



## Particle Physics Division

### Mechanical Department Engineering Note

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Title: Stored Energy in Pressurized Systems

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Abstract Summary: FESHM 5031.1 currently calls for any pressurized system with more than  $150,000 \text{ ft} \cdot \text{lb}_f$  of energy to have documentation and analysis in the form of a formal engineering note. FESHM does not provide guidance on calculating stored energy, so several proposed methods are discussed. In conclusion, a general conservative equation is provided with references to other institutions which are in agreement. The current FESHM limit is also discussed and compared to the limits of other institutions, and finally a revision is suggested.

## Stored Energy in a Compressed Gas

### ✓ Initial Discussion, Contents and References

As a result of recent confusion during several attempts to calculate the energy within a pressurized gas, I will analyze the basis and results of several different Fermilab staff proposed equations. In order to simplify initially complex equations, one or more assumptions must be made i.e. constant mass of gas, constant volume, adiabatic, isothermal, ect... I will then recommend a general method to be used at Fermilab.

1. -The PNNL [Pacific Northwest National Laboratory] equation (use proposed by Michael Geynisman with the link:

[http://www.pnl.gov/main/publications/external/technical\\_reports/PNNL-](http://www.pnl.gov/main/publications/external/technical_reports/PNNL-18696.pdf)

[18696.pdf](http://www.pnl.gov/main/publications/external/technical_reports/PNNL-18696.pdf).) It is sometimes called the Baker equation. 
$$E = \frac{P_2 V_2}{k-1} \left[ 1 - \left( \frac{P_1}{P_2} \right)^{\frac{k-1}{k}} \right]$$

2. The IPEIA [International Pressure Equipment Integrity Association] equation (use proposed by Michael Geynisnam with the link:

[http://www.ipeia.com/Misc\\_Docs/2008%20John%20Aumuller.pdf](http://www.ipeia.com/Misc_Docs/2008%20John%20Aumuller.pdf)

$$E = \frac{P_1 V_1 k}{k-1} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right]$$

3. The ASME stored energy equation (use proposed by Terry Tope):

$$E = 2.5 P_2 V_2 \left[ 1 - \frac{P_1^{0.286}}{P_2} \right]$$

4. Bill Coopers derivation:  $W = \int_{x_1}^{x_2} F dx$

5. Popular Fermilab equation:  $E = P_2 V_2$

6. The first Law of thermodynamics (use proposed by Rich Schmitt):

$\Delta E = \Delta U + \Delta K.E. + \Delta P.E. = -W - Q$ , where  $-W$  is the work done on the system and  $-Q$  is the heat leaving the system.

References were all extensively checked and in some cases equations were re-derived. I will discuss each of the above equations individually citing the laws each were derived from, assumptions made during derivation, and the practical relevance of each. Initial calculations take the same pressures and volumes used by the references in the equations above.

I will also discuss a summary of all the results, stored energy limits based on energy in the system based upon several references some of which include the above, and general recommendations (section 7).

### Numbers used:

$$P1a := 14.69 \text{ psi} = 1.013 \times 10^5 \text{ Pa}$$

$$P1g := 0 \text{ psi} = 0$$

$$P2a := 114.69 \text{ psi} = 7.908 \times 10^5 \text{ Pa}$$

$$P2g := 100 \text{ psi} = 6.895 \times 10^5 \text{ Pa}$$

$$\text{Area} := 1 \text{ ft}^2 = 0.093 \text{ m}^2$$

$$P_{\text{atm}} := 14.69 \text{ psi} = 1.013 \times 10^5 \text{ Pa}$$

$$V1 := .977 \text{ m}^3 = 34.502 \text{ ft}^3$$

$$x1 := \frac{V1}{\text{Area}} = 34.502 \text{ ft}$$

$$V2 := \frac{(P1a \cdot V1)}{P2a} = 4.421 \text{ ft}^3 = 0.125 \text{ m}^3$$

$$x2 := \frac{V2}{\text{Area}} = 4.421 \text{ ft}$$

\*V2 If isothermal

Pressures used in equations are always absolute pressures unless otherwise noted

REFPROP: (air)

