1.0 INTRODUCTION

This MathCad calculation sheet determine the pipeline expansion due to thermal loading. The theory is based on that given in OTC paper 4067, Palmer & Ling, 1981. Further, the pipeline in situ stresses are also considered herein.

2.0 ASSUMPTIONS

The following assumptions are made:
2.1) The pipeline longitudinal friction coefficient of 0.5.
2.2) The calculation is conservatively based on the maximum pipeline departure and arrival temperatures.
2.3) The pipeline is laid at the bottom of the trench and does not have a span.

3.0 MATHCAD STANDARD UNITS

$ORIGIN \equiv 1$	$rev \equiv 1$	$°C \equiv 1$	$°K \equiv 1$
$N \equiv newton$	$kN \equiv 10^{3}N$	$Te \equiv tonne$	$\% := 100^{-1}$
$kPa \equiv 10^{3}Pa$	$bar \equiv 10^{5}Pa$	$MPa \equiv 10^{6}Pa$	$GPa \equiv 10^{9}Pa$
$litre \equiv liter$	$ppm := mg \times m^{-3}$	$kmol := 10^{3}mol$	$barrel := 0.159m^{3}$
$HP \equiv 1hp$	$MJ \equiv 10^{6}joule$	$kJ := 10^{3}joule$	$kW := 10^{3}watt$
$° \equiv deg$	$knots := 0.514m \times sec^{-1}$		$ksi := 1000psi$

4.0 PIPELINE EXPANSION CALCULATIONS

4.1 Input Data

For 12"Pipeline-Operation Condition

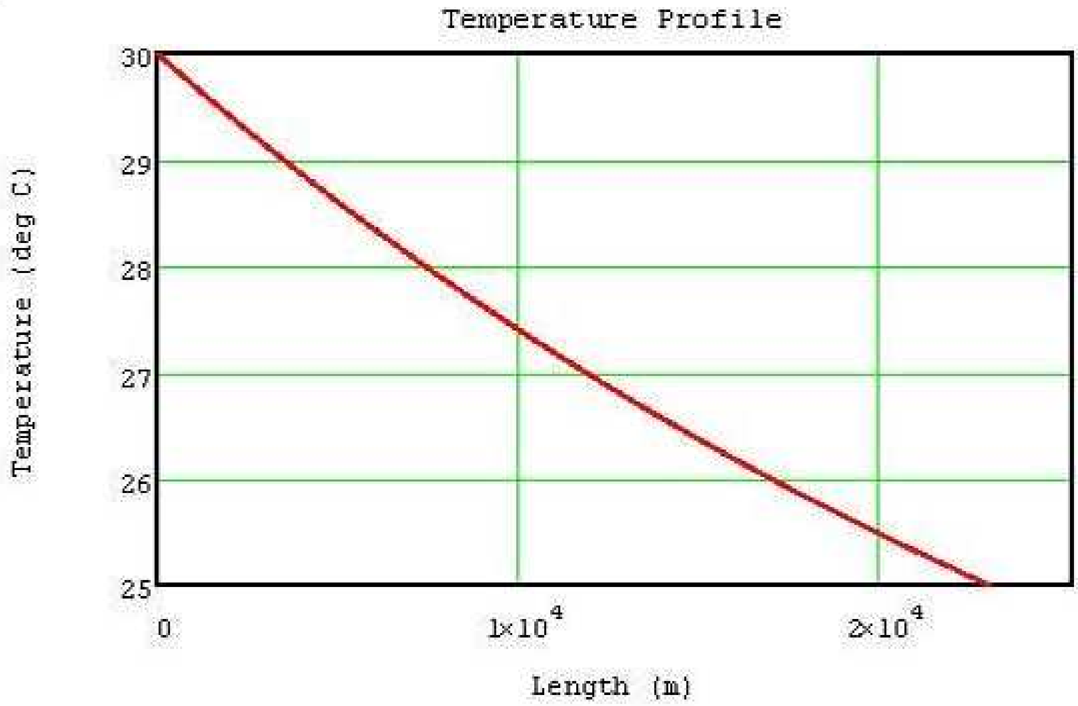
Internal pressure	$p_i := 3.7MPa$	Water Depth	$D := 3.0m$
Pipeline OD	$OD := 323.85mm$	Water density	$\rho_{sw} := 1025kg \times m^{-3}$
Pipeline wall thickness	$t := 8.8mm$	Soil/Pipeline friction coeff.	$\mu := 0.5$
Corrosion allowance	$t_{corr} := 3.0mm$	Poisson's ratio of steel	$\nu := 0.3$
Length of pipeline	$L := 23100m$	Elastic modulus of steel	$E := 207GPa$
Product departure temp	$T_{dep} := 30°C$		
Product arrival temp	$T_{arr} := 25°C$		
Ambient temp	$T_{amb} := 20.0°C$		
Pipeline weight	$w_p := 68.373kg \times m^{-1}$		
Steel thermal conductivity	$\alpha := 1.16 \times 10^{-5} \circ C^{-1}$		

