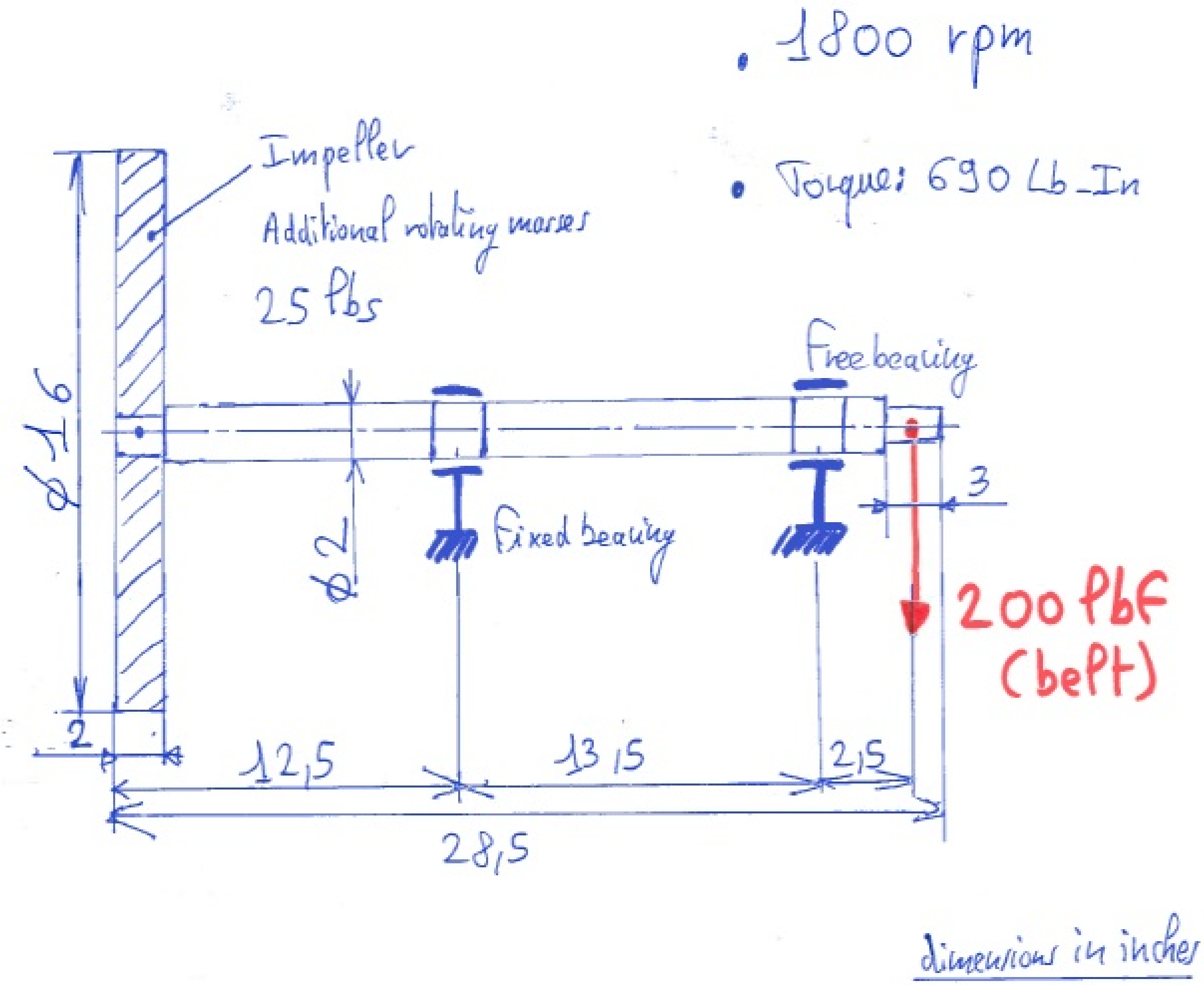
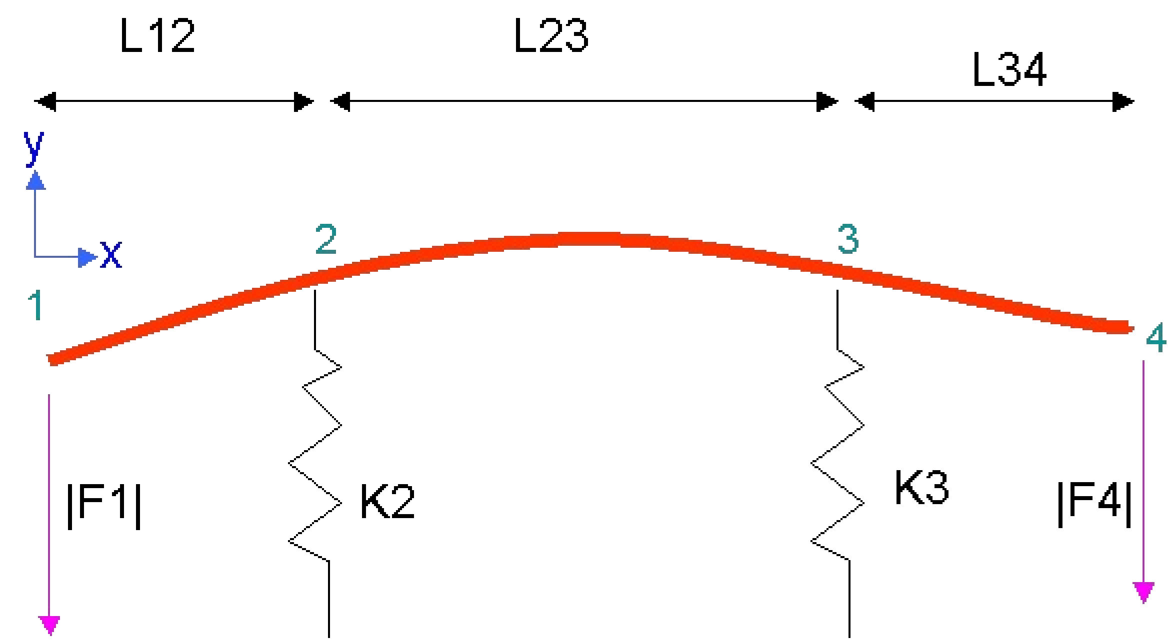
Model system to analyse Static Deflections:
Version 15: K2 = K3 =



