Longitudinal shear strength of Douglas-fir

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Weibull's theory of brittle fracture is applied to the determination of strength of Douglas-fir wood in longitudinal shear. Ultimate stresses, at a given survival probability, are derived for beams under different loading conditions. The theory allows an explanation for the difference in shear strength between beams and the standard American Society for Testing Materials shear block, as well as for the dependence of shear strength upon beam size. The theory is verified by comparing theoretical predictions and test results on Griplam nailed connections loaded parallel to the grain and shear tests on torque tubes. Very good agreement is shown. Finally, allowable shear stresses for beams under different loading conditions are derived.

Les auteurs appliquent la théorie de la rupture fragile de Weibull à la détermination de la résistance au cisaillement longitudinal du pin Douglas. Pour une certaine probabilité de survie donnée, ils calculent la contrainte de rupture pour des poutres sollicitées de diverses façons. La théorie permet d'expliquer, d'une part, les différences constatées dans la résistance en cisaillement entre les poutres mises à l'essai et le diagramme standard préconisé pour les cisaillements par les prescriptions American Society for Testing Materials, et d'autre part, la relation observée entre la résistance en cisaillement et les dimensions d'une poutre. On vérifie la théorie en comparant les résultats dérivés par des calculs et les résultats tirés d'essais de cisaillement réalisés sur des assemblages à goussets cloués du type à étrilles chargés parallèlement aux fibres et sur des pièces tubulaires sollicitées en torsion. On constate alors une très bonne concordance des deux groupes de résultats à la lumière desquels les auteurs proposent des taux de travail appropriés en cisaillement pour les poutres appelées à travailler dans différentes conditions de chargement.

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Introduction

Two problems are to be considered in studying the shear strength of timber beams: (1) the magnitude of the shear stresses induced by the applied loads and (2) the level of shear stresses for failure to occur.

Although, based on a parabolic distribution, maximum shear stresses could be approximately estimated by the simple formulae from elementary strength of materials, the magnitude of the maximum shear stress in timber beams has been computed by a formula developed by Newlin *et al.* (1934) which is supposed to take into account the effect of checks. Newlin's approach, known as the "two-beam theory", has been adopted by codes of design practice both in Canada and the United States. Based on the same theory, the codes also recommend a "worst" position for a load near the support where it must be assumed to act for the determination of shear stresses. Lately, however, both in Canada and the United States there has been a tendency towards using the simple formula $v = 3V/2A$ from elementary beam theory. From a practical point of view, the differences between Newlin's approach and the elementary beam theory are not great although, for example, Newlin's "shear force" does not satisfy conditions of beam equilibrium. Whether one approach or the other is used, the main problem in shear strength of beams is that of determining the level of shear stress that produces failure and how this level is influenced by beam geometry and the applied loads.

It has been found that the shear strength of large beams is usually lower than that obtained from the American Society for Testing Materials (ASTM 1973) standard shear block test. Furthermore, it has been observed that the strength of beams depends upon the geometry of the beam and different authors have expressed this effect in terms of different independent variables. Thus, Huggins *et al.*

(1966) used the shear span-to-depth ratio, while, more recently, Keenan and Selby (1973) used a "shear area" defined as the product of shear span and width. All these variables are, of course, related to the size of the beam and the underlying rĕsult is that the shear strength is size-dependent.

The present study shows the results of a stress analysis of beams near supports and an evaluation of the adequacy of the elementary beam theory to predict the magnitude of shear stresses. Further, it is shown that the introduction of brittle fracture concepts to shear failure of wood is useful in that it helps to explain the size-dependence observed in tests and allows the derivation of ultimate and allowable shear stresses for beams.

Stress Analysis

Consider Fig. 1. A finite element analysis was carried out for the deformations and stresses induced by the loading shown. The finite element used was a quadratic isoparametric element (Zienkiewicz 1971) and nonlinear material properties were considered. In particular, nonlinear laws were assumed for compression parallel and compression perpendicular to the grain, while purely elastic behaviour was assumed in tension and in shear.

The results of this study may be summarized as follows:

(1) High shear stresses develop near the corners A of Fig. 1, where the beam meets the support and loading plates. These shear stresses, however, diminish quite rapidly towards the mid-depth of the beam and only contribute to localized bearing-type failure.

(2) Maximum shear stresses away from the support and loading plates are conservatively estimated by the formula $v = 3V/2A$ of elementary beam theory. The formula is more exact the longer the distance a ("shear span").

(3) The distribution and magnitude of shear stresses near the mid-depth of the beam is almost independent of the type of nonlinear law assumed for compression perpendicular to the grain.

(4) Maximum bending stresses are accurately estimated by the elementary beam formulae.

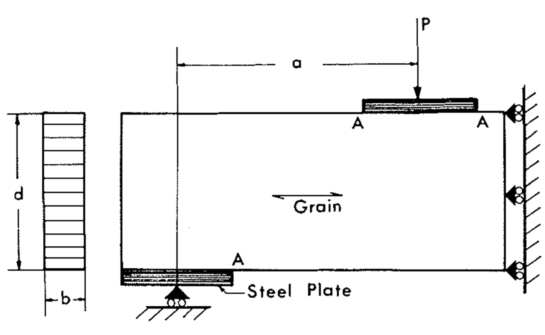

FIG. 1 Beam geometry, finite element analysis.

