops to one-third of V_1. The fault at this moment generates both negative and zero sequences that flow through the system, producing voltage drops as shown. The voltage V_2

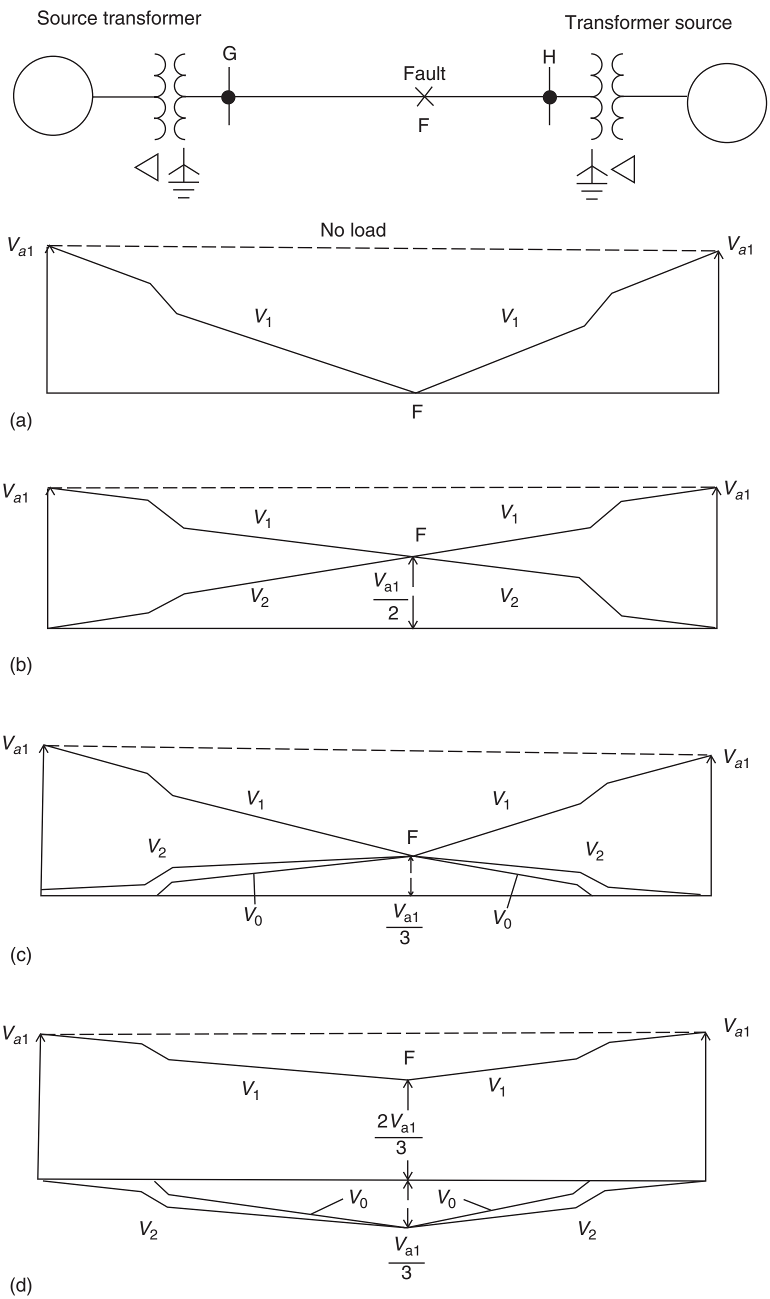

FIGURE 4.31 System sequence voltage profiles during shunt faults: (a) three-phase faults; (b) phase-to-phase faults; (c) two-phase-to-ground faults; (d) phase-to-ground faults.

becomes zero in the generators, whereas V_0 is zero at the grounded transformer neutral point.

The fault voltage for a phase-*a*-to-ground solid fault is zero and as documented in Figure 4.31d, the sum of the positive-, negative-, and zero-voltage components at the fault add to zero. Thus, the positive-sequence voltage drops to $2/3V_1$ when $Z_1 = Z_2 = Z_0$ at the fault point, where $-1/3V_2$ and $-1/3V_0$ are generated. Subsequently, they drop to zero in the generator or source for the negative sequence and to zero at the grounded transformer bank neutral.

The fundamental concept illustrated in Figure 4.31 is that positive-sequence voltage is always maximum at the generators and minimum at the fault. The negative- and zero-sequence voltages are always maximum at the fault and minimum at the generator or grounded neutral.

It is common to refer to the grounded-wye–delta or similar banks as "ground sources." This is really a misnomer, as the source of zero sequence is the unbalance, the ground fault. However, thus designating these transformers as ground sources is practical, since, by convention ground ($3I_0$) current flows up the grounded neutral, through the system, and down the fault into ground.

