

The shears and moments in foundation elements are conservative when such elements are considered rigid. However, soil pressures may be significantly underestimated when foundation flexibility is ignored. The flexibility and nonlinear response of soil and of foundation structures should be considered when the results would change.

For beams on elastic supports (for instance, strip footings and grade beams) with a point load at midspan, the beam may be considered rigid when:

$$\frac{EI}{L^4} > \frac{2}{3} k_{sv} B \quad (C4-1)$$

The above equation is generally consistent with traditional beam-on-elastic foundation limits (NAVFAC, 1986b; Bowles, 1988). The resulting soil bearing pressures are within 3% of the results, including foundation flexibility.

For rectangular plates (with plan dimensions  $L$  and  $B$ , and thickness  $t$ , and mechanical properties  $E_f$  and  $\nu_f$ ) on elastic supports (for instance, mat foundations or isolated footings) subjected to a point load in the center, the foundation may be considered rigid when:

$$4k_{sv} \sum_{m=1}^5 \sum_{n=1}^5 \frac{\sin^2\left(\frac{m \cdot \pi}{2}\right) \sin^2\left(\frac{n \cdot \pi}{2}\right)}{\left[\pi^4 D_f \left(\frac{m^2}{L^2} + \frac{n^2}{B^2}\right)^2\right] + k_{sv}} < 0.03 \quad (C4-2)$$

where:

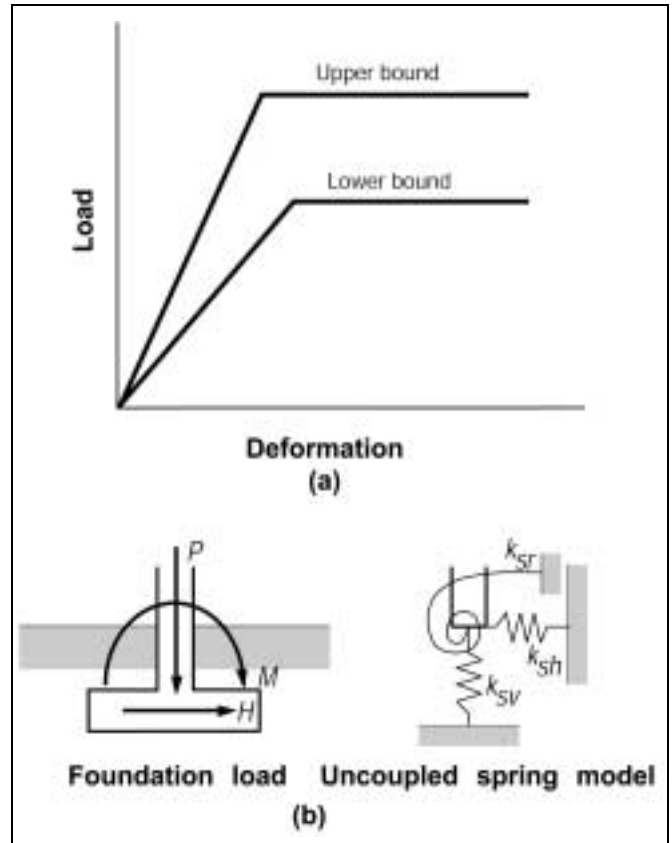
$$D_f = \frac{E_f t^3}{12(1 - \nu_f)^2} \quad (C4-3)$$

The above equation is based on Timoshenko's solutions for plates on elastic foundations (Timoshenko, 1959). The general solution has been simplified by restriction to a center load. Only the first five values of  $m$  and  $n$  (in the infinite series) are required to achieve reasonable accuracy.

#### 4.4.2.1.2 Method 1

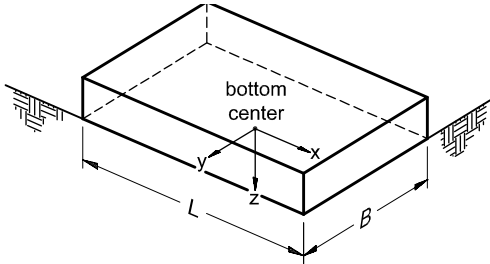
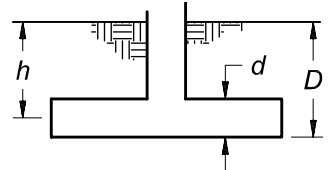
For shallow bearing footings that are rigid with respect to the supporting soil, an uncoupled spring model, as shown in Figure 4-3(b), shall represent the foundation stiffness.

