D-c System Short-circuit Current Calculations

Dec. 3, 1956

SOURCES OF SHORT-CIRCUIT CURRENT

The sources of short-circuit current in direct-current systems include motors, generators, rectifiers, batteries, electrolytic cells, and synchronous converters.

The short-circuit characteristics of motors, generators, rectifiers, batteries and synchronous converters are presented in preceding sections. The characteristics in each case include equivalent circuits to represent the particular source when calculating the initial rate of rise of the current and the maximum short-circuit current.

The characteristics for electrolytic cells are not firmly established at this time.

BUS SHORT-CIRCUIT CURRENT CALCULATION

The calculation of the short-circuit current for a bus fault can generally be done by considering each source individually, (neglecting the other sources entirely) constructing a current-time curve for each source, and adding the curves graphically to obtain the total short-circuit current. This method can be used when the bus is considered to have negligible resistance and inductance, since the short-circuit current from one source has no effect on the other sources. The short circuit itself is always assumed to have zero resistance and inductance.

The information presented in the preceding sections is adequate to make this calculation since each source of short-circuit current is considered individually.

This method of calculating the total short-circuit current, i.e., by graphic summation of the individual currents, is also applicable when calculating the short-circuit duty on a feeder circuit breaker. The resistance and inductance of the feeder circuit breaker are neglected when calculating the duty on that circuit breaker and, therefore, the calculation of the short-circuit current is usually the same as for a bus fault.

The above discussion of the calculation of bus short-circuit current is applicable when the bus is considered to have negligible resistance and inductance. This would be the case for a switchgear bus or a short, open bus. In some cases the direct-current system in a plant does not have an actual bus or if there is a bus it may be quite long. In these cases the bus short-circuit current cannot be calculated by considering each source individually. A procedure such as described in the following paragraphs for a feeder short-circuit current calculation will have to be used.

FEEDER SHORT-CIRCUIT CURRENT CALCULATION

A short circuit on a feeder will result in the current from all of the sources flowing together through some of the same system elements. Referring to Fig. 1, for the indicated short-circuit location the currents from all three sources must flow through a common circuit element which in this case is the feeder from the bus to the short-circuit location. Since the currents must share a common path, the calculation of the currents for a feeder short circuit must be a system calculation where the entire system is taken into account at all times in the calculation. This calculation is best performed with the aid of two system diagrams: an inductance diagram and a resistance diagram.

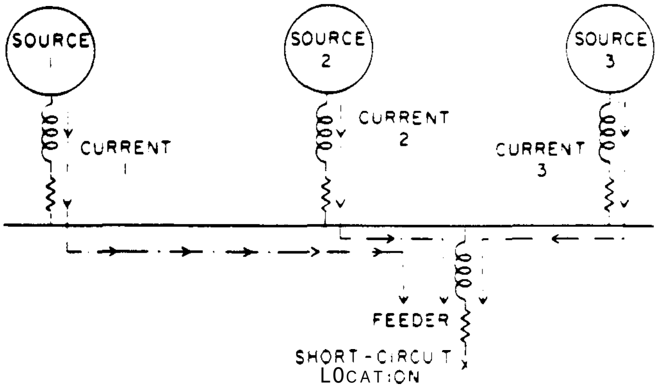

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Fig. 1. Diagram for a feeder short circuit showing the currents sharing the path through the common circuit element.

Resistance Diagram

The resistance diagram for a direct-current system is similar to a system one-line diagram and shows all of the system resistances. The resistance diagram is shown in Fig. 3 for the very simple direct-current system shown in one-line diagram form in Fig. 2. For simplicity only one feeder circuit is indicated. The values for the internal resistances of the sources of short-circuit current are determined on the basis of the short-circuit characteristics presented in preceding sections. Resistances of the other system components are determined from a knowledge of their size and composition.

This diagram is used to calculate the maximum short-circuit current for a short circuit at any point in the system. The resistance diagram is handled in the same manner as the reactance diagram in an a-c short-circuit study. The resistances can be combined in parallel or series until one equivalent system resistance is determined to represent the system from the point of short circuit back to the voltage source.

