

Fixed end Moment for Variable Section Beam: FH: RECTANGULAR SECTION



Initialization

ORIGIN ≡ 1 Count with fingers

TOL := 10⁻¹

CTOL := 10⁻¹

N := newton

ton := 1000·kgf

ksi := 70.307· $\frac{\text{kgf}}{\text{cm}^2}$

psi := $\frac{\text{ksi}}{1000}$

kip := 453.592·kgf

MPa := 10.197· $\frac{\text{kgf}}{\text{cm}^2}$

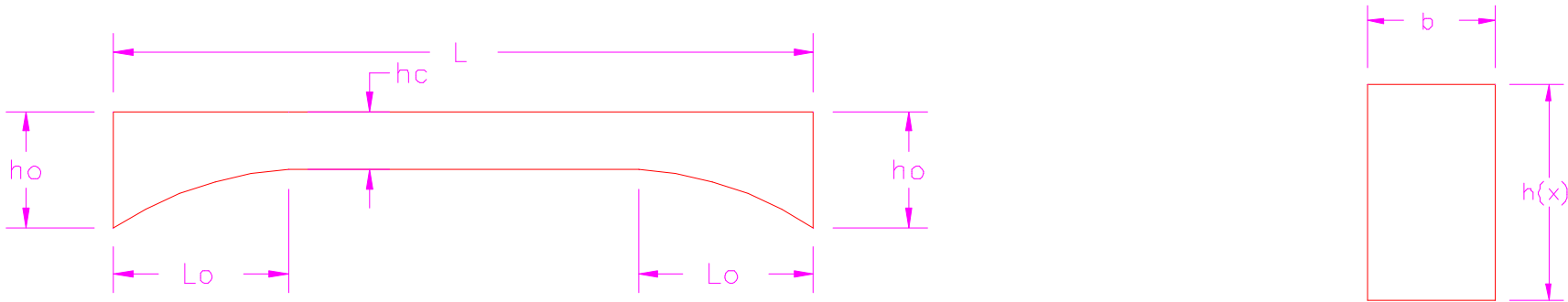
kN := 1000·N

GPa := 1000·MPa



We have one hinge at left end and fixity at right end

1. Symmetrical under set of trapecial and set of point loads



In this case the beam has 2 symmetrical corbels that follow une linear, parabolic, cubic etc curve

b := 30·cm

we keep constant

E := 20·GPa

L := 6·m

h₀ := 60·cm

going to

h_c := 30·cm

in

L₀ := 1·m

from both ends

k := 2

1 if linear soffit, 2 if parabolic, 3 if cubic etc

$w := \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \cdot \frac{\text{ton}}{\text{m}}$

each row one trapecial load
value of trapecial load at left and at right

$al := \begin{pmatrix} 0 & 6 \\ 0 & 0 \end{pmatrix} \cdot \text{m}$

abscissa of left start of load, then length with load,
sum of values in row not to exceed L

$$P := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \cdot \text{ton}$$

point loads

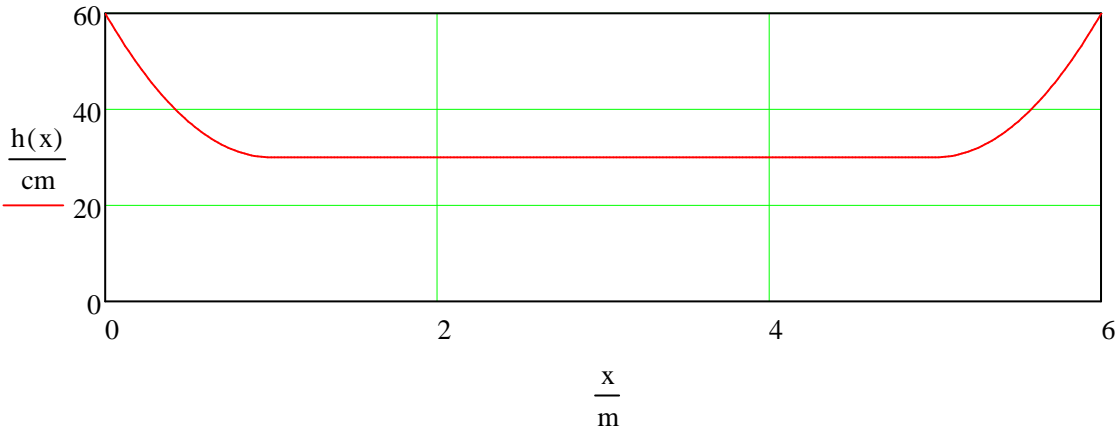
$$\underset{\text{www}}{A} := \begin{pmatrix} 1.5 \\ 3 \\ 4.5 \\ 0 \end{pmatrix} \cdot \text{m}$$