The equivalent spring constants shall be calculated as specified in Figure 4-4.



**Figure 4-3** (a) Idealized Elasto-Plastic Load-Deformation Behavior for Soils  
(b) Uncoupled Spring Model for Rigid Footings

## Chapter 4: Foundations and Geologic Site Hazards

Degree of Freedom	Stiffness of Foundation at Surface	Note
Translation along x-axis	$K_{x, sur} = \frac{GB}{2-v} \left[ 3.4 \left( \frac{L}{B} \right)^{0.65} + 1.2 \right]$	 <p style="text-align: center;">bottom center</p> <p style="text-align: center;">Orient axes such that <math>L \geq B</math></p>
Translation along y-axis	$K_{y, sur} = \frac{GB}{2-v} \left[ 3.4 \left( \frac{L}{B} \right)^{0.65} + 0.4 \frac{L}{B} + 0.8 \right]$	
Translation along z-axis	$K_{z, sur} = \frac{GB}{1-v} \left[ 1.55 \left( \frac{L}{B} \right)^{0.75} + 0.8 \right]$	
Rocking about x-axis	$K_{xx, sur} = \frac{GB^3}{1-v} \left[ 0.4 \left( \frac{L}{B} \right) + 0.1 \right]$	
Rocking about y-axis	$K_{yy, sur} = \frac{GB^3}{1-v} \left[ 0.47 \left( \frac{L}{B} \right)^{2.4} + 0.034 \right]$	
Torsion about z-axis	$K_{zz, sur} = GB^3 \left[ 0.53 \left( \frac{L}{B} \right)^{2.45} + 0.51 \right]$	
Degree of Freedom	Correction Factor for Embedment	Note
Translation along x-axis	$\beta_x = \left( 1 + 0.21 \sqrt{\frac{D}{B}} \right) \cdot \left[ 1 + 1.6 \left( \frac{hd(B+L)}{BL^2} \right)^{0.4} \right]$	 <p><math>d</math> = height of effective sidewall contact (may be less than total foundation height)  <math>h</math> = depth to centroid of effective sidewall contact</p> <p>For each degree of freedom, calculate  <math>K_{emb} = \beta K_{sur}</math></p>
Translation along y-axis	$\beta_y = \beta_x$	
Translation along z-axis	$\beta_z = \left[ 1 + \frac{1}{21} \frac{D}{B} \left( 2 + 2.6 \frac{B}{L} \right) \right] \cdot \left[ 1 + 0.32 \left( \frac{d(B+L)}{BL} \right)^{2/3} \right]$	
Rocking about x-axis	$\beta_{xx} = 1 + 2.5 \frac{d}{B} \left[ 1 + \frac{2d}{B} \left( \frac{d}{D} \right)^{-0.2} \sqrt{\frac{B}{L}} \right]$	
Rocking about y-axis	$\beta_{yy} = 1 + 1.4 \left( \frac{d}{L} \right)^{0.6} \left[ 1.5 + 3.7 \left( \frac{d}{L} \right)^{1.9} \left( \frac{d}{D} \right)^{-0.6} \right]$	
Torsion about z-axis	$\beta_{zz} = 1 + 2.6 \left( 1 + \frac{B}{L} \right) \left( \frac{d}{B} \right)^{0.9}$	