4.2 Calculations

Minimum wall thickness	$t_2 := t - t_{corr}$	$t_2 = 5.8 \times mm$
Internal over pressure	$p_d := p_i - \rho_{sw} \times g \times D$	$p_d = 3.6698 \times MPa$
Hoop stress	$\sigma_h := \dfrac{p_d \times OD}{2 \times (t_2)}$	$\sigma_h = 102.4551 \times MPa$
Pipe Internal Diameter	$ID := OD - 2t_2$	$ID = 312.25 \times mm$
Area of steel	$A_s := \pi \times (OD - t_2) \times t_2$	$A_s = 5795.2646 \times mm^2$
Internal area of pipeline	$A_i := \pi \times \dfrac{(OD - 2 \times t_2)^2}{4}$	$A_i = 76576.37 \times mm^2$
Friction force per metre	$f := \mu \times w_p \times g$	$f = 335.255 \times N \times m^{-1}$

4.2.1 Temperature Profile

$l := 0m, 20m .. L$	$\theta_1 := T_{dep} - T_{amb}$	$\theta_1 = 10 \times °C$
$\lambda := \dfrac{-L}{\ln\left[\dfrac{-(-\theta_1 + T_{dep} - T_{arr})}{\theta_1}\right]}$	$\theta(l) := \theta_1 \times e^{\frac{-l}{\lambda}}$	$\lambda = 33326.2554\, m$



$\theta(0m) + T_{amb} = 30 \times {}^\circ C \qquad \theta\left(\frac{L}{2}\right) + T_{amb} = 27.0711 \times {}^\circ C \; \theta(L) + T_{amb} = 25 \times {}^\circ C$

Determine whether pipeline is long or short, by finding strain at midpoint:

Pressure strain $\varepsilon_p := \frac{1}{E} \times \left(\frac{A_i \times p_i}{A_s} - \nu \times \sigma_h\right)$ $\varepsilon_p = 0.0001$

Temperature strain $\varepsilon_T := \alpha \times \left(\theta_1 \times e^{\frac{-L}{2\times\lambda}}\right)$ $\varepsilon_T = 0.0001$

Frictional strain $\varepsilon_f := f \times \frac{L}{2} \times \frac{1}{E \times A_s}$ $\varepsilon_f = 0.0032$

Strain at midpoint $\varepsilon_M := \varepsilon_p + \varepsilon_T - \varepsilon_f$ $\varepsilon_M = -0.0031$

If strain (ε_M) is > 0 the anchor point is at the midpoint, if the strain is < 0 then the location of the two anchor point must be found.

4.2.2 Short Pipe

Expansion of pipe at hot end

$$\Delta_{as} := \int_0^{\frac{L}{2}} \left[\frac{1}{E} \times \left[\left(\frac{A_i \times p_i}{A_s} - \nu \times \sigma_h \right) - \frac{f \times x}{A_s} \right] + \alpha \times \theta_1 \times e^{\frac{-x}{\lambda}} \right] dx \qquad \Delta_{as} = -16.4956\,m$$

Expansion of pipe at cold end

$$\Delta_{bs} := \int_{\frac{L}{2}}^{L} \left[\frac{1}{E} \times \left[\left(\frac{A_i \times p_i}{A_s} - \nu \times \sigma_h \right) - \frac{f \times (L - x)}{A_s} \right] + \alpha \times \theta_1 \times e^{\frac{-x}{\lambda}} \right] dx \qquad \Delta_{bs} = -16.8273\,m$$