The total maximum short-circuit current is then calculated by using this equivalent system resistance in the following expression.

$$I_T = \frac{E}{R_{eq}} \text{ amperes where} \qquad (1)$$

E = System voltage (volts)
R_{eq} = Equivalent system resistance (ohms)
I_T = Total maximum short-circuit current (amperes)

The current in each branch of the system can be calculated in the same manner as the branch currents are calculated in an alternating current impedance or reactance diagram.

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The calculation of the maximum short-circuit current can also be performed by using a superposition method instead of resolving the resistance diagram to one equivalent resistance as described above. The superposition method normally requires more calculating than would be involved in obtaining one equivalent resistance and, therefore, the method outlined above is usually used. A possible exception is when a power rectifier is one of the sources of short-circuit current. In this case, with the rectifier equivalent circuit having a voltage in it equal to the system voltage, the rectifier internal resistance is a variable and a trial and error solution is required to obtain an accurate value for the maximum short-circuit current. Use of the superposition method of calculating will allow a fixed value to be used for the rectifier internal resistance with a voltage higher than normal in the rectifier equivalent circuit behind this resistance. Thus, for the case where a power rectifier is included, the calculation may be easier to perform by the superposition method since it eliminates a trial and error calculation to obtain an accurate answer. The superposition method of calculating is described further in the example presented later.

The long, tedious task of resolving the system resistances or using the superposition method for a complete study of a large d-c system can be eliminated by studying the system on a d-c calculating board.

Inductance Diagram

The inductance diagram for the direct-current system is also similar to a one-line diagram and shows all of the inductances in the system. The inductance diagram is shown in Fig. 4 for the system of Fig. 2. The inductance values for the sources of short-circuit currents can be determined from the characteristics given in the preceding sections and the inductances of the other system components must be determined from a knowledge of their physical arrangement and length.

The inductance diagram can be handled in the same manner as the resistance diagram or the reactance diagram in an a-c short-circuit study. The inductances can be combined as parallel or series elements until one equivalent inductance is obtained to represent the entire system from the point of short circuit back to the voltage source.

This equivalent system inductance is then used to calculate the initial rate of rise of the total short-circuit current from the expression,

$$\frac{di_T}{dt} = \text{Rate of rise of total current} = \frac{E}{L_{eq}} \text{ (amperes per second)} \quad (2)$$

where E = System voltage (volts)

L_{eq} = Equivalent system inductance (henries)

The initial rate of rise of the current in each branch of the system can be determined from the inductance diagram in the same manner that the branch currents are calculated in a resistance network or the alternating currents are calculated in the branches of an impedance diagram.

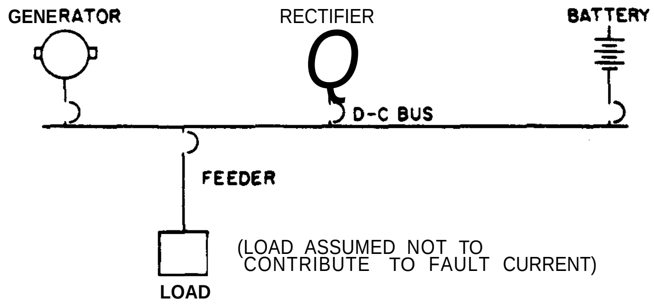

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fig. 2. One-line diagram of simple d-c system.

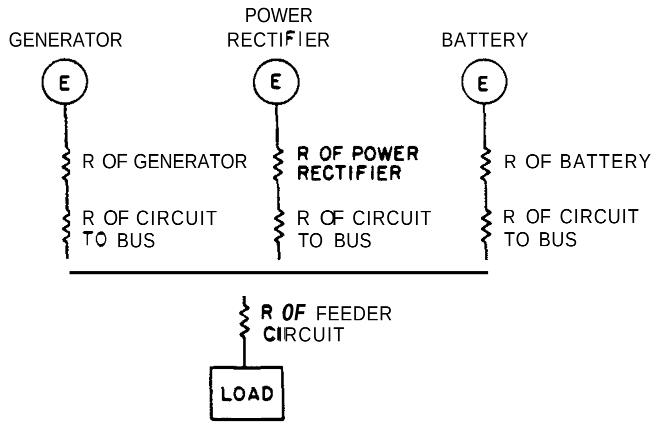

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Fig. 3. Resistance diagram for d-c system of Fig. 2.

