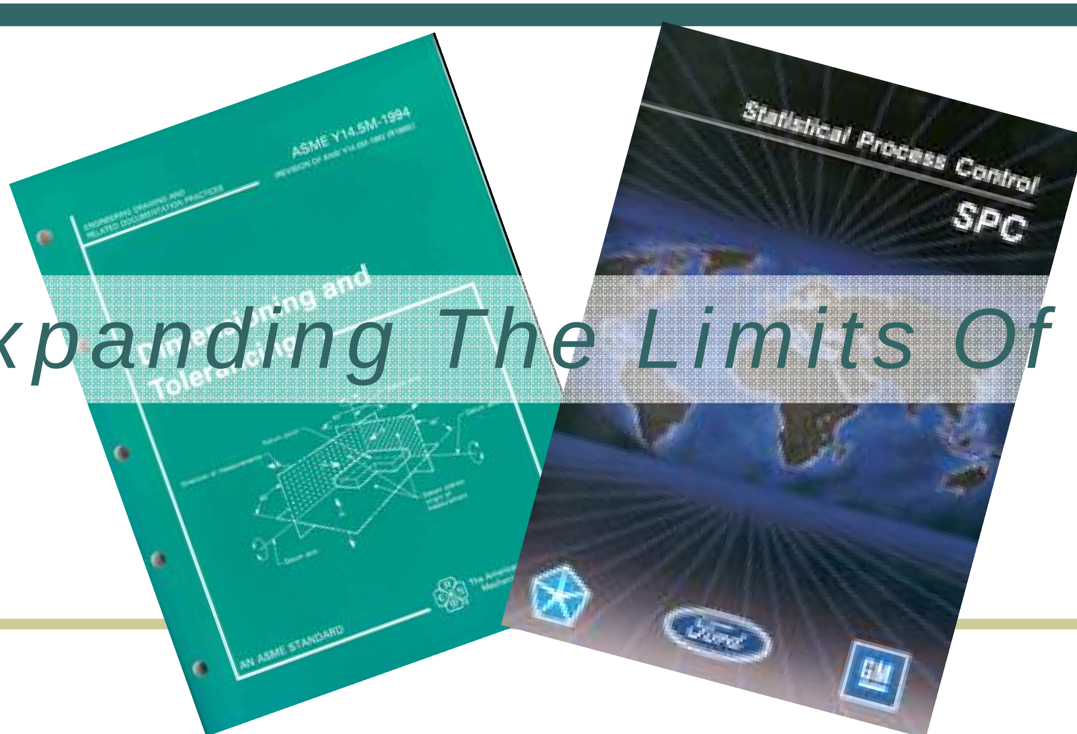
Expanding The Limits Of SPC



Process Parameter Optimization & Process Capability Prediction with Variable Tolerance Limits

Paul F. Jackson 4/25/2006

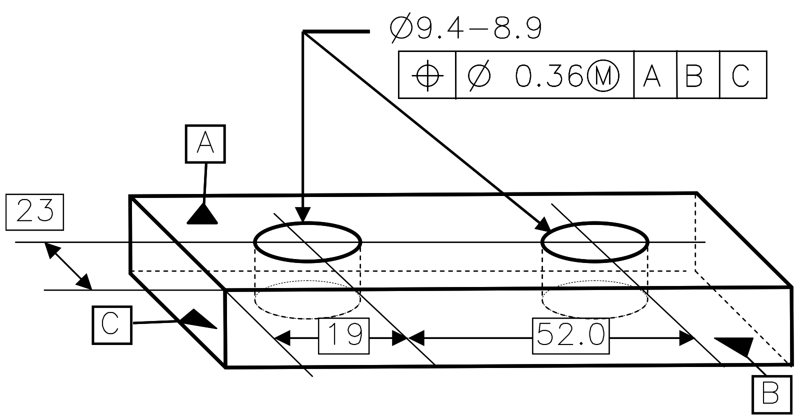
Contents

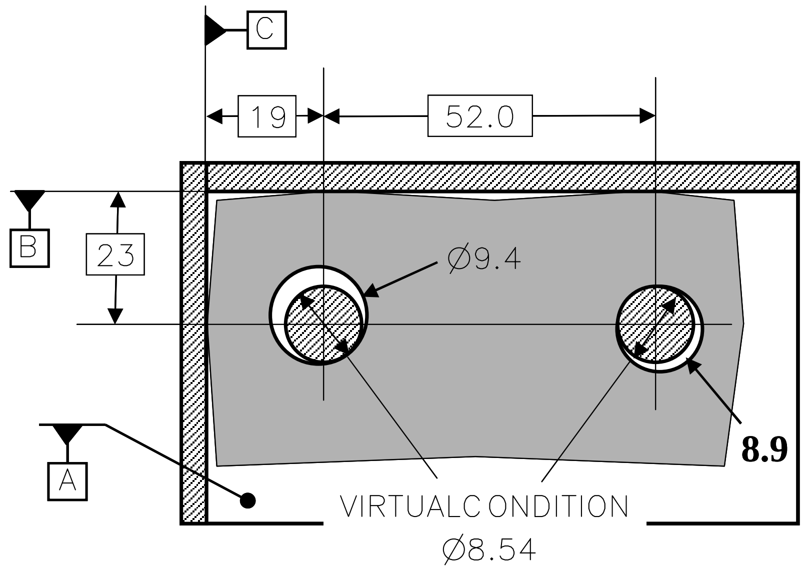
- How to recognize a variable tolerance limit.
- How discreet and continuous data is gathered for ⊕ @ Ⓜ.
- How the data is typically used in a capability analysis.
- How a variable tolerance can be visualized in a histogram.
- What statistical model reflects the probability of defect with a variable tolerance.
- Why coordinate tolerance distributions are often non-normal.
- How process potential can be examined and optimized with both constant and variable tolerances.
- How typical and proposed capability analysis methods compare relative to variable tolerances.
- What the risks are in applying the proposed analysis methods with variable tolerance distributions

this presentation does not address process control. The capability predictions herein are demonstrated assuming that the process variation is “in-control”, due solely to common cause (random) variation, and void of special cause variation.

How Can Tolerance Limits Be Variable?

Specifications with Ⓜ or Ⓛ make the tolerance variable with respect to size.

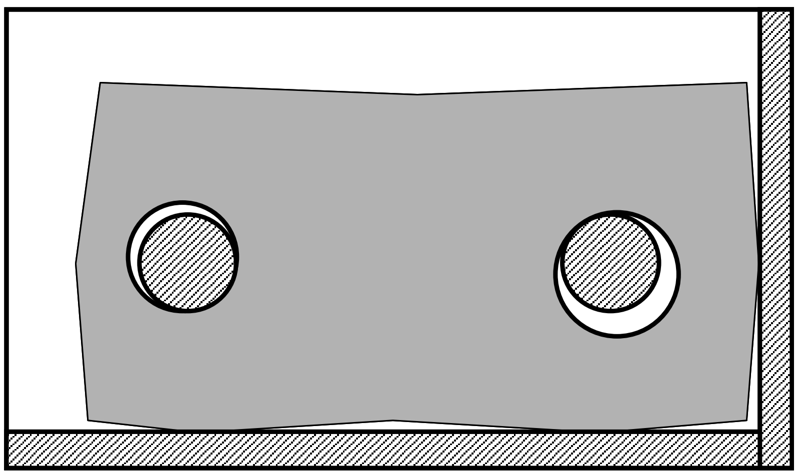




A gage built to the virtual condition of the feature will allow the larger hole "9.4" to be further off-location than the smaller MMC hole "8.9".

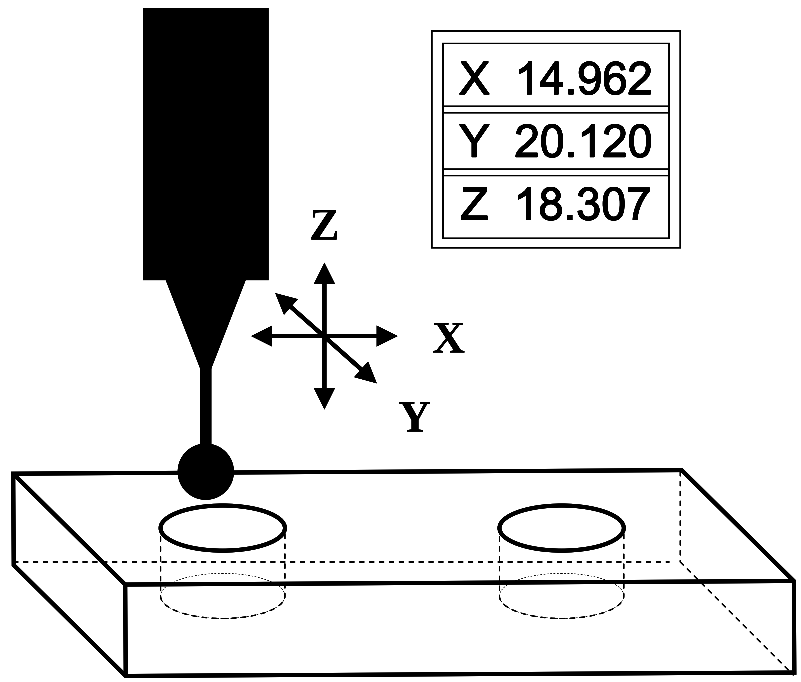
How Is Data Gathered For SPC? (Features With Variable Tolerance)

