grade practice and covered a wide range of sections, both rolled shapes and built-up sections (*Trans. Am. Soc. C. E.*, vol. lxxxiii, 1919–1920).

The tests showed that, for columns of the proportions commonly used, the effect of the variation in the steel, kinks, initial stresses, and similar defects in the column was more important than the effect of length. They also showed that the thin metal gave definitely higher strength, per unit area, than the thicker metal of the same type of section.

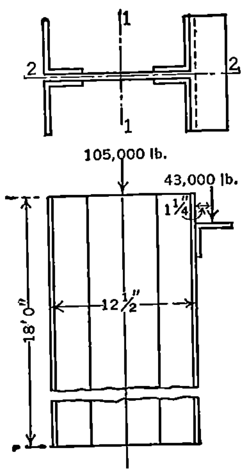

FIG. 2

Example. Design a steel column 18 ft long to carry an axial load of 105,000 lb and an additional load of 43,000 lb which comes from a crane runway, and which is 1 1/4 in. beyond the outside edge of the column.

The formula used will be the A.I.S.C. formula, 1946, $P/A = 17{,}000 - 0.485l^2/k^2$, and the shape selected will be a plate and four angles, as shown in Fig. 2.

Using 10,000 psi as a preliminary estimate, one plate $12 \times 5/16$ in. and four angles $5 \times 3 \times 3/8$ in. will have an area of 15.19 sq in. The safe load for this column will now be found. The moment of inertia I_{1-1} is 406 in.^{4}, the radius of gyration k_{1-1} is 5.17 in., and the radius of gyration k_{2-2} is 2.13 in.

Considering the safe load with respect to axis 2–2 about which neither load is eccentric,

$$P = 15.19\left(17{,}000 - 0.485\,\frac{(18 \times 12)^2}{2.13^2}\right) = 182{,}400 \text{ lb}$$

Consider now the safe load about axis 1–1, with respect to which the load of 105,000 lb is axial, and the load of 43,000 lb has an eccentricity of 7 1/2 in. The load of 43,000 lb will produce a stress due to its eccentricity, and this stress will reduce by that amount the unit load that the column can carry. The safe load will therefore be

$$P = 15.19\left(17{,}000 - 0.485\,\frac{(18 \times 12)^2}{5.17^2} - \frac{43{,}000 \times 7.5 \times 6.25}{406}\right)$$
$$= 170{,}000 \text{ lb}$$

As the load to be carried is 148,000 lb this column is satisfactory.

25. CAST-IRON COLUMNS

Disadvantages of Cast-iron Columns. Tests made on cast-iron columns show that a large factor of safety should be used when the material is employed in column construction. It is difficult to obtain a practical formula for the strength of a cast-iron column on account of the uncertainty of the quality of the casting, and the danger of hidden defects such as internal stresses due to unequal cooling of the casting, cinder or dirt, blowholes, cold shuts, and cracks on the inner surface, which cannot be discovered by external inspection. Variation in the thickness of the wall due to shifting of the core is another common defect. Brackets which are usually cast on the face of the column to support beams and girders may fail by shearing unless most skillfully designed.

Tests of full-size cast-iron columns made at the Watertown Arsenal and at Phoenixville, Pa., showed that for short columns the strength in compression is much less than the strength of small test specimens of cast iron. For short columns, failure of test columns takes place by shearing on an inclined plane; for long columns, failure takes place by rupture on the convex side. Failure of cast-iron columns is always a sudden, shattering failure.

Advantages of Cast-iron Columns. For buildings of moderate height cast-iron columns are sometimes used instead of wooden columns to save space, and instead of steel columns to save expense. The fact that they are cheap and can be obtained more quickly than steel built-up columns will prevent them from being superseded entirely. Moreover they can be cast in almost any desired shape with lugs and brackets attached for supporting beams or girders.

Design of Cast-iron Columns. Cast-iron columns are usually in the form of a hollow cylinder, as this is the most economical and reliable type. Hollow rectangular sections are permitted by some building codes and are convenient in outer walls where it is necessary to bond into the masonry. They are less reliable than the hollow round type and should not be used if anything else is possible.