Brittle Fracture and Shear Strength

Shear failure in wood with a moisture content of about 12% may be considered as brittle and the theory of brittle fracture, as developed by Weibull (Bolotin 1969), may be used to study the conditions under which failure may develop. This theory has already been used to study the tension perpendicular to grain strength of Douglas-fir in different structural applications (Barrett *et al.* 1975).

Briefly, consider a volume V of wood under a distribution of shear stresses τ. Weibull's theory allows the computation of the probability of failure of the volume V when the stresses are known. This probability is given by

$$[1] \quad F_V = 1 - \exp\left\{-\frac{1}{V^*}\int_V \left(\frac{\tau - \tau_0}{m}\right)^k dv\right\}$$

where m, k, and τ_0 are material constants, V^* is a reference volume, and τ_0 corresponds to the minimum strength of the material. Since three material constants are involved, [1] is referred to as a 'three-parameter' Weibull model. A simpler, 'two-parameter' model may be used by assuming $\tau_0 = 0$. The assumption of zero minimum strength may appear to be unrealistic, but it will be shown that, for practical purposes, both models give approximately the same results at probabilities of failure larger than or equal to 0.05. The parameter k is related to the coefficient of variation of the material for a given geometric and loading configuration. For coefficients of variation of the order of 0.20 commonly encountered with wood, k is of the order of 5.

Assume then a two-parameter model and, for simplicity, consider the reference volume V^* as a *unit volume* ($V^* = 1$ in.^{3} or 1 m^3) *under a uniform stress* τ^*. V^* and the volume

V of [1] will have the same probability of failure if the following relationship holds between the stress τ^* and the stresses τ:

$$[2] \qquad \int_V \tau^k \, dv = \tau^{*k}$$

If the system of stresses τ is expressed as

$$[3] \qquad \tau = \tau_M \theta[x,y,z]$$

where τ_M is a characteristic stress of the system, [2] may be written

$$[4] \qquad \tau_M{}^k I[k] = \tau^{*k}$$

with

$$[5] \qquad I[k] = \int_V (\theta[x,y,z])^k dv$$

Thus, if the strength of the unit volume under uniform shear is known at different levels of survival probability, and if the system of shear stresses τ is known so that [5] can be integrated, the strength of the volume V at different levels of survival probability can be computed from

$$[6] \qquad \tau_M = \tau^*/I[k]^{1/k}$$

To study the shear strength of beams, we consider V to be the total beam volume and τ_M to be the shear stress given by the elementary beam theory,

$$[7] \qquad \tau_M = 3V_M/2A$$

with V_M being the maximum shear force, in absolute value, and A the cross-sectional area of the beam. The integration of [5] requires a detailed shear stress distribution. In the cases considered herein, Eq. [5] was evaluated by direct numerical integration of finite element results.

Shear Strength of Beams

CASE I: Single Concentrated Load

This case corresponds to that of Fig. 2. The integration of [5] over the distance a can be expressed as

$$[8] \qquad I[k] = b\,d\,a\,I_a[a/d]$$

where I_a depends on k and on the ratio a/d. Figure 2 shows, for example, the variation of I_a with a/d for $k = 5$. For large a/d, the shear stresses are approximated quite closely by the parabolic distribution of elementary beam

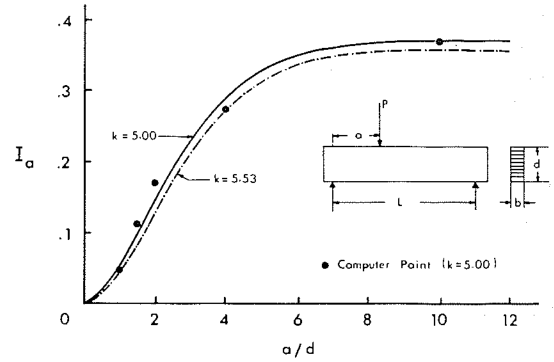

FIG. 2. Coefficient I_a $[a/d]$—concentrated load.

theory. Accordingly, I_a approaches the value obtained when the function $\theta[x,y,z]$ in [5] is made to coincide with that corresponding to the parabolic distribution. The computer results may be approximated by regression equations and, for example, the curve for $k = 5$ may be expressed as

$$[9] \qquad I_a[a/d] = 0.369\,[1.0 - \exp(-0.170\,(a/d)^{1.595})]$$

This regression curve is shown in Fig. 2. Equation [4] may now be written for the entire beam of Fig. 2:

$$[10] \qquad \tau_M{}^k\{b\,d\,aI_a[a/d] + (a/(L-a))^k\,b\,d\,(L-a)\,I_a\,[(L-a)/d]\} = \tau^{*k}$$

from where, defining

$$[11] \qquad \beta_s = \{(a/L)I_a[a/d] + (a/(L-a))^k\,((L-a)/L)\,I_a\,[(L-a)/d]\}^{-1/k}$$

and letting $V = b\ d\ L$ be the total beam volume,

$$[12] \qquad \tau_M = \beta_s\ (\tau^*/V^{1/k})$$

Equation [12] gives the mean failure shear stress for the beam, for example, if the mean failure stress for the unit volume under uniform shear is used. The factor β_s is plotted in Fig. 3 for $k = 5$. It is seen that two ratios control β_s, namely, L/d and a/L. The first is the aspect ratio for the beam and the second gives the position of the load. For a given beam, Fig. 3 shows the manner in which the shear span a influences β_s and, in turn, the shear strength of the beam.

