Consider: $$s = f(x,t) = \frac{x}{2\sqrt{\alpha t}}$$

And the temperature as function of s(x,t):

$$T(s) = (T_i - T_s) \cdot \text{erf}(s) + T_s = (T_i - T_s)\frac{2}{\sqrt{\pi}}\int_0^s e^{-s^2}\,ds + T_s$$

Where Ti is the initial temperature of the body and Ts is the surface temperature

The heat flux is defined as:

$$\dot{q_s}'' = -k\frac{\partial T(s)}{\partial x} = k(T_s - T_i)\frac{2}{\sqrt{\pi}} \cdot \frac{\partial}{\partial x}\left[\boldsymbol{\int_0^s e^{-s^2}\,ds}\right] \quad \text{(notice)}$$

The term $\frac{\partial}{\partial x}\left[\int_0^s e^{-s^2}\,ds\right]$ is calculated using the Leibniz's rule, thus:

$$\dot{q_s}'' = k(T_s - T_i)\frac{2}{\sqrt{\pi}}\left\{\int_0^{\frac{x}{2\sqrt{\alpha t}}}\frac{\partial}{\partial x}\left[e^{-s^2}\right]ds + e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2}\frac{\partial}{\partial x}\left[\frac{x}{2\sqrt{\alpha t}}\right] - e^{-(0)^2}\frac{\partial}{\partial x}[0]\right\}$$

$$= k(T_s - T_i)\frac{2}{\sqrt{\pi}} \cdot e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2} \cdot \frac{1}{2\sqrt{\alpha t}} = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}} \cdot e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2}$$

Question: how to achieve the same result expressing the temperature function in terms of x and t instead of s?

$$ds = \frac{\partial}{\partial x}\left(\frac{x}{2\sqrt{\alpha t}}\right) \cdot dx + \frac{\partial}{\partial t}\left(\frac{x}{2\sqrt{\alpha t}}\right) \cdot dt$$

$$T(x,t) = (T_i - T_S)\left\{\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2} \cdot \left[\frac{\partial}{\partial x}\left(\frac{x}{2\sqrt{\alpha t}}\right) \cdot dx + \frac{\partial}{\partial t}\left(\frac{x}{2\sqrt{\alpha t}}\right) \cdot dt\right]\right\} + T_S$$

$$= (T_i - T_S)\frac{2}{\sqrt{\pi}}\left\{\int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2} \cdot \left(\frac{1}{2\sqrt{\alpha t}}\right) \cdot dx + \int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2} \cdot \left(-\frac{x}{4\sqrt{\alpha}} \cdot t^{-\frac{3}{2}}\right) \cdot dt\right\} + T_S$$

If the previous equation was correct, then the following integrals

$$I_x = \int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2} \cdot \left(\frac{1}{2\sqrt{\alpha t}}\right) \cdot dx \qquad \text{and} \qquad I_t = \int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2} \cdot \left(-\frac{x}{4\sqrt{\alpha}} \cdot t^{-\frac{3}{2}}\right) \cdot dt$$

may be solved by substitution, placing $y = \frac{x}{2\sqrt{\alpha t}}$

$$D[y]dy = D\left[\frac{x}{2\sqrt{\alpha t}}\right]dx \qquad \rightarrow \qquad dy = \frac{1}{2\sqrt{\alpha t}} \cdot dx$$

$$I_x = \int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2} \cdot \left(\frac{1}{2\sqrt{\alpha t}}\right) \cdot dx = \int_0^y e^{-(y)^2} \cdot dy$$

$$D[y]dy = D\left[\frac{x}{2\sqrt{\alpha t}}\right]dt \qquad \rightarrow \qquad dy = -\frac{1}{4\sqrt{\alpha}} \cdot t^{-\frac{3}{2}} \cdot dt$$

$$I_t = \int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-\left(\frac{x}{2\sqrt{\alpha t}}\right)^2} \cdot \left(-\frac{x}{4\sqrt{\alpha}} \cdot t^{-\frac{3}{2}}\right) \cdot dt = \int_0^y e^{-(y)^2} \cdot dy$$

$$T(x,t) = (T_i - T_S)\frac{2}{\sqrt{\pi}}\{I_x + I_t\} + T_S = (T_i - T_S)\frac{2}{\sqrt{\pi}}\left\{\int_0^y e^{-(y)^2} \cdot dy + \int_0^y e^{-(y)^2} \cdot dy\right\} + T_S$$
$$= (T_i - T_S)\frac{2}{\sqrt{\pi}} \cdot 2 \cdot \int_0^y e^{-(y)^2} \cdot dy + T_S$$

But, if it was correct, eventually I would get a result double than simply applying the Leibniz's rule to the temperature function expressed as function od s only.

Where is the mistake?