EFFECTS OF DISSOLVED GAS ON CAVITATION INCEPTION IN FREE SURFACE FLOWS

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ABSTRACT

In a fluid flow the reduction of the local pressure below vapour pressure causes an explosive growth of nuclei which is characterised as vapour cavitation. At the same time the reduction of the partial pressure of gas in the fluid from the equilibrium to a supersaturated state produces the formation of free gas due to diffusion and consequently growth of the nucleus. This process is examined theoretically for a spherical bubble and transferred to the flow conditions in the investigated experimental set-up. The growth of a single bubble is estimated from the mass flow into the bubble using a mathematical approach based on the quasi steady bubble theory. This theory takes into account the convective diffusion of dissolved gas which increases the driving process of diffusion. The theoretical investigations are applied to cavitation nuclei which were measured in the physical experiments of a free surface flow to quantify the influence of gaseous cavitation.

NOMENCLATURE

C	Concentration	(kmol/m³)
D	Diffusion coefficient of air in water	(m²/s)
J	Mass-flux	(kmol/m²·s)
K	Gas constant	(J/kmol·K)
k	Coefficient of absorption	(m³ air/m³ H_2O·bar)
l	Length of strip in Fig. 1	(m)
M	Molar mass	(kg/kmol)
m	Mass	(kg)
R	Bubble radius	(m)
p	Static pressure	(N/m²)
T	Temperature	(K)
t	Time	(s)
U	Mean stream velocity	(m/s)
$\vec{v}$	Local fluid velocity	(m/s)
σ	Coefficient of surface tension	(N/m)

Subscripts

fl	Fluid
g	Gas
i	Increment
rel	relative
s	Saturation
v	Vapour
∞	at infinity
$_0$	Initial value in R_0 , equilibrium value in C_0

1. INTRODUCTION

Cavitation inception in a given flow configuration is generally described by the Thoma-Number. Accordingly, model tests for prototypes are carried out using the Thoma-Number as a scaling law. But several damages in prototypes have shown that the scale-up of results obtained in these model tests is severely influenced by scale effects. The factors influencing the scale-up – and thus the Thoma-Number for cavitation inception – can be divided in two global groups: factors influencing the hydrodynamic parameters (e.g. flow velocity or pressure distribution) and factors influencing the properties of the fluid (e.g. content of free or dissolved gas). Focusing on the latter parameters, in this paper the influence of the dissolved gas is investigated theoretically and compared to measurements on a free surface flow.

The scale effects induced by dissolved gas in the fluid are due to the fact that the gas can cause „gas cavitation“ which is often interpreted as „vapour cavitation“. In this case „gas cavitation“ is defined as a process in which dissolved air diffuses into gas nuclei and influences their stability. The radius of a small, spherical nucleus in equilibrium state is calculated to:

$$R = \frac{2\sigma}{p_g + p_v - p_{fl}} \qquad (1)$$

It is influenced by surface tension σ, partial pressure of gas p_g, vapour pressure p_v and the outer pressure p_{fl} of the fluid. As it can be seen from this equation, diffusion of dissolved gas will change the partial pressure of gas and thus the bubble radius. The examinations in this paper are dealing with cavitation nuclei which are observed in the free flow. Effects on a steady cavity behind a body are not investigated.

2. DIFFUSION OF DISSOLVED AIR

The mechanism of diffusion causes a molecular mass-flux J due to the behaviour of a fluid to balance differences in concentration. The direction of diffusion follows the concentration gradient, which is described by Fick's law:

$$J = -D \cdot \nabla C \tag{2}$$

The factor of proportionality is the molecular diffusion coefficient D for air in water. The mass-flux is affecting the molecular components of air which are dissolved in the fluid. If the fluid is supersaturated the gas will be present as free gas transported within the fluid. The equilibrium concentration C_O of gas in the fluid around a nucleus depends on the partial pressure p_g at its phase boundary according to Henry's law:

$$C_O = k \cdot p_g \tag{3}$$

The factor k denotes the proportionality being only a function of temperature. Dealing with a free surface flow, the reference pressure for the equilibrium concentration of the whole system is the atmospheric pressure. Assuming saturation equilibrium in the system, the fluid can be supersaturated in the vicinity of a nucleus where the equilibrium is characterised by its partial pressure of gas. Due to the concentration gradient diffusion of dissolved air into the bubble is occurring.