4.16.3 Unbalanced Currents in the Unfaulted Phases for Phase-to-Ground Faults in Loop Systems

A typical loop system is illustrated in Figure 4.32. A phase-*a*-to-ground fault occurs on the bus at station E, as shown in Figure 4.33. The fault calculation was made at no-load; therefore, the current before the fault in all three phases was zero in all parts of the system. However, fault current is shown flowing in all three phases. This is because the current distribution factors in the loop are different in the sequence networks. With X_0 not equal

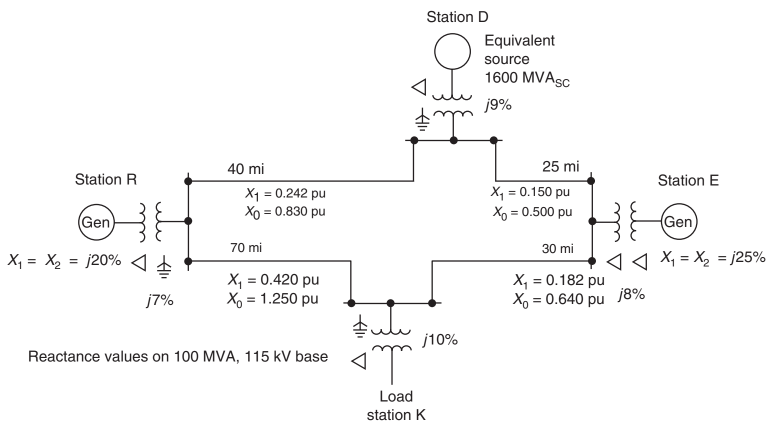

FIGURE 4.32 A typical loop-type power system.

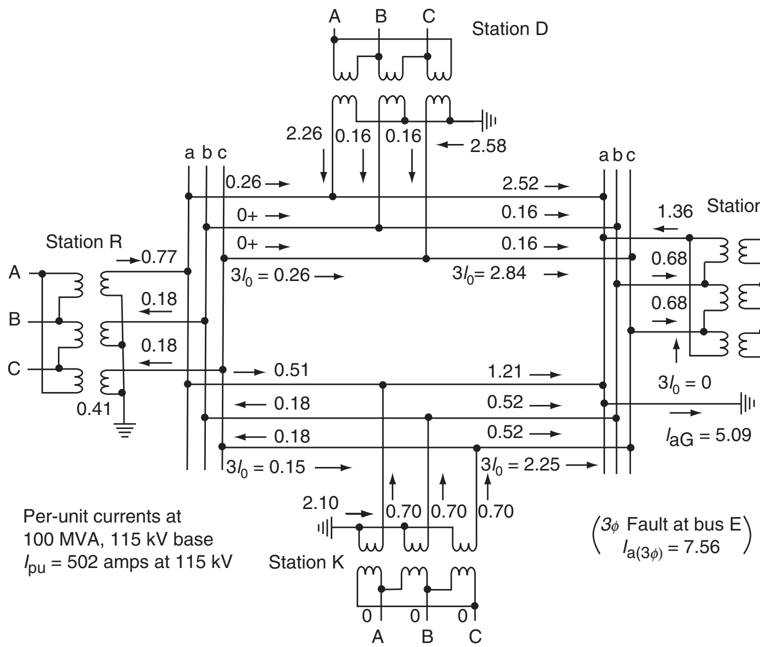

FIGURE 4.33 Currents for a phase-a-to-ground fault at station E bus in Figure 4.32.

to $X_1 = X_2$, $I_b = a^2 I_1 + aI_2 + I_0 = -I_1 + I_0$. Likewise, $I_c = -I_1 + I_0$. These are the currents flowing in phases b and c as shown in Figure 4.33.

This will always occur in any system or part of a system in which there are positive-sequence sources or zero-sequence sources at both ends. In ground fault studies, $3I_0$ values should be recorded because the ground relays are operated by $3I_0$, not the phase-fault currents, which can be quite different, as seen in Figure 4.33. Thus, there is little or no use in recording the phase values. These differences make fuse applications on loop systems quite difficult, because the fuse is operated on phase current, but the ground relays are on $3I_0$ currents.

For radial lines or feeders (positive-sequence source and a wye-grounded transformer at the same end, and no source or grounded transformer at the other end) I_b and I_c will be zero for all phase-a-to-ground faults. With the same phase and $3I_0$ ground currents, it is easier to coordinate ground relays and fuses.

4.16.4 Voltage and Current Fault Phasors for All Combinations of the Different Faults

The sequence phasors from a different perspective are presented in Figure 4.34 and Figure 4.35. The voltages and currents generated by the sources can