Simplified Model. Neglect weight of shaft



Note displacements and forces are positive in the upward direction. The applied forces shown happen to have negative values.

======Symbols=====
x = longitudinal coordinate
y = radial coordinate

V(x) = Shear due to force F1
M(x) = Moment due to force F1
T(x) = Slope (Theta) due to force F1
Y(x) = Displacement due to force F1

VV(x) = Shear due to force F2
MM(x) = Moment due to force F2
TT(x) = Slope (Theta) due to force F2
YY(x) = Displacement due to force F2

Initialization:

```
reset()
```

```
assume(L12>0); assume(L23>0); assume(L34>0); assume(F1>0);
```

```
assume(K2>0); assume(K3>0);
```

```
use(linalg)
Warning: 'htranspose' already has a value, not exported. [use]
Warning: 'transpose' already has a value, not exported. [use]
Warning: 'det' already has a value, not exported. [use]
```

==== PART 1 - Find deflections due to force F1 ==========
For the first part, we want to determine alpha11 and alpha14.
We will apply force F1 upwards at point 1 and compute the response displacements Y1 and Y4
Then alpha11 = Y1/F1 and alpha14 = Y4/F1

Note we must assume F4=0 during this part! (That's a helpful simplifier)

Find R3 based on moment balance about point 2

```
R3:=solve(F1*L12=R3*L23,R3)[1]
```

$$\frac{\mathrm{F1}\ \mathrm{L12}}{\mathrm{L23}}$$

Find R2 based on Force balance:

```
R2:=solve(F1+R2+R3=0,R2)[1]
```

$$-\mathrm{F1}-\frac{\mathrm{F1}\ \mathrm{L12}}{\mathrm{L23}}$$

Double check the moment balance about point 3 is satisfied

```
simplify(F1*(L12+L23)+R2*L23)
```

$$0$$

Double check passes!

Check forces sum to zero

```
F1+R2+R3
```

$$0$$

Check passes!