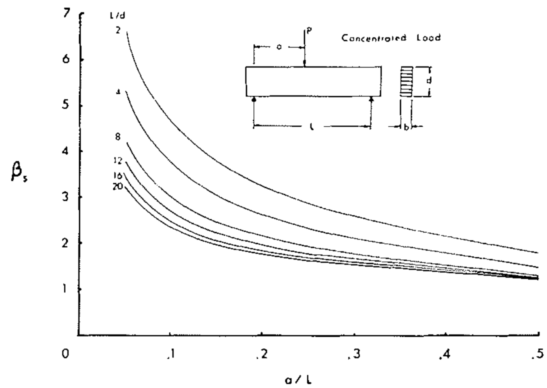

FIG. 3. Coefficient β_s—concentrated load.

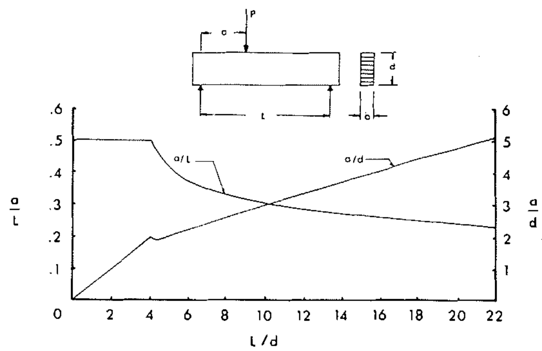

FIG. 4. Worst position a/L or a/d for a concentrated load, as a function of L/d.

CASE II: Worst Position of a Stationary Concentrated Load

The load P that a beam can carry in shear with a given probability of survival can be obtained from [12] by expressing τ_M according to [7]

$$[13] \qquad P = \beta_s \frac{2AL}{3(L-a)} \frac{\tau^*}{V^{1/k}}$$

or, defining

$$[14] \qquad \beta^* = \frac{3}{2} \frac{L-a}{\beta_s L}$$

[13] may be written

$$[15] \qquad P = \frac{A\tau^*}{\beta^* V^{1/k}}$$

When the shear span a increases, the factor β_s from Fig. 3 decreases. But so does the denominator $(L - a)$ in [13], tending to increase P. Both effects oppose each other and one may ask whether a position exists for which the load P that may be carried is a minimum. This can be answered by studying the function β^* of [14].

The value at which β^* is a maximum corresponds, from [15], to the minimum of P, and may be obtained for different aspect ratios L/d. Figure 4 shows the values a/L and a/d corresponding to the minimum P (worst position for P) for different L/d.

If a load is going to act on a beam and the position is not known *a priori*, the load must be assumed to act at a point given by a/L of Fig. 4 and the corresponding allowable stress must be obtained from the information in Fig. 3. The load must remain stationary after it has been applied, for if it moves, the effect of motion must be entered into [4], and this will be discussed in Case V.

CASE III: Uniformly Distributed Load

This case corresponds to that of Fig. 5. The integration of [5] over the entire beam volume $V = b\, d\, L$ can be expressed as

$$[16] \qquad I[k] = b\, d\, L \bar{I}_a[L/d]$$

where $\bar{I}_a$ depends on k and on the aspect ratio L/d. Figure 5 shows, for example, the variation of $\bar{I}_a$ with L/d for $k = 5$. For large L/d, the shear stresses are closely approximated by the parabolic distribution of elementary beam theory. Accordingly, $\bar{I}_a$ approaches the value obtained when the function $\theta[x,y,z]$ in [5] is that corresponding to the parabolic distribution. Again, computer results may be approximated by regression equations and, for $k = 5$,

$$[17] \qquad \bar{I}_a[L/d] = 0.0616[1.0 - \exp(-0.0022(L/d)^{2.394})]$$

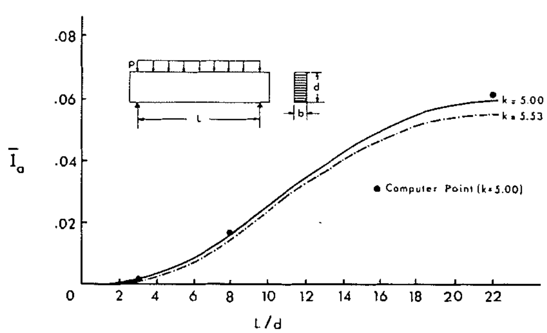

FIG. 5. Coefficient $\bar{I}_a$ $[L/d]$—uniformly distributed load.

This regression curve is shown in Fig. 5. Equation [4] may now be written

$$[18] \qquad \tau_M{}^k \, b \, d \, L \, \bar{I}_a = \tau^{*k}$$

or, defining

$$[19] \qquad \beta_U = \bar{I}_a^{-1/k}$$

the strength of the beam in shear under uniformly distributed load can be computed from

$$[20] \qquad \tau_M = \beta_U \frac{\tau^*}{V^{1/k}}$$

where τ_M is the shear stress from [7] using as V_M the maximum shear force in absolute value. The coefficient β_U is plotted in Fig. 6 for $k = 5$ and different values of the aspect ratio L/d.