$$u_1 := 336090 \frac{\text{J}}{\text{kg}}$$

$$\rho_{s1} := 1.1997 \frac{\text{kg}}{\text{m}^3}$$

$$m_1 := \rho_{s1} \cdot V1 = 1.172 \text{ kg}$$

$$u_2 := 334670 \frac{\text{J}}{\text{kg}}$$

$$\rho_{s2} := 9.3875 \frac{\text{kg}}{\text{m}^3}$$

→ For a constant mass

$$U_1 := u_1 \cdot m_1 = 3.939 \times 10^5 \text{ J}$$

$$U_2 := u_2 \cdot m_1 = 3.923 \times 10^5 \text{ J}$$

- ✓ Calculations were done in Mathcad all in SI units.

### Some Terms

- ✓ When an energy analysis is done, either a control mass or a control volume must be assumed.
  - Control mass A control mass has a constant system mass to be pressurized i.e. a closed system, piston type compression or expansion. Non constant volume.
  - control volume A control volume has a fixed volume and is pressurized by adding a mass of gas into the system. This system is more generally applicable but much more complex to analytically solve for energy. Non constant mass has many implications which will be discussed.
- ✓ Several methods imply their result is the Available energy: This is defined as the amount of energy capable of doing work on the environment, rather than the actual stored energy. It can be understood as the explosion energy.

## 1. The PNNL [Pacific Northwest National Laboratory] equation

$$E = \frac{P_2 V_2}{k - 1} \left[ 1 - \left( \frac{P_1}{P_2} \right)^{\frac{k-1}{k}} \right]$$

PNNL has been using this equation to calculate stored energy in pressure systems for over twenty years. They set a 1,000  $ft * lb_f$  stored energy limit in which every vessel with a higher stored energy needs pre approval to be designed or operated. I have re-derived the equation from thermodynamics in order to understand the basic implications and assumptions made.

The equation can be rewritten as follows if the initial volume  $V_1$  is known, or more easily measured than the final volume  $V_2$ :  $E = \frac{P_1 V_1}{k-1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right]$

### 1. Assumptions and indications during derivation:

- This is derived from the integral  $\int P dv$  which is the boundary process work done (from the first law of thermodynamics) during the compression or expansion of a gas.
  - The gas is assumed to be polytropic  $PV^n = \text{constant}$ 
    - Setting  $n = k$  assumes an ideal gas
  - This integral implies a non constant volume.
  - The original form of the solution is:  $\frac{P_1 V_1 - P_2 V_2}{1-k}$
- The process is assumed to be isentropic, adiabatic and reversible.
- ASBA (Alberta Boiler Safety Association) cites this is valid as the isentropic expansion of a confined gas.

PNNL concludes its analysis by stating this equation is acceptable for determining stored energy and over pressurization events, and cites its use at other national labs (INL, LANL, and LLNL). The author suggests that another equation is recommended by Lee's fire safety manual which is the original form of the solution with a constant volume; however this is an impossible integral and will be considered theoretically erroneous.

## 2. The IPEIA [International Pressure Equipment Integrity Association] equation

The IPEIA primer on pneumatic testing is fairly poorly put together with several contradictions within itself. The IPEIA calculates stored energy in an example with the

following: 
$$E = \frac{P_1 V_1 k}{k-1} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right]$$

In the slide previous to identifying the equation they clearly identify  $k$  as the bulk modulus of air. However, this equation is clearly the thermodynamic derivation of the shaft work in an isentropic power system. This work is in fact the correct interpretation of this equation where  $k$  is the ratio of specific heats.

The IPEIA pneumatic testing primer then goes on to state in a later slide that energy should be calculated based on ABSA (Alberta Boiler Safety Association) regulations, which then cite an LLNL link for instruction. Following the *\*links*, I arrive at the same equation studied in section 1 for the PNNL equation. It seems the IPEIA suggests method 1 is what should be used and then goes on to the method above. However with this being a valid energy equation it will be analyzed none the less.