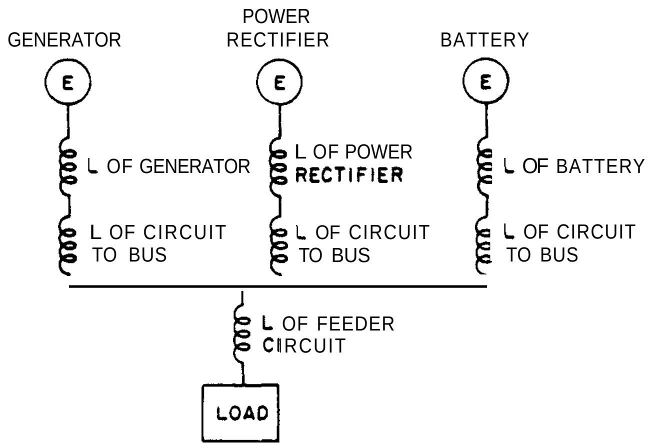

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fig. 4. Inductance diagram for d-c system of Fig. 2.

The inductance diagram can also be studied on a d-c calculating board to eliminate the longhand resolution of the system if desired. It should be noted that the study of the inductance diagram on the d-c calculating board is analogous to the study of the resistance diagram. In the case of the resistance diagram the resistances which are plugged on the d-c board are miniature representations of the actual sys-

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tem resistances and the board currents are miniature replicas of the actual system currents. In the case of the inductance diagram the resistances which are plugged on the d-c board are miniature replicas of the actual system inductances and, therefore, the magnitude of the board currents represent the magnitude of the rate of rise of the actual system currents.

This analogy can be further illustrated by noting that the equations for calculating rate of rise of current and maximum current are quite similar. That is:

$$\frac{di}{dt} = \text{Rate of rise} = \frac{E}{L}$$

$$I = \text{Current magnitude} = \frac{E}{R}$$

Current-Time Curve

Based on the maximum current, determined from the resistance diagram, and the initial rate of rise of the current, determined from the inductance diagram, the approximate current-time curve for the total short-circuit current can be constructed. This approximate curve is based on the assumption that the total current will have a current-time curve which can be represented by a simple exponential curve.

A simple exponential curve for the current in a d-c circuit composed of a voltage source, a resistance, and an inductance will have an initial slope as determined by equation (2), a maximum value as determined by equation (1), and a time constant equal to the ratio of inductance to resistance.

The time constant is given by the expression:

$$T_T = \text{Time constant for total current} = \frac{L_{eq}}{R_{eq}} \text{(sec)} \quad \textbf{(3)}$$

The time constant of a simple exponential is the time, after initiation of the transient, at which the current is equal to **63.2** percent of the maximum value. Therefore, the total short-circuit current will have a value equal to **63.2** percent of $\frac{E}{R_{eq}}$ at a time of $\frac{L_{eq}}{R_{eq}}$ seconds after the short-circuit occurs. Also, at a time equal to two time constants the current will have a magnitude equal to approximately **87** percent of the maximum value.

Thus the current-time curve for the total short-circuit current, Fig. **5**, can be constructed from a knowledge of the maximum current (equation 1), the initial rate of rise (equation **2**), and the time constant (equation 3).

Interpretation of Calculation

The current-time curve for the total short-circuit current, if constructed in the manner described in the previous paragraphs, will accurately represent the actual total short-circuit current when the parallel branches in the system all have the same ratio of inductance to resistance (commonly called L/R ratio). Referring to Fig. 2, this means that the L R ratios for the generator circuit, rectifier circuit, and battery circuit must be the same. If these ratios are all the same and the circuit parameters all remain constant the current-time curve of the total current will be a simple exponential type curve.

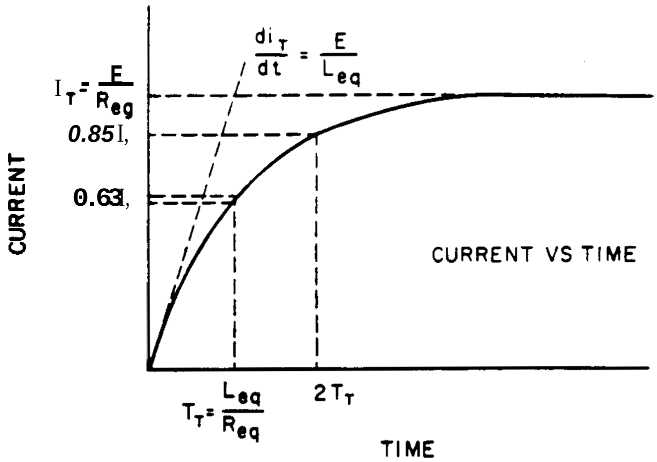

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Fig. 5. Typical time-current curve for feeder short circuit.