**Figure 4-4** Elastic Solutions for Rigid Footing Spring Constraints

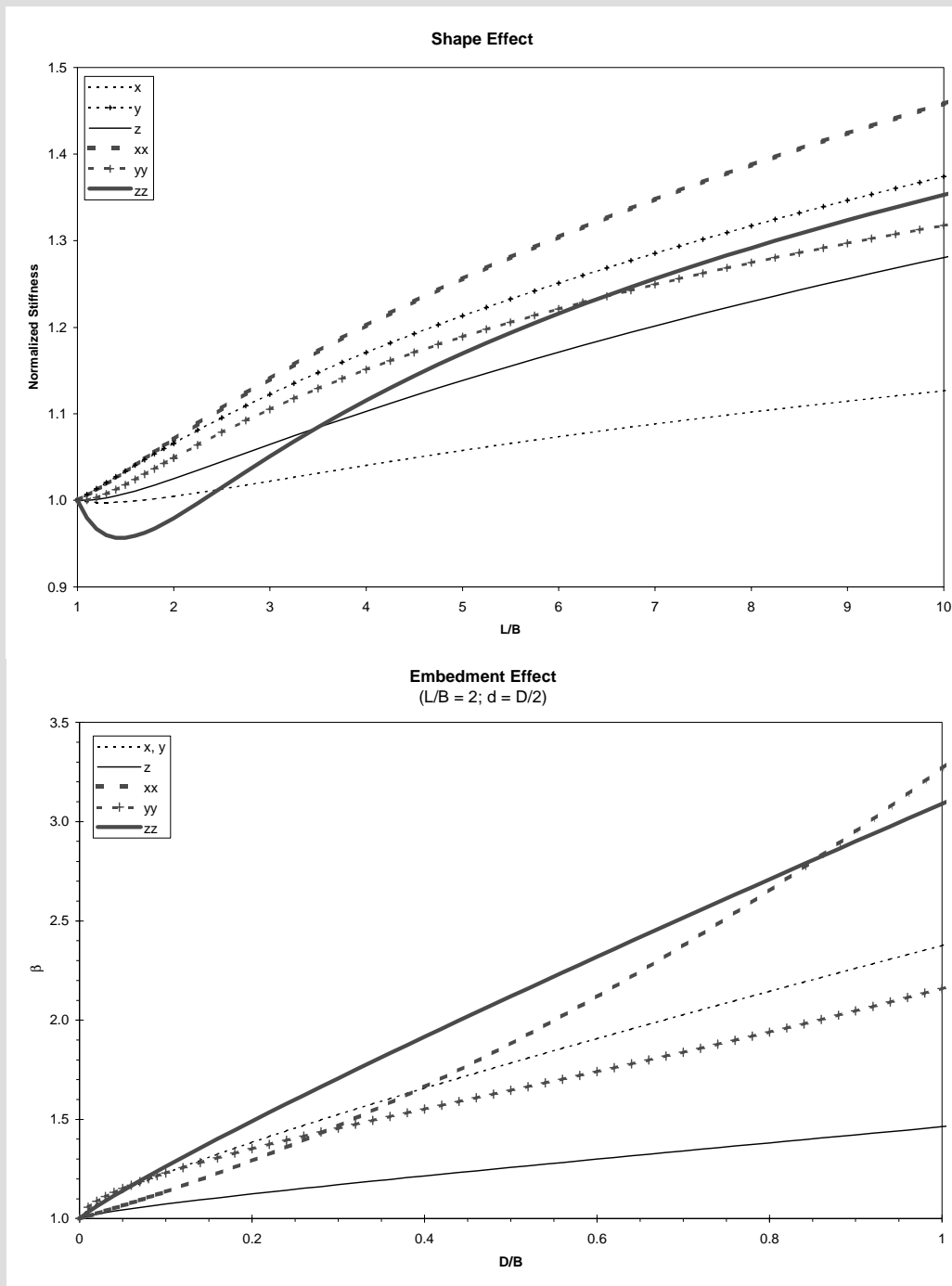
**C4.4.2.1.2 Method 1**

Researchers have developed spring stiffness solutions that are applicable to any solid basemat shape on the surface of, or partially or fully embedded in, a homogeneous halfspace (Gazetas). Rectangular foundations are most common in buildings. Therefore, the general spring stiffness solutions were adapted to the general rectangular foundation problem, which includes rectangular strip footings.

Using Figure 4-4, a two step calculation process is required. First, the stiffness terms are calculated for a foundation at the surface. Then, an embedment correction factor is calculated for each stiffness term. The stiffness of the embedded foundation is the product of these two terms. Figure C4-2 illustrates the effects of foundation aspect ratio and embedment.

According to Gazetas, the height of effective sidewall contact,  $d$ , should be taken as the average height of the sidewall that is in good contact with the surrounding soil. It should, in general, be smaller than the nominal height of contact to account for such phenomena as slippage and separation that may occur near the ground surface. Note that  $d$  will not necessarily attain a single value for all modes of oscillation. When  $d$  is taken larger than zero, the resulting stiffness includes sidewall friction and passive pressure contributions

Although frequency-dependent solutions are available, results are reasonably insensitive to loading frequencies within the range of parameters of interest for buildings subjected to earthquakes. It is sufficient to use static stiffnesses as representative of repeated loading conditions. Other formulations incorporating a wider range of variables may be found in Gazetas and Lam, et al.