*\*Links followed from IPEIA pneumatic testing primer:* On page 17 of the primer ([http://www.ipeia.com/Misc\\_Docs/2008%20John%20Aumuller.pdf](http://www.ipeia.com/Misc_Docs/2008%20John%20Aumuller.pdf)), the IPEIA states minimum requirements apply for determination of pneumatic test feasibility with the reference of the "ABSA Pressure news" Volume 7 (<http://www.absa.ca/newsletter/v07-2002/v7-iss2.pdf>). This link then states that the energy in the test fluid must be calculated using the isentropic expansion of a confined gas and proves a link to the LLBL training site (<https://www-training.llnl.gov/training/hc/5060/LiquidGas.html>). This link finally lists the same equation as PNNL. It would seem the IPEIA primer suggests use of the PNNL equation after it (incorrectly) uses the above equation for a compressor or pump.

Lastly the IPEIA primer suggests that no vessel be pneumatically tested if its  $P_{2gauge} * V_2 > 50,000 \text{ ft} * \text{lb}_f$ . This is often currently used here at Fermilab, however the current FESHM restriction for critical analysis is  $150,000 \text{ ft} * \text{lb}_f$  but does not provide guidance on calculations. This will be discussed in the last section.

### 2. Assumptions and implications of the above (top) equation derivation:

- This equation is understood as the shaft work of a power system such as a pump or compressor.
- The derivation is based on the first and second laws of thermodynamics.
  - It is the final solution to the integral  $\int V dP$ , where there is a non constant  $V$
  - It assumes a steady state and steady flow process
  - It assumes the system is isentropic, adiabatic and reversible
  - The gas is assumed to be polytropic  $PV^n = \text{constant}$

### 3. The ASME stored energy equation

ASME PCC-2-2008 (Repair of Pressure Equipment and Piping) lists the following equation as the procedure to calculate stored energy in pressure components.

$$E = 2.5P_2V_2 \left[ 1 - \frac{P_1^{0.286}}{P_2} \right]; \text{ in SI units for only air or nitrogen}$$

$$\text{Reduces to: } E = 2.5P_1V_1 \left[ 1 - \frac{P_2^{0.286}}{P_1} \right];$$

Upon further analysis of the origin of the equation, it was found in the original ASME text that this is in fact the same equation used by PNNL and other labs discussed in section 1. ASME has simply reduced the equation to plugging in the k value for air or nitrogen. The original equation is once

again:  $E = \frac{P_1V_1}{k-1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right]$ ; so further implications of the ASME standard can be taken from section 1.

**\*\*\*An important note on this equation commonly found online:** There exists a pneumatic testing primer commonly found online that references the top equation in English units as  $E =$

$P_2V_2 \left[ 1 - \frac{P_1^{0.286}}{P_2} \right]$ . **This English unit version of the equation is incorrect and missing a 360 constant at the front.** For the correct equation the original ASME text or this note should be referenced. The online primer does not include a source or author.

\*A free copy of the correct version of the original ASME PCC-2 code is available at the following link: <http://www.docstoc.com/docs/70030481/PCC-2-Repair-Standard-2008>. Fermilab also hosts a copy on their website. This correct link or the Fermilab website version should always be used in lieu of the copy I started with. I do not know where the incorrect copy originated from, so I cannot provide a link with a warning; however it is the only copy I have come across with the typo.

#### 4. Bill Coopers derivation:

Bill Cooper suggests the energy of the compressed gas be calculated with the equation:  $W = \int_{x_1}^{x_2} F dx$ ; which he integrates assuming an isothermal process ( $P_1 V_1 = P_2 V_2$ ) to arrive at

$W = P_1 V_1 \ln\left(\frac{P_2}{P_1}\right)$ . He would then later realize an error in using  $F = P(\text{absolute}) \cdot A$  when in fact it should be  $P(\text{gauge}) \cdot \text{Area}$ . Semi-coincidentally the maximum possible energy stored in an isothermal process in compressing a gas is  $W = \int_{v_1}^{v_2} P dv$ , which in fact integrates to the same thing as Cooper's original derivation:  $W = P_1 V_1 \ln\left(\frac{P_2}{P_1}\right)$ . For comparison I have also re-derived Coopers original integral.