CASE IV: N Concentrated Loads

This case is shown in Fig. 7. If V_1 is the maximum shear force in absolute value, let the stress τ_M of [7] be defined as

$$[21] \qquad \tau_M = \frac{3}{2}\frac{V_1}{A}$$

The integration required by [5] may be performed, in this case, as follows. Near supports, that is, over the segments a and c_N of Fig. 7, the information of Case I is used; between loads, the conservative assumption is made that the shear stresses can be approximated by the parabolic distribution of elementary theory, and thus, I_a of Case I for a/d approaching infinity is used. Equation [4] may be written, therefore,

$$[22] \qquad \left(1.5\frac{V_1}{A}\right)^k b \, d \, a \, I_a\left[\frac{a}{d}\right] + \sum_{i=1}^{N-1}\left(1.5\frac{V_{i+1}}{A}\right)^k b \, d \, c_i \, I_a[\infty] + \left(1.5\frac{V_{N+1}}{A}\right)^k b \, d \, c_N \, I_a\left[\frac{c_N}{d}\right] = \tau^{*k}$$

or, defining

$$[23] \qquad \beta_M = \left\{\left(\frac{a}{L}\right) I_a\left[\frac{a}{d}\right] + \sum_{i=1}^{N-1}\left(\frac{V_{i+1}}{V_1}\right)^k \times \frac{c_i}{L} I_a[\infty] + \left(\frac{V_{N+1}}{V_1}\right)^k \frac{c_N}{L} I_a\left[\frac{c_N}{d}\right]\right\}^{-1/k}$$

the shear strength of the beam, as given by the stress τ_M of [21], can be computed from

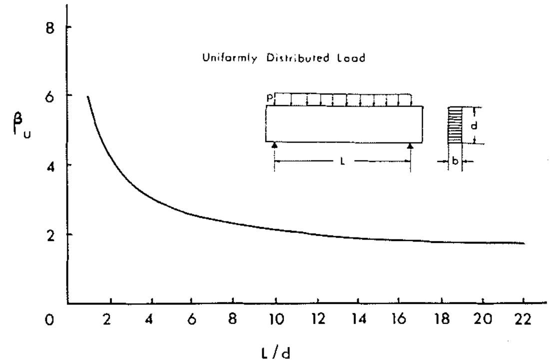

FIG. 6. Coefficient β_U—uniformly distributed load.

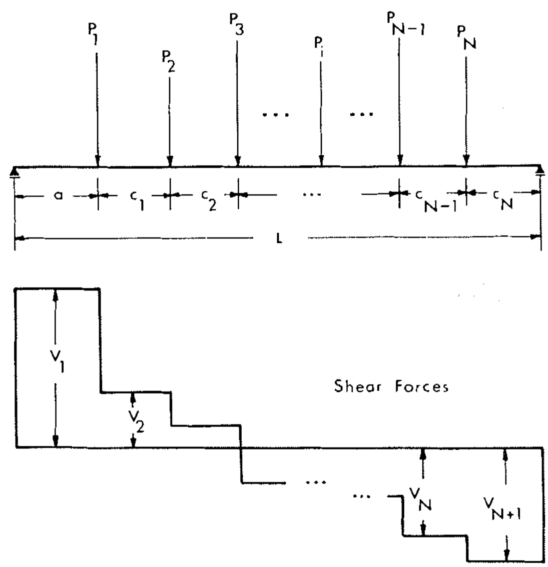

FIG. 7. Case of N concentrated loads.

$$[24] \qquad \tau_M = \beta_M \frac{\tau^*}{V^{1/k}}$$

Again, V is the total beam volume $V = b \, d \, L$. It is seen that the information developed for Case I may be used, together with the shear–force diagram, to develop [4] for any situation of concentrated loads. This includes the case of cantilevered beams, with the shear–force diagram for the cantilevered part just added to the rest of the shear–force diagram and considered in writing [22].

CASE V: Moving Single Concentrated Load

This is the situation in which a single load moves across the beam, as in the case of a truck axle travelling a bridge. In the integration of the shear stresses needed in [2], it is required to consider now the maximum shear stress that occurs at any point in the beam as the load moves over it. Consider Fig. 8. The shear force at x will be given in terms of the position of the load P by the following equations:

$$[25] \qquad V[x] = -P\xi/L \text{ if } \xi < x$$

$$[26] \qquad V[x] = P\,(L-\xi)/L \text{ if } \xi > x$$

It can be shown, from [25] and [26], that the maximum shear force at x occurs when the load P passes over that point ($\xi = x$) and that this maximum is given by

$$[27] \qquad V_{\text{max.}} = P\,(L-x)/L \text{ if } x \leqslant L/2$$

$$[28] \qquad V_{\text{max.}} = P\,x/L \text{ if } x \geqslant L/2$$

The maximum shear force $V_{\text{max.}}$ induces shear stresses at x given by

$$[29] \qquad \tau = \gamma\left[\frac{x}{d}\right]\frac{V_{\text{max.}}}{bd}\left[1 - \left(\frac{2y}{d}\right)^2\right]$$

where it has been assumed that the stresses are distributed, over the cross section, according to a parabolic law. The coefficient $\gamma\ [x/d]$ takes into account the dependence of the maximum shear stress upon the closeness of the cross section at x to the support. This coefficient was derived from the finite element analysis of Case I and may be expressed by

$$[30] \qquad \gamma\,[x/d] = 1.50\,[1.0 - \exp\,(-1.15\,x/d)]$$

Hence, the shear stresses τ to be considered in the integration of [2] are given, from [27], [28], [29], and [30] by:

if $x/L \leqslant 0.5$,

$$[31] \qquad \tau = 1.5\{1 - e^{-1.15(x/d)}\}\frac{P(L-x)}{Lbd} \times \left\{1 - \left(\frac{2y}{d}\right)^2\right\};$$

if $x/L \geqslant 0.5$,

$$[32] \qquad \tau = 1.5\{1 - e^{-1.15(L-x)/d}\}\frac{Px}{Lbd} \times \left\{1 - \left(\frac{2y}{d}\right)^2\right\}$$