abscissas from left

$$w_c := 2450 \cdot \frac{\text{kgf}}{\text{m}^3}$$

own weight of material

$$h(x) := \begin{cases} h_c & \text{if } x > L_0 \wedge x < L - L_0 \\ \text{otherwise} \\ \begin{cases} h_0 - (h_0 - h_c) \cdot \left[1 - \left(\frac{L_0 - x}{L_0} \right)^k \right] & \text{if } x \leq L_0 \\ h_0 - (h_0 - h_c) \cdot \left[1 - \left[\frac{L_0 - (L - x)}{L_0} \right]^k \right] & \text{otherwise} \end{cases} \end{cases}$$



We have only one unknown (moment at fixity) that we solve from Mohr 2nd theorem. Moments we will take respect that of horizontal tangent, which is the right fixed end in our case. Moment of M/EI area respect right end must be zero

$$I(x) \mathrel{\mathop:}= b \cdot \frac{h(x)^3}{12}$$

$$M_1\Big(w_1,w_2,a_1,l_1,L_1\Big) \mathrel{\mathop:}= \int\limits_{0\cdot m}^{L_1} \left| \begin{array}{l} w_1 + \frac{w_2-w_1}{l_1} \cdot \big(x-a_1\big) \quad \text{if } x > a_1 \wedge x < a_1 + l_1 \\ 0 \cdot \frac{\text{ton}}{\text{m}} \quad \text{otherwise} \end{array} \right. \cdot \big(L_1-x\big) \, \text{d}x$$

$$R_L\Big(w_1,w_2,a_1,l_1\Big) \mathrel{\mathop:}= \frac{M_1\Big(w_1,w_2,a_1,l_1,L\Big)}{L}$$

isostatic reaction at left for single trapecial load

$$M_w\Big(w_1,w_2,a_1,l_1,X\Big) \mathrel{\mathop:}= R_L\Big(w_1,w_2,a_1,l_1\Big) \cdot X - M_1\Big(w_1,w_2,a_1,l_1,X\Big)$$

isostatic moment of single trapecial load

$$M_P(x,P,a) \mathrel{\mathop:}= \left| \begin{array}{l} P \cdot \frac{(L-a)}{L} \cdot x \quad \text{if } x \leq a \\ P \cdot \frac{a}{L} \cdot (L-x) \quad \text{otherwise} \end{array} \right. \qquad \begin{array}{l} \text{isostatic moment of single load } P \\ \text{at abscissa } a \end{array}$$

$$ow(x) \mathrel{\mathop:}= b \cdot h(x) \cdot w_c$$

$$R_{Low} \mathrel{\mathop:}= \frac{\int\limits_{0\cdot m}^L ow(x) \cdot (L-x) \, \text{d}x}{L} \qquad R_{Low} = 0.74 \, \text{ton}$$

$$M_{ow}(X) := R_{Low} \cdot X - \int_{0 \cdot m}^X ow(x) \cdot (X - x) \, dx$$

$$M_{isostatical_of_loads}(x) := M_{ow}(x) + \sum_{j = 1}^{length(w^{\langle 1 \rangle})} M_w(w_j, 1, w_j, 2, al_j, 1, al_j, 2, x) + \sum_{i = 1}^{length(P)} M_P(x, P_i, A_i)$$

$$M_1(x, M_a) := M_a - \frac{x}{L} \cdot M_a \qquad \text{for an arbitrary moment at left end}$$

$$M_2(x, M_b) := \frac{x}{L} \cdot M_b \qquad \text{for an arbitrary moment at right end}$$

$$M(x, M_a, M_b) := M_{isostatical_of_loads}(x) + M_1(x, M_a) + M_2(x, M_b)$$

Now we seek solution to 1st and 2nd theorem of Mohr provided equations by

$$M_a := 0 \cdot m \cdot ton \qquad \text{known from definition}$$

$$M_b := -3 \cdot m \cdot ton \qquad \text{unwarranted guess for solution}$$

Given

$$\int_{0\cdot\text{m}}^L \frac{M(x, M_a, M_b)}{E \cdot I(x)} \cdot x \, dx = 0 \cdot \text{cm} \quad \text{total deflection to tangent at right in length=0*cm}$$

$$M_b := \text{Find}(M_b)$$

$M_a = 0 \text{ m} \cdot \text{ton}$
 $M_b = -15.46 \text{ m} \cdot \text{ton}$

fixity moments at left and right ends for variable section and this load

For a FEM application we need between other things to substitute effect of loads in the span by the end fixity forces in equilibrium with them. This includes so the shear actions that we pass to calculate now by remembering that by determination of end moments we have converted the problem in isostatic. Each end (negative) moment causes the respective end augment its reaction, so it depends on which is higher what end gets its reaction at support augmented.