- Coopers original (theoretically flawed) solution to  $\int_{x_1}^{x_2} F dx$ :  $W = P_1 V_1 \ln\left(\frac{P_2}{P_1}\right)$
  - Solution to maximum possible stored energy (the irreversible work) in a compressed isothermal gas:  $W = \int_{v_1}^{v_2} P dv$ :  $W = P_1 V_1 \ln\left(\frac{P_2}{P_1}\right)$
  - $\int_{x_1}^{x_2} F dx$  re-derived for comparison:  $W = P_1 A \left[ (x_2 - x_1) - x_1 \ln\left(\frac{P_1}{P_2}\right) \right]$ 
    - *This solution can be understood as the maximum possible stored energy minus the amount of energy needed to displace the atmosphere, or the "maximum amount of available energy".*
  - Assumptions and implications of the above equation derivations:
    - Ideal gas, isothermal process, yet for internal energy to exist the implication is somewhere between all or none of the calculated energy may in fact be present, which some lost as heat.
      - "True to life" energy existing in the system will be more accurately calculated for an isothermal process in section 6 (the first law of thermodynamics).
    - $(P_1 V_1 = P_2 V_2) = nRT$
    - This integral indicates there must be a non constant volume
- It's important to note that the integral  $\int_{v_1}^{v_2} P dv$ , from which the expressions are derived are the boundary work from the volume changing, not the actual stored energy. Much of the energy generated from the compression is lost in the form of heat.

## 5. Fermilab equation:

The equation  $E = P_{2gauge}V_2$ , is sometimes used within Fermilab to determine internal energy in a pressurized gas. The origins of the equation is currently unknown to me, however the IPEIA pneumatic testing primer suggests that PV must be less than 50,000 ft \* lb<sub>f</sub>.

Several different forums call this the “potential pressure energy”, a non intrinsic system property. I would tend to agree with this claim based on the self contradictions of the first law that will be addressed and explained in the next section. When this non intrinsic potential pressure energy is calculated it is always orders of magnitude less than the prior methods (sections 1-4), which may imply this does not include any type of heat energy as equations in sections 1-3 do. It is simply the potential available energy of the pressure alone in an isothermal process, not the maximum possible amount of stored energy.

- Implications of  $E = P_{2gauge}V_2$ :
  - Non intrinsic system property
  - The “potential pressure energy”
  - Does not account for any type of heat energy
  - It is a reflection of the total available energy, not the stored energy
  - $V_2$  was calculated to tabulate values using the polytropic relationship,  $PV^n = constant$ , where  $n = k$  for an ideal gas
- Polytropic (adiabatic) vs. isothermal
  - When using P\*V: isothermal
    - $V_2 = \frac{P_1V_1}{P_2}$
  - When using PV: polytropic
    - $V_2 = \sqrt[k]{\frac{P_1V_1^k}{P_2}}$
- $E = P_{2gauge} \sqrt[k]{\frac{P_1V_1^k}{P_2}}$ ; for polytropic (adiabatic) potential pressure energy
- $E = P_{2gauge} \frac{P_1V_1}{P_2}$ ; for isothermal potential pressure energy



## 6. The First Law of Thermodynamics

Equation:  $\Delta E = \Delta U + \Delta K.E. + \Delta P.E. = -W - Q$ , where  $-W$  is the work done on the system and  $-Q$  is the heat leaving the system. This was by far the most complex and confusing process analyzed.

The first law simply states the conservation of energy; the total energy change in a system is the change in the internal energy plus the change in kinetic energy plus the change in potential energy, or stated in terms of work and heat, the work done on the system minus the heat leaving the system.

