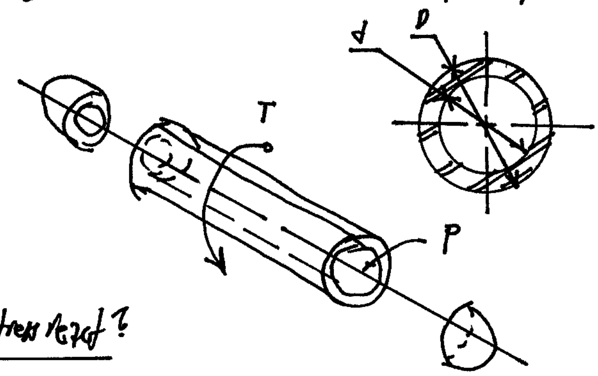
11 June 2007

$$2\sigma^2 = \sigma\times\sigma + 3\tau_{xy}^2 \quad \forall \quad \sigma = \sigma_x\hat{i} + \sigma_y\hat{j} + \sigma_z\hat{k}$$

Von Mises–Hencky Theory



pressure vessel theory: Hoop: $\sigma_x = p\dfrac{D^2+d^2}{D^2-d^2}$,

Radial: $\sigma_y = -p$,

Longitudinal: $\sigma_z = p\dfrac{d^2}{D^2-d^2}$,

· introduce a torque T along outer surface D; how does the principle stress react?

• clearly we need to examine the mathematics given $\tau_{xy} \neq 0$.

figure 1: pressure vessel geometry

torsional introduction: $\tau = \dfrac{Tc}{J} \;\forall\; c = \dfrac{D}{2}$, $J = \dfrac{\pi}{32}(D^4 - d^4)$ given $I_x = I_y \wedge J = I_x + I_y$,

$$\tau = \frac{T\left(\frac{D}{2}\right)}{\frac{\pi}{32}(D^4-d^4)} = \frac{16DT}{\pi(D^4-d^4)} \quad\vee\quad \tau_{xy} = \frac{16TD}{\pi(D^4-d^4)}.$$

torque–elastic range

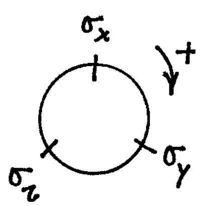


figure 2: cyclic permutation

$$2\sigma^2 = \sigma\times\sigma + 3\tau_{xy}^2$$

non-principle stress state

$$2\sigma^2 = [\sigma_x - \sigma_y]^2 + [\sigma_y - \sigma_z]^2 + [\sigma_z - \sigma_x]^2 + 3\tau_{xy}^2$$

$$2\sigma^2 = \left[p\frac{D^2+d^2}{D^2-d^2} - -p\right]^2 + \left[-p - p\frac{d^2}{D^2-d^2}\right]^2 + \left[p\frac{d^2}{D^2-d^2} - p\frac{D^2+d^2}{D^2-d^2}\right]^2 + 3\left[\frac{16TD}{\pi(D^4-d^4)}\right]^2$$

$$2\sigma^2 = \left[p\frac{D^2+d^2}{D^2-d^2} + p\frac{D^2-d^2}{D^2-d^2}\right]^2 + \left[-p\frac{D^2-d^2}{D^2-d^2} - p\frac{d^2}{D^2-d^2}\right]^2 + \left[p\frac{d^2}{D^2-d^2} - p\frac{D^2+d^2}{D^2-d^2}\right]^2 + 3\Bigg[\underbrace{\frac{16TD}{\pi(D^2-d^2)(D^2+d^2)}}_{\text{WHOA!}}\Bigg]^2,$$

$$2(D^2-d^2)^2\cdot\sigma^2 = \left(p(D^2+d^2) + p(D^2-d^2)\right)^2 + \left(-p(D^2-d^2) - pd^2\right)^2 + \left(pd^2 - p(D^2+d^2)\right)^2 + 3\left[\frac{16TD}{\pi(D^2+d^2)}\right]^2,$$

$$2(D^2-d^2)^2\sigma^2 = \left[pD^2 + \cancel{pd^2} + pD^2 - \cancel{pd^2}\right]^2 + \left[-pD^2 + \cancel{pd^2} - \cancel{pd^2}\right]^2 + \left[\cancel{pd^2} - pD^2 - \cancel{pd^2}\right]^2 + 3\left[\frac{16TD}{\pi(D^2+d^2)}\right]^2,$$

