

CAST IN PLACI

4-#6 & 9-#3  
@ 12" O.C. REINF.

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FOR LENGTH  
COPING SECT

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EL. + 8

TOP OF  
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2-3/4"  $\phi$   
AT EACH

FACE

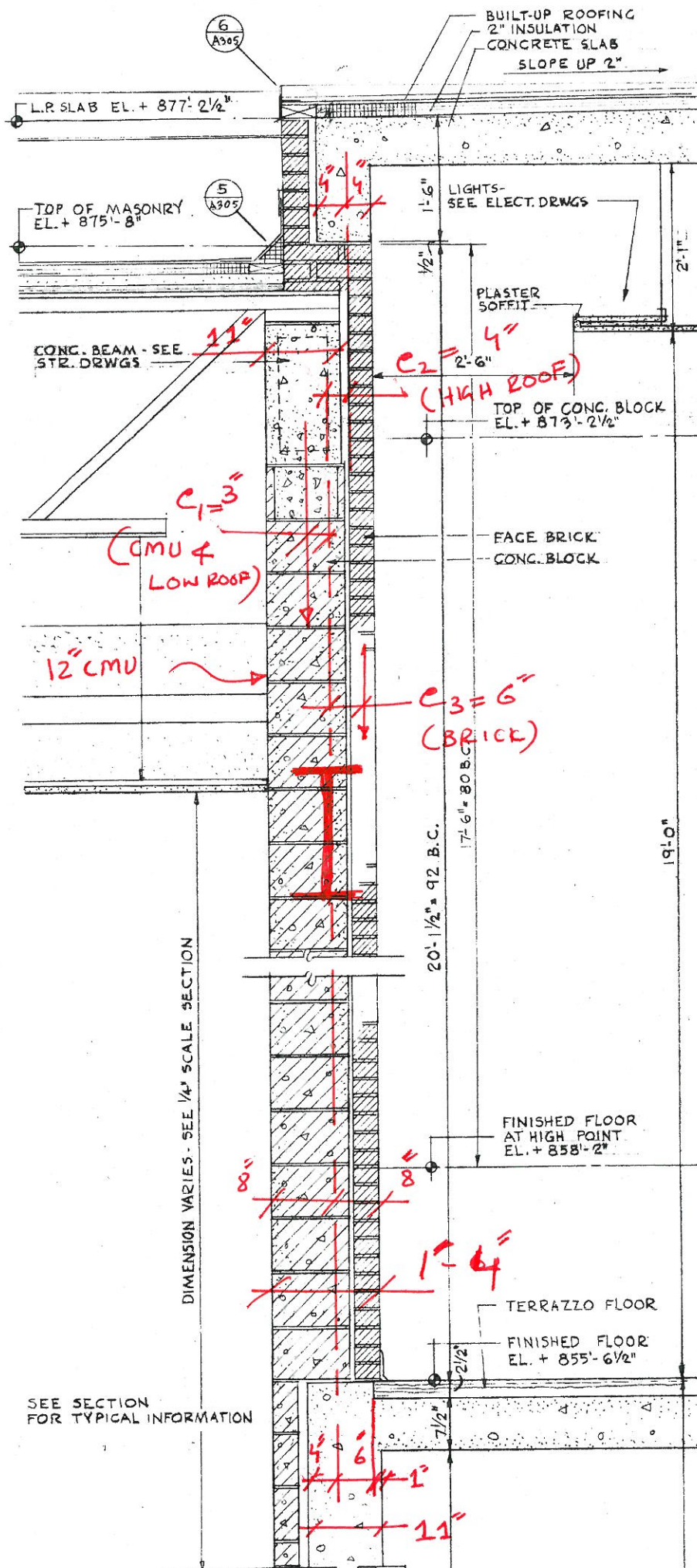
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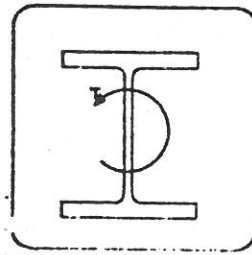
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USS  
STEEL DESIGN MANUAL  
1981

## CHAPTER 7 Torsion

(COPIED 5/11/84)

### 7.1 Introduction

A torsional moment applied to opposite ends of a member causes each cross section of the member to rotate. As illustrated in Figure 7.1, if the member is a round bar or tube, each cross section rotates in its own plane without warping, and the resistance to torsion is provided by the shear stresses, which are proportional to the distance from the centroid. However, if a member with a non-circular cross section is subjected to the same torsional moment, the cross sections not only rotate but also deform non-uniformly in the longitudinal directions so that plane transverse sections do not remain plane after twisting. This latter deformation, known as "warping," is illustrated in Figure 7.2 for a rectangular bar. If the warping is not restrained, resistance to torsion is due to a distribution of shear stress known as "St. Venant torsion." If warping is restrained, additional direct shear stresses due to bending of the component parts of the section are superposed on the St. Venant shear stresses and add effectively to the torsional resistance. Bending of the component parts also induces longitudinal direct stresses that may require consideration in design.