✓ Assumptions for control mass system analysis:

- There is no change in kinetic or potential energy
- The process is isothermal ( $\Delta T_{gas} = 0$ )
- The process has a fixed mass of gas (piston type compression).
- When dealing with an ideal gas such as air in an isothermal process, the first law begins to break down.
- $\Delta U$  in an ideal gas is a function of temperature only,  $f(T)$ .
- If there is no change in temperature of the gas (isothermal), there is no change in  $U$
- This would lead to  $\Delta E = \Delta U = -W - Q \rightarrow$  with  $\Delta U = 0$ ,  $\Delta E = 0$ ,  $W = -Q$
- $W = -Q$  implies all energy added during pressurization is perfectly converted and lost due to heat transfer.
- However if a pressure still exists in the vessel there is undoubtedly still energy no matter what the temperature.
  - In reality no gas is perfect, and there will in fact be a slight change in internal energy. REFPROP computes this as  $1,420 \frac{J}{kg} = \Delta u$
  - $\Delta E = \Delta u * mass = 1,420 \frac{J}{kg} * (\rho_1 V_1) = 1,664 J$

✓ If the system is considered adiabatic (though this deviates for the first law) still with a control mass it

will heat up with the relation:  $\frac{T_2}{T_1} = \frac{P_2}{P_1}^{\frac{k-1}{k}} \rightarrow T_2 = 1270 F (@T_1 = 700 F)$ .

- New  $u_2$  (REFPROP) =  $357,630 \frac{J}{kg}$
- $\Delta E = (new u_2 - u_1) * mass = 25,250 J$  or  $18,620 ft * lb_f$
- This is a drastic increase from the isothermal calculation above, however still low

✓ When mass is added to a set volume in order to pressurize, the energy can be approximated by multiplying the internal energy change by the average mass the system contains from start to finish:  $\Delta u * average mass$ .

- First law analysis repeatedly gives much lower stored energy values for the system, and are likely not a good reflection of the system.

## Discussion, Numerical Result Comparisons, Stored Energy Limits, and Recommendations.

### ✓ Numerical Results

Section/equation title		Equation	Result		Notes and Applicability
			Joules	ft * lb <sub>f</sub>	
1/ The PNNL equation		$E = \frac{P_1 V_1}{k-1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right]$	197,900	146,000	-Boundary process work in an isentropic <u>adiabatic</u> and reversible compression of an ideal gas. -With a well insulated system this is an accurate estimate of total <u>stored</u> energy. -Assumes a non constant V
2/ The IPEIA equation		$E = \frac{P_2 V_2 k}{k-1} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right]$	276,700	204,100	-Shaft work for a power system on an ideal gas -Isentropic, adiabatic, reversible -Applicable to steady state steady flow processes like a pump or compressor
3/ The ASME equation		$E = 2.5 P_1 V_1 \left[ 1 - \frac{P_2^{0.286}}{P_1} \right]$	197,900	146,000	-A simplification of the section 2 equation using the k value of <u>air or nitrogen</u> .
4/ The Bill Cooper derivation of isothermal compression	Original	$W = P_1 V_1 \ln \left( \frac{P_2}{P_1} \right)$	203,400	150,000	-Coopers original derivation is the correct amount of boundary work performed in compressing a volume of gas in an isothermal process.
	Re-derived	$W = P_1 A [ (x_2 - x_1) - x_1 \ln \left( \frac{P_1}{P_2} \right) ]$	117,100	86,360	-The re-derived form is the amount of available energy in the system -When considering an isothermal process on a gas, the amount of work is the <u>max possible amount of energy</u> in the system, though <u>most will be lost to heat</u> . It is irreversible work.
	Isothermal boundary work	$W = P_1 V_1 \ln \left( \frac{P_2}{P_1} \right)$	203,400	150,000	-A more accurate estimate of the actual stored energy is covered in section 6 by the first law of thermodynamics.