Discreet Data
With attribute gages



PASS / FAIL

Continuous Data
With Variables Gages

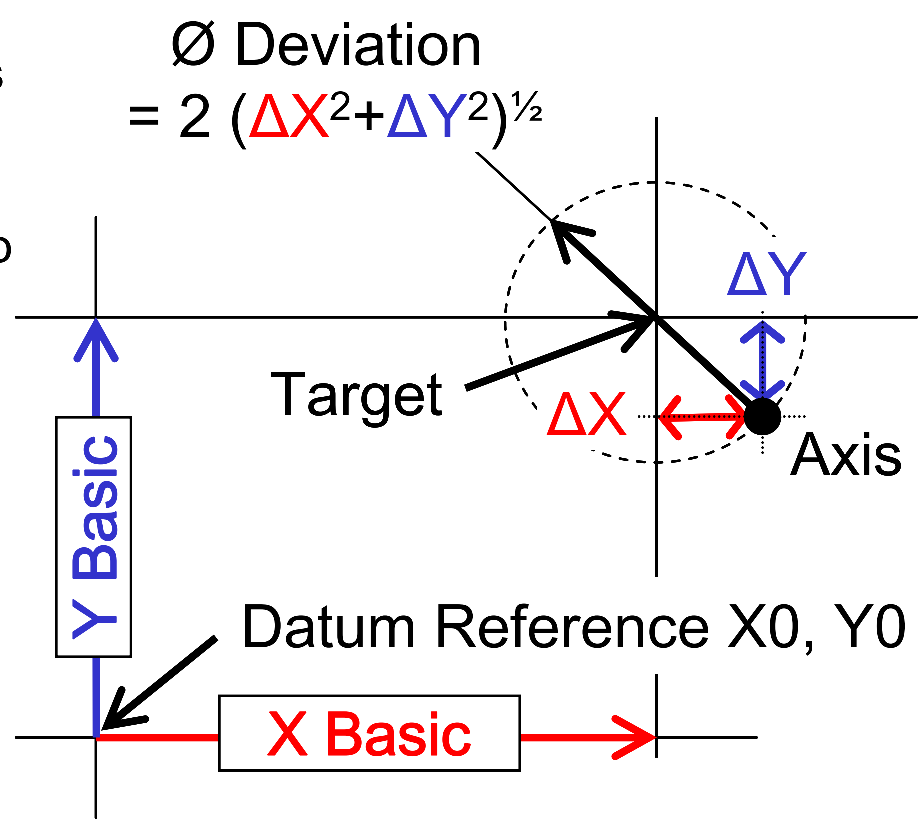


What Is Done With Continuous Coordinate Data?

Individual coordinate deviations are converted into the equivalent form as the tolerance specified so that the tolerance required to contain the deviation can be compared directly to the tolerance specified.

⌖	Ø0.36Ⓜ	A	B	C

Commonly the feature axis must reside within a diametrical or cylindrical tolerance zone so the individual coordinate deviations are converted into their resultant diametrical zone.



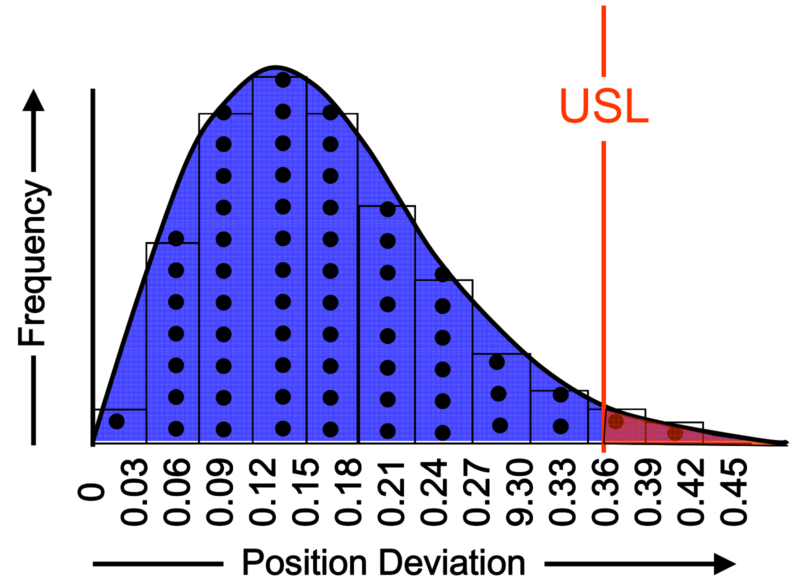
How Is Conformance Predicted?

Discreet Data

% defective figured from the ratio of position go-gage failures to the total number of parts sampled.

Continuous Data

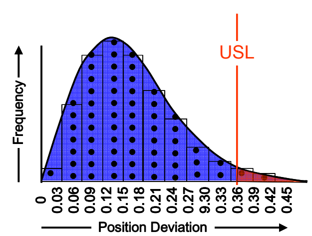
% defective figured from the area under the fitted curve > USL compared to the total area under the fitted curve.



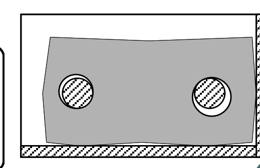
What’s The Difference?

Typical continuous data predictions of variable tolerance limits compare the position deviation to the specified limit as if it were a constant value whereas the discreet data predictions use both the specified minimum value plus the variable portion of tolerance to test acceptance.

23 Basic	Y Deviation	19 Basic	X Deviation	|⌖|Ø0.36Ⓜ| Ø Deviation	Ø Dev < Constant Tolerance (0.36)?	Hole Size 9.4~8.9	Bonus (size - MMC hole)	**Variable Tolerance** USL+Bonus	Ø Dev< Variable Tolerance (0.36+Bonus)?
23.056	0.056	**19.009**	0.009	**0.114**	**Pass**	**9.027**	0.127	0.487	**Pass**
23.109	0.109	**19.136**	0.136	**0.349**	**Pass**	**9.036**	0.136	0.496	**Pass**
23.186	0.186	**18.943**	-0.057	**0.389**	**Fail**	**9.078**	0.178	0.538	**Pass**
23.014	0.014	**19.066**	0.066	**0.135**	**Pass**	**9.069**	0.169	0.529	**Pass**
23.063	0.063	**19.290**	0.290	**0.594**	**Fail**	**9.057**	0.157	0.517	**Fail**
23.036	0.036	**19.218**	0.218	**0.443**	**Fail**	**9.049**	0.149	0.509	**Pass**
22.943	-0.057	**19.269**	0.269	**0.551**	**Fail**	**9.051**	0.151	0.511	**Fail**
23.063	0.063	**19.075**	0.075	**0.196**	**Pass**	**9.029**	0.129	0.489	**Pass**
23.075	0.075	**18.906**	-0.094	**0.241**	**Pass**	**9.057**	0.157	0.517	**Pass**
23.199	0.199	**19.063**	0.063	**0.417**	**Fail**	**9.018**	0.118	0.478	**Pass**



Constant Tolerance 0.417 > 0.36 Fail
Variable Tolerance 0.417 < 0.478 Pass
50% vs. 20% Defective!