The mass flux dm/dt causes the growth of the bubble according to the basic laws of gas thermodynamics. Using the gas density inside the bubble as proportional to the partial pressure of gas yields (Baur [1])

$$\frac{dR}{dt} = \frac{\dfrac{KT}{4\pi R^2 M} \cdot \dfrac{dm}{dt} - \dfrac{R}{3} \cdot \dfrac{dp_{fl}}{dt}}{p_{fl} - p_v + \dfrac{4\sigma}{3R}} \tag{4}$$

where K is the general gas coefficient and M the molar mass. Now the approach has to focus on the determination of the change of mass due to diffusion processes.

3. MATHEMATICAL APPROACH OF CONVECTIVE DIFFUSION

To quantify the change of mass dm/dt the transport equation (5) of gas, which governs the concentration distribution, has to be taken into consideration

$$\frac{\partial C}{\partial t} + \vec{v} \cdot \nabla C = D \cdot \nabla^2 C \tag{5}$$

with the boundary conditions

$$C = C_\infty \quad \text{at infinity} \tag{6}$$

$$C = C_0 \quad \text{at the bubble surface.} \tag{7}$$

According to Parkin and Kermeen [2] the convective part of the conventional advection-diffusion-equation is applied, using a velocity vector which indicates a convective transport of dissolved gas. Looking at the different influence factors of diffusion, it was proved in reference [2] that convective diffusion is of much higher importance for the mass flow than molecular diffusion. In this terms convective diffusion means the diffusion of dissolved gas under the influence of the convective transport of fluid (Fig. 1). This is represented by the velocity U_{rel} of the fluid relative to the velocity of the bubble.

The authors [2] presumed a steady state diffusion process with

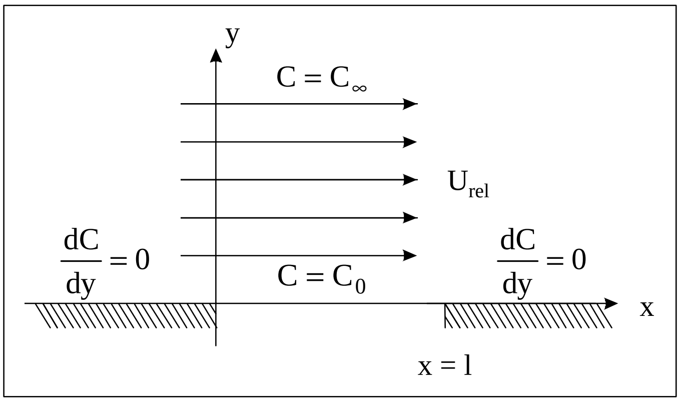

Fig. 1: Model of convective diffusion by Parkin and Kermeen [2]

no severe effect of $\partial C / \partial t$ and developed an approximation theory with the boundary conditions of Figure 1. The bubble surface is taken as a strip of length l. At the right and left side of it the concentration gradient is zero. The fluid is transported with the homogeneous velocity U_{rel} parallel to the surface. The length l can be taken as

$$l^2 = \pi R^2 \tag{8}$$

in order to use the whole bubble surface to be active for diffusion. This finally yields the change of mass where D is again the molecular diffusion coefficient [2]:

$$\frac{dm}{dt} = 4(C_\infty - C_0)R\left(\sqrt{U_{rel} D R \sqrt{\pi}} - D\frac{\sqrt{\pi}}{2}\right) \tag{9}$$

C_∞ is in sense of equation (3) associated with atmospheric pressure. The combination of equation (4) and (9) yields a differential equation which is now solved by a finite differences approach in order to receive the time dependent bubble radius:

$$R_{i+1} = R_i + \left(\frac{dR}{dt}\right)_i \cdot \Delta t \tag{10}$$

Within a time interval Δt the growth is taken to be quasi constant and dependent on the global parameters T, p_v, p_∞, C_∞, D, σ, U_{rel}, and the quasi constant parameters for each time step p_{fl}, p_g, C_0 and ρ_g. At the same time an initial bubble radius R_0 has to be applied.