**Figure C4-2** (a) Foundation Shape Effect  
(b) Foundation Embedment Effect

#### 4.4.2.1.3 Method 2

For shallow bearing foundations that are not rigid with respect to the supporting soils, a finite element representation of linear or nonlinear foundation behavior using Winkler models shall be used. Distributed vertical stiffness properties shall be calculated by dividing the total vertical stiffness by the area. Uniformly distributed rotational stiffness properties shall be calculated by dividing the total rotational stiffness of the footing by the moment of inertia of the footing in the direction of loading. Vertical

and rotational stiffnesses shall be decoupled for a Winkler model. It shall be permitted to use the procedure illustrated in Figure 4-5 to decouple these stiffnesses.

#### C4.4.2.1.3 Method 2

The stiffness per unit length in these end zones is based on the vertical stiffness of a  $B \times B/6$  isolated footing. The stiffness per unit length in the middle zone is equivalent to that of an infinitely long strip footing.

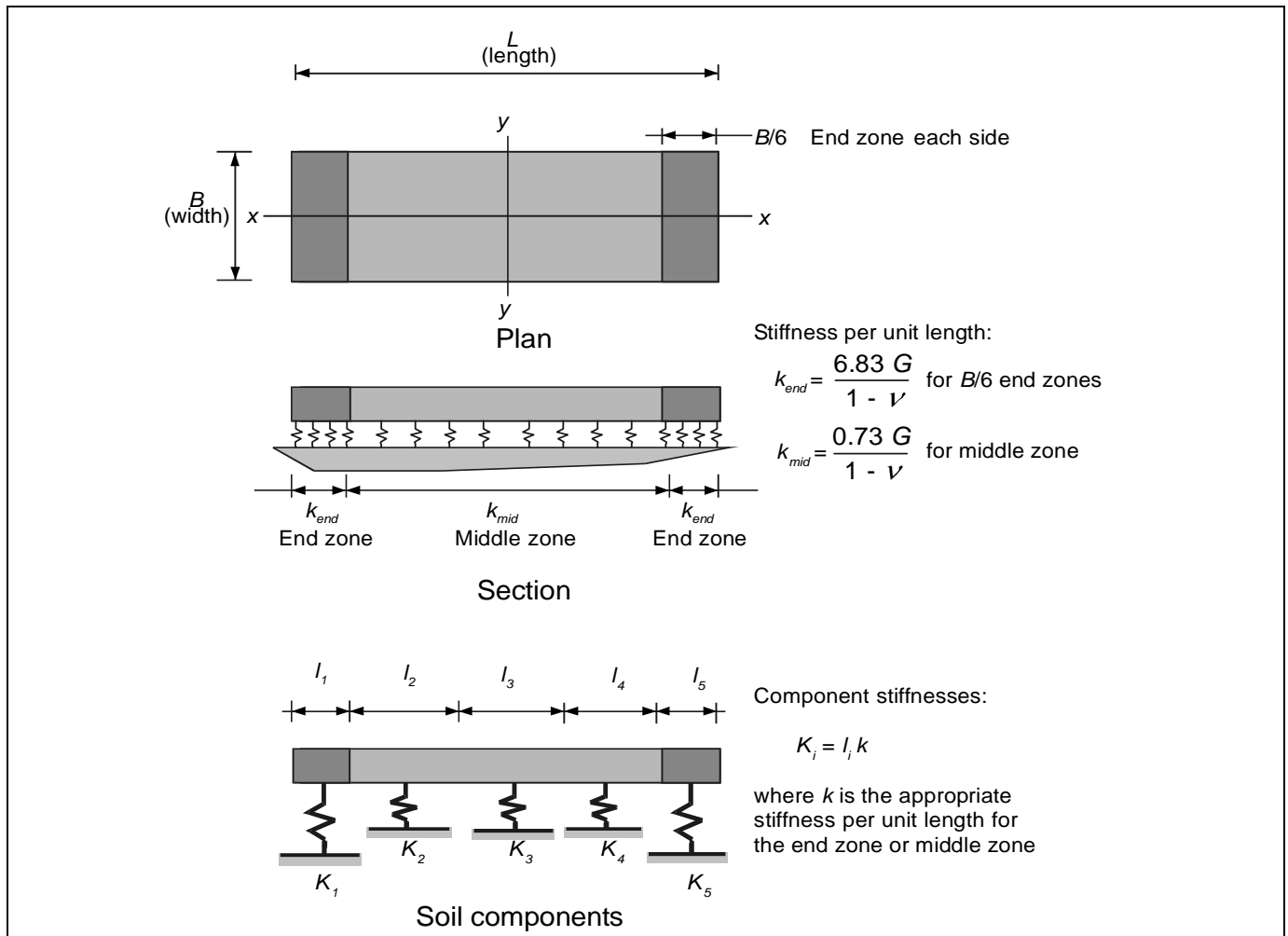


Figure 4-5 Vertical Stiffness Modeling for Shallow Bearing Footings

#### 4.4.2.1.4 Method 3

For shallow bearing foundations that are flexible relative to the supporting soil, based on approved theoretical solutions for beams or plates on elastic supports, the foundation stiffness shall be permitted to be calculated by a decoupled Winkler model using a unit subgrade spring coefficient. For flexible foundation systems, the unit subgrade spring coefficient,  $k_{sv}$ , shall be calculated by Equation (4-6).

$$k_{sv} = \frac{1.3G}{B(1-\nu)} \quad (4-6)$$

where:

- $G$  = Shear modulus
- $B$  = Width of footing
- $\nu$  = Poisson's ratio

#### 4.4.2.1.5 Capacity Parameters

The vertical expected capacity of shallow bearing foundations shall be determined using the procedures of Section 4.4.1.

In the absence of moment loading, the expected vertical load capacity,  $Q_c$ , of a rectangular footing shall be calculated by Equation (4-7).

$$Q_c = q_c BL \quad (4-7)$$

where:

- $q_c$  = Expected bearing capacity determined in Section 4.4.1
- $B$  = Width of footing
- $L$  = Length of footing

The moment capacity of a rectangular footing shall be calculated by Equation (4-8):

$$M_c = \frac{LP}{2} \left( 1 - \frac{q}{q_c} \right) \quad (4-8)$$