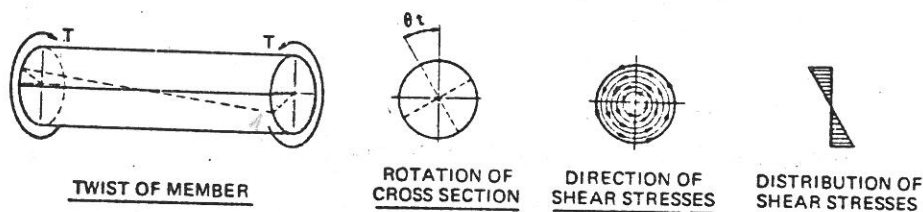


FIGURE 7.1 ROUND BAR IN TORSION

Solutions for six sets of conditions are given in Figures 7.9 and 7.10. The solutions are in terms of hyperbolic functions, which can be found in mathematics handbooks, and the torsional flange bending constant "a" given by

$$a = \sqrt{EC_w/JG} = d/2 \sqrt{EI_y/JG} \quad (7.18)$$

where  $C_w$  is a torsional warping constant.\* Values of both  $C_w$  and  $J$  are tabulated in the AISC Manual<sup>2</sup>; values of "a" can be easily calculated for most rolled beam sections. Equations for the maximum total angle of twist are given in the Figures 7.9 and 7.10.

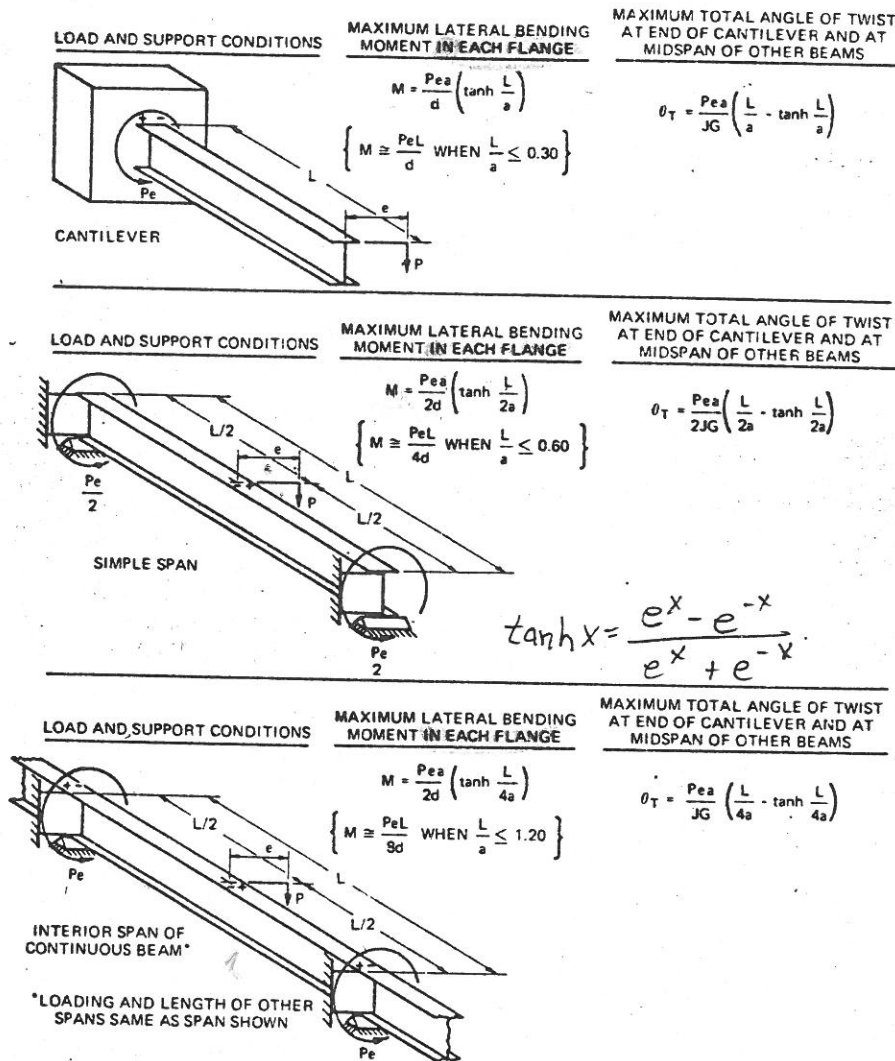


FIGURE 7.9 MAXIMUM LATERAL BENDING MOMENT AND TOTAL ANGLE OF TWIST FOR BEAMS IN TORSION FROM CONCENTRATED LOADS

\*Equations for  $C_w$  for various cross sections are given in Reference 4 where the symbol  $\Gamma$  replaces  $C_w$ . Values for most shapes are also tabulated in the AISC Manual.



