A COMPARISON OF FRAME STABILITY ANALYSIS METHODS IN ANSI/AISC 360-05

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ANSI/AISC 360-05 *Specification for Structural Steel Buildings* (AISC, 2005a) includes three prescriptive approaches for stability analysis and design. Table 2-1 in the 13th Edition AISC *Steel Construction Manual* (AISC, 2005b) provides a comparison of the methods and design options associated with each. A fourth approach, referred to as the Simplified Method, is also presented in the Manual (see page 2-12) and on the AISC *Basic Design Values* cards. These four methods are illustrated in this paper in order to give the reader a general understanding of the differences between them:

1. The Second-Order Analysis Method (Section C2.2a)
2. The First-Order Analysis Method (Section C2.2b)
3. The Direct Analysis Method (Appendix 7)
4. The Simplified Method (Manual page 2-12; AISC *Basic Design Values* cards)

Two simple unbraced frames are used in this paper. The one-bay frame shown in Figure 1 has a rigid roof element spanning between a flagpole column (Column A) and leaning column (Column B). Drift is not limited for this frame, which results in a higher ratio of second-order drift to first-order drift, and allows illustration of the detailed requirements in each method for the calculation of K-factors, notional loads, and required and available strengths. The three-bay frame shown in Figure 2 has rigid roof elements spanning between two flagpole columns (Columns D and E) and two leaning columns (Columns C and F). This frame is used with a drift limit of $L/400$ to illustrate the simplifying effect a drift limit can have on the analysis requirements in each method.

Although these example frames are not realistic frames, the results obtained are representative of the impact of second-order elastic and inelastic effects on strength requirements in real frames, particularly when the number of moment connections is reduced. The loads shown in Figures 1 and 2 are from the controlling LRFD load combination and the corresponding designs are performed using LRFD. The process is essentially identical for ASD, where ASD load combinations are used with $\alpha = 1.6$ as a multiplier, when required in each method, to account for the second-order effects at the ultimate load level.

When it is required to include second-order effects, the B_1-B_2 amplification is used with a first-order analysis throughout this paper. A direct second-order analysis is straightforward and could have been used instead of the B_1-B_2 amplification.

THE ONE-BAY FRAME

A trial shape is selected using a first-order analysis without consideration of drift limits or second-order effects. Thereafter, that trial shape is used as the basis for comparison of the four methods discussed above.

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Selection of Trial Shape Based Upon Strength Consideration Only

Based upon the loading shown in Figure 1, the first-order axial force, strong-axis moment, and design parameters for Column A are:

$P_u = 200$ kips $M_{ux} = (20 \text{ kips})(15 \text{ ft}) = 300$ kip-ft
$K_x = 2.0$, $K_y = 1.0$ $C_b = 1.67$
$L_x = L_y = 15$ ft $L_b = 15$ ft

Note that $K_x = 2.0$, the theoretical value for a column with a fixed base and pinned top, is used rather than the value of 2.1 recommended for design in AISC 360-05 Commentary Table C-C2.2. The value of 2.0 is used because it is consistent with the formulation of the lateral stiffness calculation below. Note also that the impact of the leaning column on K_x is ignored in selecting the trial size, although it will be considered in subsequent sections when K_x cannot be taken equal to 1 for Column A. Out of the plane of the frame, K_y is taken as 1.0.

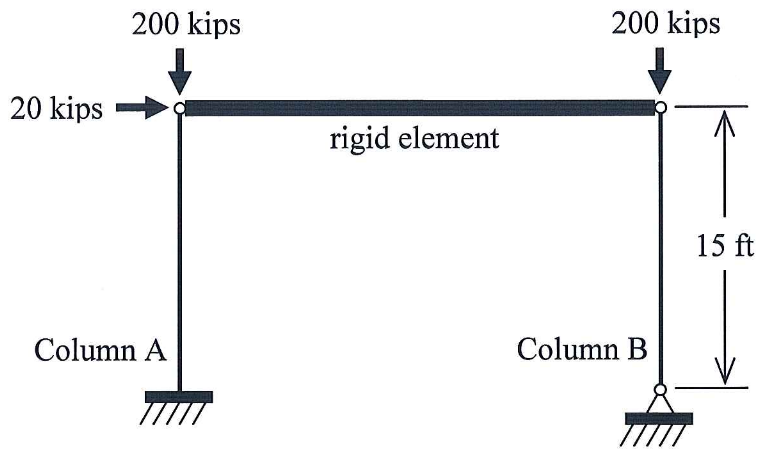

Figure 1. One-bay unbraced frame used in examples.

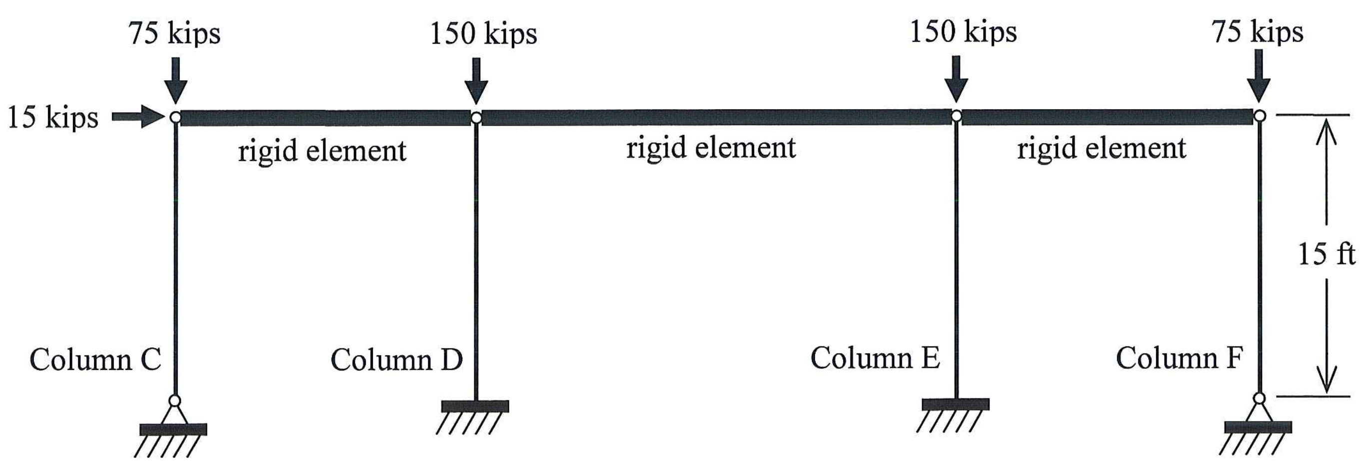

Figure 2. Three-bay unbraced frame used in examples.