5/ The Fermi equation	$P_{2gauge}V_2$	Constant mass	155,200	114,500	<p>-The simple P x V equation must be looked at as a control volume being filled with gas, as well as a control mass being compressed</p> <p><u>-The P*V2 equation is a valid measure of the non intrinsic available potential pressure energy in the system.</u></p> <p>-Neither account for heat energy, giving a good indication of actual isothermal stored energy</p> <p>-It is obvious that due to the much larger pressurized volume there is much more energy, however being the only equation that encompasses a larger volume, it is <u>not comparable to the previous sections' results.</u></p>
	$\frac{3}{2}P_{2gauge}V_2$		232,800	171,750	
	$P_{2gauge}V_1$	Constant volume (*not comparable to other values)	673,600	496,800	
6/ First law of thermodynamics	$\begin{aligned}\Delta E &= \Delta U \\ &+ \Delta K.E. + \Delta P.E. \\ &= -W - Q; \\ \Delta E &= \Delta U\end{aligned}$	Constant mass-isothermal	1,664	1,228	<p>-See section 2 for details and self contradictions of the first law.</p> <p>-First law values were computed for different scenarios using changes in internal energy and masses or mass averages.</p> <p>-They are a good representation of actual internal energy changes of the systems.</p> <p>-Notice all are relatively low values compared to other methods, especially the isothermal values. Previous isothermal values are a measure of the max possible stored energy rather than actual.</p>
		Constant volume-isothermal	7,344	5,417	
		Constant mass-adiabatic	25,250	18,620	
		Constant volume-adiabatic	111,400	82,170	

✓ **Stored Energy Limits**

Limit		Source	Intention	Notes
J	$ft * lb_f$			
203,400	150,000	Fermilab FESHM 5031.1	Engineering note must be done if stored energy exceeds this limit.	Not a pneumatic testing limit. This is comparable to the stored energy in a 50 gallon standard air compressor
271,000	200,000	ASME BCC-2-2008	Limit for pneumatic testing	
100,000	75,000	LBL	Over this limit of stored energy is considered a high hazard	Documentation and training must be done for design and operation
67,990	50,000	IPEIA	Pneumatic testing limit, bound to the P*V equation.	This limit is bound to the simple P*V equation (the potential pressure energy) which will always give a relatively lower result
1,356	1,000	PNNL	Greater than this stored energy requires pre-approval for design and/or operation. This limit is comparable to the amount of stored energy in a bicycle tire.	This limit was conceived concerning standard operating and design. It is not a pneumatic test limit and is intended to protect workers in everyday situations. PNNL admits it is a very low limit and is aware of one 10x higher at another lab.  $1,000 ft * lb_f$ is 5x less energy than in a standard car tire.

✓ **Comparison of Values (all in  $ft * lb_f$ )**

Equation		25 psi	50 psi	75 psi	100 psi	125 psi	150 psi
1. PNNL		60,000	96,360	123,700	146,000	165,000	181,800
2. IPEIA		83,900	134,700	172,900	204,100	230,700	254,200
3. ASME		<b>60,000</b>	<b>96,360</b>	<b>123,700</b>	<b>146,000</b>	<b>165,000</b>	<b>181,800</b>
4. Work derivation (isothermal)	Original work	72,550	108,200	132,100	150,000	164,400	176,400
	Available energy	26,570	51,790	71,020	86,360	99,090	109,900
5. P*V (polytropic)		<b>61,080</b>	<b>86,180</b>	<b>102,400</b>	<b>114,500</b>	<b>124,300</b>	<b>136,200</b>
P*V (isothermal)		45,980	56,430	61,050	63,660	65,330	66,500
3/2 P*V (polytropic)		91,620	129,270	153,600	171,750	186,450	204,300
3/2 P*V (isothermal)		68,970	84,645	91,575	95,490	97,995	99,750
6. First Law		*Given the large difference in values for the 1st law, results will not be compared					

PNNL/ASME: The PNNL and ASME equation are the most widely accepted methods for calculating stored energy. They are based on the same concept as #4, the isothermal boundary work derivation, assuming a polytropic relationship and an adiabatic process rather than isothermal.

IPEIA: The IPEIA pneumatic test primer uses the work/energy equation for a steady state steady flow power system, which incorporates kinetic energy. In the case of a static system it is generally not applicable

Work derivation: The original work derivation is a measure of the process boundary work in an isothermal compression. The amount of boundary work done is a measure of the energy expended in compression (the irreversible work), not the stored energy. Much of the work energy expended is lost to heat. Therefore this can be thought to represent the maximum possible amount of energy in the system, though most will be lost.