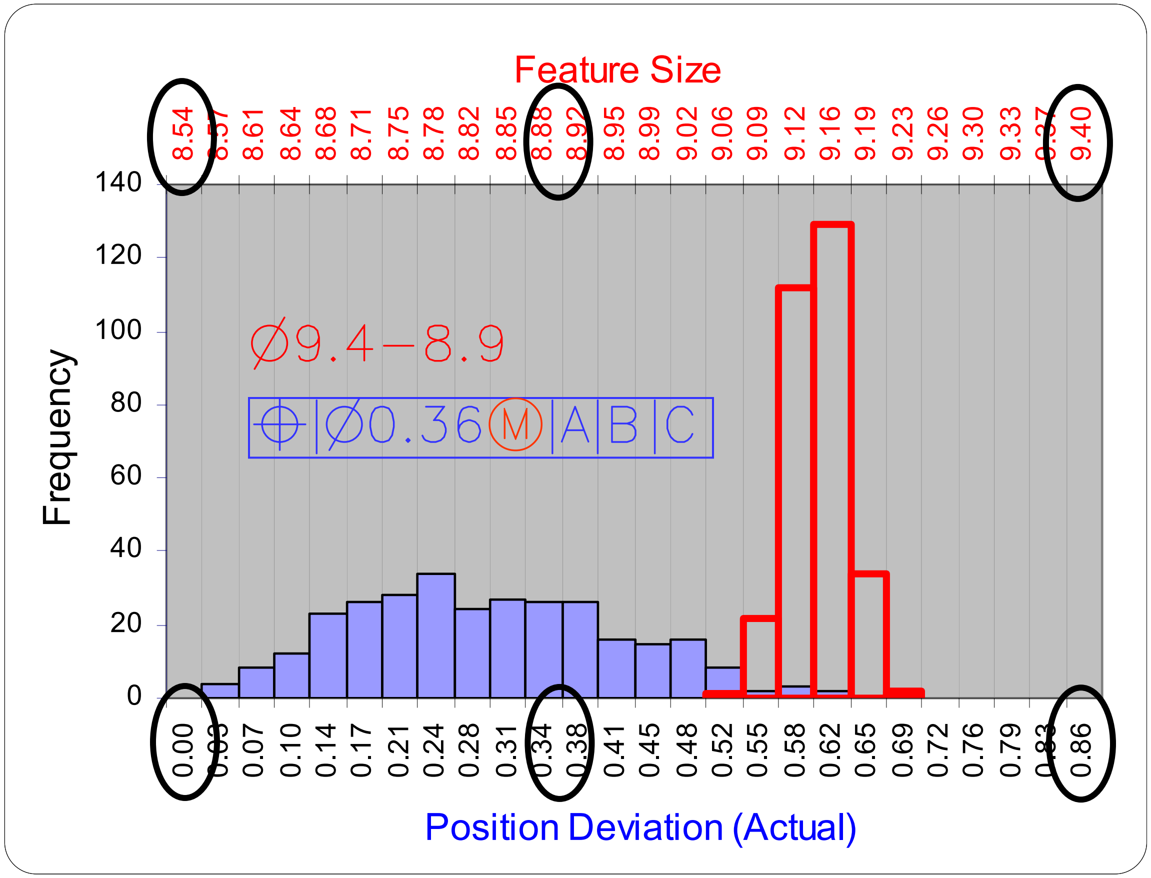
Can A Histogram Show A Variable Tolerance?

Both size and position are plotted on the same histogram.
Feature sizes align with their respective position tolerances.

Virtual Condition
8.54 = 0.0 Position

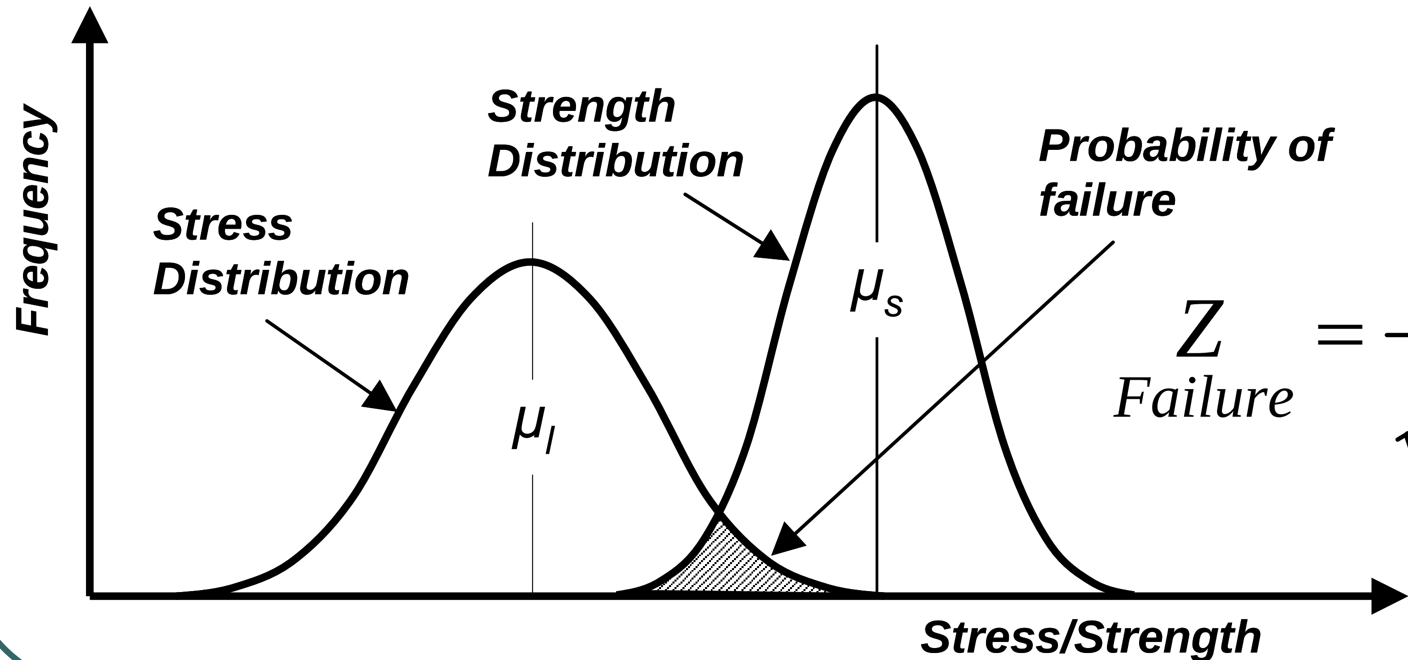
MMC
8.9 = 0.36 Position

LMC
9.4 = 0.86 Position



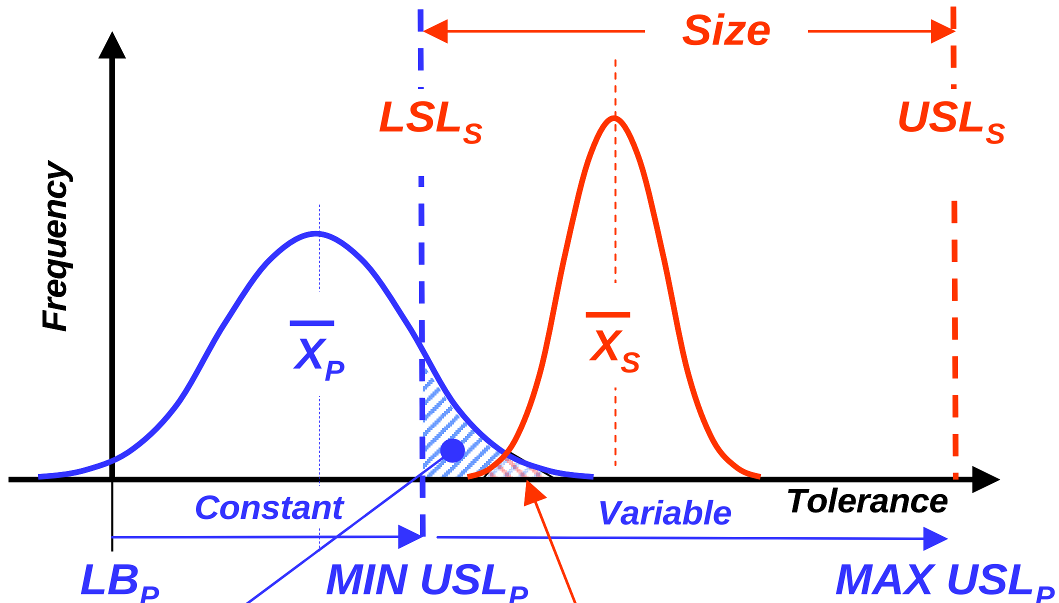
Figuring The Probability Of A Defect With A Variable Tolerance?

The classic reliability distribution model for stress vs. strength parallels the adjacent intersecting distribution analysis needed with the variable tolerance. With both distributions "normal" the Z value of the probability of failure is:



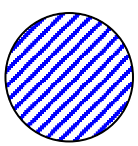
$$Z_{Failure} = \frac{\mu_S - \mu_I}{\sqrt{\sigma_S^2 + \sigma_I^2}}$$

Z_{Upper} (Constant Tolerance) Vs. Z_{Upper} (Variable Tolerance)



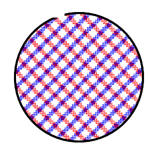
$$Z_{\text{Constant Tolerance}} = \frac{USL_P - \bar{X}_P}{\hat{\sigma}_P}$$

$$Z_{\text{Variable Tolerance}} = \frac{USL_P + (\bar{X}_S - LSL_S) - \bar{X}_P}{\sqrt{\hat{\sigma}_S^2 + \hat{\sigma}_P^2}}$$