A simple rule of thumb for trial beam-column selection is to use an equivalent axial force equal to P_u plus $24/d$ times M_u, where d is the nominal depth of the column (Geschwindner et al., 1994). Using d = 14 in. for a W14, the equivalent axial force is 714 kips and an ASTM A992 W14×90 is selected as the trial shape.

The lateral stiffness of the frame depends upon Column A only and is:

$$k = 3EI / L^3 = 3\ (29{,}000\ \text{ksi})\ (999\ \text{in.}^4) / (15\ \text{ft} \times 12\ \text{in./ft})^3 = 14.9\ \text{kips/in.}$$

The corresponding first-order drift of the frame is:

$$\Delta_{1st} = (20\ \text{kips}) / (14.9\ \text{kips/in.}) = 1.34\ \text{in.}$$

Note that this is a very flexible frame with $\Delta_{1st} / L = 1.34 / (15\ \text{ft} \times 12\ \text{in./ft}) = 1/134$.

Design by Second-Order Analysis (Section C2.2a)

Design by second-order analysis is essentially the traditional effective length method with an additional requirement for a minimum lateral load. It is permitted when the ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, is equal to or less than 1.5, and requires the use of:

1. A direct second-order analysis or a first-order analysis with B_1-B_2 amplification.
2. The nominal frame geometry with a minimum lateral load (a "notional load") N_i = $0.002Y_i$, where Y_i is the total gravity load on level i from LRFD load combinations (or 1.6 times ASD load combinations). This notional load is specified to capture the effects of initial out-of-plumbness up to the AISC *Code of Standard Practice* maximum value of 1:500. In this method, N_i is not applied when the actual lateral load is larger than the calculated notional load.
3. The nominal stiffnesses EA and EI.
4. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier on ASD load combinations ensures that the drift level is consistent for LRFD and ASD when determining second-order effects. The forces and moments obtained in this analysis are then divided by 1.6 for ASD member design.

When the ratio of second-order drift to first-order drift, which is given by B_2, is equal to or less than 1.1, K = 1.0 can be used in the design of moment frames. Otherwise, for moment frames, K is determined from a sidesway buckling analysis. Section C2.2a(4) indicates that for braced frames, K = 1.0.

For the example frame given in Figure 1, the minimum lateral load based upon the total gravity load, Y_i, is:

$$Y_i = 200\ \text{kips} + 200\ \text{kips} = 400\ \text{kips}$$
$$N_i = 0.002\ Y_i = 0.002\ (400\ \text{kips}) = 0.8\ \text{kips}$$

Because this notional load is less than the actual lateral load, it need not be applied. For a load combination that did not include a lateral load, the notional load would need to be included in the analysis.

For Column A, using first-order analysis and B_1-B_2 amplification:

$P_{nt} = 200$ kips, $P_{lt} = 0$ kips
$M_{nt} = 0$ kip-ft, $M_{lt} = 300$ kip-ft

For P-δ amplification, since there are no moments associated with the no-translation case, there is no need to calculate B_1. For P-Δ amplification, the first-order drift ratio is determined from the calculated drift of 1.34 in. Thus,

$$\Delta_{1st} / L = (1.34 \text{ in.}) / (15 \text{ ft} \times 12 \text{ in./ft}) = 0.00744$$

For moment frames, $R_m = 0.85$ and from Equation C2-6b with $\Delta_H = \Delta_{1st}$ and $\Sigma H = 20$ kips,

$$\Sigma P_{e2} = R_m \Sigma H / (\Delta_{1st} / L) = 0.85 \text{ (20 kips)} / (0.00744) = 2{,}280 \text{ kips}$$

For design by LRFD, $\alpha = 1.0$ and ΣP_{nt} is the sum of the gravity loads. Thus,

$$\alpha \Sigma P_{nt} / \Sigma P_{e2} = 1.0 \text{ (200 kips + 200 kips)} / 2{,}280 \text{ kips} = 0.175$$

From Equation C2-3, the amplification is:

$$\begin{aligned} B_2 &= 1 / (1 - \alpha \Sigma P_{nt} / \Sigma P_{e2}) \geq 1.0 \\ &= 1 / (1 - 0.175) \geq 1.0 \\ &= 1.21 \end{aligned}$$

Since $B_2 = 1.21$, the second-order drift is less than 1.5 times the first-order drift. Thus, the use of this method is permitted. Since $B_2 > 1.1$, K cannot be taken as 1.0 for column design in the moment frame with this method. Thus, K must be calculated, including the leaning-column effect. Several approaches are available in the Commentary to include this effect. A simple approach that uses the ratio of the load on the leaning columns to the load on the stabilizing columns had been provided in previous Commentaries and is used here (Lim and McNamara, 1972):

$$\Sigma P_{leaning} / \Sigma P_{stability} = \text{(200 kips) / (200 kips)} = 1$$

$$K_x^* = K_x (1 + \Sigma P_{leaning} / \Sigma P_{stability})^{1/2} = 2.0 (1 + 1)^{1/2} = 2.83$$

The amplified axial force (Equation C2-1b) and associated design parameters for this method are:

$P_r = P_{nt} + B_2 P_{lt} = 200$ kips $+ 1.21$ (0 kips) $= 200$ kips
$K_x^* = 2.83$, $K_y = 1.0$
$L_x = L_y = 15$ ft