The popular P\*V calculation and the modified 3/2 \*P\*V (<http://www.khanacademy.org/video/proof--u--3-2-pv-or-u--3-2-nrt?playlist=Chemistry>, internal energy of an ideal gas, referenced in many textbooks as well) equation can be thought of as a non intrinsic property of the system, or the potential available pressure energy. Notice for polytropic adiabatic work the values are similar to the ASME equation for <100psi. For 100 psi or more the polytropic 3/2PV converges to the ASME values. Isothermal PVs are much lower than the work derivation, since the work derivation represents something like a maximum possible value rather than the reversible work. Therefore the isothermal and polytropic PV value might be thought of as the actual available potential pressure energy of the system.

## ✓ Recommendations

### Calculation:

For general calculations of potential energy in systems containing pressurized gas, the PNNL/ASME equation is recommended. This equations derivation is based upon the process boundary work done during the isentropic expansion of a simple compressible (polytropic) gas. The implications of this derivation include the heat energy effects in the system, as well as pressure. These in combination should at the very least provide a conservative result, though the actual reversible and available energy may actually be closer to  $P^*V$ . This is due to the fact that the integral  $\int_{v_1}^{v_2} P dv$  is equal to 0 if the volume is constant i.e. if a vessel is being pressurized by adding mass rather than a compression. This being realized, when an isentropic reversible adiabatic process is assumed the work reduces to  $\int_{P_1}^{P_2} V dP$  (second law of thermodynamics), which at a constant volume reduces to simple  $V(P_2 - P_1)$ . This can be considered the pressure boundary work in a constant volume vessel. Though the  $V^*P$  (polytropic for insulated systems, and isothermal for non insulated systems) value may in fact be a truer value at a constant volume pressurization, I recommend using the PNNL/ASME equation. This equation has precedent over the others and is a much more widely accepted method of calculating stored energy as a limiting factor. The conservative value will provide tighter safety constraints. Institutions using the isentropic equation for a confined gas include: ASME, PNNL, LBL, LLNL, and the ABSA.

### General Equation Forms:

$$-E = \frac{P_2 V_2}{k-1} \left[ 1 - \left( \frac{P_1}{P_2} \right)^{\frac{k-1}{k}} \right]; \text{ If } V_2 \text{ and } P_2 \text{ are known}$$

$$-E = \frac{P_1 V_1}{k-1} \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right]; \text{ If } V_1 \text{ and desired } P \text{ are known}$$

$$\frac{P_1 V_1 - P_2 V_2}{1-k}; \text{ Most general form, if } P_2, V_1 \text{ and } V_2 \text{ are known}$$

$$-E = 2.5 P_1 V_1 \left[ 1 - \frac{P_2^{0.286}}{P_1} \right]; \text{ Simplification for air or Nitrogen, only in metric units}$$

### Stored energy limits:

FESHMs current limit is  $150,000 \text{ ft} * \text{lb}_f$ . Greater than this stored energy requires a formal engineering note. When comparisons to other limits are made, the current  $150,000 \text{ ft} * \text{lb}_f$  limit clearly coincides better with a pneumatic testing limit, rather than a limit for formal documentation. PNNLs stored energy limit for a system to require documentation is currently  $1,000 \text{ ft} * \text{lb}_f$ , which is comparable to the amount of energy in a bike tire. Though this limit seems extraordinarily low compared to the pneumatic testing limit; when the system is in routine use with bystanders in close proximity, certainly even a failure with this relatively low energy could in fact be hazardous. It is for this reason that I will recommend the FESHM 5031.1 be altered, and the  $150,000 \text{ ft} * \text{lb}_f$  limit be changed to a pneumatic testing limit (this could possibly be raised to ASMEs limit of  $200,000 \text{ ft} * \text{lb}_f$ ). A new engineering documentation limit should then be established at a much lower energy such as PNNL uses. I believe PNNLs limit to be extremely conservative at  $1,000 \text{ ft} * \text{lb}_f$ , and could see a limit being set to  $5,000 \text{ ft} * \text{lb}_f$  or higher with a very minimal risk ( $5,000 \text{ ft} * \text{lb}_f$  is about the stored energy in a car tire).