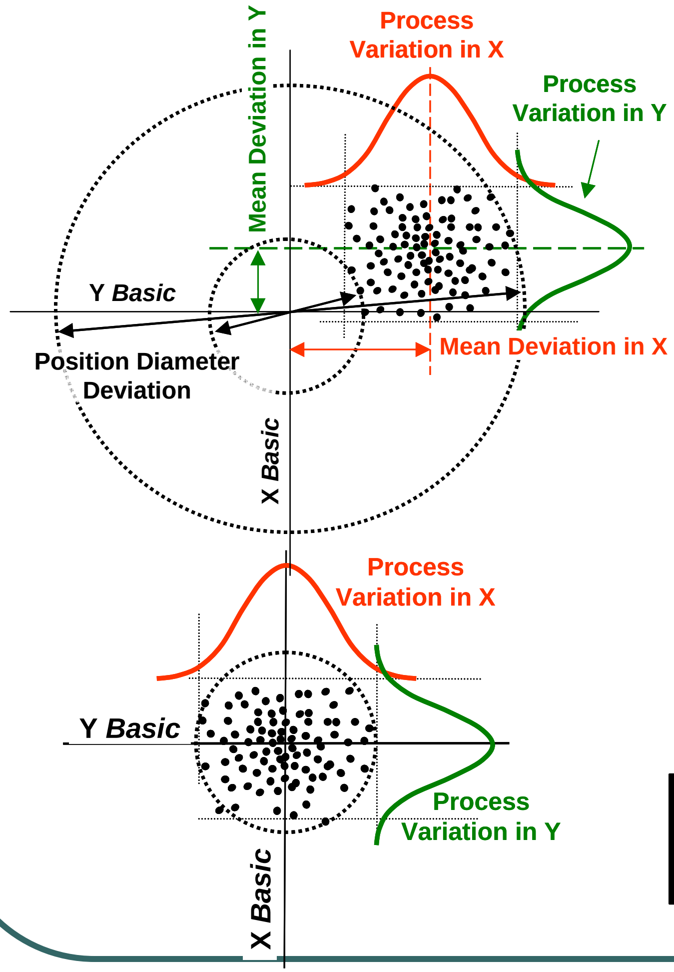


⌖ | Ø0.36 | A | B | C

⌖ | Ø0.36 Ⓜ | A | B | C



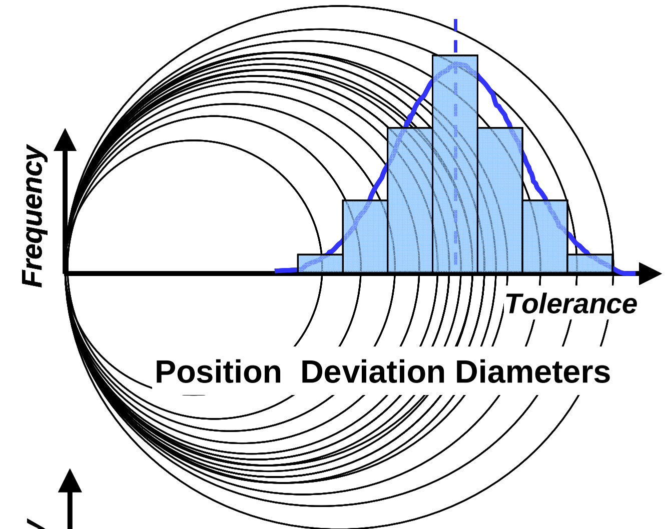
Why Are Coordinate Position Distributions Often Non-normal?

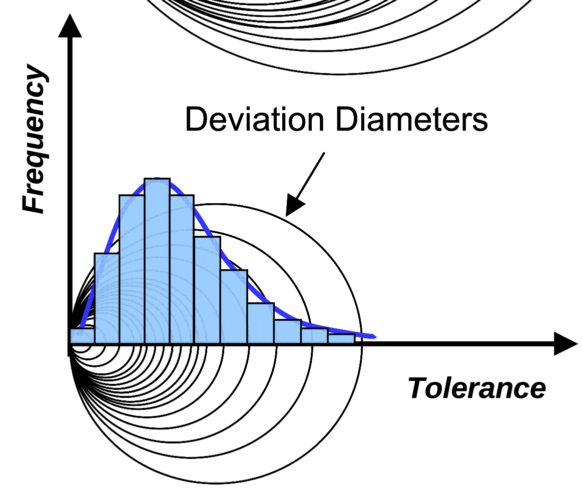


Off Target Normal

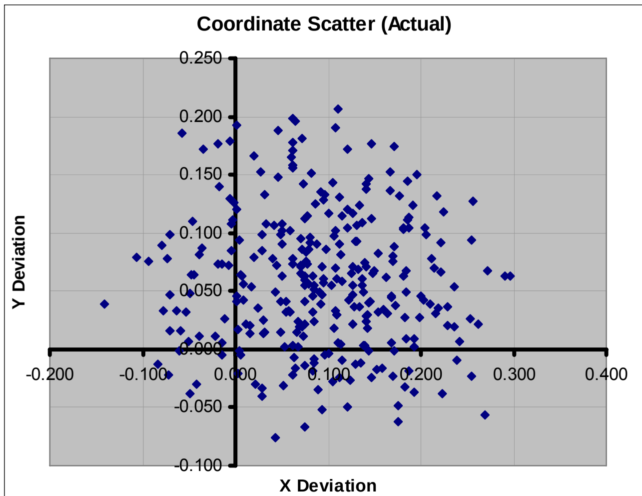
The diameter of the deviation is always a positive value so equivalent radial deviations in any polar direction from the target will have the same value. A well centered cluster with deviations surrounding the target will produce a more skewed distribution.

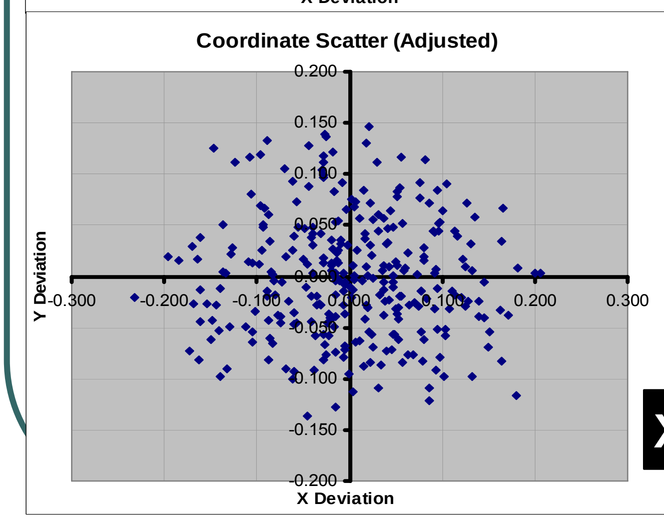
On Target Skewed





Analyzing Process Potential (Pp) With Coordinate Position Tolerances?

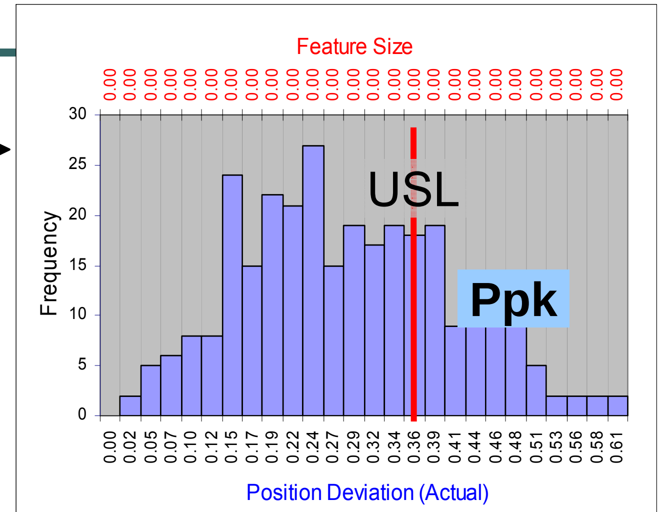


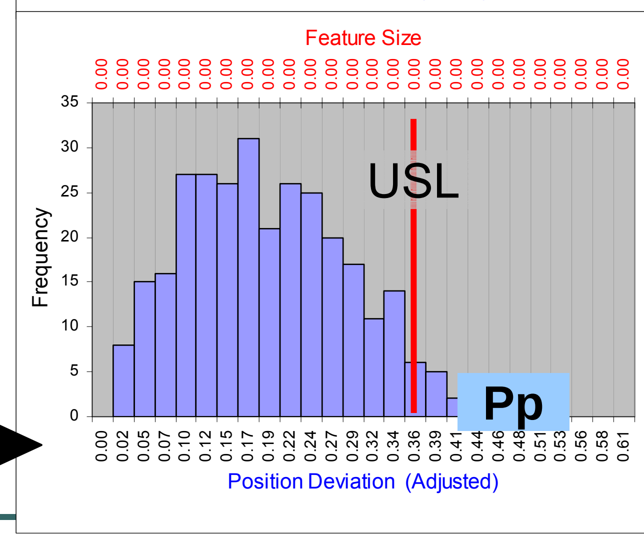


X&Y means actual

Many claim that Process Potential (Pp) for a constant (RFS) unilateral tolerance cannot be predicted but by centering the X and Y means of the coordinate distribution at its basic targets and re-computing the position deviations Pp can be estimated.