The amplified moment (Equation C2-1a) and associated design parameters for this method are:

$M_{rx} = B_1 M_{nt} + B_2 M_{lt}$ = (0 kip-ft) + 1.21 (300 kip-ft) = 363 kip-ft
$C_b = 1.67$
$L_b = 15$ ft

Based upon these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14x90 are:

$P_c = \phi_c P_n = 721$ kips
$M_{cx} = \phi_b M_{nx} = 573$ kip-ft

To determine which interaction equation is applicable, the ratio of the axial required strength to axial available strength must be determined.

$$\frac{P_r}{P_c} = \frac{200 \text{ kips}}{721 \text{ kips}} = 0.277 \geq 0.2$$

Thus, Equation H1-1a is applicable.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.277 + \frac{8}{9}\left(\frac{363 \text{ kip-ft}}{573 \text{ kip-ft}}\right) = 0.840$$

The W14×90 is adequate since $0.840 \leq 1.0$.

Design by First-Order Analysis (Section C2.2b)
The first-order analysis method is permitted when:

1. The ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, is equal to or less than 1.5.
2. The column axial force $\alpha P_r \leq 0.5 P_y$, where $\alpha = 1.0$ for LRFD, 1.6 for ASD.

This method requires the use of:

1. A first-order analysis.
2. The nominal frame geometry with an additional lateral load $N_i = 2.1(\Delta/L)Y_i \geq 0.0042 Y_i$, applied in all load cases.
3. The nominal stiffnesses EA and EI.
4. B_1 as a multiplier on the total moment in beam-columns.
5. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier on ASD load combinations ensures that the drift level is consistent for LRFD and ASD when determining the notional loads. The forces and moments obtained in this analysis are then divided by 1.6 for ASD member design.

For all frames designed with this method, $K = 1.0$.

For the example frame given in Figure 1, the additional lateral load is based on the first-order drift ratio, Δ/L, and the total gravity load, Y_i. Thus, with $\Delta = \Delta_{1st}$,

$$\Delta_{1st} / L = (1.34 \text{ in.}) / (15 \text{ ft} \times 12 \text{ in./ft}) = 0.00744$$
$$Y_i = 200 \text{ kips} + 200 \text{ kips} = 400 \text{ kips}$$
$$\begin{aligned} N_i &= 2.1(\Delta_{1st} / L)Y_i \geq 0.0042Y_i \\ &= 2.1\ (0.00744)\ (400 \text{ kips}) \geq 0.0042\ (400 \text{ kips}) \\ &= 6.25 \text{ kips} \end{aligned}$$

It was previously determined in the illustration of design by second-order analysis example that the second-order drift is less than 1.5 times the first-order drift. Additionally,

$$\alpha P_r = 1.0\ (200 \text{ kips}) = 200 \text{ kips}$$

and

$$0.5P_y = 0.5F_yA_g = 0.5\ (50 \text{ ksi})\ (26.5 \text{ in.}^2) = 663 \text{ kips}$$

Since $\Delta_{2nd} < 1.5\Delta_{1st}$ and $\alpha P_r < 0.5P_y$, the use of this method is permitted.

The loading for this method is the same as that shown in Figure 1, except for the addition of a notional load of 6.25 kips coincident with the lateral load of 20 kips shown, resulting in a column moment M_u of 394 kip-ft.

This moment must be amplified by B_1 as determined from Equation C2-2. The Euler buckling load is calculated with $K_1 = 1.0$. Thus,

$$P_{e1} = \pi^2 EI / (K_1 L)^2 = \pi^2\ (29{,}000 \text{ ksi})\ (999 \text{ in.}^4) / (1.0 \times 15 \text{ ft} \times 12 \text{ in./ft})^2 = 8{,}830 \text{ kips}$$

Since the moment on one end of the column is zero, the moment gradient term is:

$$C_m = 0.6 - 0.4\ (M_1 / M_2) = 0.6 - 0.4\ (0 / 394) = 0.6$$

From Equation C2-2,

$$\begin{aligned} \alpha P_r / P_{e1} &= 1.0\ (200 \text{ kips}) / (8{,}830 \text{ kips}) \\ &= 0.0227 \end{aligned}$$

$$\begin{aligned} B_1 &= C_m / (1 - \alpha P_r / P_{e1}) \geq 1.0 \\ &= 0.6 / (1 - 0.0227) \geq 1.0 \\ &= 0.614 \geq 1.0 \end{aligned}$$

Thus,

$$B_1 = 1.0$$

The axial force and associated design parameters for this method are:

$P_r = 200$ kips
$K_x = K_y = 1.0$
$L_x = L_y = 15$ ft

The amplified moment and associated design parameters for this method are:

$M_{rx} = B_1 M_u = 1.0$ (394 kip-ft) = 394 kip-ft
$C_b = 1.67$
$L_b = 15$ ft

Based upon these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14x90 are:

$P_c = \phi_c P_n = 1{,}000$ kips
$M_{cx} = \phi_b M_{nx} = 573$ kip-ft

To determine which interaction equation is applicable, the ratio of the axial required strength to axial available strength must be determined.

$$\frac{P_r}{P_c} = \frac{200 \text{ kips}}{1{,}000 \text{ kips}} = 0.200 \geq 0.2$$

Thus, Equation H1-1a is applicable.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.200 + \frac{8}{9}\left(\frac{394 \text{ kip-ft}}{573 \text{ kip-ft}}\right) = 0.811$$

The W14×90 is adequate since $0.811 \leq 1.0$.

Design by Direct Analysis (Appendix 7)

The direct analysis method is permitted for any ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, and required when this ratio exceeds 1.5. It requires the use of:

1. A direct second-order analysis or a first-order analysis with B_1-B_2 amplification.
2. The nominal frame geometry with an additional lateral load of $N_i = 0.002Y_i$, where Y_i is the total gravity load on level i from LRFD load combinations, or 1.6 times ASD load combinations.
3. The reduced stiffnesses EA^* and EI^* (including in B_1-B_2 amplification, if used).
4. LRFD load combinations, or ASD load combinations multiplied by 1.6. This multiplier ensures that the drift level is consistent for LRFD and ASD when determining second-order effects. The forces and moments obtained in this analysis are then divided by 1.6 for ASD member design.