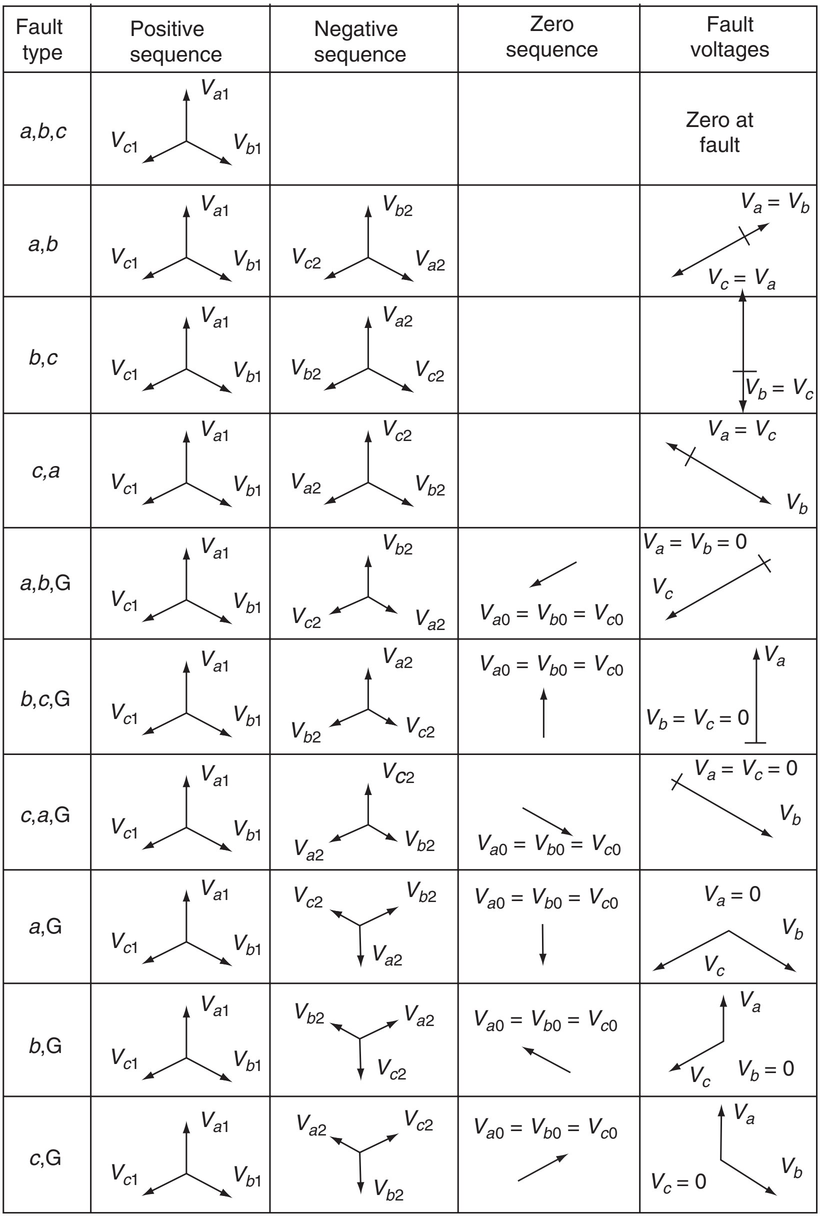

FIGURE 4.34 Sequence voltages and the voltage at the fault point for the various fault types. Solid faults with $Z_1 = Z_2 = Z_0$ for simplicity. Magnitudes are not to scale.

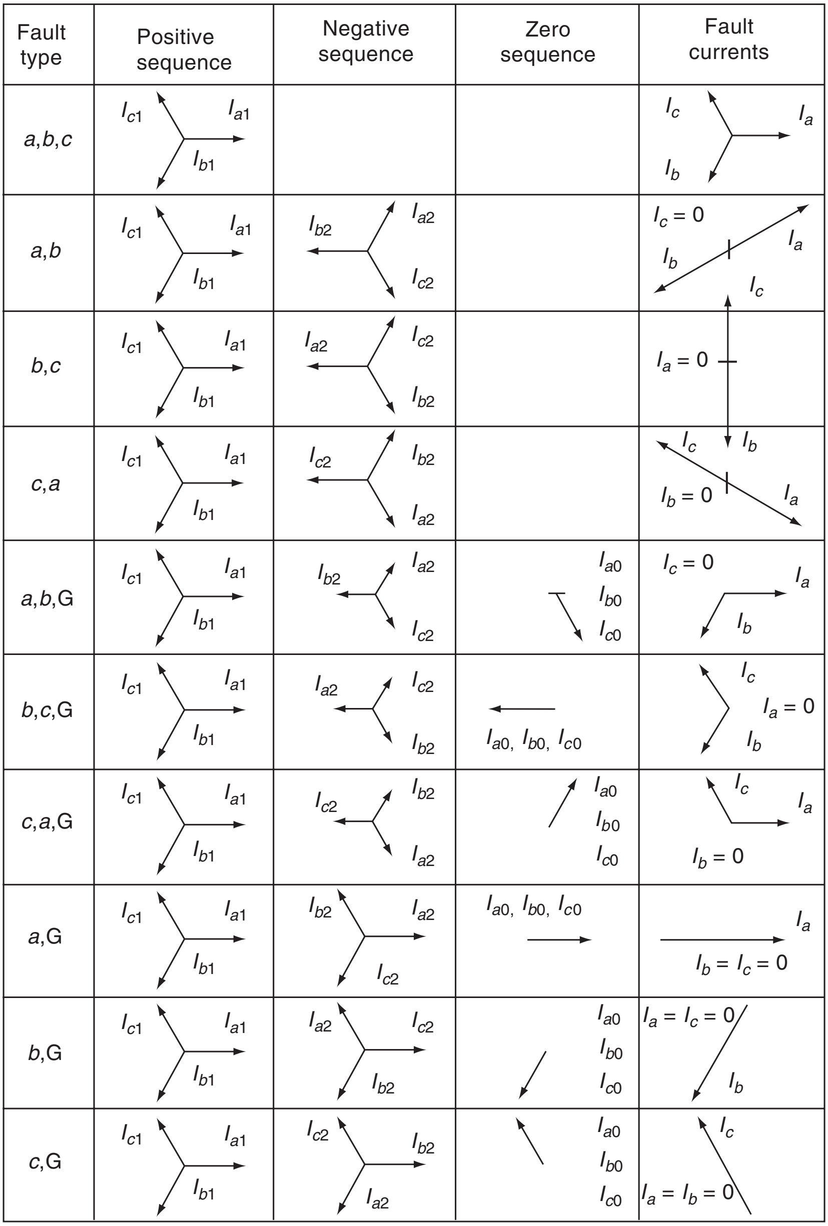

FIGURE 4.35 Sequence currents and the fault current for the various fault types: Solid faults with $Z_1 = Z_2 = Z_0$ for simplicity. Magnitudes are not to scale.

only be positive sequence by design, and nothing else. Yet the unbalanced faults require unbalanced quantities. How can this difference be resolved to satisfy both requirements: balanced quantities by the generators and unbalanced quantities at the faults? The resolution can be considered as the function of the negative-sequence quantities and for ground faults the zero-sequence quantities. This can be seen as indicated in Figure 4.34 and Figure 4.35. Considering the voltages as shown in Figure 4.34, the voltage developed by the source or generator is the same for all faults. For three-phase faults no transition help is required because these faults are symmetrical; hence, there are no negative or zero sequences. For the phase-to-phase faults, negative sequence appears to provide the transition. Note that for the several combinations, *ab*, *bc*, and *ca* phases, the negative sequence is in different positions to provide the transition. The key is that for, say, *ab* fault, phase *c* will be essentially normal, so V_{c1} and V_{c2} are basically in phase to provide this normal voltage. Correspondingly, for a *bc* fault, V_{a1} and V_{a2} are essentially in phase, and so on.