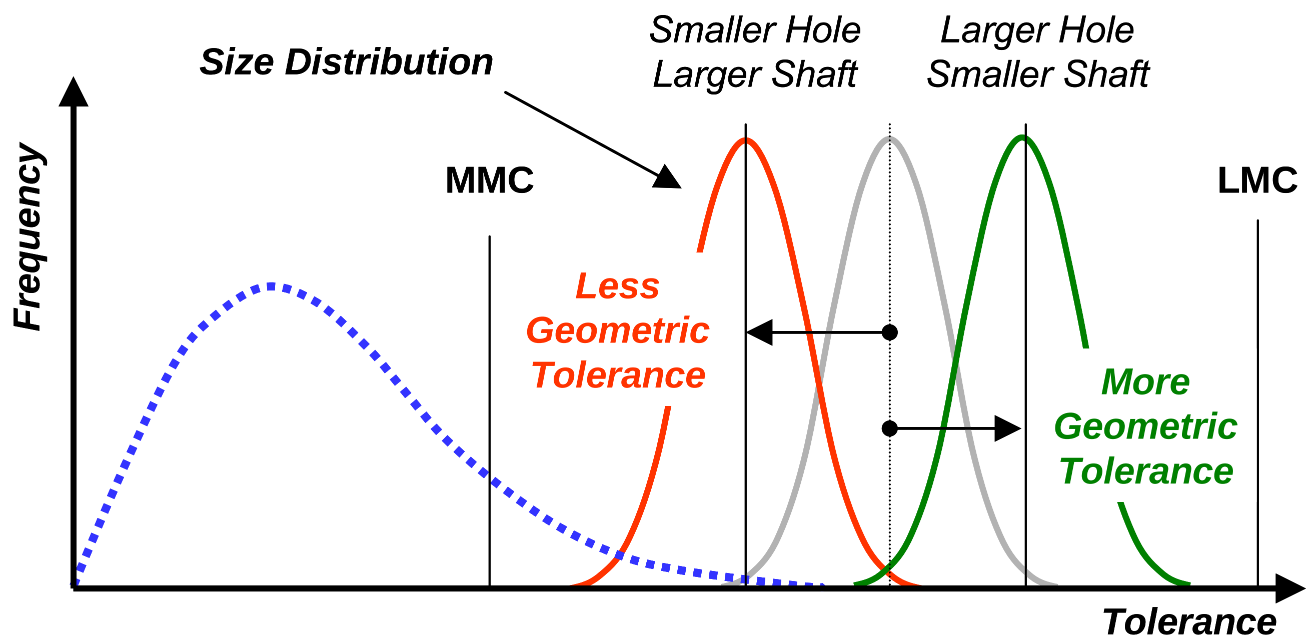
X&Y means centered



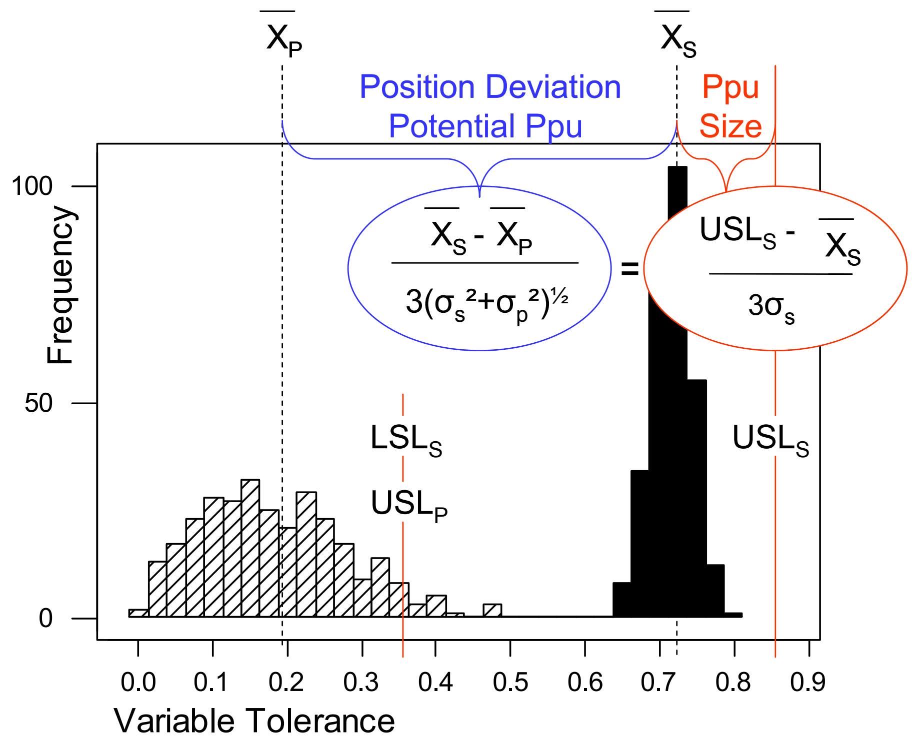


Pp for a variable tolerance?

The process potential of a variable tolerance distribution is not only dependent upon the centrality of the coordinate distributions but it is also dependent upon the target for feature size. As the mean feature size moves away from the position deviation distribution variable tolerances increases consequently as it moves toward the tolerance decreases.



Figuring Optimum Feature Size For A Variable Tolerance Specification?



The % defective can be minimized for both size and position simultaneously by setting the equations for Zu or Ppu of the size and variable tolerance equal to each other and solving for the corresponding value of mean feature size or mean variable tolerance.

Since the corresponding size and position are different scalar values one must convert to solve for the other.

$$\underset{(=\text{Size})}{\overline{X}_P} = \underset{Size}{MMC} - \underset{Position}{USL} + \underset{Position}{\overline{X}_P} \qquad \underset{(=\text{Position})}{\overline{X}_S} = \underset{Size}{\overline{X}_S} - \underset{Size}{MMC} + \underset{Position}{USL}$$

$$\underset{\substack{Upper\\Position}}{Z} = \frac{\overline{X}_S - \overline{X}_P}{\sqrt{\hat{\sigma}_S^2 + \hat{\sigma}_P^2}} = \frac{USL_S - \overline{X}_S}{\hat{\sigma}_S} = \underset{\substack{Upper\\Size}}{Z}$$

$$\overline{X}_S = \frac{\hat{\sigma}_S \times \overline{X}_P + \sqrt{\hat{\sigma}_S^2 + \hat{\sigma}_P^2} \times USL_S}{\hat{\sigma}_S + \sqrt{\hat{\sigma}_S^2 + \hat{\sigma}_P^2}}$$

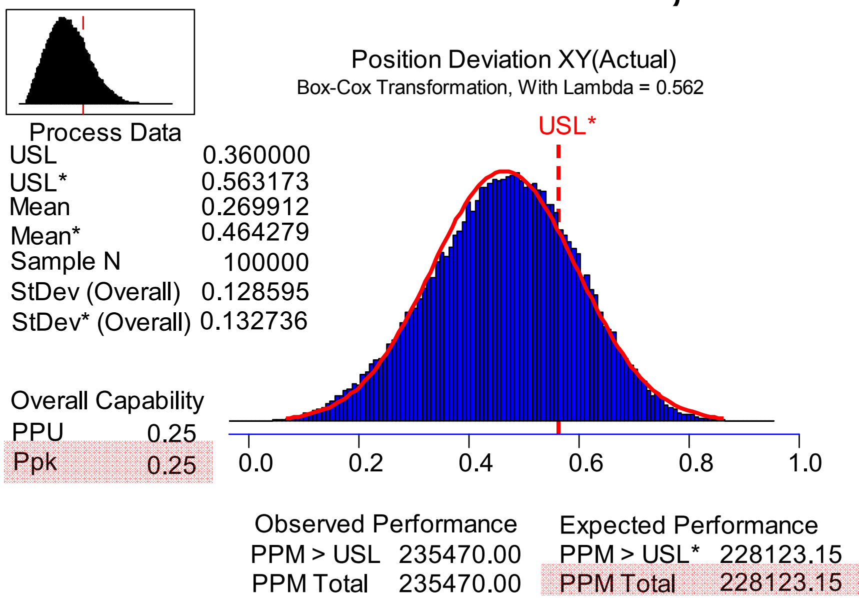
Typical Process Capability Results
Position (Non-normal Transformation)

Ppk ⊕ (Variable "bonus" Ignored)

Box-Cox Transformation (Actual)

Position **Ppu = 0.25**

Position **PPM Defective 228,123**

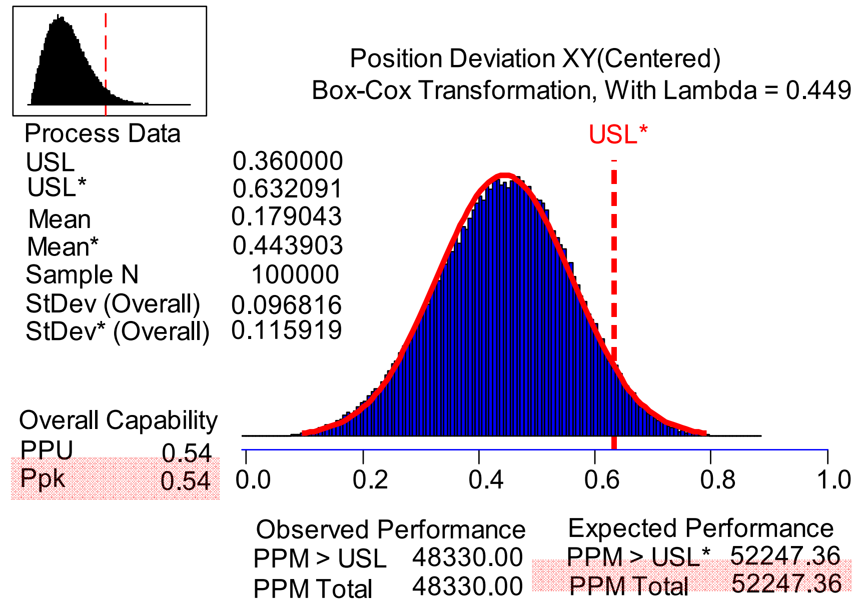


Pp ⊕ (Not Typically Considered)