The design will usually be governed by building laws, and the following limitations, which are usually specified, should be adhered to: Minimum thickness of wall, 1/2 to 3/4 in.; minimum diameter of column, 6 in.; maximum length, $20 \times$ diameter or $70 \times$ least radius of gyration. When it becomes necessary to use a wall thickness greater than 2 in. or a diameter over 18 in. it will be cheaper and more satisfactory to abandon

cast iron and substitute a steel column. Within the above limitations the ratio of length to radius of gyration should be made as small as possible; that is, if a choice may be made of two columns of equal sectional area but of different diameter and, therefore, different wall thicknesses, the one with the larger diameter will be the stronger.

Allowable safe loads on cast-iron columns may be calculated as follows

$$\frac{P}{A} = 9000 - 40\,\frac{l}{k} \tag{15}$$

where P = the safe load, pounds; A = area of section, square inches; l = length, inches; k = least radius of gyration.

SHAFTS

26. DEFINITIONS

Torsional Stress. A bar is under torsional stress when it is held fast at one end and a force acts at the other end to twist the bar. In a round bar, Fig. 1, with a constant force acting, the straight line a–b becomes the helix ad, and a radial line in the cross section, ob, moves to the position od. The angle bad remains constant while the angle bod increases with the length of the bar. Each cross section of the bar tends to shear off the one adjacent to it, and in any cross section the shearing stress at any point is normal to a radial line drawn through the point. Within the shearing proportional limit a radial line of the cross section remains straight after the twisting force has been applied, and the unit shearing stress at any point is proportional to its distance from the axis.

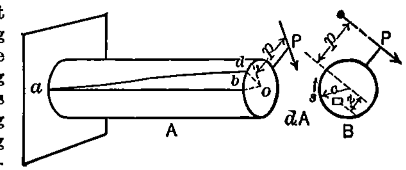

FIG. 1

The Twisting Moment, T, is equal to the product of the resultant, P, of the twisting forces, and its distance from the axis, p.

The Resisting Moment, T_r, in torsion, is equal to the sum of the moments of the unit shearing stresses acting along a cross section with respect to the axis of the bar. If dA is an elementary area of the section at a distance of z units from the axis of a circular shaft (Fig. 1B), and c is the distance from the axis to the outside of the cross section where the unit shearing stress is τ, then the total shearing force acting on dA is $(\tau z/c)\,dA$, its moment with respect to the axis is $(\tau z^2/c)\,dA$, and the sum of all the moments of the unit shearing stresses on the cross section is $\int (\tau z^2/c)\,dA$. In this expression the factor $\int z^2\,dA$ is the polar moment of inertia of the section with respect to the axis. Denoting this by J, the resisting moment may be written $\tau J/c$.

The Polar Moment of Inertia of a surface about an axis through its center of gravity and perpendicular to the surface is the sum of the products obtained by multiplying each elementary area by the square of its distance from the center of gravity of its surface; it is equal to the sum of the moments of inertia taken with respect to two axes in the plane of the surface at right angles to each other passing through the center of gravity. It is represented by J, inches^4. For the cross section of a round shaft,

$$J = (1/32)(\pi d^4) \quad \text{or} \quad 1/2\,\pi r^4 \tag{1}$$

for a hollow shaft

$$J = (1/32)\,[\pi(d^4 - d_1^4)] \tag{2}$$

where d is the outside and d_1 the inside diameter, in inches, or

$$J = 1/2\,[\pi(r^4 - r_1^4)] \tag{3}$$

where r is the outside and r_1 the inside radius, in inches.

The Polar Radius of Gyration, k_p, is also sometimes used in formulas; it is defined as the radius of a circumference along which the entire area of a surface might be concentrated and have the same polar moment of inertia as the distributed area. For a solid circular section,

$$k_p^2 = (1/8)d^2 \tag{4}$$

for a hollow circular section,

$$k_p^2 = (1/8)(d^2 + d_1^2) \tag{5}$$