$$2(D^2-d^2)^2\sigma^2 = (2pD^2)^2 + (-pD^2)^2 + (-pD^2)^2 + 3\left[\frac{16TD}{\pi(D^2+d^2)}\right]^2,$$

$$2(D^2-d^2)^2\sigma^2 = 4p^2D^4 + p^2D^4 + p^2D^4 + 3\left[\frac{16TD}{\pi(D^2+d^2)}\right]^2,$$

$$2(D^2-d^2)^2\sigma^2 = 6p^2D^4 + 3\left[\frac{16TD}{\pi(D^2+d^2)}\right]^2,$$

$$2(D^2-d^2)^2\sigma^2 = \frac{6p^2D^4\cdot\pi^2(D^2+d^2)^2 + 3\cdot 256T^2D^2}{\pi^2(D^2+d^2)^2},$$

$$\sigma^2 = \frac{6p^2D^4\left(\pi^2(D^2+d^2)^2\right) + 768T^2D^2}{2\pi^2(D^2+d^2)^2(D^2-d^2)^2},$$

$$\sigma^2 = \frac{3P^2D^4\left[\pi^2(D^2+d^2)^2\right] + 384T^2D^2}{\pi^2\left((D^2+d^2)(D^2-d^2)\right)^2},$$

$$\therefore \quad \underline{\underline{\sigma^2 = \frac{3P^2D^4\left(\pi^2(D^2+d^2)^2\right) + 384T^2D^2}{\pi^2(D^4-d^4)^2}}}.$$ <u>stress w/ shear and torque</u>

<u>case 1:</u> <u>$T \equiv 0 \Rightarrow$ general case of Von Mises–Hencky equation.</u>

$$\sigma^2 = \frac{3P^2D^4\left[\pi^2(D^2+d^2)^2\right] + \cancel{384T^2D^2}^{=0}}{\pi^2(D^4-d^4)^2} = \frac{3P^2D^4\left(\cancel{\pi^2(D^2+d^2)^2}\right)}{\cancel{\pi^2(D^2+d^2)^2}\cdot(D^2-d^2)^2},$$

$$\sigma^2 = \frac{3P^2D^4}{(D^2-d^2)^2} = \frac{3P^2}{\left(\frac{D^2-d^2}{D^2}\right)^2} = \frac{3P^2}{\left(1-(\frac{d}{D})^2\right)^2}, \text{ and set } K = \frac{D}{d} \text{ so that}$$

$$\sigma^2 = \frac{3P^2}{\left(1-(\frac{1}{K})^2\right)^2} \equiv \frac{3P^2K^4}{(K^2-1)^2} \Rightarrow \underline{\underline{\sigma = \sqrt{3}\,P\left(\frac{K^2}{K^2-1}\right)}}.$$ <u>Von Mises–Hencky Eqt.^n</u>

<u>case 2:</u> <u>dimensional consistency.</u>

$$A = 3P^2D^4\left[\pi^2(D^2+d^2)^2\right] = psi^2\cdot in^4\cdot in^4 = \frac{lbf^2}{\cancel{in^4}}\cdot\cancel{in^4}\cdot in^4 = lbf^2\cdot in^4$$

$$B = \left[384T^2\cdot D^2\right] = (in\cdot lbf)^2\cdot in^2 = in^4\cdot lbf^2$$

$$C = \pi^2(D^4-d^4)^2 = (in^4)^2 = in^8$$

$$\therefore \quad \sigma^2 = \frac{A+B}{C} = \frac{in^4\cdot lbf^2 + in^4\cdot lbf^2}{in^8} = \frac{in^4\cdot lbf^2}{in^8} = \frac{lbf^2}{in^4} = \left(\frac{lbf}{in^2}\right)^2 \equiv psi^2,$$

$$\therefore \quad \underline{\underline{[\sigma] \equiv psi}}$$ <u>acceptable units of measure–imperial</u>

12 June 2009

problem: pressure vessel theory with external longitudinal load.