where:

- $P$  = Vertical load on footing
- $q = \frac{P}{BL}$  = vertical bearing pressure

$B$  = Width of footing (parallel to the axis of bending)

$L$  = Length of footing in the direction of bending

$q_c$  = Expected bearing capacity determined in Section 4.4.1

The lateral capacity of shallow foundations shall be calculated using established principles of soil mechanics and shall include the contributions of traction at the bottom and passive pressure resistance on the leading face. Mobilization of passive pressure shall be calculated using Figure 4-6.

#### C4.4.2.1.5 Capacity Parameters

For rigid footings subject to moment and vertical load, contact stresses become concentrated at footing edges, particularly as uplift occurs. The ultimate moment capacity,  $M_c$ , is dependent upon the ratio of the vertical load stress,  $q$ , to the expected bearing capacity,  $q_c$ . Assuming that contact stresses are proportional to vertical displacement and remain elastic up to the expected bearing capacity,  $q_c$ , it can be shown that uplift will occur prior to plastic yielding of the soil when  $q/q_c$  is less than 0.5. If  $q/q_c$  is greater than 0.5, then the soil at the toe will yield prior to uplift. This is illustrated in Figure C4-3.

For footings subjected to lateral loads, the base traction strength is given by  $V = C + N \mu$ ; where  $C$  is the effective cohesion force (effective cohesion stress,  $c$ , times footing base area),  $N$  is the normal (compressive) force and  $\mu$  is the coefficient of friction. If included, side traction is calculated in a similar manner. The coefficient of friction is often specified by the geotechnical consultant. In the absence of such a recommendation,  $\mu$  may be based on the minimum of the effective internal friction angle of the soil and the friction coefficient between soil and foundation from published foundation references. The ultimate passive pressure strength is often specified by the geotechnical consultant in the form of passive pressure coefficients or equivalent fluid pressures. The passive pressure problem has been extensively investigated for more than 200 years. As a result, countless solutions and recommendations exist. The method used should, at a minimum, include the contributions of internal friction and cohesion, as appropriate.