The following exceptions apply as alternatives in item 2 above:

a. If the out-of-plumb geometry of the structures is used, the notional loads can be omitted.
b. When the ratio of second-order drift to first-order drift is equal to or less than 1.5, the notional load can be applied as a minimum lateral load, not an additional lateral load. Note that the unreduced stiffnesses, EA and EI, are used in this comparison.
c. When the actual out-of-plumbness is known, it is permitted to adjust the notional loads proportionally.

For all frames designed with this method, $K = 1.0$.

It was previously determined in the illustration of design by second-order analysis example that the second-order drift is less than 1.5 times the first-order drift (note that this check is properly made using the unreduced stiffnesses, EA and EI). Thus, the notional load can be applied as a minimum lateral load, and that minimum is:

$$Y_i = 200 \text{ kips} + 200 \text{ kips} = 400 \text{ kips}$$
$$N_i = 0.002\, Y_i = 0.002\,(400 \text{ kips}) = 0.8 \text{ kips}$$

Because this notional load is less than the actual lateral load, it need not be applied. For a load combination that did not include a lateral load, the notional load would need to be included in the analysis.

For Column A, using first-order analysis and B_1-B_2 amplification:

$$P_{nt} = 200 \text{ kips},\ P_{lt} = 0 \text{ kips}$$
$$M_{nt} = 0 \text{ kip-ft},\ M_{lt} = 300 \text{ kip-ft}$$

To determine the second-order amplification, the reduced stiffness, EI^*, must be calculated.

$$\alpha P_r = 1.0\,(200 \text{ kips}) = 200 \text{ kips}$$

and

$$0.5P_y = 0.5F_yA_g = 0.5\,(50 \text{ ksi})\,(26.5 \text{ in.}^2) = 663 \text{ kips}$$

Thus, since $\alpha P_r < 0.5P_y$, $\tau_b = 1.0$ and

$$EI^* = 0.8\tau_b EI = 0.8EI$$

For P-δ amplification, since there are no moments associated with the no-translation case, there is no need to calculate B_1. For P-Δ amplification, the reduced stiffness EI^* must be used to determine the first-order drift. Since $EI^* = 0.8EI$, the first-order drift based upon EI^* is 25 percent larger than that calculated previously. Thus,

$\Delta_{1st} = 1.25\ (1.34\ \text{in.}) = 1.68\ \text{in.}$

The first-order drift ratio is determined from the amplified drift of 1.68 in.

$\Delta_{1st} / L = (1.68\ \text{in.}) / (15\ \text{ft} \times 12\ \text{in./ft}) = 0.00933$

For moment frames, $R_m = 0.85$ and from Equation C2-6b with $\Delta_H = \Delta_{1st}$ and $\Sigma H = 20$ kips,

$\Sigma P_{e2} = R_m \Sigma H / (\Delta_{1st} / L) = 0.85\ (20\ \text{kips}) / (0.00933) = 1{,}820\ \text{kips}$

For design by LRFD, $\alpha = 1.0$ and ΣP_{nt} is the sum of the gravity loads. Thus,

$\alpha \Sigma P_{nt} / \Sigma P_{e2} = 1.0\ (200\ \text{kips} + 200\ \text{kips}) / 1{,}820\ \text{kips} = 0.220$

From Equation C2-3, the amplification is:

$$\begin{aligned} B_2 &= 1 / (1 - \alpha \Sigma P_{nt} / \Sigma P_{e2}) \geq 1.0 \\ &= 1 / (1 - 0.220) \geq 1.0 \\ &= 1.28 \end{aligned}$$

It is worth noting that use of the reduced axial stiffness, $EA^* = 0.8EA$, in members that contribute to lateral stability is also required in this method. However, due to the characteristics of the structures chosen for this example, there are no axial deformations that impact the stability of the structure.

The amplified axial force (Equation C2-1b) and associated design parameters for this method are:

$$\begin{aligned} &P_r = P_{nt} + B_2 P_{lt} = 200\ \text{kips} + 1.28\ (0\ \text{kips}) = 200\ \text{kips} \\ &K_x = K_y = 1.0 \\ &L_x = L_y = 15\ \text{ft} \end{aligned}$$

The amplified moment (Equation C2-1a) and associated design parameters for this method are:

$$\begin{aligned} &M_{rx} = B_1 M_{nt} + B_2 M_{lt} = (0\ \text{kip-ft}) + 1.28\ (300\ \text{kip-ft}) = 384\ \text{kip-ft} \\ &C_b = 1.67 \\ &L_b = 15\ \text{ft} \end{aligned}$$

Based upon these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14x90 are:

$$\begin{aligned} &P_c = \phi_c P_n = 1{,}000\ \text{kips} \\ &M_{cx} = \phi_b M_{nx} = 573\ \text{kip-ft} \end{aligned}$$

To determine which interaction equation is applicable, the ratio of the axial required strength to axial available strength must be determined.

$$\frac{P_r}{P_c} = \frac{200 \text{ kips}}{1{,}000 \text{ kips}} = 0.200 \geq 0.2$$

Thus, Equation H1-1a is applicable.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.200 + \frac{8}{9}\left(\frac{384 \text{ kip-ft}}{573 \text{ kip-ft}}\right) = 0.796$$

The W14×90 is adequate since $0.796 \leq 1.0$.

The Simplified Method

This method is provided in the AISC *Basic Design Values* Cards and the 13th Edition Steel Construction Manual (AISC 2005b), and excerpted as shown in Figure 3. This simplified method is derived from the effective length method (Design by Second-Order Analysis; Section C2.2a) using B_1-B_2 amplification with B_1 taken equal to B_2. Note that the user note in Section C2.1b says that B_1 may be taken equal to B_2 as long as B_1 is less than 1.05. However, it is also conservative to take B_1 equal to B_2 any time B_1 is less than B_2. Although it cannot universally be stated that B_1 is always equal to or less than B_2, this is the case for typical framing. It is left to engineering judgment to confirm that this criterion is satisfied when applying the simplified method.