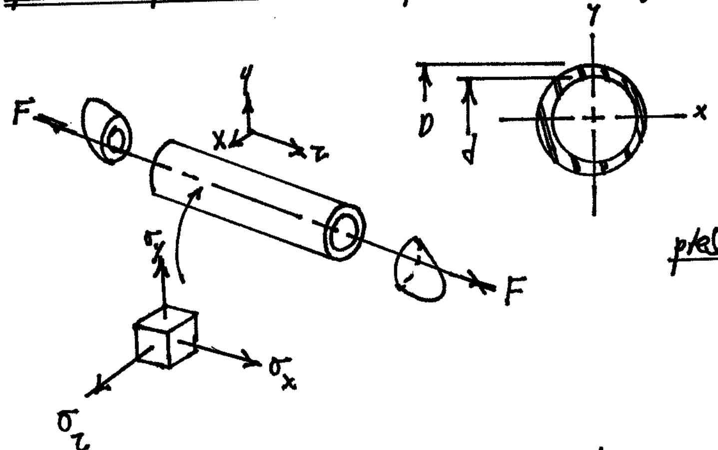


figure 1: pressure vessel geometry and cross-section.

$$2\cdot\nabla^2 = \nabla\times\nabla + 3\tau_{xy}^2$$ Von Mises-Hencky Model

set $\nabla = \sigma_x\hat{i} + \sigma_y\hat{j} + \sigma_z\hat{k}$ as the stress vector gradient,

pressure vessel theory: HOOP $\sigma_x = p\,\dfrac{D^2+d^2}{D^2-d^2}$

RADIAL $\sigma_y = -p$

LONGITUDINAL $\sigma_z = p\,\dfrac{d^2}{D^2-d^2}$

- clearly for external load F, the normal wall load is $\sigma = \dfrac{F}{A}$,

$$\sigma_z' = \frac{F}{\frac{\pi}{4}(D^2-d^2)} \equiv \frac{4F}{\pi(D^2-d^2)},$$

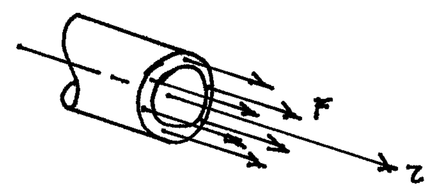
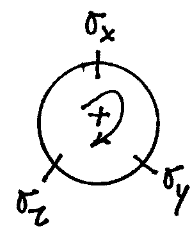


figure 2: normal load parallel to z-axis (longitudinal)

- then the "modified" longitudinal component is:

$$\sigma_z'' = \sigma_z + \sigma_z'$$ combined loading (longitudinal)



- noting the cyclic permutation, positive to the right, the result of this would become:

$$2\cdot\sigma^2 = (\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z'')^2 + (\sigma_z''-\sigma_x)^2 + 3\tau_{xy},$$

- set $\tau_{xy} \equiv 0$ for principle stress as per Mohr's Circle. noting vectorial sense, $\pm F$,

$$2\cdot\sigma^2 = \left[p\,\frac{D^2+d^2}{D^2-d^2} - -p\right]^2 + \left[-p - \left(p\frac{d^2}{D^2-d^2} \pm \frac{4F}{\pi(D^2-d^2)}\right)\right]^2 + \left[\left(p\frac{d^2}{D^2-d^2} \pm \frac{4F}{\pi(D^2-d^2)}\right) - p\,\frac{D^2+d^2}{D^2-d^2}\right]^2,$$

$$2\cdot\sigma^2 = \left[p\,\frac{(D^2+d^2)+(D^2-d^2)}{(D^2-d^2)}\right]^2 + \left[p\,\frac{-(D^2-d^2)-\left(d^2\pm\frac{4F}{\pi p}\right)}{(D^2-d^2)}\right]^2 + \left[p\,\frac{\left(d^2\pm\frac{4F}{\pi p}\right)-(D^2+d^2)}{(D^2-d^2)}\right]^2,$$

$$\frac{2(D^2-d^2)^2}{p^2}\sigma^2 = \left[D^2+\cancel{d^2}+D^2-\cancel{d^2}\right]^2 + \left[-D^2+\cancel{d^2}-\cancel{d^2}\mp\frac{4F}{\pi p}\right]^2 + \left[\cancel{d^2}\pm\frac{4F}{\pi p}-D^2-\cancel{d^2}\right]^2,$$

$$\frac{2(D^2-d^2)^2}{p^2}\sigma^2 = 4D^4 + \left(-D^2\mp\frac{4F}{\pi p}\right)^2 + \left(\pm\frac{4F}{\pi p}-D^2\right)^2,$$ "!" "!"