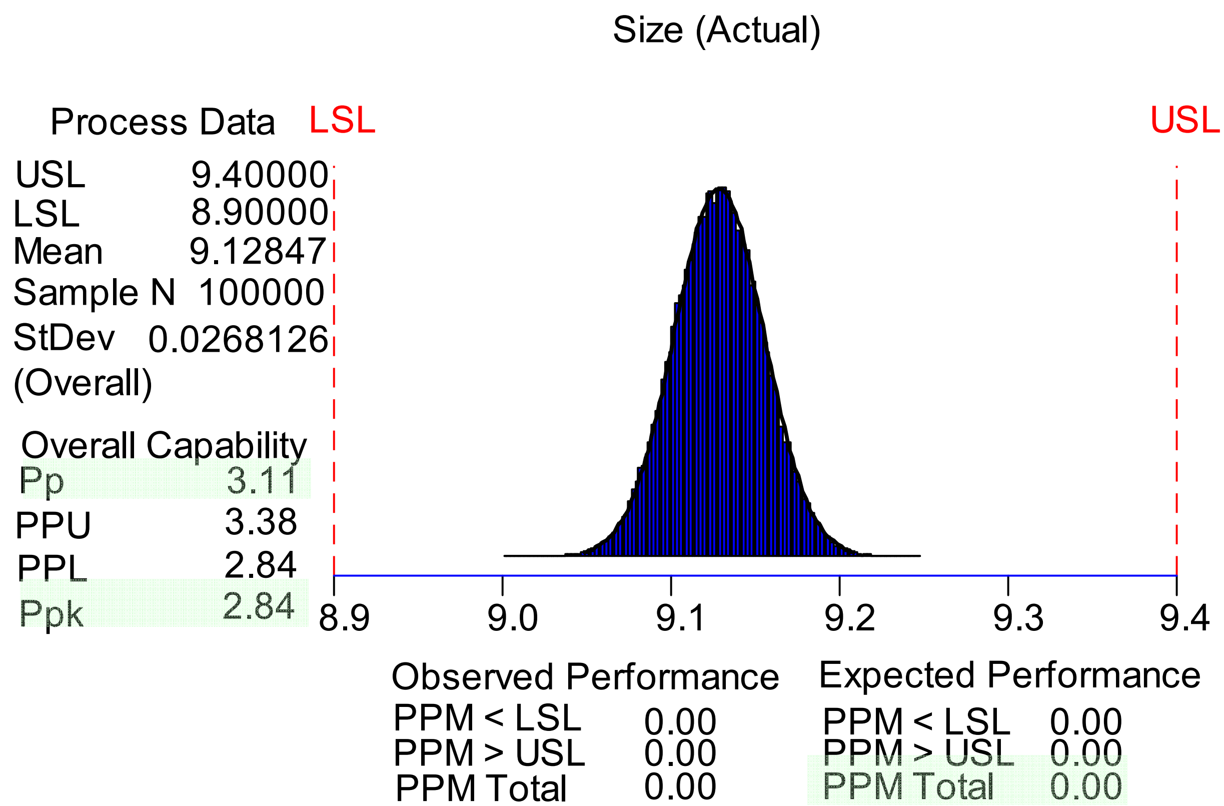
Box-Cox Transformation (Potential)

Position **Pp = 0.54**

Position **PPM Defective 52,247**



Typical Process Capability Results Size



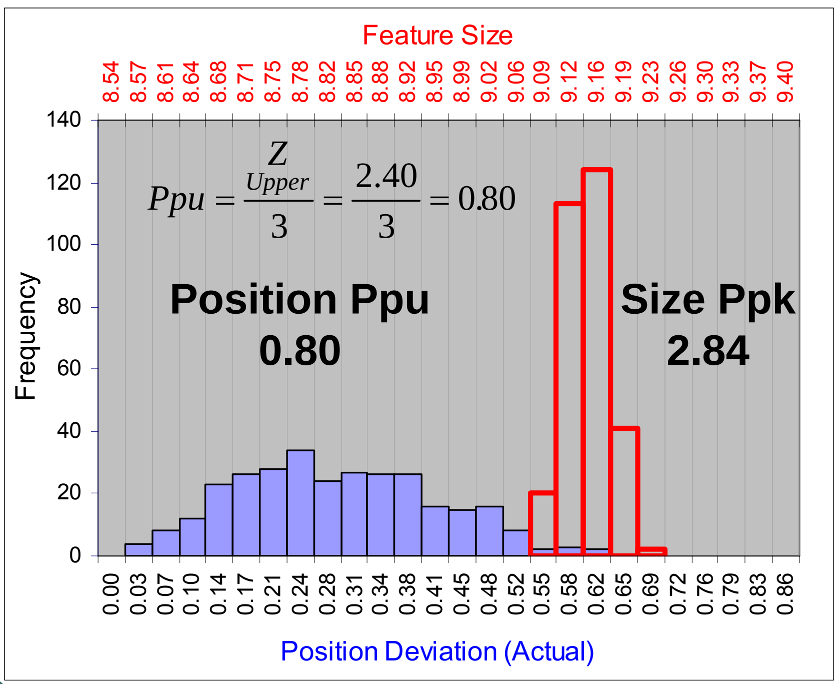
Figuring the Capability of a Variable Tolerance

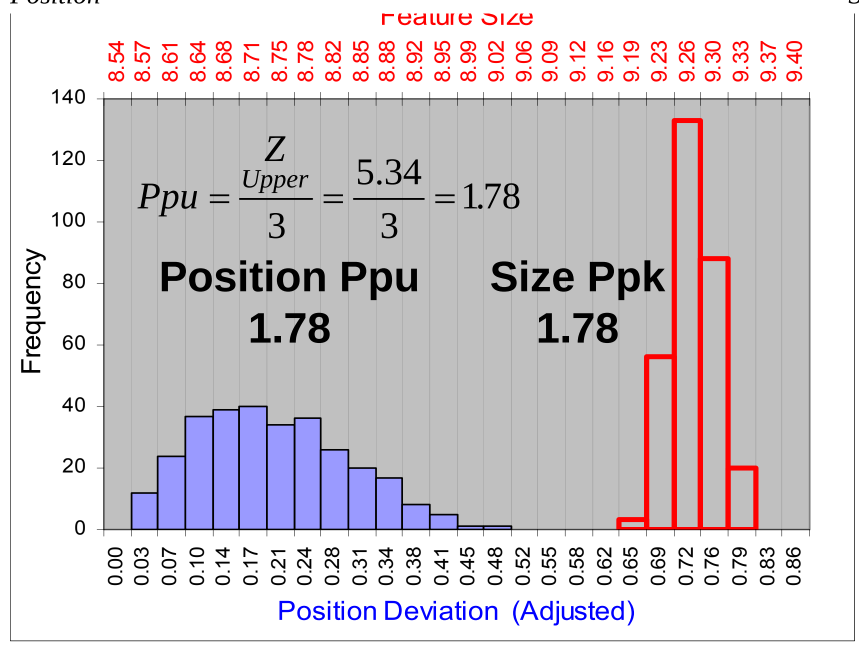
Position Ⓜ & Size (Actual) — Potential Pos & Size (Optimum)

$$Z_{\substack{Upper\\Position}} = \frac{\overline{X}_S - \overline{X}_P}{\sqrt{\hat{\sigma}_S^2 + \hat{\sigma}_P^2}} \qquad Z_{\substack{Upper\\Position}} = \frac{0.588 - 0.270}{\sqrt{.0268^2 + .1296^2}} = 2.40$$