This method is permitted when the ratio of second-order drift, Δ_{2nd}, to first-order drift, Δ_{1st}, is equal to or less than 1.5 as with the Design by Second-Order Analysis method. It allows the use of a first-order analysis based upon nominal stiffnesses, *EA* and *EI*, with a minimum lateral load $N_i = 0.002Y_i$, where Y_i is the total gravity load on level *i* from LRFD load combinations or ASD load combinations. The 1.6 multiplier on ASD load combinations is not used at this point but its effect is included in the determination of the amplification multiplier upon entering the table.

The ratio of total story gravity load (times 1.0 in LRFD, 1.6 in ASD) to the story lateral load is used to enter the table in Figure 3. The second-order amplification multiplier is determined from the value in the table corresponding to the calculated load ratio and design story drift limit. While linear interpolation between tabular values is permitted, it is important to note that the tabular values have, in essence, only two significant digits. Accordingly, the value determined should not be calculated to more than one decimal place. The tabular value is used to amplify all forces and moments in the analysis.

When the ratio of second-order drift to first-order drift is equal to or less than 1.1, $K = 1.0$ can be used in the design of moment frames. Otherwise, for moment frames, K is determined from a sidesway buckling analysis. For braced frames, $K = 1.0$.

For the example frame given in Figure 1, the minimum lateral load is:

$Y_i = 200 \text{ kips} + 200 \text{ kips} = 400 \text{ kips}$
$N_i = 0.002\ Y_i = 0.002\ (400 \text{ kips}) = 0.8 \text{ kips}$

Simplified Method

Step 1. Perform first-order analysis. Use 0.2% of total story gravity load as minimum lateral load in all load combinations.
Step 2. Establish the design story drift limit and determine the lateral load required to produce it.
Step 3. Determine the ratio of the total story gravity load to the lateral load determined in Step 2. For ASD, multiply by 1.6.
Step 4. Multiply first-order results by the tabular value. K=1, except for moment frames when the tabular value is greater than 1.1.

Design Story Drift Limit	Ratio from Step 3 (times 1.6 for ASD, 1.0 for LRFD)										
	0	5	10	20	30	40	50	60	80	100	120
H/100	1	1.1	1.1	1.3	1.4				When ratio exceeds 1.5, simplified method requires a stiffer structure.		
H/200	1	1	1.1	1.1	1.2	1.3	1.3	1.4			
H/300	1	1	1	1.1	1.1	1.2	1.2	1.3	1.4	1.5	
H/400	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.3	1.4
H/500	1	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.3

K=1

Figure 3. Simplified Method from AISC *Basic Design Values* Cards

Because this notional load is less than the actual lateral load, it need not be applied. For a load combination that did not include a lateral load, the notional load would need to be included in the analysis.

The actual first-order drift of the trial frame corresponds to a drift ratio of $L/134$ and the load ratio is:

$$1.0 \times (200 \text{ kips} + 200 \text{ kips}) / (20 \text{ kips}) = 20$$

Entering the table in the column for a load ratio of 20, the corresponding multiplier for a drift ratio of $H/134$ is 1.3 (determined by interpolation to one decimal place). Since this multiplier is less than 1.5, $\Delta_{2nd} < 1.5\Delta_{1st}$ and the use of this method is permitted. However, since the multiplier is greater than 1.1, K cannot be taken as 1.0 for column design in the moment frame with this method. Thus, K must be calculated, including the leaning column effect. Using the same approach as previously discussed (Lim and McNamara, 1972):

$$\Sigma P_{leaning} / \Sigma P_{stability} = (200 \text{ kips}) / (200 \text{ kips}) = 1$$

$$K_x{}^* = K_x\,(1 + \Sigma P_{leaning} / \Sigma P_{stability})^{1/2} = 2.0\,(1 + 1)^{1/2} = 2.83$$

The amplified axial force (with the full axial force amplified by B_2) and associated design parameters for this method are:

$$P_r = 1.3P_u = 1.3\,(200 \text{ kips}) = 260 \text{ kips}$$
$$K_x{}^* = 2.83,\ K_y = 1.0$$
$$L_x = L_y = 15 \text{ ft}$$

The amplified moment (with the full moment amplified by B_2) and associated design parameters for this method are:

$M_{rx} = 1.3M_u = 1.3$ (300 kip-ft) = 390 kip-ft
$C_b = 1.67$
$L_b = 15$ ft

Based upon these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14x90 are:

$P_c = \phi_c P_n = 721$ kips
$M_{cx} = \phi_b M_{nx} = 573$ kip-ft

To determine which interaction equation is applicable, the ratio of the axial required strength to axial available strength must be determined.

$$\frac{P_r}{P_c} = \frac{260 \text{ kips}}{721 \text{ kips}} = 0.361 \geq 0.2$$

Thus, Equation H1-1a is applicable.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.361 + \frac{8}{9}\left(\frac{390 \text{ kip-ft}}{573 \text{ kip-ft}}\right) = 0.966$$

The W14×90 is adequate since $0.966 \leq 1.0$.

Summary for the One-Bay Frame
All methods illustrated in the foregoing sections produce similar designs. The results are tabulated below for comparison, where the result of the beam-column interaction equation are given for each method. A lower interaction equation result for the same column shape signifies a prediction of higher strength.

Method	**Interaction Equation**
Second-Order	0.840
First-Order	0.811
Direct Analysis	0.796
Simplified	0.966

In this example, the direct analysis method predicts the highest strength, while the simplified method predicts the lowest strength. This would be expected since the Direct Analysis Method was developed as the most accurate approach while the simplified method was developed to produce a quick yet conservative solution.

The designs compared above are based upon strength with no consideration of drift limitation, except to the extent that the actual drift impacts the magnitude of the second-order effects. The usual drift limits of approximately $L/400$ will necessitate framing members and configurations with more lateral stiffness than this frame provides. Hence, the designer may find that a frame configured for drift first will often require no increase in member size for strength, including second-order effects. This will be explored further with the three-bay frame.