$$\frac{2(D^2-d^2)^2}{p^2}\sigma^2 = 4D^4 + \left(D^4 \mp \frac{8F}{\pi p}D^2 + \frac{16F^2}{\pi^2p^2}\right) + \left(\frac{16F^2}{\pi^2p^2} \pm 2\,\frac{4F}{\pi p}(-D^2) + D^4\right),$$

$$\frac{2(D^2-d^2)^2}{p^2}\sigma^2 = 4D^4 + D^4 \mp \frac{8F}{\pi p}D^2 + \frac{16F^2}{\pi^2 p^2} + \frac{16F^2}{\pi^2 p^2} \mp \frac{8F}{\pi p}D^2 + D^4,$$

$$\frac{2(D^2-d^2)^2}{p^2}\sigma^2 = 6D^4 \mp \frac{16F}{\pi p}D^2 + \frac{32F^2}{\pi^2 p^2},$$

$$\sigma^2 = \left(3p^2D^4 \mp \frac{8F}{\pi}pD^2 + \frac{16F^2}{\pi^2}\right)\frac{1}{(D^2-d^2)^2},$$

$$\sigma^2 = \frac{3\pi^2p^2D^4 \mp 8\pi FpD^2 + 16F^2}{\pi^2(D^2-d^2)^2}.$$

stress w/ longitudinal load (external).

Case 1: external load is zero $\Rightarrow$ general case for Von Mises-Hencky Model.

$$\sigma^2 = \frac{3\pi^2p^2D^4 \mp \cancel{8\pi FpD^2}^{=0} + \cancel{16F^2}^{=0}}{\pi^2(D^2-d^2)^2} = \frac{3\cancel{\pi^2}p^2D^4}{\cancel{\pi^2}(D^2-d^2)^2} = \sqrt{3}^2 p^2\left[\frac{D^2}{D^2-d^2}\right]^2,$$

set $k = \frac{D}{d}$ $\therefore$ $$\sigma^2 = 3p^2\frac{D^4}{(D^2-d^2)^2} = 3p^2\left[\frac{\left(\frac{D^2}{d^2}\right)}{\frac{D^2}{d^2}-1}\right]^2 = 3p^2\left(\frac{k^2}{k^2-1}\right)^2,$$

$$\therefore\ \sigma = \sqrt{3}\,p\,\frac{k^2}{k^2-1}$$

Von Mises-Hencky Equation — $F \equiv 0$.

Case 2: dimensional consistency.

$$[3\pi^2p^2D^4] = psi^2\cdot in^4 = \left(\frac{lbf}{in^2}\right)^2 in^4 = \frac{lbf^2}{in^4}\cdot in^4 \equiv lbf^2,$$

$$[8\pi FpD^2] = lbf\cdot psi\cdot in^2 = lbf\cdot\frac{lbf}{\cancel{in^2}}\cdot\cancel{in^2} = lbf^2,$$

$$[16F^2] = lbf^2.$$

$$\sigma^2 = \frac{3\pi^2p^2D^4 \mp 8\pi FpD^2 + 16F^2}{\pi^2(D^2-d^2)^2} = \frac{lbf^2+lbf^2+lbf^2}{(in^2)^2} = \frac{lbf^2}{in^4} \equiv psi^2$$

units balance out $[\sigma] \equiv psi$.

therefore, the equation correctly gives the appropriate dimensional consistency (Buckingham-Pi Theory).

• for clarity, denote axial sense of the force vector as $F>0$ tensile, $F<0$ compression,

$$\sigma^2 = \frac{3\pi^2P^2D^4 - 8\pi FPD^2 + 16F^2}{\pi^2(D^2-d^2)^2} \quad \vee \quad \sigma = \frac{\sqrt{3\pi^2P^2D^4 - 8\pi FPD^2 + 16F^2}}{\pi(D^2-d^2)},$$

which cannot be disassembled by multiplication of two monomials under the radical.

i.e. $(3\pi PD^2 - 4F)(\pi PD^2 - 4F) = 3\pi^2P^2D^4 - 12\pi PFD^2 - 4\pi FPD^2 + 16F^2$
$= 3\pi^2P^2D^4 - 16\pi PFD^2 + 16F^2$

$(3\pi PD^2 - 8F)(\pi PD^2 - 2F) = 3\pi^2P^2D^4 - 6\pi PFD^2 - 8\pi FPD^2 + 16F^2$
$= 3\pi^2P^2D^4 - 14\pi PFD^2 + 16F^2$

* factors for number 16 $\longrightarrow$

1	2	4	8	16
16	8	4	2	1

, the "-8" middle term is not possible!