The two-phase-to-ground faults are similar; for *ab*-G faults, the uninvolved phase *c* quantities V_{c1}, V_{c2}, and V_{c0} combine to provide the uncollapsed phase *c* voltage. In the figure, these are shown in phase and at half magnitude. In actual cases, there will be slight variations because the sequence impedances do not have the same magnitude or phase angle.

For single-phase-*a*-to-ground faults, the negative (V_{z2})- and zero (V_{a0})-sequence voltages add to cancel the positive-sequence V_{a1}, which will be zero at a solid fault. Correspondingly, for a phase *b* fault, V_{b2} and V_{b0} oppose V_{b1}, and similarly for the phase *c* fault.

The same concept is applied to the sequence currents, as shown in Figure 4.35. The positive-sequence currents are shown in the same for all faults and for 90° lag (*X*-only system) relative to the voltages in Figure 4.34. These will vary depending on the system constants, but the concepts illustrated are valid. Moreover, for three-phase faults, no transition help is required; hence, there is no negative- or zero-sequence involvement.

For phase-to-phase faults negative sequence provided the necessary transition, with the unfaulted phase-sequence currents in opposition to provide either a zero or a low current. Thus, for the *ab* fault, I_{c1} and I_{c2} are in opposition.

Similarly for two-phase-to-ground faults; for an *ab*-G fault, I_{c2} and I_{c0} tend to cancel I_{c1} and so on. For single-phase-to-ground faults the faulted phase components tend to add to provide a large fault current, because $I_{a1} + I_{a2} + I_{a0} = I_a$.

4.17 SUMMARY

A question often asked is "Are the sequence quantities real or only useful in mathematical concepts?" This has been debated for years, and in a sense they are both accepted. Yes, they are real; positive sequence certainly because it is generated, sold, and consumed; zero sequence because it flows in the neutral, ground, and deltas; and negative sequence, for example, cannot be measured directly by an ammeter or voltmeter. Networks are available and commonly used in protection to measure V_2 and I_2, but these are designed to solve the basic equations for those quantities.

In either event, analyses of symmetrical components is an extremely valuable and powerful tool. Protection engineers automatically tend to think in its terms when evaluating and solving unbalanced situations in the power system.

It is important to always remember that any sequence quantity cannot exit in only one phase; this is a three-phase concept. If any sequence is in one phase, it must be in all three phases, according to the fundamental definitions of Equation 4.2 through Equation 4.4.

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Appendix 4.1 Short-Circuit MVA and Equivalent Impedance

Quite often short-circuit MVA data are supplied for three-phase and single-phase-to-ground faults at various buses or interconnection points in a power system. The derivation for this and conversion into system impedances are as follows:

THREE-PHASE FAULTS

$$\mathrm{MVA_{SC}} = 3\phi \text{ fault-short-circuit MVA} = \frac{\sqrt{3}\mathrm{I}_{3\phi}\mathrm{kV}}{1000}, \tag{A4.1-1}$$

where $I_{3\phi}$ is the total three-phase fault current in amperes and kV is the system line-to-line voltage in kilovolts. From this,

$$I_{3\phi} = \frac{1000\ \mathrm{MVA_{SC}}}{\sqrt{3}\mathrm{kV}}, \tag{A4.1-2}$$

$$Z_{\Omega} = \frac{V_{\mathrm{LN}}}{I_{3\phi}} = \frac{1000\ \mathrm{kV}}{\sqrt{3}I_{3\phi}} = \frac{\mathrm{kV}^2}{\mathrm{MVA_{SC}}}. \tag{A4.1-3}$$

Substituting Equation 2.15, which is

$$Z_{\mathrm{pu}} = \frac{\mathrm{MVA_{base}}Z_{\Omega}}{\mathrm{kV}^2}, \tag{2.15}$$

the positive-sequence impedance to the fault location is

$$Z_1 = \frac{\mathrm{MVA_{base}}}{\mathrm{MVA_{SC}}}\mathrm{pu}, \tag{A4.1-4}$$

$Z_1 = Z_2$ for all practical cases. Z_1 can be assumed to be X_1 unless X/R data are provided to determine an angle.

SINGLE-PHASE-TO-GROUND FAULTS

$$\frac{\text{MVA}_{\phi\text{GSC}=\phi}\text{G fault-short-circuit MVA} = \sqrt{3}I_{\phi\text{G}}\text{kV}}{1000}, \tag{A4.1-5}$$

where $I_{\phi\text{G}}$ is the total single-line-to-ground fault current in amperes and kV is the system line-to-line voltage in kilovolts.

$$I_{\phi\text{G}} = \frac{1000\ \text{MVA}_{\phi\text{GSC}}}{\sqrt{3}\text{kV}}. \tag{A4.1-6}$$

However,

$$I_{\phi\text{G}} = I_1 + I_2 + I_0 = \frac{3V_{\text{LN}}}{Z_1 + Z_2 + Z_0} = \frac{3V_{\text{LN}}}{Z_g}, \tag{A4.1-7}$$

where $Z_g = Z_1 + Z_2 + Z_0$. From Equation A4.1-3 and Equation A4.1-7,

$$Z_g = \frac{3\text{kV}^2}{\text{MVA}_{\phi\text{GSC}},} \tag{A4.1-8}$$

$$Z_g = \frac{3\text{MVA}_{\text{base}}}{\text{MVA}_{\phi\text{GSC}}}\ \text{pu}. \tag{A4.1-9}$$

Therefore, $Z_0 = Z_g - Z_1 - Z_2$, or in most practical cases, $X_0 = X_g - X_1 - X_2$ because the resistance is usually small in relation to the reactance.