THE THREE-BAY FRAME

For the frame shown in Figure 2, a trial shape is selected using a first-order drift limit of $L/600$ under a service level lateral load of 10 kips. Thereafter, that trial shape is used as the basis for comparison of the four methods used previously for the one-bay frame.

Selection of Trial Shape Based Upon the Drift Limit Only

For the dimensions shown in Figure 2:

$$L/600 = (15 \text{ ft} \times 12 \text{ in./ft}) / 600 = 0.300 \text{ in.}$$

The lateral stiffness of the frame depends upon Columns D and E only and is:

$$k = 2 \times 3EI / L^3 = 2 \times 3 \text{ (29,000 ksi) } (I) / (15 \text{ ft} \times 12 \text{ in./ft})^3 = 0.0298(I)$$

With the service level lateral load on the frame of 10 kips:

$$0.0298(I) \geq (10 \text{ kips}) / (0.300 \text{ in.})$$

Thus, I_{req} = 1,120 in.^{4} and an ASTM A992 W14×109 is selected as the trial shape with I_x = 1,240 in.^{4}.

The actual lateral stiffness of the frame is:

$$k = 2 \times 3EI / L^3 = 2 \times 3 \text{ (29,000 ksi) (1,240 in.}^4) / (15 \text{ ft} \times 12 \text{ in./ft})^3 = 37.0 \text{ kips/in.}$$

The corresponding first-order drift of the frame under the LRFD lateral load of 15 kips is:

$$\Delta_{1st} = (15 \text{ kips}) / (37.0 \text{ kips/in.}) = 0.405 \text{ in.}$$

The first-order axial force, strong-axis moment, and design parameters for Columns D and E are:

$P_u = 150$ kips	$M_{ux} = (15 \text{ kips}) (15 \text{ ft}) / 2 = 113$ kip-ft
$K_x = 2.0$, $K_y = 1.0$	$C_b = 1.67$
$L_x = L_y = 15$ ft	$L_b = 15$ ft

Note that K_x = 2.0, the theoretical value for a column with a fixed base and pinned top, is used rather than the value of 2.1 recommended for design in AISC 360-05 Commentary Table C-C2.2. The value of 2.0 is used because it is consistent with the formulation of the lateral stiffness calculation below. Note also that the impact of the leaning column on K_x is ignored in selecting the trial size, although it will be considered in subsequent sections when K_x cannot be taken equal to 1 for Column A. Out of the plane of the frame, K_y is taken as 1.0.

Design by Second-Order Analysis (Section C2.2a)

For the example frame given in Figure 2, the minimum lateral load is:

Y_i = 75 kips + 150 kips + 150 kips + 75 kips = 450 kips
$N_i = 0.002\ Y_i$ = 0.002 (450 kips) = 0.9 kips

Because this notional load is less than the actual lateral load, it need not be applied.

For Columns D and E, using first-order analysis and B_1-B_2 amplification:

P_{nt} = 150 kips, P_{lt} = 0 kips
M_{nt} = 0 kip-ft, M_{lt} = 113 kip-ft

For P-δ amplification, since there are no moments associated with the no-translation case, there is no need to calculate B_1. For P-Δ amplification, the first-order drift ratio is determined from the calculated drift of 0.405 in. Thus,

Δ_{1st} / L = (0.405 in.) / (15 ft × 12 in./ft) = 0.00225

For moment frames, R_m = 0.85 and from Equation C2-6b with $\Delta_H = \Delta_{1st}$ and ΣH = 15 kips,

$\Sigma P_{e2} = R_m \Sigma H / (\Delta_{1st} / L)$ = 0.85 (15 kips) / (0.00225) = 5,670 kips

For design by LRFD, α = 1.0 and ΣP_{nt} is the sum of the gravity loads. Thus,

$\alpha \Sigma P_{nt} / \Sigma P_{e2}$ = 1.0 (75 kips + 150 kips + 150 kips + 75 kips) / 5,670 kips = 0.0794

From Equation C2-3, the amplification is:

$B_2 = 1 / (1 - \alpha \Sigma P_{nt} / \Sigma P_{e2}) \geq 1.0$
$= 1 / (1 - 0.0794) \geq 1.0$
$= 1.09$

Since B_2 = 1.09, the second-order drift is less than 1.5 times the first-order drift. Thus, the use of this method is permitted. Since $B_2 < 1.1$, K can be taken as 1.0 for column design in the moment frame with this method.

The amplified axial force (Equation C2-1b) and associated design parameters for this method are:

$P_r = P_{nt} + B_2 P_{lt}$ = 150 kips + 1.09 (0 kips) = 150 kips
$K_x = K_y = 1.0$
$L_x = L_y$ = 15 ft

The amplified moment (Equation C2-1a) and associated design parameters for this method:

$M_{rx} = B_1 M_{nt} + B_2 M_{lt}$ = (0 kip-ft) + 1.09 (113 kip-ft) = 123 kip-ft
$C_b = 1.67$

$L_b = 15$ ft

Based upon these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14x109 are:

$P_c = \phi_c P_n = 1{,}220$ kips
$M_{cx} = \phi_b M_{nx} = 720$ kip-ft

To determine which interaction equation is applicable, the ratio of the axial required strength to axial available strength must be determined.

$$\frac{P_r}{P_c} = \frac{150 \text{ kips}}{1{,}220 \text{ kips}} = 0.123 < 0.2$$

Thus, Equation H1-1b is applicable.

$$\frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} = \frac{0.123}{2} + \frac{123 \text{ kip-ft}}{720 \text{ kip-ft}} = 0.232$$

The W14×109 is adequate since $0.232 \leq 1.0$.