4.2.3 Long Pipe

$x := 0.3 \times L$

Given $\qquad x = \frac{1}{f} \times \left[A_i \times p_i - A_s \times \left[\nu \times \sigma_h - E \times \alpha \times \left(\theta_1 \times e^{\frac{-x}{\lambda}} \right) \right] \right]$

Length to anchor point at hot end $\qquad L_a := \begin{cases} \frac{L}{2} & \text{if } \varepsilon_M > 0 \\ \text{Find}(x) & \text{otherwise} \end{cases} \qquad L_a = 720.0128\,m$

$x := .75 \times L$

Given $\qquad x = \frac{1}{f} \times \left[A_s \times \left[\nu \times \sigma_h - E \times \alpha \times \left(\theta_1 \times e^{\frac{-x}{\lambda}} \right) \right] - A_i \times p_i + f \times L \right]$

Length to anchor point at cold end $\qquad L_b := \begin{cases} \frac{L}{2} & \text{if } \varepsilon_M > 0 \\ \text{Find}(x) & \text{otherwise} \end{cases} \qquad L_b = 22575.360$

Expansion of pipe at hot end

$$\Delta_{al} := \int_0^{L_a} \left[\frac{1}{E} \times \left[\left(\frac{A_i \times p_i}{A_s} - \nu \times \sigma_h \right) - \frac{f \times x}{A_s} \right] + \alpha \times \theta_1 \times e^{\frac{-x}{\lambda}} \right] dx \qquad \Delta_{al} = 0.0733\,m$$

Expansion of pipe at cold end

$$\Delta_{b1} := \int_{L_b}^{L} \left[\frac{1}{E} \times \left[\left(\frac{A_i \times p_i}{A_s} - \nu \times \sigma_h \right) - \frac{f \times (L - x)}{A_s} \right] + \alpha \times \theta_1 \times e^{\frac{-x}{\lambda}} \right] dx \qquad \Delta_{b1} = 0.0382\,m$$

4.2.4 Results

$$L_A := \begin{cases} \frac{L}{2} & \text{if } \varepsilon_M \ge 0 \\ L_a & \text{otherwise} \end{cases} \qquad L_B := \begin{cases} \frac{L}{2} & \text{if } \varepsilon_M \ge 0 \\ L_b & \text{otherwise} \end{cases}$$