Write the Shear function as integration of applied forces

```
V(x):=simplify(F1*heaviside(x)+R2*heaviside(x-L12)+R3*heaviside(x-(L12+L23)
```

$$\frac{\mathrm{F1}\ (\mathrm{L23}\ \mathrm{heaviside}(x)-\mathrm{L23}+\mathrm{L12}\ \mathrm{heaviside}(\mathrm{L12}-x)+\mathrm{L23}\ \mathrm{heaviside}(\mathrm{L12}-x)-\mathrm{L12}\ \mathrm{heaviside}(\mathrm{L12}+\mathrm{L23}}{\mathrm{L23}}$$

Write Moment as integration of shear, with unknown integ constant "M0"

```
M(x):=simplify(int(V(x),x))+M0
```

$$\mathrm{M0}-\mathrm{F1}\ x-\mathrm{F1}\ \mathrm{heaviside}(\mathrm{L12}-x)\ (\mathrm{L12}-x)+\mathrm{F1}\ x\ \mathrm{heaviside}(x)-\frac{\mathrm{F1}\ \mathrm{L12}\ \mathrm{heaviside}(\mathrm{L12}-x)\ (\mathrm{L12}-x)}{\mathrm{L23}}+ -$$

Solve M0 based on B.C. that moment is zero at/beyond point 3

```
M0:=solve(   (M(x)|x=L12+L23)  = 0,    M0)[1]
```

$$0$$

As a double check, make sure the moment at left end is also zero:

```
M(x)|x=0
```

$$\frac{\mathrm{F1}\ \mathrm{L12}\ (\mathrm{L12}+\mathrm{L23})}{\mathrm{L23}}-\frac{\mathrm{F1}\ \mathrm{L12}^2}{\mathrm{L23}}-\mathrm{F1}\ \mathrm{L12}$$

Check passes

Solve slope T(x) (slope) by integrating M(x) and multiplying 1/EI, with integ const "T0"

```
T(x):=int(M(x),x)/EI+T0
```

$$T0 + \frac{\frac{F1\,x^2\,\mathrm{heaviside}(x)}{2} - \frac{F1\,x^2}{2} + \frac{F1\,\mathrm{heaviside}(L12-x)\,(L12-x)^2}{2} - \frac{F1\,L12\,\mathrm{heaviside}(L12+L23-x)\,(L12+L23-x)^2}{2\,L23} + \frac{F1\,L12\,\mathrm{heaviside}}{2}}{EI}$$

```
T(x):=simplify(T(x))
```

$$T0 + \frac{\frac{F1\,x^2\,\mathrm{heaviside}(x)}{2} - \frac{F1\,x^2}{2} + \frac{F1\,\mathrm{heaviside}(L12-x)\,(L12-x)^2}{2} - \frac{F1\,L12\,\mathrm{heaviside}(L12+L23-x)\,(L12+L23-x)^2}{2\,L23} + \frac{F1\,L12\,\mathrm{heaviside}}{2}}{EI}$$

```
//T(x):=simplify(1/E1/I1*int(M(x),x))+T0
```

Solve Y(x) as integral of T(x), with integration const "Y0"

```
Y(x):=int(T(x),x) + Y0
```

$$Y0 + T0\,x - \frac{F1\,x^3}{6\,EI} + \frac{F1\,x^3\,\mathrm{heaviside}(x)}{6\,EI} - \frac{F1\,\mathrm{heaviside}(L12-x)\,(L12-x)^3}{6\,EI} - \frac{F1\,L12\,\mathrm{heaviside}(L12-x)\,(}{6\,EI\,L23}$$

```
Y(x):=simplify(Y(x))
```

$$Y0 + T0\,x - \frac{F1\,x^3}{6\,EI} + \frac{F1\,x^3\,\mathrm{heaviside}(x)}{6\,EI} - \frac{F1\,\mathrm{heaviside}(L12-x)\,(L12-x)^3}{6\,EI} - \frac{F1\,L12\,\mathrm{heaviside}(L12-x)\,(}{6\,EI\,L23}$$