Different L/R ratios in the various parallel branches of the system will result in different rates of current build-up in the branches and the total current will not follow a simple exponential curve. The total current curve will then be **a** complex exponential curve and its precise calculation is quite difficult. In the usual case the current-time curve calculated by assuming the current to follow a simple exponential curve will be as accurate as is required.

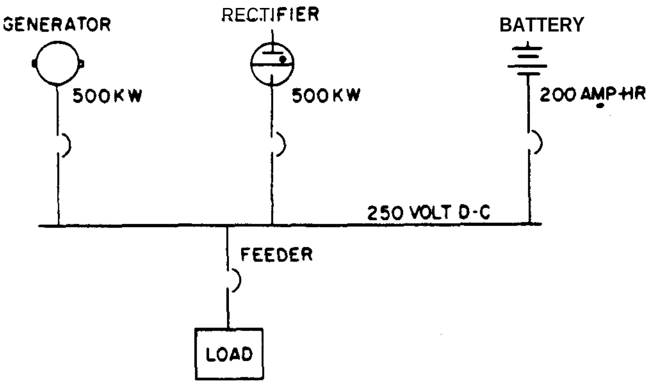
Branch Currents

The current-time curve for the current in each branch circuit will also follow a simple exponential curve when the L/R ratios for the parallel branches are all the same. This simple exponential curve will have an initial rate of rise as calculated from the inductance diagram and will have a maximum value as determined from the resistance diagram.

When the L/R ratios of the parallel branches are not the same the currents in the branch circuits will not follow simple exponential curves. Dissimilar L R ratios means that the currents in the various branches tend to build up at different rates and thus the interaction between the currents will cause the current-time curves to be complex exponential curves.

The effect of dissimilar L R ratios can be illustrated by considering a simple circuit composed of two sources of short-circuit current and one feeder, with the short circuit located at the end of the feeder. Assume one source to have a high L **R** ratio and the other source to have a low L R ratio.

D-c System Short-circuit Current Calculations



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System Data:

De Generator—500 kw; 250-volt; 2000 amperes; 600-rpm, constant-speed; shunt-wound; no pole-face windings.

Power Rectifier—500 kw; 250-volt; 2000 amperes; double-wye connected transformer secondary; 60-cycle a-c system; a-c impedance (z_a) = 0.076 per unit; a-c impedance (Z_a) = 0.022 ohms; a-c reactance (X_a) = 0.0217 ohms; a-c resistance (R_a) = 0.0036 ohms.

Battery—200 ampere-hour on 8 hr. boria; 25 amperes on 8 hr. basis; 250-volt; inductance of battery cell connections = 0.04 millihenries; resistance of battery cell connections = 0.01 ohms.

Load—Assumed to bo of o typo which cannot contribute to the short-circuit current.

Generator Circuit—Resistance = 0.0003 ohms; Inductance = 0.004 millihonrir.

Rectifier Circuit—Resistance = 0.0003 ohms; Inductance = 0.004 millihenries.

Battery Circuit—Resistance = 0.05 ohms; Inductance = 0.016 millihonrir.

Feeder Circuit—Rristonco = 0.005 ohms; Inductance = 0.010 millihenries.

Fig. 6. One-line diagram and system data for example calculation.

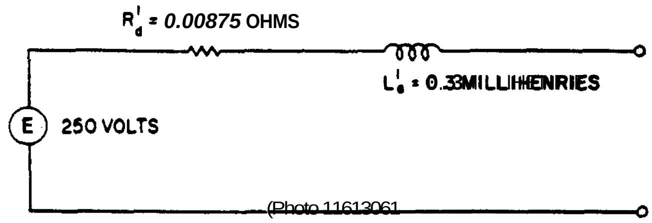

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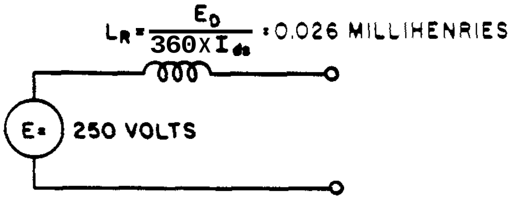
Fig. 7. Equivalent circuit for 500-kw generator.