$$R_{LP}(P, a) := P \cdot \frac{(L - a)}{L} \quad \text{isostatic left reaction of single load P at abscissa a}$$

$$R_{1_isostatical_of_loads} := R_{Low} + \sum_{j=1}^{\text{length}(w^{(1)})} R_L(w_{j,1}, w_{j,2}, al_{j,1}, al_{j,2}) + \sum_{i=1}^{\text{length}(P)} R_{LP}(P_i, A_i)$$

$$W(j) := \int_{0\cdot\text{m}}^L \left| \begin{array}{l} w_{j,1} + \frac{w_{j,2} - w_{j,1}}{al_{j,2}} \cdot (x - al_{j,1}) \quad \text{if } x > al_{j,1} \wedge x < al_{j,1} + al_{j,2} \\ 0 \cdot \frac{\text{ton}}{\text{m}} \quad \text{otherwise} \end{array} \right. dx$$

$R_a := R_{1_isostatical_of_loads} + \frac{-(M_a - M_b)}{L}$
 $R_b := \int_{0\cdot\text{m}}^L ow(x) \, dx + \sum_{j=1}^{\text{length}(w^{(1)})} W(j) + \sum_{i=1}^{\text{length}(P)} P_i - R_a$

Reaction at left end

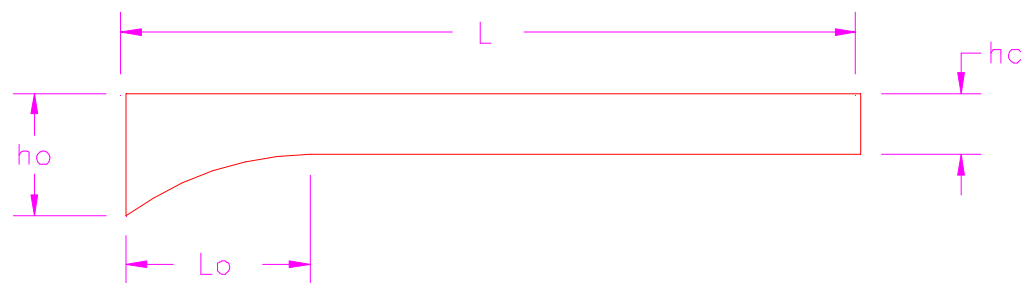
Reaction at right end

$$R_a = 5.66 \text{ ton}$$

$$R_b = 10.81 \text{ ton}$$

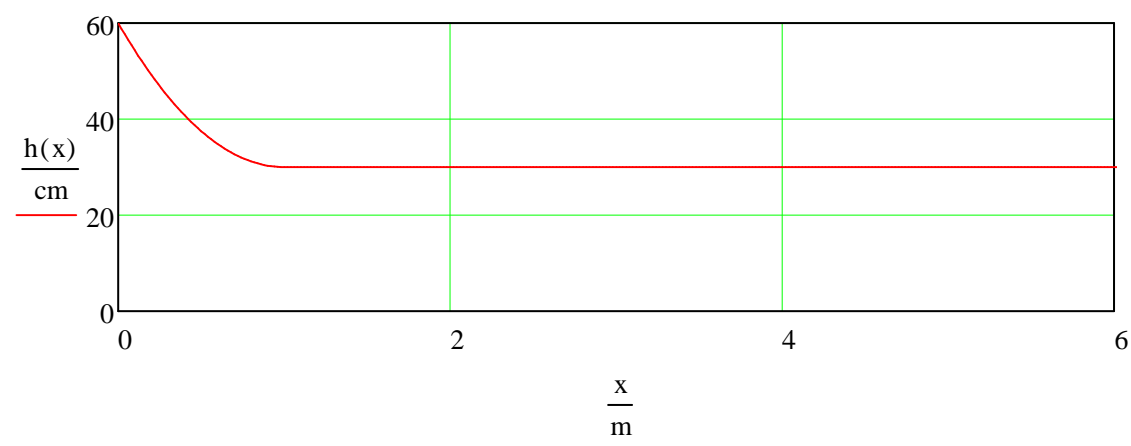
Now follow the same but with the corbel of length L_0 only at left end