Example

A short-circuit study indicates that at bus X in the 69 kV system,

$\text{MVA}_{\text{SC}} = 594$ MVA

$\text{MVA}_{\phi\text{GSC}} = 631$ MVA

on a 100 MVA base.

Thus, the total reactance to the fault is

$$X_1 = X_2 = \frac{100}{594} = 0.1684\ \text{pu}$$

$$X_g = \frac{300}{631} = 0.4754\ \text{pu},$$

$X_0 = 0.4754 - 0.1684 - 0.1684 = 0.1386$ pu, all values on a 100 MVA 69 kV base.

Appendix 4.2 Impedance and Sequence Connections for Transformer Banks

TWO-WINDING TRANSFORMER BANKS

Typical banks are shown in Figure A4.2-1. H is the high-voltage winding and L the low-voltage winding. These designations can be interchanged as required. Z_T is the transformer leakage impedance between the two windings. It is normally designated in per unit or percent by the transformer and stamped on the transformer nameplate. Unless otherwise specified, this value is on the self-cooled kVA or MVA rating at the rated voltages.

It can be measured by shorting one winding and applying voltage to the other winding. This voltage should not cause the transformer to saturate. From Figure A4.2-2,

$$Z_T = \frac{V}{I} = Z_H + \frac{Z_L Z_e}{Z_L + Z_e}. \tag{A4.2-1}$$

Because unsaturated Z_e is very large compared with Z_L, the term $Z_L Z_e / Z_L + Z_e$ approaches and is approximately equal to Z_L, so that for practical purposes

$$Z_T = \frac{V}{I} = Z_H + Z_L. \tag{A4.2-2}$$

Z_T is measured in practice by circulating rated current (I_R), through one winding with the other shorted and measuring the voltage (V_W) required to circulate this rated current. Then.

$$Z_T = \frac{V_W}{I_R}\ \Omega. \tag{A4.2-3}$$

This test can be done for either winding or as convenient. On the measured side, the base impedance will be

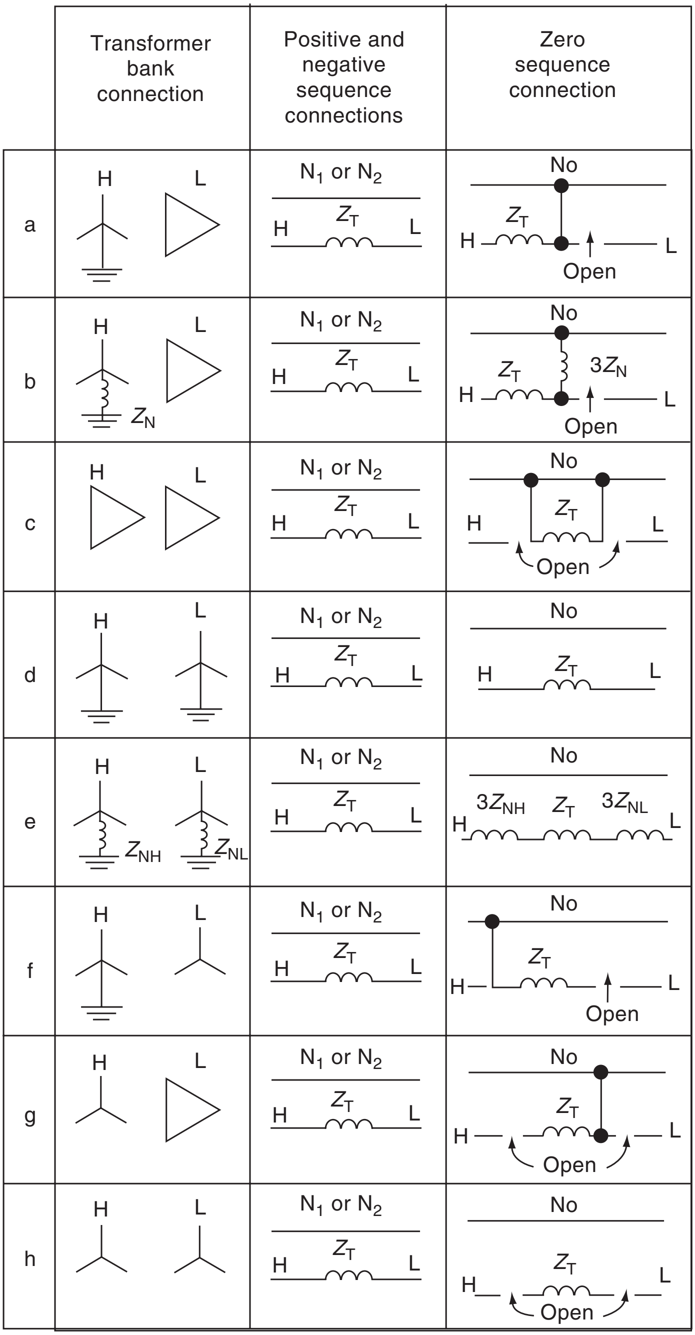

FIGURE A4.2-1 Sequence connections for typical two-winding transformer banks.

$$Z_B = \frac{V_R}{I_R}\ \Omega, \tag{A4.2-4}$$

where V_R and I_R are the rated voltage and current, respectively.