Solve Y0, T0 using force/displacement equations at points 2 and 3

```
eq1:=(Y(x)|(x=L12)) = -R2/K2
```

$$Y0 + L12\,T0 + \frac{F1\,L12\,L23^2}{6\,EI} = \frac{F1 + \frac{F1\,L12}{L23}}{K2}$$

```
eq2:=(Y(x)|(x=L12+L23)) = - R3/K3
```

$$Y0 + T0\,(L12 + L23) = -\frac{F1\,L12}{K3\,L23}$$

```
Y0T0soln:=simplify(solve({eq1,eq2},{Y0,T0}));
```

$$\left\{\left[T0 = -\frac{F1\,(-K2\,K3\,L12\,L23^3 + 6\,EI\,K3\,L23 + 6\,EI\,K2\,L12 + 6\,EI\,K3\,L12)}{6\,EI\,K2\,K3\,L23^2},\ Y0 = \frac{F1\,(6\,EI\,K2\,L12^2 + (}{}\right.\right.$$

```
Y0:=simplify(Y0|Y0T0soln)
```

$$\frac{F1\,(6\,EI\,K2\,L12^2 + 6\,EI\,K3\,L12^2 + 6\,EI\,K3\,L23^2 - K2\,K3\,L12^2\,L23^3 + 12\,EI\,K3\,L12\,L23 - K2\,K3\,L12\,L23}{6\,EI\,K2\,K3\,L23^2}$$

```
T0:=simplify(T0|Y0T0soln)
```

$$-\frac{F1\,(-K2\,K3\,L12\,L23^3 + 6\,EI\,K3\,L23 + 6\,EI\,K2\,L12 + 6\,EI\,K3\,L12)}{6\,EI\,K2\,K3\,L23^2}$$

```
simplify(lhs(eq1)-rhs(eq1))
```
0

```
simplify(lhs(eq2)-rhs(eq2))
```
0

==== Part 2 - find equations for VV, MN, TT, YY from symmetry and similar approach as part 1

```
// Find R2 based on moment balance about point 3
R2:=solve(F4*L34=R2*L23,R2)[1]:
// Find R2 based on Force balance
R3:=solve(F4+R2+R3=0,R3)[1]:
VV(x):=simplify(F4*heaviside(x-(L12+L23+L34))+R2*heaviside(x-L12)+R3*heavis
MM(x):=simplify(int(VV(x),x))+MM0:
MM0:=solve(  (MM(x)|x=L12) = 0,   MM0)[1]:
TT(x):=int(MM(x),x)/EI+TT0:
TT(x):=simplify(TT(x)):
YY(x):=int(TT(x),x) + YY0:
YY(x):=simplify(YY(x)):
eq1:=(YY(x)|(x=L12)) = -R2/K2:
eq2:=(YY(x)|(x=L12+L23)) = - R3/K3:
YY0TT0soln:=simplify(solve({eq1,eq2},{YY0,TT0})):
YY0:=simplify(YY0|YY0TT0soln):
TT0:=simplify(TT0|YY0TT0soln):
```

Assign values in MKS system

```
L12:=11.5/39.37
```
0.2921005842

```
L23:=13.5/39.37
```
0.3429006858

```
L34:=2.5/39.37
```
0.063500127

```
Ltot:=L12+L23+L34
```
0.698501397

```
D1:=2/39.37 // diameter
```
0.0508001016

```
F1:=-25*4.44
```