Parkin and Kermeen [2] did not take into account the effects of surface tension and vapour pressure in the bubble when applying their theory to an observed experiment. The theory was re-examined by Wijngaarden [3] who focused on this aspect but assumed a negligible equilibrium concentration of gas at the bubble surface. In this paper the latter is taken from equations (1) and (3) and used in equation (9).

With this mathematical approach a tool is given to calculate theoretically the effect of convective diffusion on the bubble growth in any flow condition.

4. EXPERIMENTAL INVESTIGATION

The observed flow field, to which the mathematical approach is applied, is part of a free surface flow with a small sill fixed to the bottom of a rectangular test section according to Figure 2. In the shear layer behind the sill, cavitation inception can be observed in the indicated region of interest. The boundary parameters for the diffusion processes which have to be measured are system temperature and system pressure and the initial radius of the observed nucleus.

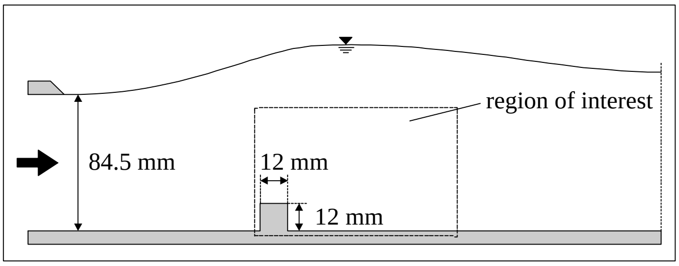

Fig. 2: Observed flow condition in cavitation tunnel

By digital image processing the growth of nuclei could be measured from video images of an area behind the sill. The images were recorded by a high speed video camera (1000 Hz) of type *Kodak Ektapro Motion Analyser*. Using a threshold of grey value, a bubble can be separated in the image from the surrounding fluid. More about this method applied in cavitation experiments can be found in Leucker and Rouvé [4]. The radius of the nucleus can be estimated with the mentioned technique. From a sequence of images the actual time dependent bubble growth can be determined. The result of a measurement is shown in Figure 3. The bubble is identified in this example by two different thresholds in the grey value distribution.

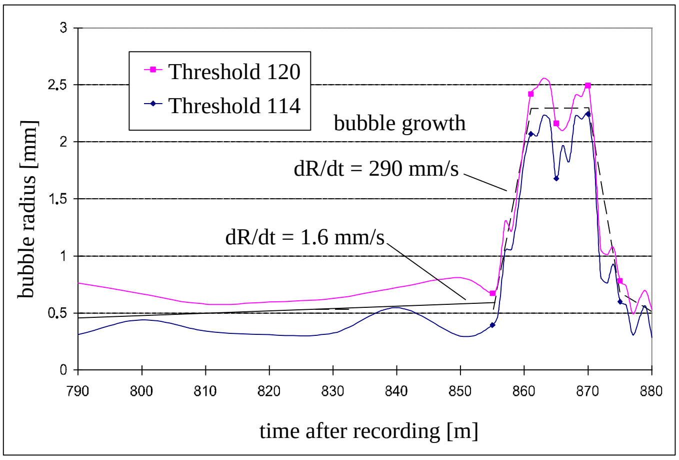

Fig. 3: Observed bubble growth in experiment

The plot indicates the time dependent change of the radius while the straight line characterises its idealisation taking into account the different thresholds. At the time 855 ms after recording the bubble of the initial radius 0.54 mm suddenly explodes to a size of approx. 2.3 mm. The growth changes from 1.6 mm/s to 290 mm/s

Besides the change in bubble radius, the co-ordinates of the bubble centre for every image is measured in order to get an information about the way the bubble follows by passing the sill. This is important to investigate the missing parameters for the theoretical approach as it is mentioned later.

5. COMPARISON WITH THEORETICAL BUBBLE GROWTH

The measured bubble growth is now to be compared with the theoretical growth. In this paper the state variations in the pressure field are regarded for a time, which is long enough that diffusion of dissolved air and saturation equilibrium must be taken into account but short enough that effects of inertia and friction can be neglected.