Then, the per-unit impedance from Equation 2.1 is

$$Z_T = \frac{Z_T(\Omega)}{Z_B(\Omega)} = \frac{V_W I_R}{I_R V_R} = \frac{V_W}{V_R}\ \text{pu}. \tag{A4.2-5}$$

For three-phase–type transformer units, the nameplate should specify this Z_T, usually in percent, on the three-phase kVA (MVA) rating, and the kV line-to-line voltages. When several kVA (MVA) ratings are specified, the normal rating, without fans, pumps, and such (lowest) rating, should be used as one of the impedance bases.

For individual single-phase transformers, the transformer impedance is normally specified on a single-phase kVA (MVA) and the rated winding voltages (kV) of the transformer. When three such units are used in three-phase systems, then the three-phase kVA (MVA) and line-to-line voltage (kV) bases are required, as outlined in Chapter 2.

Thus, when three individual single-phase transformers are connected in the power system, the individual nameplate percent or per-unit impedance will be the Z_T leakage impedance, but only on the three-phase kVA (MVA) base, and the system line-to-line kV.

Example: Impedance of Single-Phase Transformers in Three-Phase Power Systems

Consider single-phase transformers, each with a nameplate rating of 20 MVA, 66.5 kV: 13.8 kV, $X = 10\%$. Considering the individual transformer alone, its leakage reactance is

$$X_T = 0.10 \text{ pu on 20 MVA, 66.5 kV or} \tag{A4.2-6}$$

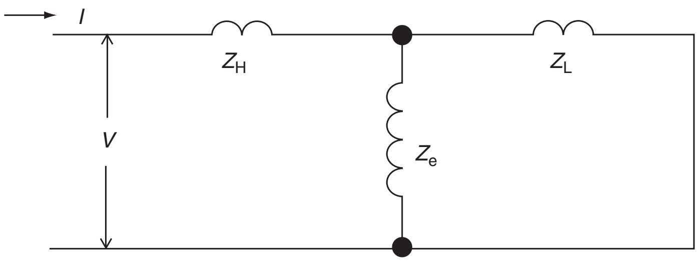

FIGURE A4.2-2 Simplified equivalent diagram for a transformer: Z_H and Z_L are the components of the transformer leakage impedance and Z_e is the exciting impedance. All values are in per unit or primary H side ohms.

$$X_T = 0.10 \text{ pu on 20 MVA, 13.8 kV.}$$

Converting these to actual ohms using Equation A4.2-5, $V_{WH} = 0.10\ V_{RH} = 0.10 \times 66{,}500 = 6650$ volts on the high side, where $I_{RH} = 20{,}000/66.5 = 300.75$ amperes primary.

Then, we obtain from Equation A4.2-3,

$$X_{TH} = \frac{6650}{300.75} = 22.11\ \Omega \text{ primary} \tag{A4.2-7}$$

or on the secondary side, $V_{WL} = 0.10 \times 13{,}800 = 1380$ V, and $I_{RL} = 20{,}000/\ 13.8 = 1449.28$ amperes secondary.

$$X_{TL} = \frac{1380}{1449.28} = 0.952\ \Omega \text{ secondary.} \tag{A4.2-8}$$

Check:

$$\left(\frac{66.5}{13.8}\right)^2 \times 0.952 = 22.11\ \Omega \text{ primary.}$$

Now consider two possible applications of three of these individual transformers to a power system. These are intended to demonstrate the fundamentals; do not consider if the transformer windings are compatible or suitable for the system voltages shown.

Case 1

Connect the high-voltage windings in wye to a 115 kV system and the low-voltage windings in delta to a 13.8 kV system. As indicated previously, the leakage impedance of this transformer bank for this application is

$$X_T = 0.10 \text{ pu on 60 MVA, 115 kV,}$$

$$X_T = 0.10 \text{ pu on 60 MVA, 13.8 kV.} \tag{A4.2-9}$$

Now Let us check this. From Equation 2.17,

$$X_{TH} = \frac{115^2 \times 0.10}{60} = 22.11\ \Omega \text{ primary,}$$

$$X_{TL} = \frac{13.8^2 \times 0.10}{60} = 0.317\ \Omega \text{ secondary.} \tag{A4.2-10}$$

It will be noted that the individual transformer reactance per Equation A4.2-8 is 0.952 Ω, but this is the reactance across the 13.8 kV because of the delta connection. The equivalent wye impedance can be determined by the product of the two-delta branches on either side of the desired wye branch divided by the sum of the three-delta branches. Thus, the wye equivalent is

$$\frac{(0.952)0.952}{(3)0.952} = \frac{0.952}{3} = 0.317\ \Omega \text{ as before}$$

Check

$$\left(\frac{115}{13.8}\right)^2 \times 0.317 = 22.1\ \Omega \text{ primary}$$

Case 2

Connect the high-voltage windings in delta to a 66.5 kV system and the low-voltage side in wye to a 24 kV system. Now the transformer bank impedance for this system application is

$$X_T = 0.10 \text{ pu on 60 MVA, 66.5 kV} \quad \text{or}$$

$$X_T = 0.10 \text{ pu on 60 MVA, 24 kV.} \tag{A4.2-11}$$

Now to check this by converting to ohms, using Equation 2.17

$$X_{TH} = \frac{66.5^2 \times 0.10}{60} = 7.37\ \Omega \text{ primary} \tag{A4.2-12}$$

is obtained.