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(7.18)

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an be easily calculated for  
maximum total angle of twist

MAXIMUM TOTAL ANGLE OF TWIST  
AT END OF CANTILEVER AND AT  
MIDSPAN OF OTHER BEAMS

$$\theta_T = \frac{Pe a}{JG} \left( \frac{L}{a} - \tanh \frac{L}{a} \right)$$

LOAD AND SUPPORT CONDITIONS

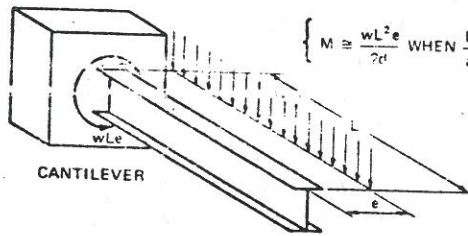
MAXIMUM LATERAL BENDING  
MOMENT IN EACH FLANGE

MAXIMUM TOTAL ANGLE OF TWIST  
AT END OF CANTILEVER AND AT  
MIDSPAN OF OTHER BEAMS

$$M = \frac{wLe a}{d} \left( \tanh \frac{L}{a} + \frac{1}{\cosh \frac{L}{a}} - \frac{a}{L} \right)$$

$$\left\{ M \approx \frac{wL^2 e}{2d} \text{ WHEN } \frac{L}{a} \leq 0.30 \right\}$$

$$\theta_T = \frac{wLe a}{JG} \left( \frac{L}{2a} - \tanh \frac{L}{a} - \frac{1}{\cosh \frac{L}{a}} + \frac{a}{L} \right)$$



LOAD AND SUPPORT CONDITIONS

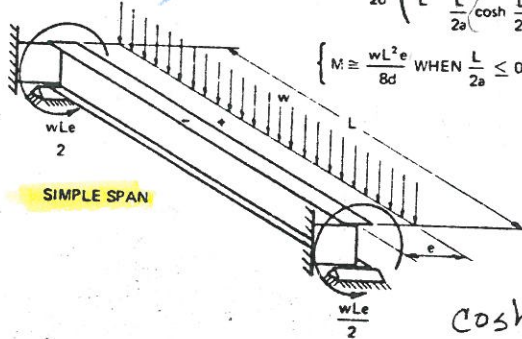
MAXIMUM LATERAL BENDING  
MOMENT IN EACH FLANGE

MAXIMUM TOTAL ANGLE OF TWIST  
AT END OF CANTILEVER AND AT  
MIDSPAN OF OTHER BEAMS

$$M = \frac{wLe a}{2d} \left( \frac{2a}{L} - \frac{1}{\cosh \frac{L}{2a}} \right)$$

$$\left\{ M \approx \frac{wL^2 e}{8d} \text{ WHEN } \frac{L}{2a} \leq 0.60 \right\}$$

$$\theta_T = \frac{wLe a}{2JG} \left( \frac{L}{4a} - \frac{2a}{L} + \frac{1}{\cosh \frac{L}{2a}} \right)$$



T = END TORQUE

$$= \frac{wLe}{2}$$

$$M = \frac{Ta}{d} \left[ \frac{2a}{L} - \frac{1}{\cosh \frac{L}{2a}} \right]$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

MAXIMUM TOTAL ANGLE OF TWIST  
AT END OF CANTILEVER AND AT  
MIDSPAN OF OTHER BEAMS

$$\theta_T = \frac{Pe a}{2JG} \left( \frac{L}{2a} - \tanh \frac{L}{2a} \right)$$

LOAD AND SUPPORT CONDITIONS

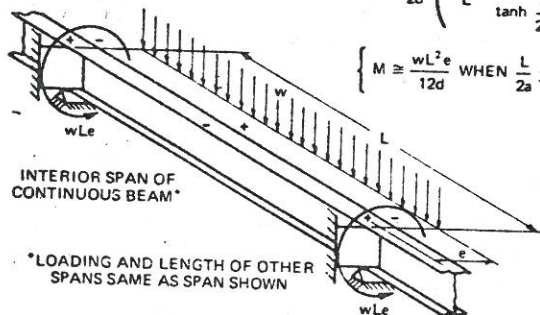
MAXIMUM LATERAL BENDING  
MOMENT IN EACH FLANGE

MAXIMUM TOTAL ANGLE OF TWIST  
AT END OF CANTILEVER AND AT  
MIDSPAN OF OTHER BEAMS

$$M = \frac{wLe a}{2d} \left( \frac{2a}{L} + \frac{1}{\tanh \frac{L}{2a}} \right)$$

$$\left\{ M \approx \frac{wL^2 e}{12d} \text{ WHEN } \frac{L}{2a} \leq 0.60 \right\}$$

$$\theta_T = \frac{wLe a}{2JG} \left( \frac{L}{4a} - \tanh \frac{L}{4a} \right)$$



\*LOADING AND LENGTH OF OTHER  
SPANS SAME AS SPAN SHOWN

FIGURE 7.10 MAXIMUM LATERAL BENDING MOMENT AND TOTAL ANGLE OF TWIST FOR BEAMS IN TORSION FROM UNIFORM LOADS