Design by First-Order Analysis (Section C2.2b)
For the example frame given in Figure 2, the additional lateral load (with $\Delta = \Delta_{1st}$) is:

$\Delta_{1st} / L = (0.405 \text{ in.}) / (15 \text{ ft} \times 12 \text{ in./ft}) = 0.00225$
$Y_i = 75 \text{ kips} + 150 \text{ kips} + 150 \text{ kips} + 75 \text{ kips} = 450 \text{ kips}$
$N_i = 2.1(\Delta_{1st} / L)Y_i \geq 0.0042Y_i$
$= 2.1\ (0.00225)\ (450 \text{ kips}) \geq 0.0042\ (450 \text{ kips})$
$= 2.13$ kips

It was previously determined in the illustration of design by second-order analysis example that the second-order drift is less than 1.5 times the first-order drift. Additionally,

$\alpha P_r = 1.0\ (150 \text{ kips}) = 150$ kips

and

$0.5P_y = 0.5F_yA_g = 0.5\ (50 \text{ ksi})\ (32.0 \text{ in.}^2) = 800$ kips

Since $\Delta_{2nd} < 1.5\Delta_{1st}$ and $\alpha P_r < 0.5P_y$, the use of this method is permitted.

The loading for this method is the same as shown in Figure 2, except for the addition of a notional load of 2.13 kips coincident with the lateral load of 15 kips shown, resulting in a moment M_u of 128 kip-ft in each column.

This moment must be amplified by B_1 as determined from Equation C2-2. The Euler buckling load is calculated with $K_1 = 1.0$. Thus,

$$P_{e1} = \pi^2 EI / (K_1 L)^2 = \pi^2 \text{ (29,000 ksi) (1,240 in.}^4\text{) / (1.0} \times \text{15 ft} \times \text{12 in./ft)}^2 = \text{11,000 kips}$$

Since the moment on one end of the column is zero, the moment gradient term is:

$$C_m = 0.6 - 0.4\,(M_1 / M_2) = 0.6 - 0.4\,(0 / 128) = 0.6$$

From Equation C2-2,

$$\alpha P_r / P_{e1} = 1.0 \text{ (150 kips) / (11,000 kips)}$$
$$= 0.0136$$

$$B_1 = C_m / (1 - \alpha P_r / P_{e1}) \geq 1.0$$
$$= 0.6 / (1 - 0.0136) \geq 1.0$$
$$= 0.608 \geq 1.0$$

Thus,

$$B_1 = 1.0$$

The axial force and associated design parameters for this method are:

$$P_r = 150 \text{ kips}$$
$$K_x = K_y = 1.0$$
$$L_x = L_y = 15 \text{ ft}$$

The amplified moment and associated design parameters for this method are:

$$M_{rx} = B_1 M_u = 1.0 \text{ (128 kip-ft)} = 128 \text{ kip-ft}$$
$$C_b = 1.67$$
$$L_b = 15 \text{ ft}$$

Based upon these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14x109 are:

$$P_c = \phi_c P_n = 1{,}220 \text{ kips}$$
$$M_{cx} = \phi_b M_{nx} = 720 \text{ kip-ft}$$

To determine which interaction equation is applicable, the ratio of the axial required strength to axial available strength must be determined.

$$\frac{P_r}{P_c} = \frac{150 \text{ kips}}{1{,}220 \text{ kips}} = 0.123 < 0.2$$

Thus, Equation H1-1b is applicable.

$$\frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} = \frac{0.123}{2} + \frac{128 \text{ kip-ft}}{720 \text{ kip-ft}} = 0.239$$

The W14×109 is adequate since 0.239 ≤ 1.0.

Direct Analysis Method (Appendix 7)
It was previously determined in the illustration of design by second-order analysis example that the second-order drift is less than 1.5 times the first-order drift (note that this check is properly made using the unreduced stiffness *EI*). Thus, the notional load can be applied as minimum lateral load, and that minimum is:

Y_i = 75 kips + 150 kips + 150 kips + 75 kips = 450 kips
$N_i = 0.002\ Y_i$ = 0.002 (450 kips) = 0.9 kips

Because this notional load is less than the actual lateral load, it need not be applied.

For Columns D and E, using first-order analysis and B_1-B_2 amplification:

P_{nt} = 150 kips, P_{lt} = 0 kips
M_{nt} = 0 kip-ft, M_{lt} = 113 kip-ft

To determine the second-order amplification, the reduced stiffness, EI^*, must be calculated.

αP_r = 1.0 (150 kips) = 150 kips

and

$0.5P_y = 0.5F_yA_g$ = 0.5 (50 ksi) (32.0 in.^{2}) = 800 kips

Thus, since $\alpha P_r < 0.5P_y$, $\tau_b = 1.0$ and

$$EI^* = 0.8\tau_b EI = 0.8EI$$

For P-δ amplification, since there are no moments associated with the no-translation case, there is no need to calculate B_1. For P-Δ amplification, the reduced stiffness EI^* must be used to determine the first-order drift. Since $EI^* = 0.8EI$, the first-order drift based upon EI^* is 25 percent larger than that calculated previously. Thus,

Δ_{1st} = 1.25 (0.405 in.) = 0.506 in.

The first-order drift ratio is determined from the amplified drift of 0.506 in.

Δ_{1st} / L = (0.506 in.) / (15 ft x 12 in./ft) = 0.00281

For moment frames, $R_m = 0.85$ and from Equation C2-6b with $\Delta_H = \Delta_{1st}$ and $\Sigma H = 15$ kips,

$$\Sigma P_{e2} = R_m \Sigma H / (\Delta_{1st} / L) = 0.85\ (15\ \text{kips}) / (0.00281) = 4{,}540\ \text{kips}$$

For design by LRFD, $\alpha = 1.0$ and ΣP_{nt} is the sum of the gravity loads. Thus,

$$\alpha \Sigma P_{nt} / \Sigma P_{e2} = 1.0\ (75\ \text{kips} + 150\ \text{kips} + 150\ \text{kips} + 75\ \text{kips}) / 4{,}540\ \text{kips} = 0.0991$$

From Equation C2-3, the amplification is:

$$\begin{aligned} B_2 &= 1 / (1 - \alpha \Sigma P_{nt} / \Sigma P_{e2}) \geq 1.0 \\ &= 1 / (1 - 0.0991) \geq 1.0 \\ &= 1.11 \end{aligned}$$

It is worth noting that use of the reduced axial stiffness, $EA^* = 0.8EA$, in members that contribute to lateral stability is also required in this method. However, due to the characteristics of the structures chosen for this example, there are no axial deformations that impact the stability of the structure.