Now this primary winding of 22.11 ohms (Equation A4.2-7) is connected across the 66.5 kV system because of the delta. Accordingly, the equivalent wye reactance is $22.11 \times 22.11/3 \times 22.11 = 7.37$ ohms line-to-neutral on the primary side.

On the secondary side, $X_{TL} = 24^2 \times 0.10/60 = 0.952\ \Omega$ is secondary as Equation A4.2-8.

Check

$$\left(\frac{66.5}{24}\right)^2 \times 0.952 = 7.37\ \Omega \text{ primary.}$$

The connections of two-winding transformers in the sequence networks are documented in Figure A4.2-1. Note that the connections for positive and negative sequences are all the same and are independent of the bank connections. This is not true for the zero sequence with different connections for each type of bank.

Neutral impedance is shown for several connections. If the blanks are solid-grounded, the neutral impedance is zero, and the values shown are shorted-out in the system and are zero-sequence diagrams.

THREE-WINDING AND AUTOTRANSFORMER BANKS

Typical banks are shown in Figure A4.2-3. H, M, and L are the high-, medium-, and low-voltage windings. These designations can be interchanged as required. Normally, the manufacturer provides the leakage impedance between the windings as Z_{HM}, Z_{HL}, and Z_{ML}, usually on different kVA or MVA ratings and at the rated winding voltages.

To use these impedances in the sequence networks, they must be converted to an equivalent wye-type network, as shown. This conversion is

$$Z_H = \tfrac{1}{2}(Z_{HM} + Z_{HL} - Z_{ML}), \tag{A4.2-13}$$

$$Z_M = \tfrac{1}{2}(Z_{HM} + Z_{ML} - Z_{HL}), \tag{A4.2-14}$$

$$Z_L = \tfrac{1}{2}(Z_{HL} + Z_{ML} - Z_{HM}). \tag{A4.2-15}$$

It is easy to remember this conversion, for the equivalent wye value is always half the sum of the leakage impedances involved, minus the one that is not involved. For example, Z_H is half of Z_{HM}, both involving H and minus Z_{ML} that does not involve H.

After determining Z_H, Z_M, and Z_L, a good check is to see if they add up as $Z_H + Z_M = Z_{HM}, \ldots$. If these values are not available, they can be measured as described for a two-winding transformer. For the three-winding or autotransformers: Z_{HM} is the impedance looking into H winding with M shorted, L open; Z_{HL} is the impedance looking into H winding, with L shorted, M open; Z_{ML} is the impedance looking into M winding, with L shorted, H open.

This equivalent wye is a mathematical network representation valid for determining currents and voltages at the transformer terminals or in the associated system. The wye point has no physical meaning. Quite often, one of the values will be negative and should be used as such in the network. It does not represent a capacitor.

The positive- and negative-sequence connections are all the same and independent of the actual bank connections. However, the connections for the zero-sequence network are all different and depend on the transformer bank connections. If the neutrals are solidly grounded, then the Z_N and $3Z_N$ components shown are shorted-out in the system and sequence circuits.