The molecular diffusion coefficient for equation (9) is chosen according to Parkin and Kermeen [2] with $D = 2{\cdot}10^{-9}$ m²/s. With this parameter there is still the need for the knowledge of the velocity of the fluid relative to the velocity of the bubble. To cover this demand data of Laser-Doppler-Measurements were used. At a rectangular grid around the sill mean velocities and RMS-values were measured. On this basis also the steady pressure field can be deduced.

To take into account the effect of convective diffusion not the absolute velocity but the flow velocity relative to the nucleus has to be presumed. The relative velocity was roughly estimated according to Figure 4 where the vertical velocity gradient from the LDV measurements and the bubble radius results in a velocity difference.

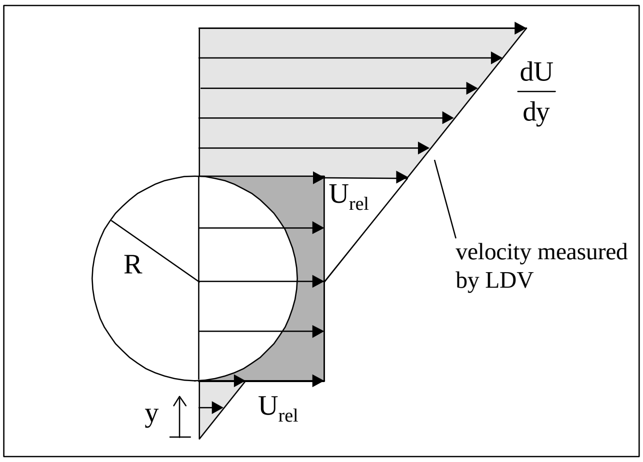

Fig. 4: Estimation of the fluid velocity relative to the bubble

This is given by:

$$U_{rel} = \frac{dU}{dy} \cdot R \qquad (11)$$

Furthermore, the actual pressure of the ambient fluid which is required in equation (4) is taken from the Lagrangian pressure distribution which can be obtained from the way through the calculated time averaged pressure field.

Taking all these parameters into account, it is possible to theoretically determine the change of bubble radius due to diffusion processes while passing through the shear layer above the sill (Fig. 5). The initial radius is taken to be 0.54 mm as it was observed in the experiment.

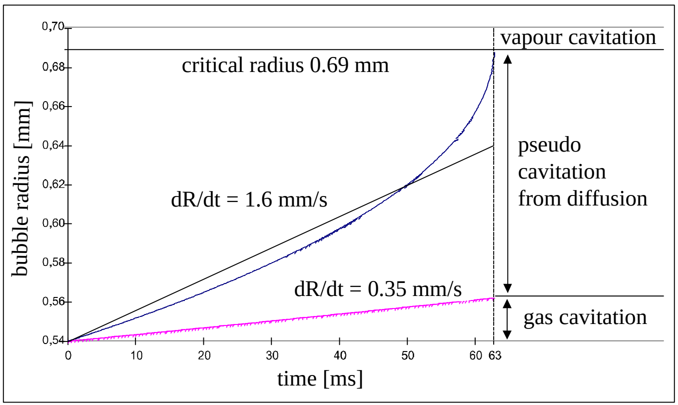

Fig. 5: Theoretical growth of the nucleus in the experiment

In Figure 5 three different curves are displayed, the lower one is representing the bubble growth due to pure gas cavitation. The other two curves additionally cover the effect of the static equilibrium diameter due to surface tension while the straight line is a linear approximation. Thus, the upper curves have to be associated with the definition of pseudo cavitation. Therefore the shown effect is called pseudo cavitation from diffusion.

It can be seen that gas cavitation in the case of a relative velocity of the fluid of 0.16 m/s as determined with equation (11) causes a constant growth of 0.35 mm/s. Furthermore, the upper plot of pseudo cavitation demonstrates that the growth of the radius increases steadily and reaches after 63 ms an infinite value. In this point vapour cavitation is occurring which could be also observed in the experiment when a bubble growth of 290 mm/s was measured. The average growth of the curve is approximated with 1.6 mm/s agreeing well with the initial phase of the observed bubble. Even the theoretical approach with equation (4) indicates a critical radius R when the gradient dR/dt is infinite and the bubble becomes unstable due to vapour cavitation.