The result of the integration can be expressed as

$$[33] \qquad \tau_M = \beta_{\text{mov}}\frac{\tau^*}{V^{1/k}}$$

with the stress τ_M defined as

$$[34] \qquad \tau_M = 3P/2A$$

The coefficient β_{mov} is determined from

$$[35] \qquad \beta_{\text{mov}} = \left\{\int_{-1}^{1}\frac{(1-\eta^2)^k}{2}\,d\eta \times \int_{-1}^{1}\frac{1}{2}\{1 - e^{-1.15L(1+\xi)/4d}\}^k \times \left(\frac{3-\xi}{4}\right)^k d\xi\right\}^{-1/k}$$

and, together with [33] and [34], it allows the computation of the maximum load that can travel across the beam for it to have a given survival probability in shear. Another approach to this calculation is to consider the load P at the worst position as a stationary load, according to Case II, and modify β_s of Case I by a factor α to obtain the same result as that of [33]. Thus, if for the aspect ratio L/d of the given beam the worst position of a stationary load is at a/L, one has, from Case I,

$$[37] \qquad \frac{3}{2}\frac{P}{A}\frac{L-a}{L} = \alpha\beta_s\frac{\tau^*}{V^{1/k}}$$

from which, by introducing [33] and [34], it can be concluded that

$$[37] \qquad \alpha = \left(\frac{\beta_{\text{mov}}}{\beta_s}\right)\left(1 - \frac{a}{L}\right)$$

The factor α depends upon the ratio L/d as shown in Fig. 8. It is apparent that, for most beams, the value of τ_M of [12] computed for the worst position of the stationary load must

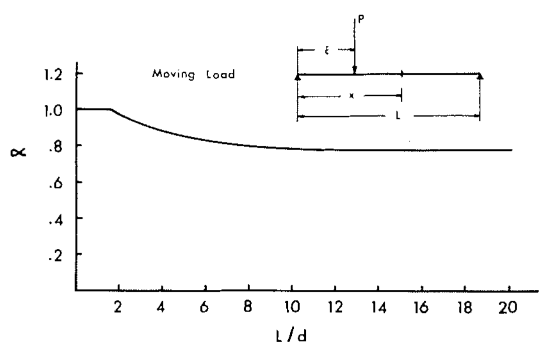

FIG. 8. Factor α—moving concentrated load.

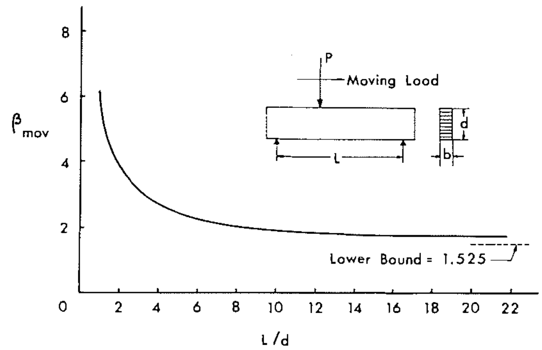

FIG. 9. Coefficient β_{mov}—moving concentrated load.

be multiplied by approximately 0.8 if the load is moving across the beam. The coefficient β_{mov} is shown in Fig. 9 for $k = 5$.

Shear Strength of the ASTM Specimen

Equation [2] may also be evaluated for the shear stress distribution in the shear block test specimen (ASTM 1973). A finite element analysis was carried out for this case and the integration of [2] was performed numerically from the finite element results. The calculation was carried out for several values of k, and the results may be summarized as follows:

$$[38] \qquad \tau_0 = \frac{\tau^*}{\beta_t}$$

where

$$[39] \qquad \begin{cases} \beta_t = 1.333 + 0.336\,(k - 4) & \text{if } 4 \leqslant k \leqslant 8 \\ \beta_t = 2.678 + 0.251\,(k - 8) & \text{if } 8 \leqslant k \leqslant 10 \end{cases}$$

If P is the applied load, τ_0 is defined as the average stress over the sheared area A_s:

$$[40] \qquad \tau_0 = P/A_s$$

Unit Volume Strength and Evaluation of Parameters *k* and *m*

It is very difficult to experimentally determine the unit volume strength τ^* under a situation of uniform shear. Equations [12], [20], [24], [33], and [38] can be used to predict the median strength of a particular beam configuration, if the value of unit volume strength $\tau^*[0.5]$ corresponding to a survival probability of 0.5 is used. Thus, the value $\tau^*[0.5]$ may be obtained by fitting the theoretical model to the results of experiments by considering the mean of failure stresses for a given configuration. If N such configurations are available, with V_i and τ_i being, respectively, the volume and the mean failure stress for the ith configuration, the parameters k and $\tau^*[0.5]$ can be found by minimization of the following sum of squares:

$$[41] \qquad S = \left(\tau_0 - \frac{\tau^*[0.5]}{\beta_t}\right)^2 + \sum_{i=1}^{N} \left(\tau_i - \beta_i \frac{\tau^*[0.5]}{V_i^{1/k}}\right)^2$$

where τ_0 is the mean strength of the ASTM specimen. The coefficient β_i must adopt the form corresponding to the test configuration, that is, β_s, β_U, β_{mov}, etc.