As shown in Figure 4-6, the force-displacement response associated with passive pressure resistance is highly nonlinear. However, for shallow foundations, passive pressure resistance generally accounts for much less than half of the total strength. Therefore, it is adequate to characterize the nonlinear response of shallow foundations as elastic-perfectly plastic using the initial, effective stiffness and the total expected strength. The actual behavior is expected to fall within the upper and lower bounds prescribed in this standard.

#### 4.4.2.2 Pile Foundations

A pile foundation shall be defined as a deep foundation system composed of one or more driven or cast-in-place piles and a pile cap cast-in-place over the piles, which together form a pile group supporting one or more load-bearing columns, or a linear sequence of pile groups supporting a shear wall.

The requirements of this section shall apply to piles less than or equal to 24 inches in diameter. The stiffness characteristics of single large-diameter piles or drilled shafts larger than 24 inches in diameter shall comply with the requirements of Section 4.4.2.3.

##### 4.4.2.2.1 Stiffness Parameters

The uncoupled spring model shown in Figure 4-3(b) shall be used to represent the stiffness of a pile foundation where the footing in the figure represents the pile cap. In calculating the vertical and rocking springs, the contribution of the soil immediately beneath the pile cap shall be neglected. The total lateral stiffness of a pile group shall include the contributions of the piles (with an appropriate modification for group effects) and the passive resistance of the pile cap. The lateral stiffness of piles shall be based on classical methods or on analytical solutions using approved beam-column pile models. The lateral stiffness contribution of the pile cap shall be calculated using the passive pressure mobilization curve in Figure 4-6.

Pile group axial spring stiffness values,  $k_{sv}$ , shall be calculated using Equation (4-9).

$$k_{sv} = \sum_{n=1}^N \frac{A E}{L} \quad (4-9)$$

where:

- $A$  = Cross-sectional area of a pile
- $E$  = Modulus of elasticity of piles
- $L$  = Length of piles
- $N$  = Number of piles in group

The rocking spring stiffness values about each horizontal pile cap axis shall be computed by modeling each pile axial spring as a discrete Winkler spring. The rotational spring constant,  $k_{sr}$ , (moment per unit rotation) shall be calculated using Equation (4-10).

$$k_{sr} = \sum_{n=1}^N k_{vn} S_n^2 \quad (4-10)$$

where:

- $k_{vn}$  = Axial stiffness of the  $n$ th pile
- $S_n$  = Distance between  $n$ th pile and axis of rotation.

##### C4.4.2.2.1 Stiffness Parameters

As the passive pressure resistance may be a significant part of the total strength, and deep foundations often require larger lateral displacements than shallow foundations to mobilize the expected strength, it may not be appropriate to base the force-displacement response on the initial, effective stiffness alone. Instead, the contribution of passive pressure should be based on the passive pressure mobilization curve provided in Figure 4-6.

Although the effects of group action and the influence of pile batter are not directly accounted for in the form of the above equations, it can be reasonably assumed that the latter effects are accounted for in the range of uncertainties that must be considered in accordance with Section 4.4.1.

##### 4.4.2.2.2 Capacity Parameters

The axial load capacity of piles in compression and tension shall be determined using the procedures in Section 4.4.1. The axial capacity in tension shall not exceed the tensile load capacity of the pile cap and splice connections.

The moment capacity of a pile group shall be determined assuming a rigid pile cap. Lower-bound moment capacity shall be based on triangular distribution of axial pile loading and lower-bound axial capacity of the piles. Upper-bound moment capacity shall be based on a rectangular distribution of axial pile load using full, upper-bound axial capacity of the piles.

The lateral capacity of a pile group shall include the contributions of the piles (with an appropriate modification for group effects) and the passive resistance of the pile cap. The lateral capacity of the piles shall be calculated using the same method used to calculate the stiffness. The lateral capacity of the pile

cap, due to passive pressure, shall be calculated using established principles of soil mechanics. Passive pressure mobilization shall be calculated using Figure 4-6.

#### C4.4.2.2.2 Capacity Parameters

The lateral capacity of a pile cap should be calculated in the same way that the capacity of a shallow foundation is computed, except that the contribution of base traction should be neglected. Section C4.4.2.1.5 provides a more detailed description of the calculation procedure.