-111.0

```
F4:=-200*4.44
```

-888.0

```
E1:=2.3E11
```

$2.3\ 10^{11}$

```
I1:=float(PI)*D1^4/64
```

0.0000003269100125

```
EI:=E1*I1
```

75189.30287

```
K2:=1E10
```

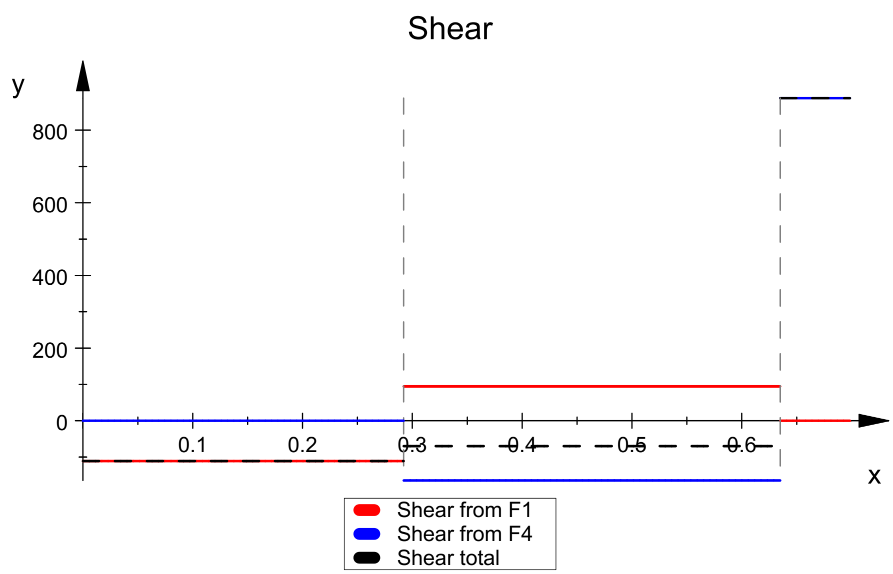
10000000000.0

```
K3:=K2
```

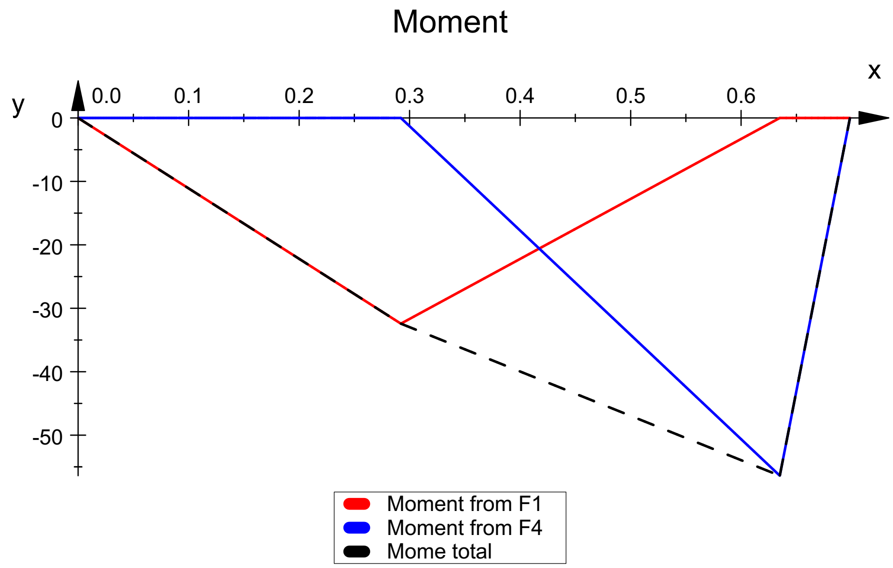
10000000000.0

======================Plot========

```
plot(plot::Function2d(V(x),x=0..Ltot,Color=RGB::Red,Legend = "Shear from F1
plot::Function2d(VV(x),x=0..Ltot,Color=RGB::Blue,Legend = "Shear from F4"),
plot::Function2d(V(x)+VV(x),x=0..Ltot,Color=RGB::Black,LineStyle = Dashed,I
,Header="Shear")
```