The expression of [41] is highly nonlinear in k, as this parameter enters in the coefficients β_i, for example. For any k, however, $\tau^*[0.5]$ corresponding to the minimum of [41] is given by

$$[42] \qquad \tau^*[0.5] = \left\{\sum_{i=1}^{N} \frac{\beta_i \tau_i}{V_i^{1/k}} + \frac{\tau_0}{\beta_t}\right\} \div \left\{\sum_{i=1}^{N} \left(\frac{\beta_i}{V_i^{1/k}}\right)^2 + \left(\frac{1}{\beta_t}\right)^2\right\}$$

The value of k corresponding to the minimum of [41] can be determined by an iterative procedure and $\tau^*[0.5]$ can then be computed from [42].

For a unit volume under uniform shear τ^*, [1] becomes

$$[43] \qquad F_V = 1 - e^{-(\tau^*/m)^k}$$

and, using $F_V = 0.5$ and $\tau^*[0.5]$, the value of m is found to be

$$[44] \qquad m = \frac{\tau^*[0.5]}{[-\ln(0.5)]^{1/k}}$$

Finally, using [44], the value of τ^* for a survival probability of 0.95 can be obtained from [43]. The result may be expressed as follows:

$$[45] \qquad \tau^*[0.95] = \tau^*[0.5]\left[\frac{\ln(0.95)}{\ln(0.5)}\right]^{1/k}$$

In obtaining the parameters $\tau^*[0.5]$ and k by least-squares fitting, it is important that a good estimate for the mean strength be available for each configuration tested. Thus, it is

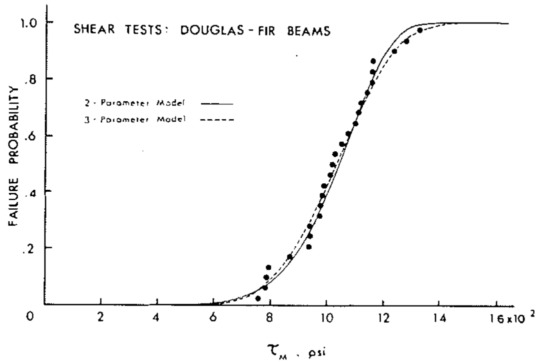

FIG. 10. Comparison of two and three parameter Weibull regressions with experimental beam test results.

necessary that a rather large sample be considered for each configuration and that, in the case of beams, most of the beams in the sample actually fail in shear. If only a few do so, while all others fail in bending, it is apparent that only the weaker portion of the shear population would have been tested.

The parameters k and m can also be obtained from a direct fitting of [1] to the experimental data for a given test configuration. Figure 10 shows such a fit for 27 tests with shear failures in a glued-laminated Douglas-fir beam with cross-sectional dimensions $b = 3.0$ in (0.076 m) and $d = 10.5$ in. (0.267 m). The beams were loaded with a symmetrical two-point loading system and the ratio a/d was 2.0. The broken curve corresponds to a simpler two-parameter model. It is apparent that, for the number of replications in this test, it is difficult to distinguish which of the two models gives the best fit of the data. In fact, from a practical point of view, stress levels at different probabilities of failure given by both models can be considered to be identical. The two-parameter model (solid curve) assumes zero minimum strength and therefore goes through the origin. The three-parameter model assumes a non-zero minimum strength and, therefore, the two models can give very different answers at small probabilities of failure. The differences are normally small at the 0.05 probability of failure level at which allowable stresses are referred.

To determine the parameters k and $\tau^*[0.5]$ by means of [41], five beam configurations were tested. Thirty replications were carried out for each configuration, for a total of 150 tests. All beams were loaded with a symmetrical two-point loading system and all had a ratio $a/d = 2.0$. All beams were loaded to collapse which occurred by either shear or bending. These tests were performed by the Structural Laboratory of the Civil Engineering Department of the University of Alberta (Longworth 1975). Table 1 shows the dimensions of each beam configuration, the beam volume under shear stresses, the number of shear failures obtained in the sample of 30 replications, and the mean shear failure stress.

The data from these beams were used in [41], together with the mean strength of the ASTM specimen for Douglas-fir at about 12% moisture content (Kennedy 1965), to determine the values of k and $\tau^*[0.5]$.

Equations [44] and [45] were used to determine m and $\tau^*[0.95]$. The results obtained for unit volumes of 1 in.³ and 1m³ are as follows:

$$[46]\quad \left\{\begin{array}{ll} V^* = 1\ \text{in}^3 & V^* = 1\ \text{m}^3 \\ k = 5.53 & k = 5.53 \\ m = 2700\ \text{psi} & m = 2540\ \text{kN/m}^2 \\ \tau^*[0.5] = 2526\ \text{psi} & \\ & \tau^*[0.5] = 2377\ \text{kN/m}^2 \\ \tau^*[0.95] = 1578\ \text{psi} & \\ & \tau^*[0.95] = 1485\ \text{kN/m}^2 \end{array}\right.$$

Figure 11 shows the correlation between mean test results and the theoretical strengths for a probability of survival of 0.5.