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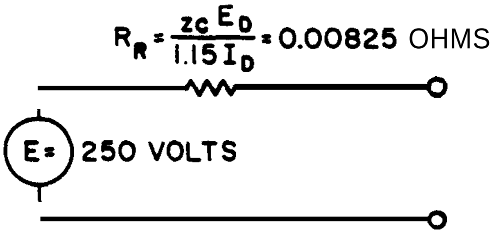
a. (for calculating initial rate of rise)



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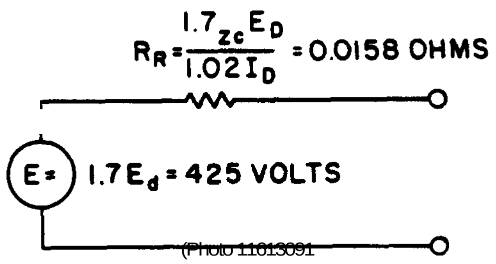
b. (for approximate calculation of maximum current)



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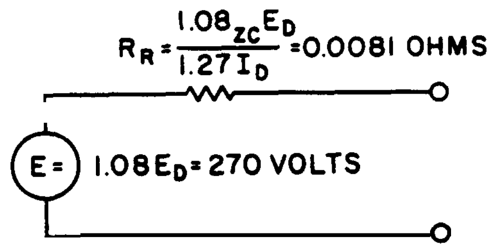
c. (for superposition mothod of calculating maximum current when tho current is expected to exceed 19,750 amperes)



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d. (for superposition method of calculating maximum current when tho current is expected to bo less thon 19,750 amperes)

Fig. 8. Equivalent circuits for 500-kw rectifier.

The source with the low L/R ratio would try to force its current into the fault quickly while the source with the high L/R ratio would feed current into the short circuit at a slower rate. However, the low L/R source, in quickly forcing its current, would build up a voltage in the common circuit element (the feeder) which would cause the high L/R source to feed current into the short circuit at an even slower rate than would be indicated by its L/R ratio alone. The net result of this interaction between the currents is that the current from the low L/R source would have a current-time curve with a hump in it and the current from the high L/R source would have a current-time curve with a dip in it.

The degree of deviation in the branch circuit current-time curves depends on the degree of difference between the L/R ratios. If it becomes necessary to develop the complete current-time curve for the branch circuits a judicious attempt must be made to estimate the effect of dissimilar L/R ratios.

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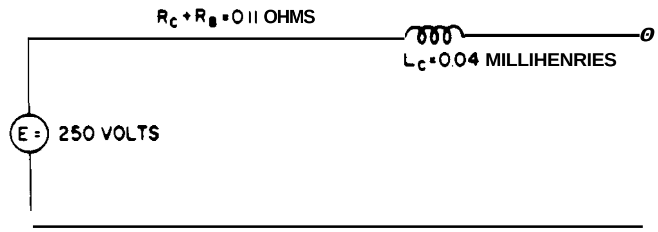

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Fig. 9. Equivalent circuit for battery.

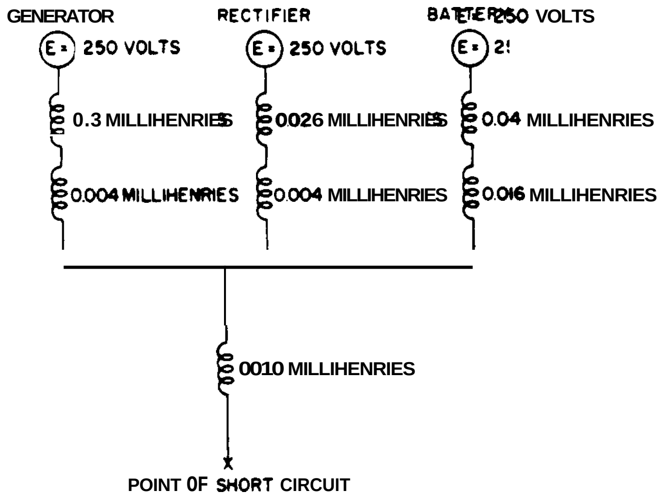

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Fig. 10. Inductance diagram for example calculation.