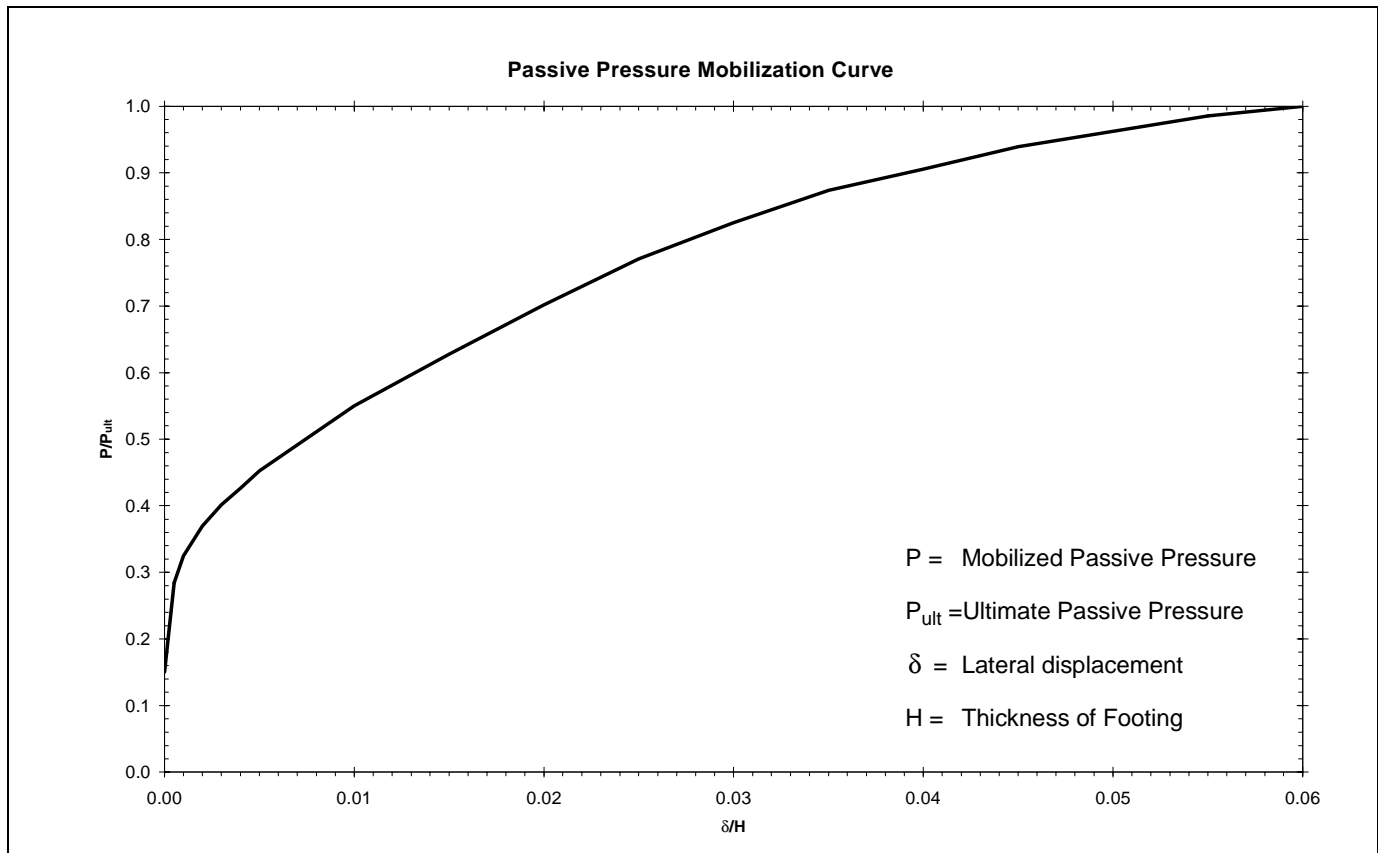
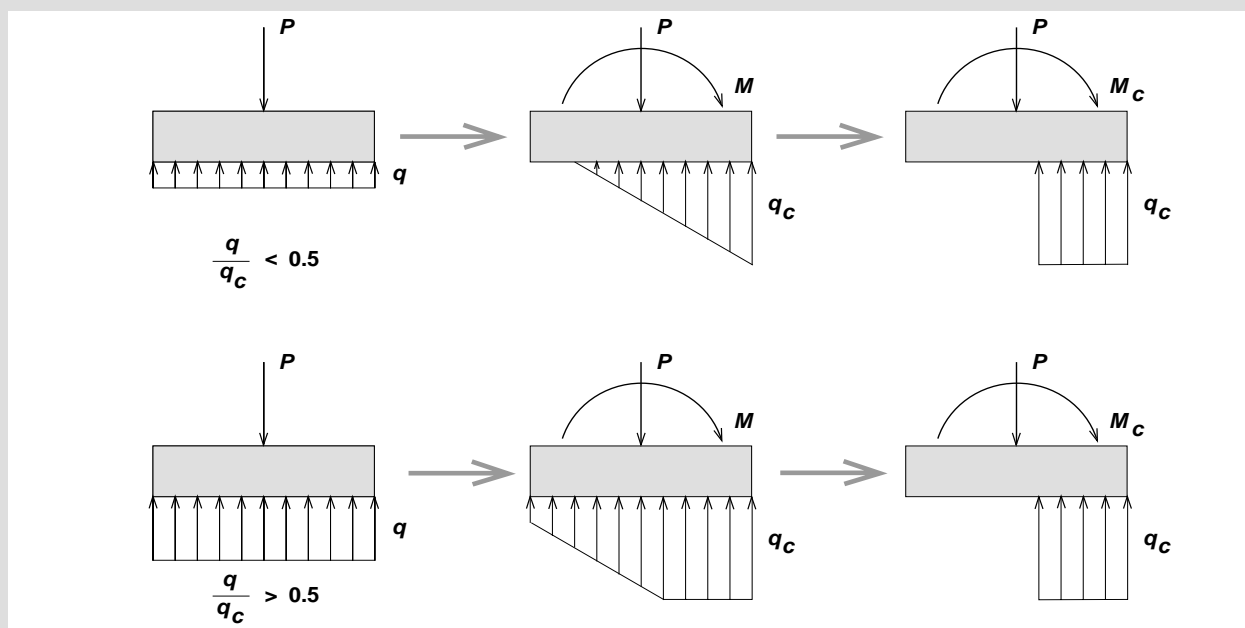