Having determined the parameter k of [46], the values of β_s, β_U, β_{mov} and α can now be evaluated for $k = 5.53$ by using the following regression results for I_a and $\bar{I}_a$ corresponding to $k = 5.53$:

$$[47]\quad \left\{\begin{array}{l} I_a[a/d] = \\ \qquad 0.354\,[1 - \exp(-0.139\,(a/d)^{1.753})] \\ \bar{I}_a[L/d] = \\ \qquad 0.0548\,[1 - \exp(-0.0013\,(L/d)^{2.641})] \end{array}\right.$$

These regression curves are plotted, respectively, in Figs. 2 and 5.

TABLE 1. Beam tests results (all glued-laminated Douglas-fir; moisture content = 12%)

Width (in.)	Depth (in.)	Length (in.)	Volume under shear (in.^{3})†	Number of shear failures	Mean failure shear stress (psi)*	Coefficient of variation
3.0	4.5	32.0	243.0	25/30	1280.4	0.109
3.0	10.5	68.0	1323.0	27/30	1027.1	0.145
8.75	10.5	68.0	3858.8	26/30	813.8	0.138
3.0	28.5	176.0	9747.0	24/30	684.2	0.137
5.0	28.5	176.0	16245.0	28/30	705.4	0.151

NOTE: 1 in. = 25.4 mm; 1 psi = 6.9 kN/m^2.
*Mean over the number of shear failures in the 30 replications.
†Volume = $2b\,d\,a$.

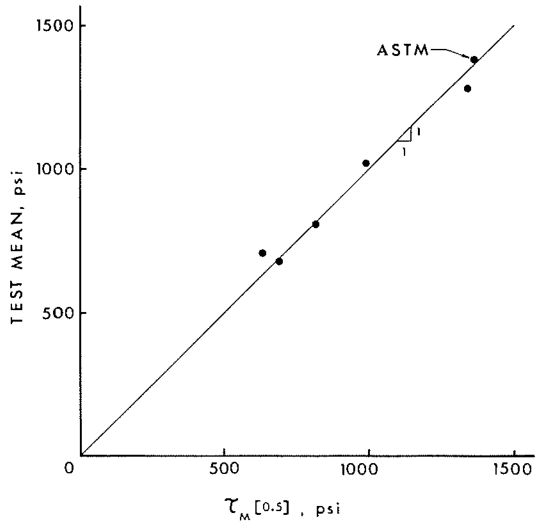

FIG. 11. Comparison of theoretical and test results.

Experimental Verification

The suitability of the theoretical model which incorporates the constants k = 5.53 and $\tau^*[0.5]$ = 2526 psi derived from the beam tests must be evaluated using other shear strength results. It is hypothesized that k and $\tau^*[0.5]$ are material constants which can be used to predict shear strength independently of shear stress distribution. A verification of the theoretical model can be obtained by examining strength data obtained from other tests where shear was the mode of failure.

For this purpose, results of Griplam nail connection, torque tube, and beam tests will be considered. An examination of failure modes of Griplam nail connections showed that failure occurred by nail bending or by wood failure in shear around the nail cluster (Foschi and Longworth 1975). Keenan and Selby (1973) and Madsen (1972) have used torque tubes to determine the shear strength of clear Douglas-fir. Good agreement between the predicted and measured mean strength of Griplam nail connections and the clear torque is shown in Fig. 12. Keenan and Selby (1973) also cite a number of references from which beam strength results were obtained for evaluating their shear area model. A comparison between the predicted strength using [24] and test values is also shown in Fig. 12. The test values are generally lower than the predicted strengths. The tests cited were generally designed to assess bending strength and, accordingly, less than 25% of beams tested actually failed in shear. Mean strength results from these tests must, therefore, be lower than would be observed in tests where the majority of the

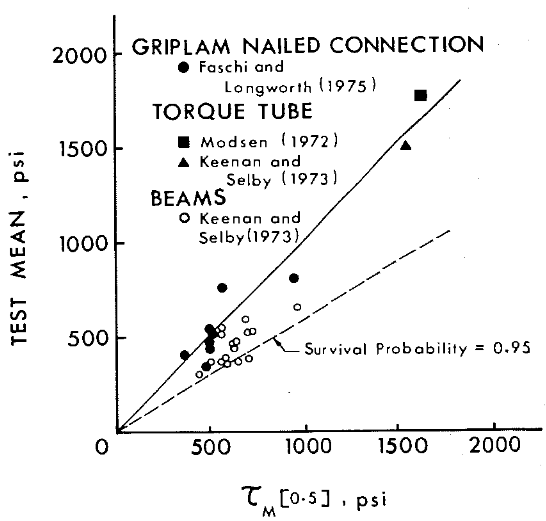

FIG. 12. Experimental verification of model.

beams failed in shear. However, the majority of these beam results do fall above the predicted strength level corresponding to a 0.95 survival probability.

Allowable Stresses

Allowable shear stresses in the case of a single concentrated load, for example, can be obtained by using [12] with $k = 5.53$ and $\tau^*[0.95]$ from [46]. The stress thus obtained will correspond to a 0.95 probability of survival, and allowable stresses can finally be obtained by dividing by a capacity factor $\phi_{0.95}$ to account for duration of load effects and the possibility of overload. Of course, the factor β_s in [12] should also be determined for $k = 5.53$.