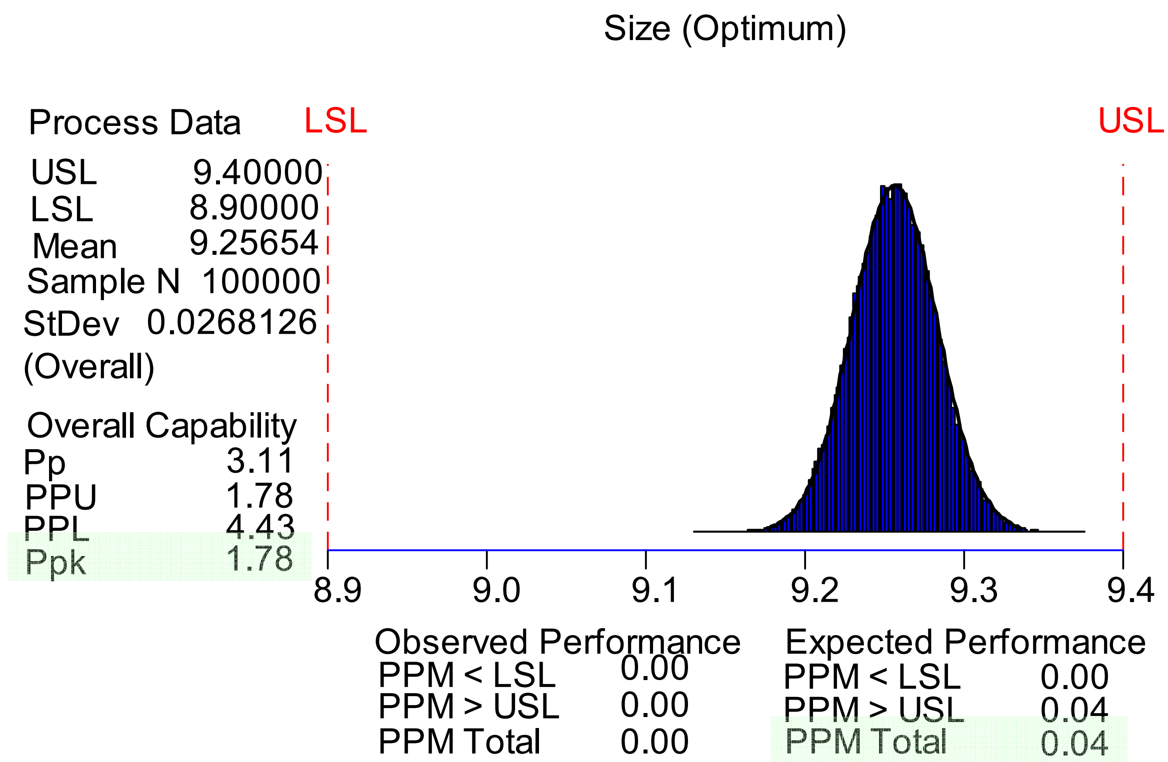
$$\overline{X}_S = \frac{.0268 \times 8.719 + \sqrt{.0268^2 + .0968^2} \times 9.4}{.0268 + \sqrt{.0268^2 + .0968^2}} = 9.257$$

$$Z_{\substack{Upper\\Position}} = \frac{0.717 - 0.179}{\sqrt{.0268^2 + .0968^2}} = 5.34 = \frac{9.4 - 9.257}{.0268} = Z_{\substack{Upper\\Size}}$$





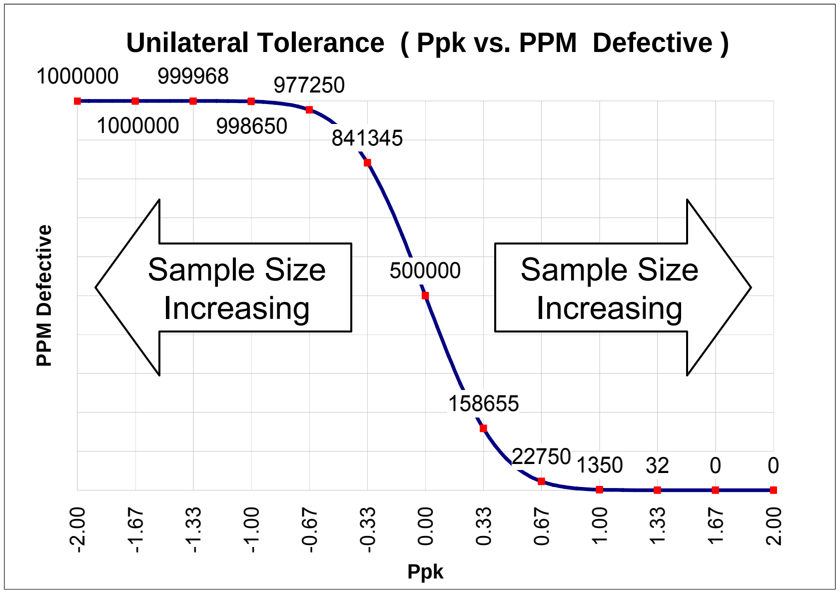
Targeting size to minimize PPM defective of size & variable position simultaneously

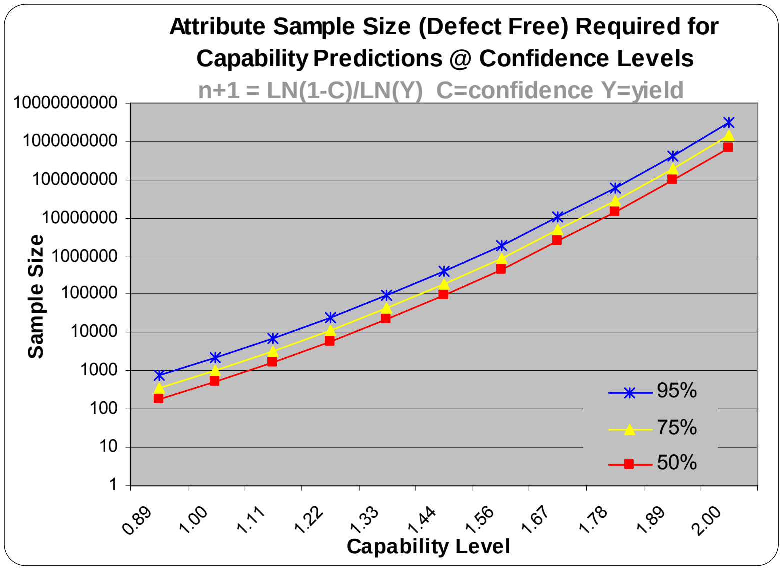


How do the continuous data predictions compare with discreet data predictions?

Attribute predictions are typically unreliable when there are not significant differences between the portion conforming and non-conforming, For typically required levels of process capability very large samples are required.

How big must that sample be?



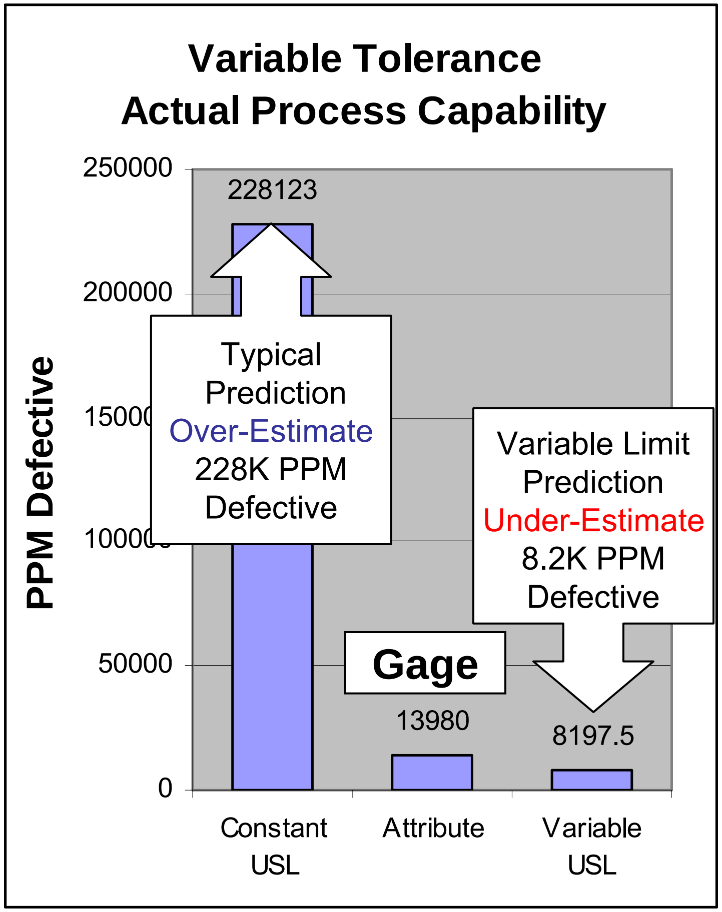


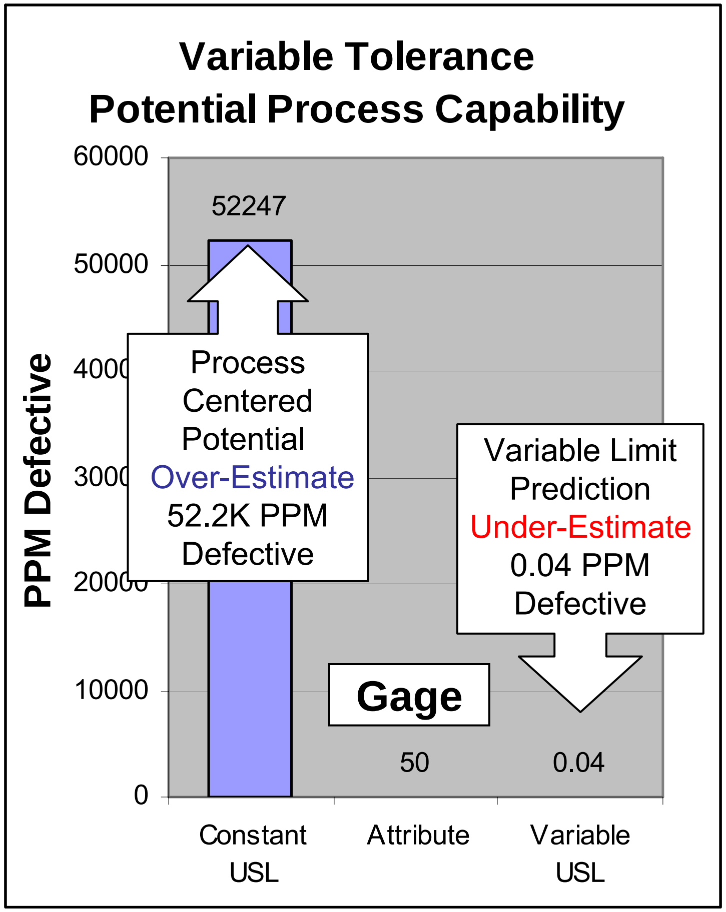
Monte Carlo Simulation of an Attribute Gage

100,000 random - normally distributed values for x-dev, y-dev, and size were generated. The X & Y values were converted to position deviations and each instance was evaluated as a variable tolerance.