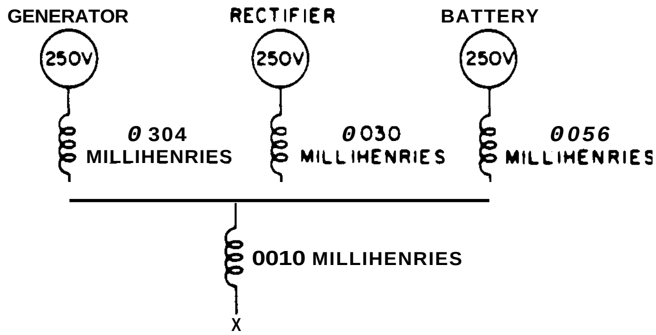
EXAMPLE SYSTEM SHORT-CIRCUIT CURRENT CALCULATION

Assume a simple direct-current system as shown in one-line diagram form in Fig. 6. It is desired to calculate the current-time curve for the short-circuit current which is available at the point where the load is connected to the feeder. In this example the load is assumed to be incapable of contributing to the short-circuit current; however, the contribution of motors to the short-circuit current should always be included in an actual system study.

Based on the data given in Fig. 6 the equivalent circuits for the generator, rectifier, and battery are as shown in Fig. 7, 8, and 9 respectively. Reference should be made to the appropriate Section (.171, .172, or .173) for the procedure involved in determining these equivalent circuits.

In Fig. 7, R_d' (the effective transient armature circuit resistance in ohms) is equal to r_d' (the effective transient armature circuit resistance in per unit) multiplied by the system voltage divided by the rated armature current of the generator, or $R_d' = r_d' \frac{E}{I_{gen}}$

In Fig. 8, equivalent circuit (a) is to be used for calculating the initial rate of rise of the short-circuit current and circuits (b), (c), and (d) are for calculating the maximum short-circuit current,



(a)

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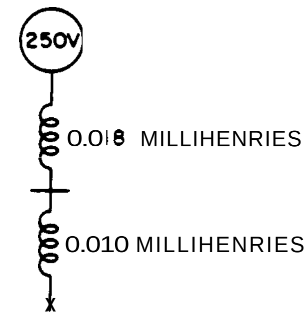

(b)

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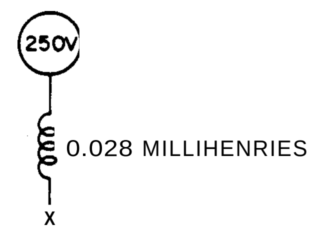

(c)

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Fig. 11. Simplification of inductance diagram.

Calculation of Initial Rate of Rise of Short-circuit Current

The inductance diagram for this system is shown in Fig. 10. The resolution of the inductance diagram into one equivalent inductance is performed in several steps. Adding the internal inductance and the circuit inductance (series elements) of each short-circuit current source results in the diagram shown in Fig. 11(a). Paralleling the inductances of the three sources results in the diagram of Fig. 11(b) and adding the series elements gives the one equivalent inductance shown in Fig. 11(c).

The initial rate of rise of the current is then:

$$\frac{di_T}{dt} = \frac{250 \text{ volts}}{0.028 \text{ millihenries}} = 8.9 \times 10^6 \text{ smperes second}$$

D-c System Short-circuit Current Calculations

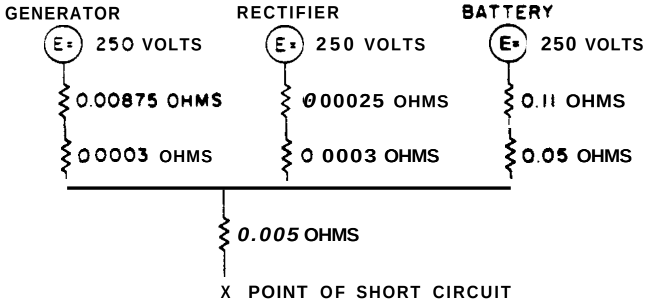

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fig. 12. Resistance diagram for example calculation when current is calculated by approximate mothod.