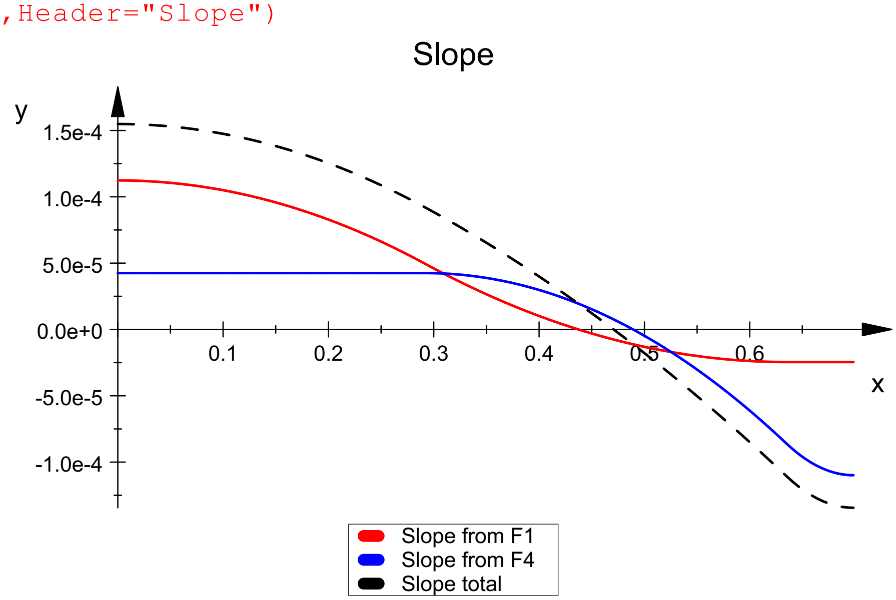

```
plot(plot::Function2d(M(x),x=0..Ltot,Color=RGB::Red,Legend = "Moment from I
plot::Function2d(MM(x),x=0..Ltot,Color=RGB::Blue,Legend = "Moment from F4")
plot::Function2d(M(x)+MM(x),x=0..Ltot,Color=RGB::Black,LineStyle = Dashed,I
,Header="Moment")
```




```
T(x):=simplify(T(x)|K23=K23): // needed simplification for plot to work
```

```
TT(x):=simplify(TT(x)|K23=K23): // needed simplification for plot to work
```

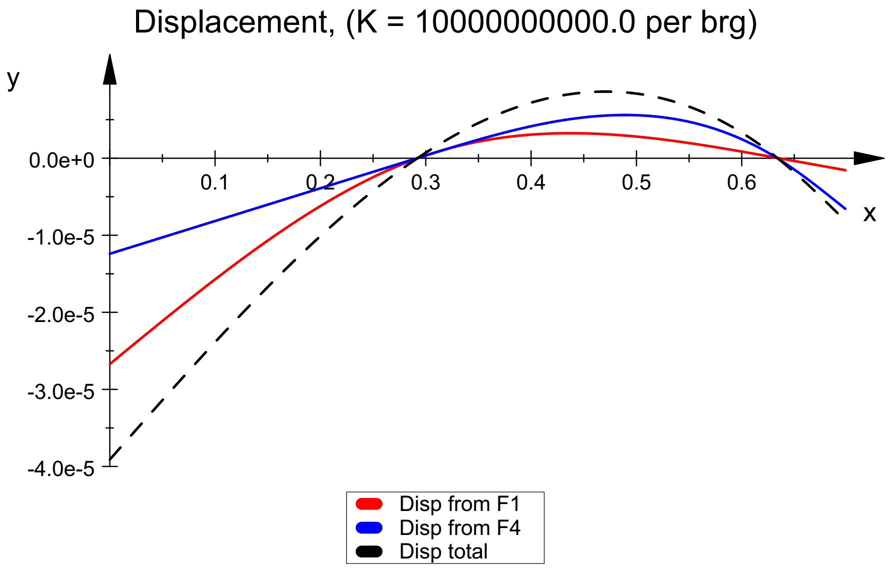
```
plot(plot::Function2d(T(x),x=0..Ltot,Color=RGB::Red,Legend = "Slope from F1
plot::Function2d(TT(x),x=0..Ltot,Color=RGB::Blue,Legend = "Slope from F4"),
plot::Function2d(T(x)+TT(x),x=0..Ltot,Color=RGB::Black,LineStyle = Dashed,I
,Header="Slope")
```




```
Y(x):=simplify(Y(x))|L23=L23:  // required for plot to work
```

```
YY(x):=simplify(YY(x))|L23=L23:  // required for plot to work
```

```
plot(plot::Function2d(Y(x),x=0..Ltot,Color=RGB::Red,Legend = "Disp from F1'
plot::Function2d(YY(x),x=0..Ltot,Color=RGB::Blue,Legend = "Disp from F4"),
plot::Function2d(Y(x)+YY(x),x=0..Ltot,Color=RGB::Black,LineStyle = Dashed,I
,Header="Displacement, (K = ".expr2text(K2)." per brg)")
```



Displacement, (K = 10000000000.0 per brg)
y
x
0.0e+0
-1.0e-5
-2.0e-5
-3.0e-5
-4.0e-5
0.1
0.2
0.3
0.4
0.5
0.6
Disp from F1
Disp from F4
Disp total