2. Asymmetric with corbel at left end and fixity at right



We need only to re-state geometry and the rest is the same

$$h(x) := \begin{cases} h_c & \text{if } x > L_0 \\ h_0 - (h_0 - h_c) \cdot \left[1 - \left(\frac{L_0 - x}{L_0} \right)^k \right] & \text{otherwise} \end{cases}$$



$$\textcolor{green}{I}(x) \mathrel{\mathop:}= b \cdot \frac{h(x)^3}{12}$$

We are not re-stating loads so please ensure that the wanted ones figure atop the sheet

$$\textcolor{green}{M}_1\big(w_1,w_2,a_1,l_1,L_1\big) \mathrel{\mathop:}= \int\limits_{0\cdot\text{m}}^{L_1} \left| \begin{array}{l} w_1 + \frac{w_2-w_1}{l_1} \cdot \big(x-a_1\big) \quad \text{if } x > a_1 \wedge x < a_1+l_1 \\ 0 \cdot \frac{\text{ton}}{\text{m}} \quad \text{otherwise} \end{array} \right. \cdot \big(L_1-x\big) \, \text{d}x$$

$$\textcolor{green}{R}_L\big(w_1,w_2,a_1,l_1\big) \mathrel{\mathop:}= \frac{M_1\big(w_1,w_2,a_1,l_1,L\big)}{L}$$

isostatic reaction at left for single trapecial load

$$\textcolor{green}{M}_x\big(w_1,w_2,a_1,l_1,X\big) \mathrel{\mathop:}= R_L\big(w_1,w_2,a_1,l_1\big) \cdot X - M_1\big(w_1,w_2,a_1,l_1,X\big)$$

isostatic moment of single trapecial load

$$M_P(x,P,a) := \begin{cases} P \cdot \frac{(L-a)}{L} \cdot x & \text{if } x \leq a \\ P \cdot \frac{a}{L} \cdot (L-x) & \text{otherwise} \end{cases}$$

isostatic moment of single load P
at abscissa a

$$ow(x) := b \cdot h(x) \cdot w_c$$

$$R_{Low} := \frac{\int_{0.m}^L ow(x) \cdot (L-x) \, dx}{L}$$

$R_{Low} = 0.73 \text{ ton}$

$$M_{ow}(X) := R_{Low} \cdot X - \int_{0.m}^X ow(x) \cdot (X-x) \, dx$$

$$M_{isostatical_of_loads}(x) := M_{ow}(x) + \sum_{j=1}^{length(w^{<1>})} M_w(w_{j,1},w_{j,2},al_{j,1},al_{j,2},x) + \sum_{i=1}^{length(P)} M_P(x,P_i,A_i)$$

$$M_1(x,M_a) := M_a - \frac{x}{L} \cdot M_a$$

for an arbitrary moment at left end

$$M_2(x,M_b) := \frac{x}{L} \cdot M_b$$

for an arbitrary moment at right end

$$M(x,M_a,M_b) := M_{isostatical_of_loads}(x) + M_1(x,M_a) + M_2(x,M_b)$$

Now we seek solution to 1st and 2nd theorem of Mohr provided equations by

$$M_a := 0 \cdot \text{m} \cdot \text{ton}$$

$$M_b := -3 \cdot \text{m} \cdot \text{ton}$$

unwarranted guess for solution

Given

$$\int_{0 \cdot \text{m}}^L \frac{M(x, M_a, M_b)}{E \cdot I(x)} \cdot x \, dx = 0 \cdot \text{cm}$$

total deflection to tangent at right in length=0*cm

$$M_b := \text{Find}(M_b)$$

$M_a = 0 \text{ m} \cdot \text{ton}$

$M_b = -12.79 \text{ m} \cdot \text{ton}$

fixity moments at left and right ends for variable section and this load

For a FEM application we need between other things to substitute effect of loads in the span by the end fixity forces in equilibrium with them. This includes so the shear actions that we pass to calculate now by remembering that by determination of end moments we have converted the problem in isostatic. Each end (negative) moment causes the respective end augment its reaction, so it depends on which is higher what end gets its reaction at support augmented.

$$R_{LP}(P, a) := P \cdot \frac{(L - a)}{L}$$

isostatic left reaction of single load P at abscissa a

$$R_{\text{isostatical_of_loads}} := R_{Low} + \sum_{j = 1}^{\text{length}(w^{(1)})} R_L(w_j, 1, w_j, 2, al_j, 1, al_j, 2) + \sum_{i = 1}^{\text{length}(P)} R_{LP}(P_i, A_i)$$

$$W(j) := \int_{0 \cdot m}^L \left| \begin{array}{l} w_{j,1} + \frac{w_{j,2} - w_{j,1}}{a_{j,2}} \cdot (x - a_{j,1}) \quad \text{if } x > a_{j,1} \wedge x < a_{j,1} + a_{j,2} \\ 0 \cdot \frac{\text{ton}}{\text{m}} \quad \text{otherwise} \end{array} \right. dx$$

$$R_a := R_{1_isostatical_of_loads} + \frac{-(M_a - M_b)}{L} \qquad R_b := \int_{0 \cdot m}^L ow(x) \, dx + \sum_{j=1}^{\text{length}(w^{(1)})} W(j) + \sum_{i=1}^{\text{length}(P)} P_i - R_a$$