$$\underline{\underline{\sigma = \frac{\sqrt{3\pi^2P^2D^4 - 8\pi FPD^2 + 16F^2}}{\pi(D^2-d^2)}}}$$

stress w/ longitudinal force (axial)

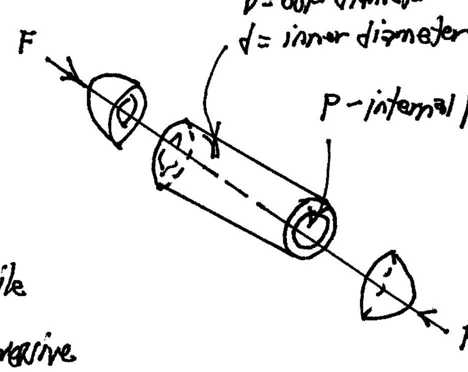


figure I: compressive longitudinal force.

corollary: significance of middle term (important).

hypothesis: there exists a compressive force capable of nullifying longitudinal stress.

$$\sigma_z'' = \sigma_z + \sigma_z' = P\frac{d^2}{D^2-d^2} + \frac{4F}{\pi(D^2-d^2)} \quad \forall \quad F = \begin{cases} >0 \Rightarrow \text{tensile} \\ <0 \Rightarrow \text{compressive} \end{cases}$$

$$\sigma_z'' = 0 \ \ni. \quad P\frac{d^2}{\cancel{D^2-d^2}} = \frac{-4F}{\pi\cancel{(D^2-d^2)}}$$

$\underline{F = \frac{-\pi}{4}Pd^2 < 0 \Rightarrow \text{compressive force}}$ (null longitudinal stress)

$$\therefore \quad \sigma = \frac{\sqrt{3\pi^2P^2D^4 - 8\pi\left(-\frac{\pi}{4}Pd^2\right)PD^2 + 16\left(-\frac{\pi}{4}Pd^2\right)^2}}{\pi(D^2-d^2)} = \frac{\sqrt{3\pi^2P^2D^4 + 2\pi^2P^2D^2d^2 + \pi^2P^2d^4}}{\pi(D^2-d^2)},$$

$$\therefore \quad \sigma_{min} = \frac{\sqrt{3\pi^2 p^2 D^4 + 2\pi^2 p^2 D^2 d^2 + \pi^2 p^2 d^4}}{\pi(D^2 - d^2)}$$ minimum wall element stress

(inner diameter)^2

minimum wall element stress $\forall$ $F = \frac{-\pi}{4} p d^2$ $\quad \sigma_{min}$

i.e.

$$\frac{d\sigma}{dF} = \frac{1}{\pi(D^2 - d^2)} \cdot \frac{1}{2}\sqrt{3\pi^2 p^2 D^4 - (8\pi p D^2)F + 16F^2}^{-1} \cdot (-8\pi p D^2 + 16 \cdot 2F) = 0,$$

(outer diameter)^2

$$-8\pi p D^2 + 32F = 0 \quad \vee \quad F = \frac{-8\pi}{32} p D^2 = \frac{-\pi}{4} p D^2.$$

$$\sigma = \frac{\sqrt{3\pi^2 p^2 D^4 - 8\pi p D^2\left(-\frac{\pi}{4} p D^2\right) + 16\left(-\frac{\pi}{4} p D^2\right)^2}}{\pi(D^2 - d^2)} = \frac{\sqrt{3\pi^2 p^2 D^4 + 2\pi^2 p^2 D^4 + \pi^2 p^2 D^4}}{\pi(D^2 - d^2)},$$

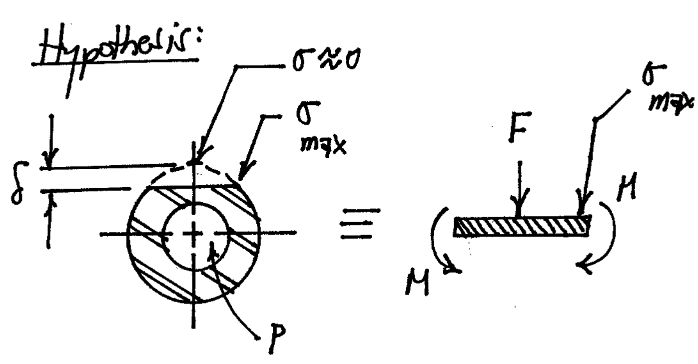
$$\sigma = \frac{\sqrt{6\pi^2 p^2 D^4}}{\pi(D^2 - d^2)} = \frac{\sqrt{6}\,\cancel{\pi} p D^2}{\cancel{\pi}(D^2 - d^2)} = \sqrt{6}\, p \frac{\left(\frac{D}{d}\right)^2}{\left(\frac{D}{d}\right)^2 - 1} = \sqrt{6}\, p \frac{K^2}{K^2 - 1} \quad \forall \ \sqrt{6} = \sqrt{2} \cdot \sqrt{3}$$