It was proved in reference [1] that the bubble growth due to the effect of convective diffusion as examined here is more than three orders of magnitude higher than molecular diffusion.

6. INFLUENCE OF SUPERSATURATED FLUID

The boundary conditions for the simulated bubble growth include the system pressure of 0.4 bar as it was measured in the experiments. Associated with this pressure the content of dissolved gas in the fluid is presumed to be in equilibrium which means the fluid is saturated with an air content of 0.87 ppm.

In order to get an idea about the influence of supersaturated conditions in the flow the simulation was performed with a range of different dissolved air contents with the other boundary conditions remaining unchanged. The result is shown in Figure 6.

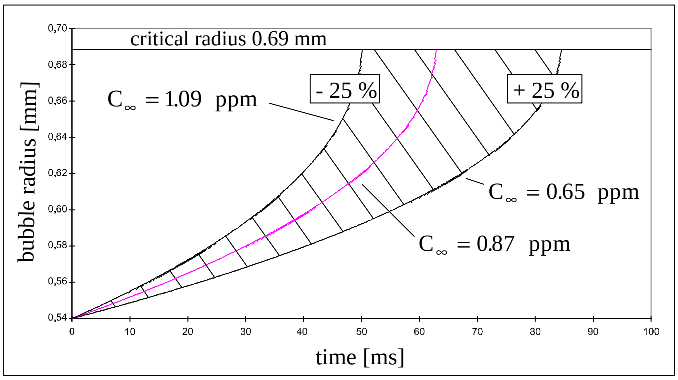

Fig. 6: Bubble growth at under- and supersaturated gas content

A supersaturated flow with a degree of 125 % gas content would yield to the growth of the same bubble to the critical radius within a time period of only 48 ms. In other words, under the same flow conditions in a 25 % supersaturated flow the nuclei would tend to vapour cavitation already at a smaller size of approx. 0.53 mm only, taking into consideration the same time period to grow. Compared to the examined degree of saturation these effects are small and considered to be negligible with respect to the main causing factor of vapour cavitation which is the hydrodynamic pressure in the fluid.

Actual measurements of the air content in the cavitation tunnel showed that the system rather tends to under-saturated conditions. Similar determinations as made above can be expressed for under-saturated flow conditions respectively.

7. CONCLUSIONS

From the comparison of the theoretical and observed bubble behaviour the following statements can be made:

1. The bubble growth in the initial phase of the observed sequence can be recalculated by using the theory of convective diffusion. The boundary conditions for the mathematical approach are only rough estimates but adequate for the simulation and realistic.

2. The observed explosion of the nucleus after a certain time must be caused by vapour cavitation. The mathematical approach yields the development of a critical radius after this time when the bubble becomes unstable.

Taking into account the difference between gas cavitation and pseudo cavitation due to diffusion processes, the theory of the convective diffusion shows that pure gas cavitation in the observed flow condition is without a severe effect on the bubble growth. But the effect of pseudo cavitation which comes along from surface tension at the interface with the fluid can cause a critical radius. Then the bubble becomes unstable and vapour cavitation occurs as it happens in the experiment. It is a question of the residence time of the nucleus in the critical flow field whether diffusion causes growth to a critical size.

The driving force for the convective diffusion is the relative velocity. As investigated by other authors (e.g. Parkin and

Ravindra [5]) in flow conditions where the fluid is transported along a steady cavity, the gas cavitation can be of a magnitude higher due to the increased relative motion.

Obviously, in the given flow condition the main parameter which is significant for an increased bubble growth due to diffusion processes, i.e. dissolved gas content in the fluid, can influence the cavitation inception if it is not controlled in cavitation experiments. The dissolved gas content is related to the quality of the fluid, which is in most cases investigated during the experiment. Usually the self-induced process of the formation of an equilibrium state does not allow for under- or supercritical flow conditions which exceed the equilibrium with more than 25 %. The effect within this range is small. Though, in theory the quality of the fluid can decide whether a nucleus tends to vapour cavitation. The mathematical approach on the basis of some approximations could be made in this paper.

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