Reaction at left end

Reaction at right end

$$R_a = 6.1 \text{ ton}$$

$$R_b = 10.3 \text{ ton}$$

3. Asymmetric with corbel at left end, fixity at left end and hinge at right end

With clever use this completes determination of the fixed end moment whichever the case of these configurations

Statement of $h(x)$ and inertia is the same than previous case but hinge and fixity ends are interchanged

We are not re-stating loads so please ensure that the wanted ones figure atop the sheet

$$M_1(w_1, w_2, a_1, l_1, L_1) := \int_{0 \cdot m}^{L_1} \left| \begin{array}{l} w_1 + \frac{w_2 - w_1}{l_1} \cdot (x - a_1) \quad \text{if } x > a_1 \wedge x < a_1 + l_1 \\ 0 \cdot \frac{\text{ton}}{\text{m}} \quad \text{otherwise} \end{array} \right. \cdot (L_1 - x) \, dx$$

$$\textcolor{green}{R_L}(w_1,w_2,a_1,l_1) := \frac{M_1(w_1,w_2,a_1,l_1,L)}{L}$$

isostatic reaction at left for single trapecial load

$$\textcolor{green}{M_L}(w_1,w_2,a_1,l_1,X) := R_L(w_1,w_2,a_1,l_1)\cdot X - M_1(w_1,w_2,a_1,l_1,X)$$

isostatic moment of single trapecial load

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isostatic moment of single load P
at abscissa a

$$\textcolor{green}{ow}(x) := b\cdot h(x)\cdot w_c$$

$$\textcolor{green}{R_{Low}} := \frac{\int\limits_{0\cdot m}^L ow(x)\cdot(L-x) \, dx}{L}$$

$R_{Low} = 0.73 \, \text{ton}$

$$\textcolor{green}{M_{ow}}(X) := R_{Low}\cdot X - \int\limits_{0\cdot m}^X ow(x)\cdot(X-x) \, dx$$

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$$\textcolor{green}{M_1}(x,M_a) := M_a - \frac{x}{L}\cdot M_a$$

for an arbitrary moment at left end

$$M_2(x, M_b) := \frac{x}{L} \cdot M_b \quad \text{for an arbitrary moment at right end}$$

$$M(x, M_a, M_b) := M_{\text{isostatical_of_loads}}(x) + M_1(x, M_a) + M_2(x, M_b)$$

Now we seek solution to 1st and 2nd theorem of Mohr provided equations by

$$M_b := 0 \cdot \text{m} \cdot \text{ton} \quad \text{since right end hinge}$$

$$M_a := -3 \cdot \text{m} \cdot \text{ton} \quad \text{unwarranted guess for current unknown}$$

Given

$$\int_{0 \cdot \text{m}}^L \frac{M(x, M_a, M_b)}{E \cdot I(x)} \cdot (L - x) \, dx = 0 \cdot \text{cm} \quad \text{total deflection to tangent at left in length=0*cm}$$

$$M_a := \text{Find}(M_a)$$

$$M_a = -15.49 \text{ m} \cdot \text{ton}$$

$$M_b = 0 \text{ m} \cdot \text{ton}$$

fixity moments at left and right ends for variable section and this load

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isostatic left reaction of single load P at abscissa a

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$$W(j) := \int_{0.m}^L \left| \begin{array}{l} w_{j,1} + \frac{w_{j,2}-w_{j,1}}{al_{j,2}} \cdot (x-al_{j,1}) \text{ if } x > al_{j,1} \wedge x < al_{j,1} + al_{j,2} \\ 0 \cdot \frac{ton}{m} \text{ otherwise} \end{array} \right. dx$$

$$R_a := R_{1_isostatical_of_loads} + \frac{-(M_a-M_b)}{L}$$

$$R_b := \int_{0.m}^L ow(x) \, dx + \sum_{j=1}^{length(w^{<1>})} W(j) + \sum_{i=1}^{length(P)} P_i - R_a$$

Reaction at left end

Reaction at right end

$$R_a = 10.81 \, ton$$

$$R_b = 5.58 \, ton$$