It is possible to correlate the factors β_s, for example, corresponding to $k = 5.53$ and to $k = 5.0$. Thus, it is possible to use [12], [20], [24], and [33] with $k = 5.0$ and correct the factors to account for the fact that k is 5.53 and not 5.0. This correlation analysis was performed over 5616 different beam configurations, varying the width from 3.0 to 12 in. (0.076 to 0.305 m); the depth from 9 to 72 in. (0.229 to 1.830 m); the spans from 8 to 60 ft (2.44 to 18.29 m); and considering both uniformly distributed and single concentrated loads. In the latter case, the load was positioned at different points along the span. From all the different beam configurations thus obtained, it can be concluded that, with a high degree of linear correlation, ($r_2 = 0.996$),

[48] $$\beta_s[5.53]/V^{1/5.53} = \{0.116 + 1.125\,\beta_s[5.0]\}/V^{1/5.0}$$

where $\beta_s[5.53]$ corresponds to $k = 5.53$ and $\beta_s[5.0]$ corresponds to $k = 5.0$ and it is given in Fig. 3. The same relationship can be used for β_U, β_m, β_{mov} for [20], [24], and [33].

Allowable stresses for normal duration of load can thus be expressed as follows:

[49] $$\begin{cases} \tau_a = (0.116 + 1.125\,\beta) \times \dfrac{1578}{\phi_{0.95}\ \sqrt[5]{V}}\ \text{psi},\ (V \text{ in in.}^3) \\ \text{or } \tau_a = (0.094 + 0.911\,\beta) \times \dfrac{1485}{\phi_{0.95}\ \sqrt[5]{V}}\ \text{kN/m}^2,\ (V \text{ in m}^3) \end{cases}$$

V is the total beam volume and the coefficient β is given by

(1) $\beta = \beta_s$ of Fig. 3, single concentrated load;

(2) $\beta = \beta_U$ of Fig. 6, uniformly distributed load;

(3) $\beta = \beta_M$ of [23] for several concentrated loads;

(4) $\beta = \beta_{mov}$ of [33] and Fig. 9, for single moving load.

Very little information is available on duration of load effects in shear, but, as a first approximation, it would be reasonable to assume that the same factors apply for shear as for bending. Thus, a factor of 1.62 should be applied to reduce short-term strength to normal load-duration strength. If an overload factor of 1.3 is considered, a total reduction factor $\phi_{0.95} = 2.1$ should be taken to obtain, from short-term strength values, those applicable to normal duration of load.

Implications

The CSA-086 (1970) code recommends a uniform allowable shear stress of 165 psi for air-dry glued-laminated Douglas-fir. According to the results presented here, this allowable should vary with beam size and with type of loading. The following are examples illustrating the implications of the present study.

Consider a beam of the following dimensions: $L = 20$ ft $= 240$ in. (6.096 m); $b = 6$ in. (0.152 m); $d = 24$ in. (0.610 m); and $\phi_{0.95} = 2.1$. Thus, $L/d = 10$. Consider now the following load cases:

(1) *Single Concentrated Load, $a/L = 0.10$*

Using [9], [11], and [12] (or Fig. 3), $\beta_s = 2.80$ and thus, from [49], the allowable shear stress is $\tau_a = 303.52$ psi (2094.3 kN/m^2) for normal load duration. The allowable load is, therefore, $P_a = 32.4$ kips (144.2 kN).

(2) *Single Concentrated Load, $a/L = 0.30$*

$\beta_s = 1.69$, $\tau_a = 187.47$ psi (1293.5 kN/m^2). The allowable load is, therefore, $P_a = 25.7$ kips (114.4 kN).

(3) *Single Concentrated Load, $a/L = 0.50$*

$\beta_s = 1.25$, $\tau_a = 141.47$ psi (976.1 kN/m^2). The allowable load is: $P_a = 27.1$ kips (120.6 kN).

(4) *Worst Position for Single Concentrated Load*

For $L/d = 10$, from Fig. 4, $a/L = 0.30$ corresponds to the worst position for a single concentrated load. This result is verified by the calculations (1) through (3), as the minimum load P_a occurs for $a/L = 0.30$.

(5) *Single Concentrated Moving Load*

From Fig. 8, for $L/d = 10$, $\alpha = 0.80$. Thus, using the result of (2) and (4), $P_a = 0.80 \times 25.7$ kips $= 20.6$ kips (91.7 kN).

(6) *Uniformly Distributed Load*

From Fig. 6, $\beta_U = 2.076$, which, using [20], gives the allowable stress $\tau_a = 277.83$ psi (1572 kN/m^2).

The total allowable load is, therefore, $P_a = 43.7$ kips (194.5 kN).

Conclusions

The distribution of shear stresses in beams near their supports has been studied and the stress analysis has been used in conjunction with a brittle-fracture model developed by Weibull to predict the probability of failure of beams of different geometry and under different loading conditions. It has been shown that this model allows a rational interpretation of size effects in shear, as well as of the difference in strength values between small shear block and beam tests. Allowable stresses have been derived for different loading conditions, and the predictions of the model have been verified with test results on Griplam nailed connections and torque tubes. It is apparent, therefore, that the model presented allows the calculation of ultimate short-term shear strength values for very different structural components. This unifying characteristic is an encouraging result, and it represents a feature previously lacking in timber design.

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