Von Mises-Hencky

$$\sigma_{max} = \sqrt{2}\left(\sigma_{princ}\right).$$ maximum wall element stress - buckling impending.

maximum wall element stress @ $F = \frac{-\pi}{4} p D^2$ $\quad \sigma_{max}$

Hypothesis:



1. the failure is the result of pressurization and not metallurgical variances,
2. flat deviation by internal pressure resulted in a bend moment at outer edges of the cut,
3. bending stress thereby resulting, were sufficient to induce a crack through threaded ports,
4. hoop stress by internal pressurization and the compressive longitudinal force CONTRIBUTED to the failure but where not the root cause.
5. crack propagation between threaded ports and rupture.

Application to Problem:

total volume: $V = 341.42\ in^3$

$341.42\ in^3 = \frac{\pi}{4}(D^2 - 3.55^2)(34.916\ in)\ in^2$,

$\therefore\ d \approx 3.55\ in\ \ni.\ V = \frac{\pi}{4}(D^2 - d^2)L\ \vee$

$\underline{\underline{D = 5.009\ \text{Ø}}}$ equivalent OD to volume

$p = 30.0\ ksi\ \wedge\ k = \frac{D}{d} = \frac{5.009\ \cancel{in}}{3.550\ \cancel{in}} = 1.41096$

$\sigma = \sqrt{3}\,p\left(\frac{k^2}{k^2-1}\right) = \sqrt{3}(30{,}000\ psi)\frac{1.41096^2}{1.41096^2 - 1} = \underline{\underline{104{,}406\ psi}}$ Von Mises-Hencky Model

application of "maximum longitudinal load" by discussion pg6 gives:

$F_{max} = -\frac{\pi}{4}Pd^2 = -\frac{\pi}{4}(30{,}000\ psi)(3.550\ in)^2 = \underline{\underline{296{,}939\ lbf}}$ allowable compressive load

$\therefore\ \sigma = \sqrt{2}\,\sigma_{Von Mises} = \sqrt{2}(104{,}406\ psi) = \underline{\underline{147{,}652\ psi}}$

pressure vessel theory (thick wall, triaxial)

allowable compressive load = 297 kips F

resulting stress (wall) = 148 ksi σ

NOTE: surface is uninterrupted and void of radical geometry change; use this as a criteria for the estimate of Electronics Insert, GP7600.

anticipated compressive load

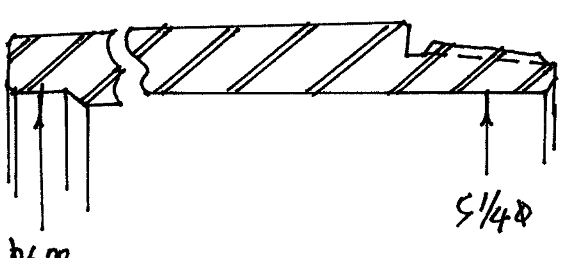


figure 4: GP7600 Lower Housing Geometry

$P = \frac{F}{A}\ \ni.\ F \approx (30{,}000\ psi)\frac{\pi}{4}(6.00^2 - 5.25^2)\ in^2$

$\underline{\underline{F = 198{,}804\ lbf}}$ differential pressure axial load

$\therefore\ 198{,}804\ lbf < 296{,}939\ lbf \Rightarrow$ acceptable

$FOS \approx \frac{296{,}939\ \cancel{lbf}}{198{,}804\ \cancel{lbf}} = 1.49$ w/o geometry variances

approximate factor of safety (best case) = 1.49

Analysis of Flat Plate Deflection [Roark's Formulae for Stress & Strain, Table 26 case 1 & 8]

$$\delta_{max} = \frac{-\alpha q b^4}{E t^3} \ \wedge \ \sigma_{ctr} = \frac{\beta q b^2}{t^2} \ \forall \ (\alpha, \beta) = \text{system constants.}$$

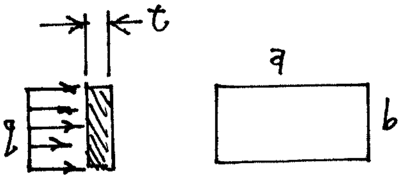


figure 5: flat plate - simple support

Roark's — case 1a - simple support (4 typ); case 8a - fixed edge (4 typ)

$b = 2.380$ in, $a = 25.435 - 0.512 - 0.40$ in $= 24.523$ in.