Attribute Gage	Attribute Gage
Variable Position with (X&Y Actual) & Size (Actual)	**Variable Position with X&Y (Centered) & Size (Optimum)**
1,398 Failed / **100,000 Sampled**	**5 Fail** / **100,000 Sampled**
PPM Defective 13,980	**PPM Defective 50**

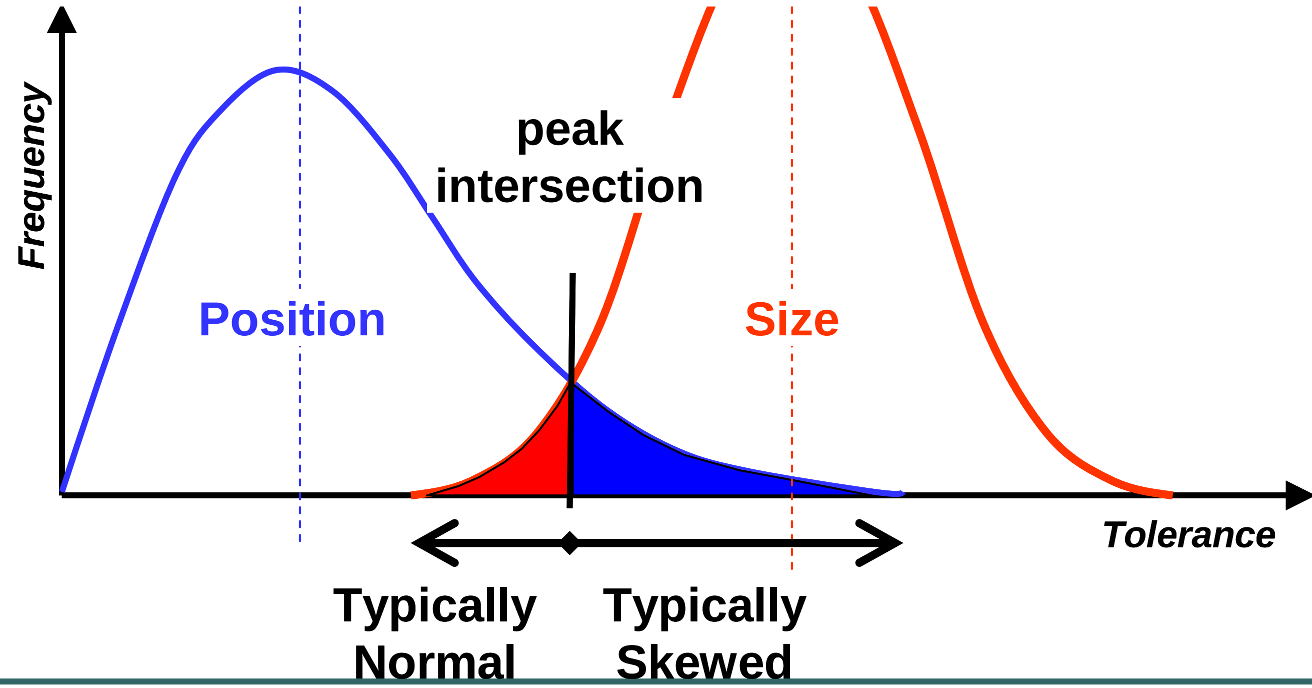
What is the difference in the predictions?





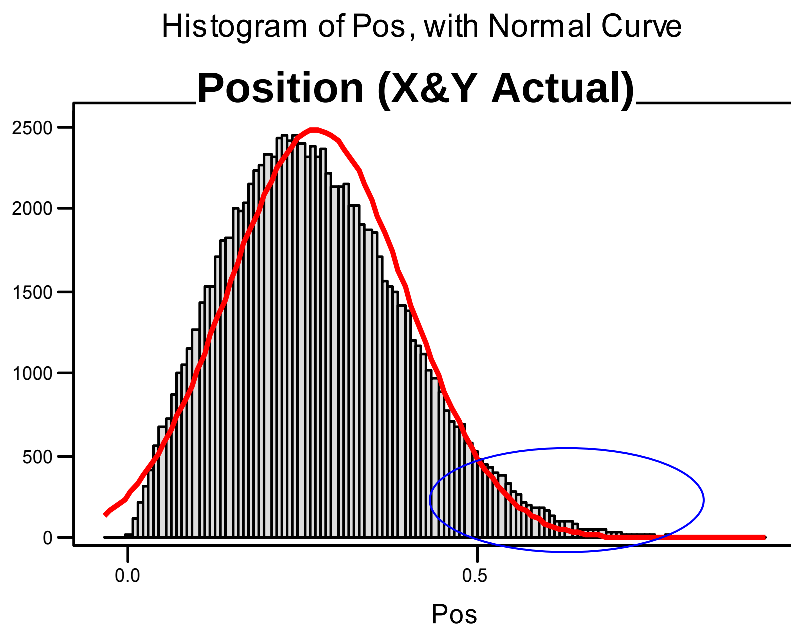
What is the probability of a defect of a variable geometric tolerance?

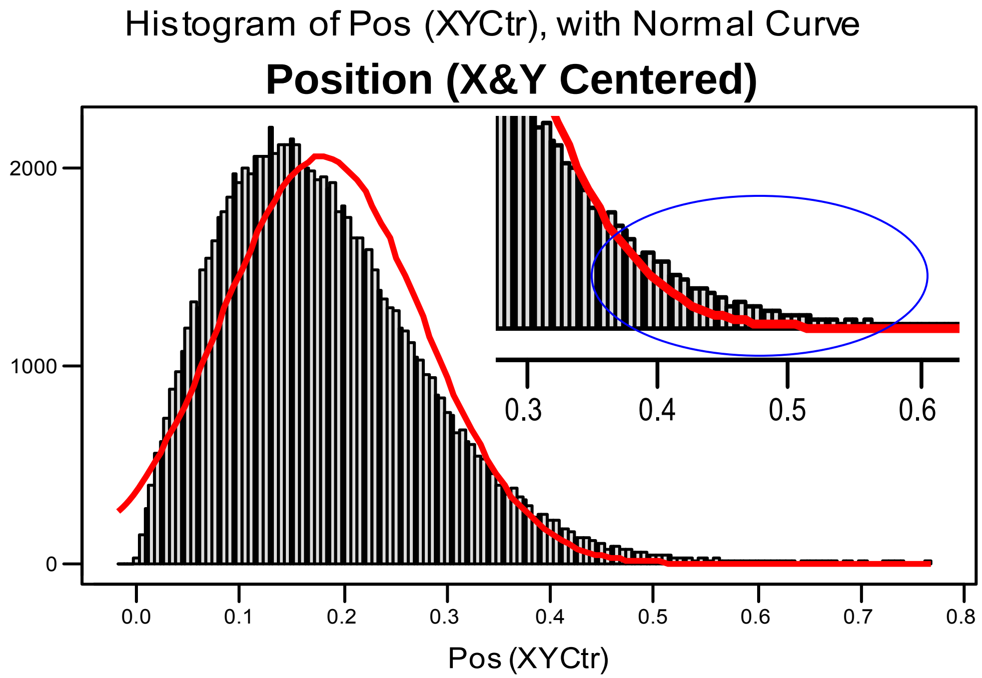
The probability of a defect with a variable tolerance can be visualized as the intersecting area relative to the combined area of adjacent distributions. The shape of that intersecting area is a composite reflection of the tails of both distributions back-to-back at the peak of the intersection.



What are the risks?
(Predicting variable tolerance capability with both intersecting distributions assumed "normal")

Typical 4 and 5 sigma 1.33-1.67 Ppk customer targeted capability requirements ensure that the area of a position distribution curve intersecting with the distribution for size that could be considered for conformance to specification will be limited to the distribution's tail.





Typically skewed position distributions fitted with normal distribution curves show that occurrence frequencies in the tail areas will be slightly underestimated.