$\boxed{L_A = 720.0128\,m}$ $\boxed{L_B = 22575.3603\,m}$

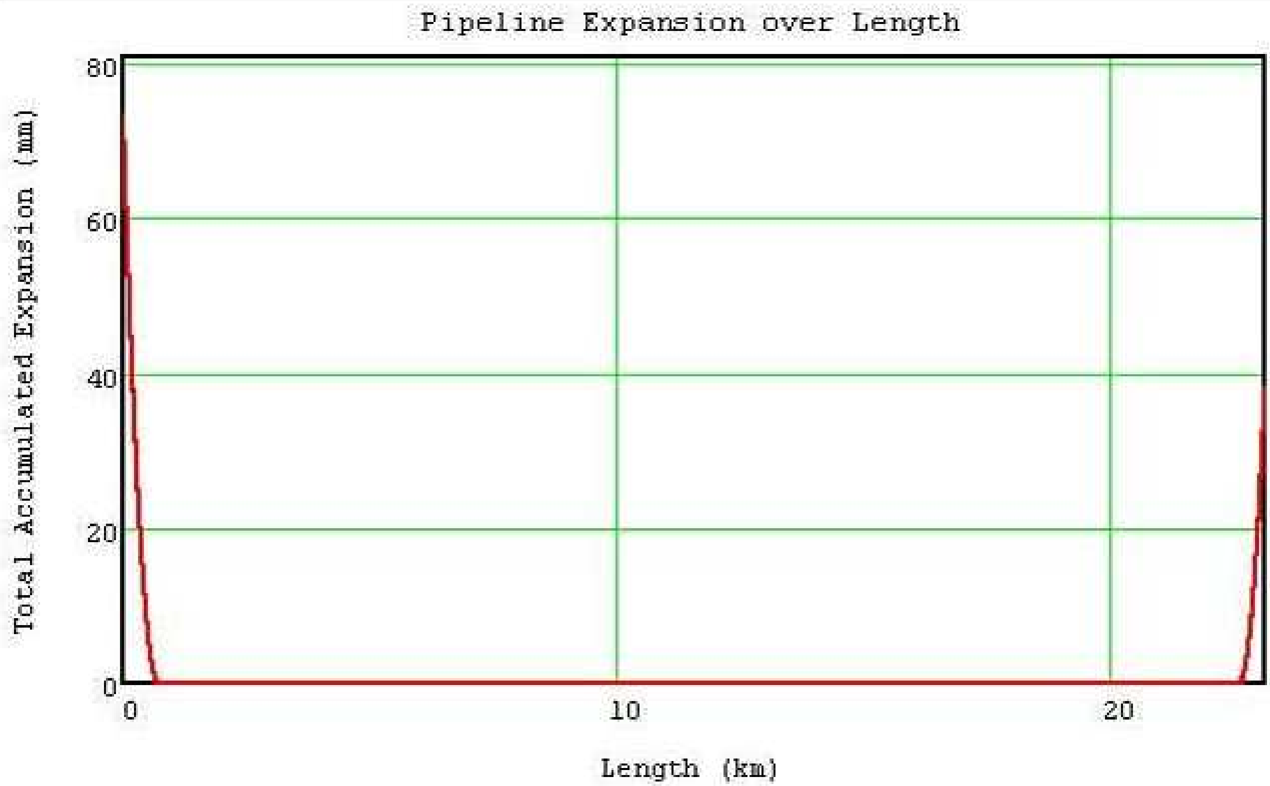
$$\Delta_A := \begin{cases} \Delta_{as} & \text{if } \varepsilon_M \ge 0 \\ \Delta_{al} & \text{otherwise} \end{cases} \qquad \Delta_B := \begin{cases} \Delta_{bs} & \text{if } \varepsilon_M \ge 0 \\ \Delta_{bl} & \text{otherwise} \end{cases}$$

$\boxed{\Delta_A = 73.33 \times mm}$ $\boxed{\Delta_B = 38.219 \times mm}$

$$\Delta_S(L_S) := \begin{cases} -\displaystyle\int_{0}^{L_S} \left[\frac{1}{E}\times\left[\left(\frac{A_i\times p_i}{A_s}-\nu\times\sigma_h\right)-\frac{f\times x}{A_s}\right]+\alpha\times\theta_1\times e^{\frac{-x}{\lambda}}\right]dx+\Delta_{as} & \text{if } L_S\le\frac{L}{2} \\ -\displaystyle\int_{L_S}^{L} \left[\frac{1}{E}\times\left[\left(\frac{A_i\times p_i}{A_s}-\nu\times\sigma_h\right)-\frac{f\times(L-x)}{A_s}\right]+\alpha\times\theta_1\times e^{\frac{-x}{\lambda}}\right]dx+\Delta_{bs} & \text{otherwise} \end{cases}$$

$$\Delta_L(L_L) := \begin{cases} -\displaystyle\int_{0m}^{L_L} \left[\frac{1}{E}\times\left[\left(\frac{A_i\times p_i}{A_s}-\nu\times\sigma_h\right)-\frac{f\times x}{A_s}\right]+\alpha\times\theta_1\times e^{\frac{-x}{\lambda}}\right]dx+\Delta_{al} & \text{if } L_L\le L_a \\ -\displaystyle\int_{L_L}^{L} \left[\frac{1}{E}\times\left[\left(\frac{A_i\times p_i}{A_s}-\nu\times\sigma_h\right)-\frac{f\times(L-x)}{A_s}\right]+\alpha\times\theta_1\times e^{\frac{-x}{\lambda}}\right]dx+\Delta_{bl} & \text{if } L_L\ge L_b \\ 0m & \text{otherwise} \end{cases}$$

$$\Delta(L_T) := \begin{cases} \Delta_S(L_T) & \text{if } \varepsilon_M\ge 0 \\ \Delta_L(L_T) & \text{otherwise} \end{cases}$$



Pipeline Expansion over Length
Total Accumulated Expansion (mm)
0
20
40
60
80
0
10
20
Length (km)