- adjust for shoulder thickness RHS, 0.362" ∴ $a \equiv 24.885$ in.
- assume uniform loading, $q \approx 30{,}000$ psi, over uniform thickness $t \approx 0.397$ in,
- from FEA work, $\delta_{max} \approx -0.007$ in.
- look at each case for simple support and fixed edge, 4 places typ.

$$\frac{a}{b} = \frac{24.885\text{ in}}{2.380\text{ in}} = 10.4588, \text{ so } \alpha\left(\tfrac{a}{b}=\infty\right) = 0.1421,\ \beta\left(\tfrac{a}{b}=\infty\right) = 0.7500.$$

$$-0.007\text{ in} = -0.1421\,\frac{q(2.380\text{ in})^4}{(28.9\times10^6\text{ psi})(0.397\text{ in})^3} \rightleftharpoons q \approx 2776\text{ psi}$$ uniform load equivalence

centre of long axis NOT at the edge!

$$\sigma_{ctr} = 0.7500\,\frac{(2776\text{ psi})(2.380\text{ in})^2}{(0.397\text{ in})^2} = 74{,}826\text{ psi}$$ estimated plate support conditions (simple loaded)

- but the plate is rigid fixed, $\alpha\left(\tfrac{a}{b}=\infty\right) = 0.0284$, $\beta_2\left(\tfrac{a}{b}=\infty\right) = 0.25 \ \wedge \ \beta_1\left(\tfrac{a}{b}=\infty\right) = 0.5000$.

centre line of flat: $$\delta_{max} = \alpha\frac{q b^4}{E t^3} \ \therefore \ 0.007\text{ in} = 0.0284\,\frac{q(2.380\text{ in})^4}{(28.9\times10^6\text{ psi})(0.397\text{ in})^3},\ q = 13{,}890\text{ psi}$$

$$\sigma_{ctr} = \beta_2\frac{q b^2}{t^2} = 0.25\,\frac{(13{,}890\text{ psi})(2.380\text{ in})^2}{(0.397\text{ in})^2} = 124{,}800\text{ psi}$$

$$\sigma_{edge} = \beta_1\frac{q b^2}{t^2} \ \forall \ \frac{\beta_1}{\beta_2} \equiv 2.0 \ \therefore \ \sigma_{edge} = 249{,}600\text{ psi.}$$

comparison to FEA: $\sigma \approx 286208$ psi @ flat bottom pocket,

$$\varepsilon = \frac{\sigma_{FEA} - \sigma_{theory}}{\sigma_{FEA}} \times 100 = \frac{286208 - 249600}{286208} \times 100 = 12.8\%\text{ error}$$

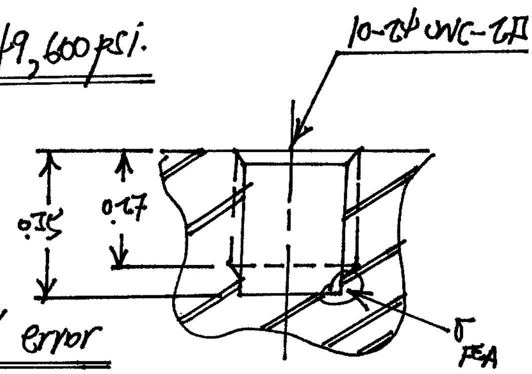
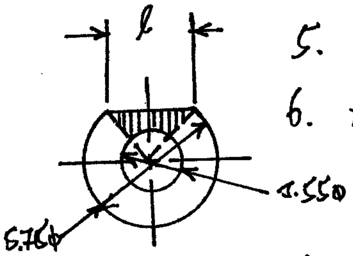


figure 6: anchor board port

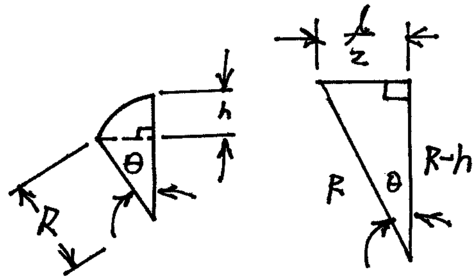
- larger than expected errors are due to departure in theory relative to practice. (theoretical assumptions)

<u>observations:</u>
1. computation assumes the classical case, flat plate of isotropic homogeneous material,
2. deflections remain small; δ_{max} < half the plate thickness,
3. plate thickness is not more than 1/4 the least transverse dimension,
4. we depart from flat surface below the plate; see figure 7 on cross-sectional profile,
5. acceptable to account for "equivalent loading" to account for plate thickness variations,
6. FEA on deformation is accurate to the model.