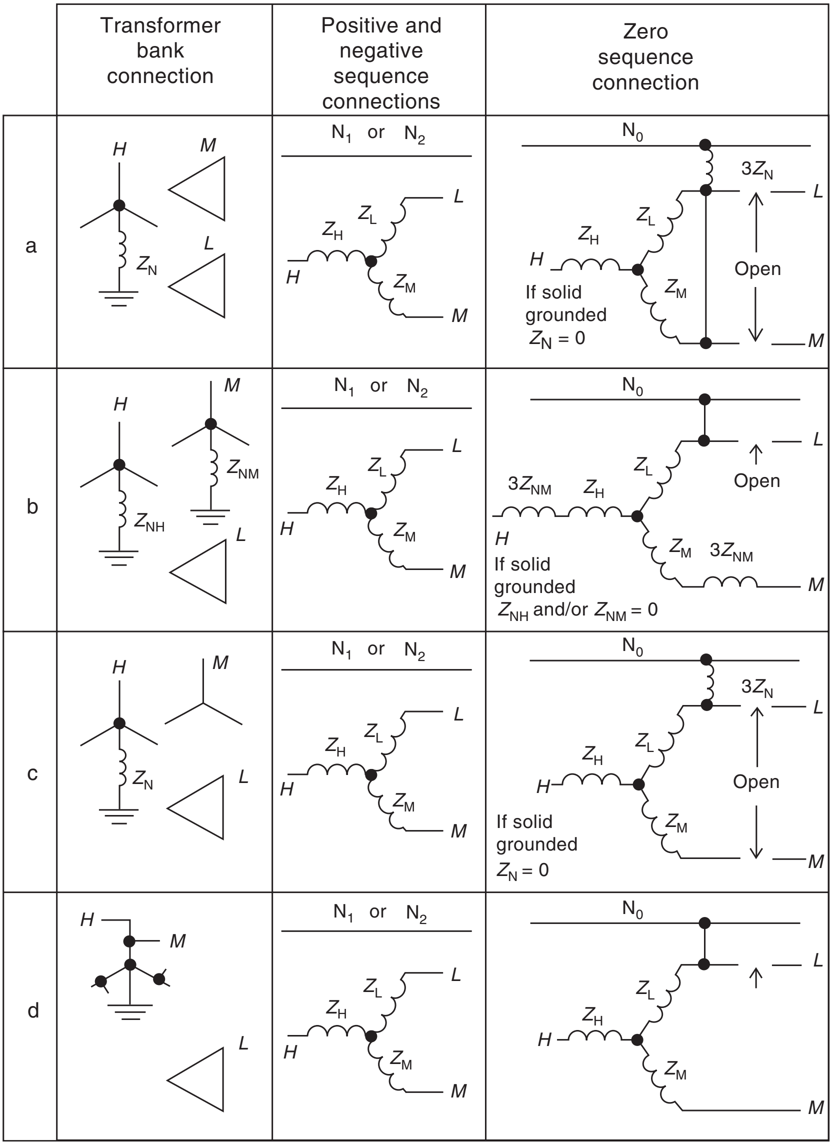

FIGURE A4.2-3 Sequence connections for typical three-winding and autotransformer banks.

Appendix 4.3 Sequence Phase Shifts through Wye–Delta Transformer Banks

As has been indicated, positive and negative sequences pass through the transformer bank, and in the sequence networks, the impedance is the same, independent of the bank connection. This is shown in Figure A4.2-1 and Figure A4.2-3. In these networks the phase shift is ignored, but if currents and voltages are transferred from one side of the transformer bank to the other, these phase shifts must be taken into account. This appendix will document these relations. For this the standard ANSI connections are shown in Figure A4.3-1.

From Figure A4.3-1a, all quantities are phase-to-neutral values, and in amperes or volts; for per unit, $N = 1$, $n = 1/\sqrt{3}$.

$$I_{\mathrm{A}} = n(I_a - I_c) \text{ and } V_a = n(V_{\mathrm{A}} - V_{\mathrm{B}}).$$

For positive sequence (see Equation 4.2),

$$\begin{aligned} I_{\mathrm{A}1} &= n(I_{a1} - aI_{a1}) = n(1 - a)I_{a1} \\ &= \sqrt{3}nI_{a1}\,\underline{/-30^\circ} = NI_{a1}\,\underline{/-30^\circ}, \end{aligned} \tag{A4.3-1}$$

$$\begin{aligned} V_{a1} &= n(V_{\mathrm{A}1} - a^2V_{\mathrm{A}1}) = n(1 - a^2)V_{\mathrm{A}1} \\ &= \sqrt{3}nV_{\mathrm{A}1}\,\underline{/+30^\circ} = NV_{\mathrm{A}1}\,\underline{/+30^\circ}. \end{aligned} \tag{A4.3-2}$$

For negative sequence (see Equation 4.3),

$$\begin{aligned} I_{\mathrm{A}2} &= n(I_{a1} - a^2I_{a1}) = n(1 - a^2)I_{a2} \\ &= \sqrt{3}nI_{a2}\,\underline{/+30^\circ} = NI_{a2}\,\underline{/+30^\circ}, \end{aligned} \tag{A4.3-3}$$

$$V_{a2} = n(V_{\mathrm{A}2} - aV_{\mathrm{A}2}) = n(1 - a)V_{\mathrm{A}2}, \tag{A4.3-4}$$

$$= \sqrt{3}nV_{\mathrm{A}2}\,\underline{/-30^\circ} = NV_{\mathrm{A}2}\,\underline{/-30^\circ}. \tag{A4.3-5}$$

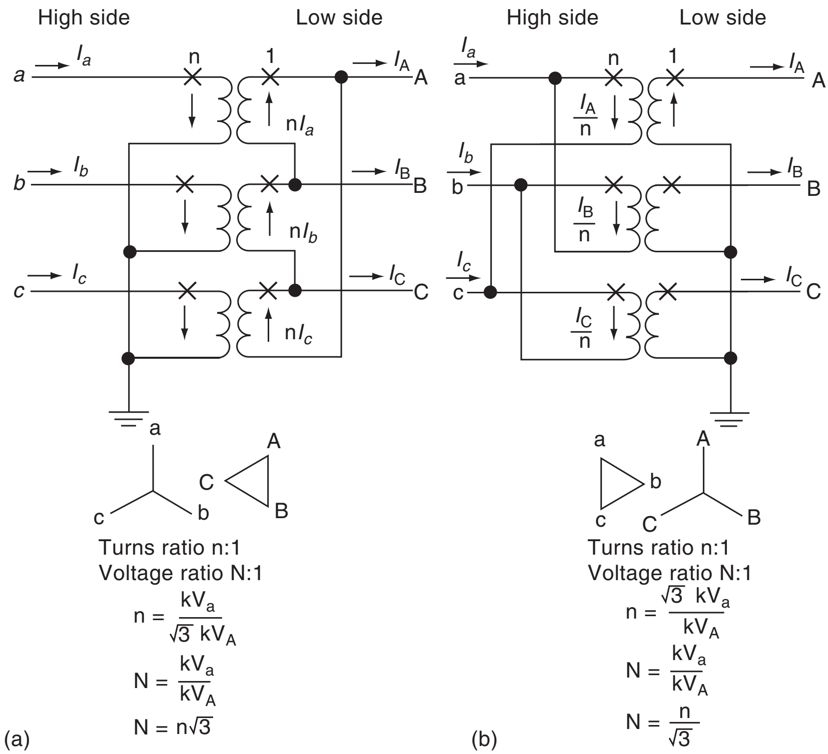

FIGURE A4.3-1 ANSI-connected wye–delta transformer banks: the high-voltage side phase *a* leads the low-voltage side phase *a* for both connections illustrated: (a) wye (star) on high side; (b) delta on high side.