The amplified axial force (Equation C2-1b) and associated design parameters for this method are:

$$\begin{aligned} P_r &= P_{nt} + B_2 P_{lt} = 150\ \text{kips} + 1.11\ (0\ \text{kips}) = 150\ \text{kips} \\ K_x &= K_y = 1.0 \\ L_x &= L_y = 15\ \text{ft} \end{aligned}$$

The amplified moment (Equation C2-1a) and associated design parameters for this method are:

$$\begin{aligned} M_{rx} &= B_1 M_{nt} + B_2 M_{lt} = (0\ \text{kip-ft}) + 1.11\ (113\ \text{kip-ft}) = 125\ \text{kip-ft} \\ C_b &= 1.67 \\ L_b &= 15\ \text{ft} \end{aligned}$$

Based upon these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14x109 are:

$$\begin{aligned} P_c &= \phi_c P_n = 1{,}220\ \text{kips} \\ M_{cx} &= \phi_b M_{nx} = 720\ \text{kip-ft} \end{aligned}$$

To determine which interaction equation is applicable, the ratio of the axial required strength to axial available strength must be determined.

$$\frac{P_r}{P_c} = \frac{150\ \text{kips}}{1{,}220\ \text{kips}} = 0.123 < 0.2$$

Thus, Equation H1-1b is applicable.

$$\frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} = \frac{0.123}{2} + \frac{125 \text{ kip-ft}}{720 \text{ kip-ft}} = 0.235$$

The W14×109 is adequate since $0.235 \leq 1.0$.

The Simplified Method

For the example frame given in Figure 2, the minimum lateral load is:

Y_i = 75 kips + 150 kips + 150 kips + 75 kips = 450 kips
$N_i = 0.002\ Y_i$ = 0.002 (450 kips) = 0.9 kips

Because this notional load is less than the actual lateral load, it need not be applied.

The 15-kip lateral load produces slightly less drift than that corresponding to the design story drift limit because the W14×109 has I = 1,240 in.^{4} (versus the 1,120 in.^{4} required to limit drift to L/400). The lateral load required to produce the design story drift limit is:

15 kips × (1,240 in.^{4}) / (1,120 in.^{4}) = 16.6 kips

The load ratio is then:

1.0 × (75 kips + 150 kips + 150 kips + 75 kips) / (16.6 kips) = 27.1

Entering the table in the row for H/400, the corresponding multiplier for a load ratio of 27.1 is 1.1 (determined by interpolation to one decimal place). Since this multiplier is less than 1.5, $\Delta_{2nd} < 1.5\Delta_{1st}$ and the use of this method is permitted. Additionally, since the multiplier is equal to 1.1, K can be taken as 1.0 for column design in the moment frame with this method.

The amplified axial force (with the full axial force amplified by B_2) and associated design parameters for this method are:

$P_r = 1.1P_u$ = 1.1 (150 kips) = 165 kips
$K_x = K_y = 1.0$
$L_x = L_y$ = 15 ft

The amplified moment (with the full moment amplified by B_2) and associated design parameters for this method are:

$M_{rx} = 1.1M_u$ = 1.1 (113 kip-ft) = 124 kip-ft
$C_b = 1.67$
L_b = 15 ft

Based upon these design parameters, the axial and strong-axis flexural available strengths of the ASTM A992 W14x109 are:

$P_c = \phi_c P_n = 1{,}220$ kips
$M_{cx} = \phi_b M_{nx} = 720$ kip-ft

To determine which interaction equation is applicable, the ratio of the axial required strength to axial available strength must be determined.

$$\frac{P_r}{P_c} = \frac{165 \text{ kips}}{1{,}220 \text{ kips}} = 0.135 < 0.2$$

Thus, Equation H1-1b is applicable.

$$\frac{P_r}{2P_c} + \frac{M_{rx}}{M_{cx}} = \frac{0.135}{2} + \frac{124 \text{ kip-ft}}{720 \text{ kip-ft}} = 0.240$$

The W14×109 is adequate since $0.240 \leq 1.0$.

Summary for the Three-Bay Frame
As before, all methods produce similar designs. The result of the beam-column interaction equation for each method is:

Method	Interaction Equation
Second-Order	0.232
First-Order	0.239
Direct Analysis	0.235
Simplified	0.240

In this example, the interaction equations predict values that are so close to each other that there is no practical difference in the results.

CONCLUSIONS

The following conclusions can be drawn from the foregoing examples:

1. If conservative assumptions are acceptable, the easiest method to apply is the Simplified Method, particularly when the drift limit is such that K can be taken equal to 1.
2. None of the analysis methods in AISC 360-05 is particularly difficult to use. The First-Order Analysis Method and Direct Analysis Method both eliminate the need to calculate K, which can be a tedious process based upon assumptions that are rarely satisfied in real structures. Nonetheless, those who prefer to continue to use the approach of past specifications, the Effective Length Method, can do so provided they incorporate the new requirement of a minimum lateral load in all load combinations.
3. Second-order effects and leaning columns have a significant impact on strength requirements, but usual drift limits such as $L/400$ sometimes can result in framing that requires no increase in member size for strength. For frames with little or no lateral load and/or heavy floor loading, it is more likely that stability will control, regardless of the drift

limits. This should not be taken as a blanket indication that the use of a drift limit eliminates the need to consider stability effects. Rather, it simply means that drift-controlled designs may be less sensitive to second-order effects because the framing is naturally stiffer and provides reserve strength. Drift limits also result in significant simplification of the analysis requirements when the increased framing stiffness allows more frequent use of the simplifications allowed in the various methods, such as the use of $K = 1$.

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