<u>figure 7: model departure</u>

<u>figure 8: mesh evaluation</u>



$$\cos\theta = \frac{R-h}{R} \Rightarrow R\cos\theta = R-h$$

$$\therefore\ h = R(1-\cos\theta).$$

$$\sin\theta = \frac{\left(\frac{\ell}{2}\right)}{R} \Rightarrow \ell = 2R\sin\theta.$$

$$\ell = 2.380\text{ in} \wedge 2R \equiv D = 5.750\text{ in}$$

$$m\angle\theta = \sin^{-1}\left(\frac{2.380\cancel{\text{in}}}{5.750\cancel{\text{in}}}\right) = 24.45088^\circ \approx 0.42675\text{ rads.}$$

$$\sin\theta = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + (-1)^{2n+1}\frac{x^{2n+1}}{(2n+1)!}$$ <u>Taylor Series @ origin</u>

- assume degree polynomial, $0.42675 = x - \frac{x^3}{3!} \Rightarrow x_1 \approx 1.4087 \vee \cancel{x_{2,3} = 0.7043 \pm 1.1496i}$ extraneous
- assume fifth degree polynomial, $0.42675 = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$$x_1 = 0.4367,\quad \cancel{x_{2,3} = 2.6609 \pm 1.7479},\quad \cancel{x_{4,5} = -2.8777 \pm 1.8433}$$

extraneous extraneous

<u>FEA Model Mesh Density:</u>

global size elements = 0.45482286 inch

tolerance = 0.02274114 3 in

<u>could of picked a slightly smaller element size</u>

<u>FEA variance(s) — minor percentage error 4.9%</u>

$$\varepsilon = \frac{0.45482 - 0.4367}{0.45482} \times 100 = 4.8593\%$$

<u>mesh size/density error contribution to theoretical deviation (12.8%)</u>

recommendations: (design review) D0090764 (101605487) CP7600 Electronics Insert.

1. move away from DRILL & TAP anchor supports for the electronic circuit boards: voltage, clever driver, r-s interface, control module. stud weld posting(s) would offer better chances in mitigating crack sites.
2. need to lessen the pocket depth, thus adding more material above the bore. the geometry on the circuit board is fixed, height restrictions will challenge this effort.
3. introduce HTSR - "heat treat stress relief" following maching of the piece. massive material removal as a consequence to minimize operational times introduced "residual stresses" in the material. this compounds the material situation since machine operations "loads" the walls.
4. material investigation and possible change to high stress alloy.

   e.g. Carpenter 465 ? perhaps QA/QC on pH 17-4 H900 to higher grade material?
5. modification of anchor port profiles to "cone" or "spherical" bottom may reduce stress levels, but primary mode of failure was crack initiation as a result of bending in the flats due to high bore pressure.
   (a) material uncertainty contributed to FOS ≤ 1,
   (b) machine operations for bottom anchor port profile, drill and tap depth variations, etc also contributed to design uncertainties,
   (c) hysterisis effect of pre loaded stresses & the behavior in material would be much less than that anticipated!
6. decommission this piece to lower operational pressure without suggested changes.

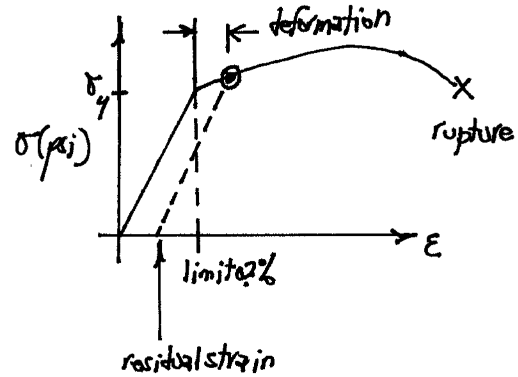


figure 9: effect in plastic deformation

K. Hueston
Nisku Technology - 17 June 2009.