Figure 4-6 Passive Pressure Mobilization Curve





**Figure C4-3** Idealized Concentration of Stress at Edge of Rigid Footings Subjected to Overturning Moment

#### 4.4.2.3 Drilled Shafts

The stiffness and capacity of drilled shaft foundations and piers of diameter less than or equal to 24 inches shall be calculated using the requirements for pile foundations specified in 4.4.2.2. For drilled shaft foundations and piers of diameter greater than 24 inches, the capacity shall be calculated based on the interaction of the soil and shaft where the soil shall be represented using Winkler type models specified in Section 4.4.2.2.

##### C4.4.2.3 Drilled Shafts

When the diameter of the shaft becomes large (> 24 inches), the bending and the lateral stiffness and strength of the shaft itself may contribute to the overall capacity. This is obviously necessary for the case of individual shafts supporting isolated columns.

#### 4.4.3 Foundation Acceptability Criteria

The foundation soil shall comply with the acceptance criteria specified in this section. The structural components of foundations shall meet the appropriate requirements of Chapters 5 through 8. The foundation soil shall be evaluated to support all actions, including vertical loads, moments, and lateral forces applied to the soil by the foundation.

#### 4.4.3.1 Simplified Rehabilitation

The foundation soil of buildings for which the Simplified Rehabilitation Method is selected in accordance with Section 2.3.1 shall comply with the requirements of Chapter 10.

#### 4.4.3.2 Linear Procedures

The acceptance criteria for foundation soil analyzed by linear procedures shall be based on the modeling assumptions for the base of the structure specified in Section 4.4.3.2.1 or 4.4.3.2.2.

##### 4.4.3.2.1 Fixed Base Assumption

If the base of the structure is assumed to be completely rigid, the geotechnical components shall be classified as deformation-controlled. Component actions shall be determined by Equation (3-18). Acceptance criteria shall be based on Equation (3-20),  $m$ -factors for geotechnical components shall not exceed 3, and the use of upper-bound component capacities shall be permitted. A fixed base assumption shall not be used for buildings being rehabilitated to the Immediate Occupancy Performance Level that are sensitive to base rotations or other types of foundation movement.