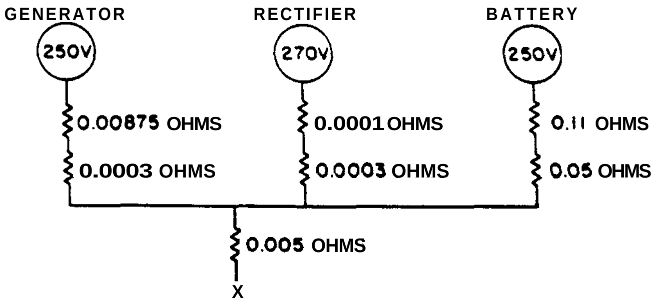

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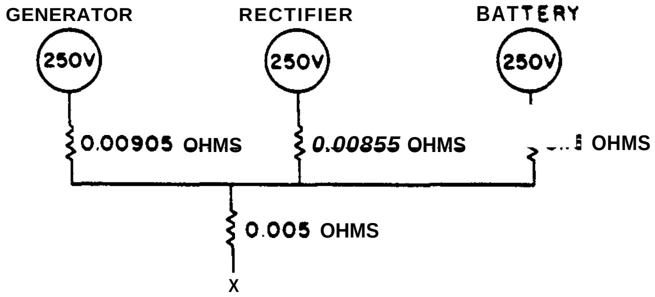
Fig. 14. Resistance diagram for superposition method of calculating the current in the example.



(a)

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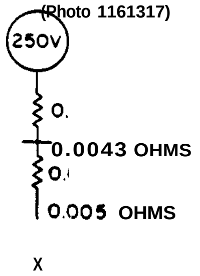

(b)

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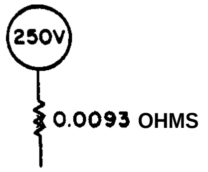

(c)

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fig. 13. Simplification of resistance diagram of Fig. 12.

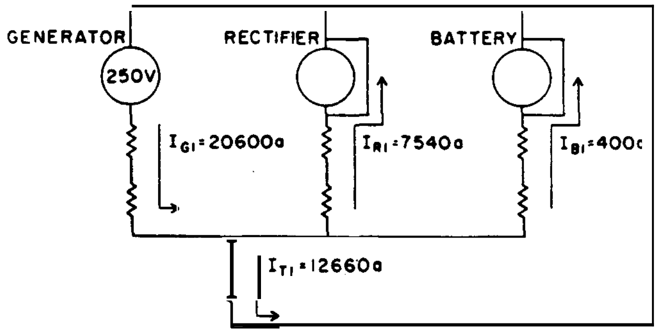

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Fig. 15(a). Step in superposition mothod.

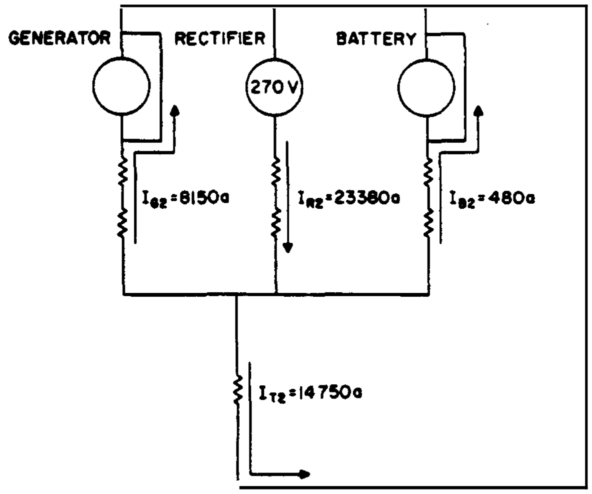

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Fig. 15(b). Step in superposition method.

Calculation of Maximum Short-circuit Current

USING APPROXIMATE VALUE FOR RECTIFIER RESISTANCE

The resistance diagram for the system is shown in Fig. 12. Note that the circuit used for the rectifier is the equivalent circuit of Fig. 8(b).

Adding the series elements in each source circuit results in the diagram of Fig. 13(a). Paralleling the three sources results in the diagram of Fig. 13(b) and adding the two series elements gives the single equivalent resistance in Fig. 13(c).

The maximum short-circuit current is then

$$I_T = \frac{250 \text{ volts}}{0.0093 \text{ ohms}} = 26{,}900 \text{ amperes}$$

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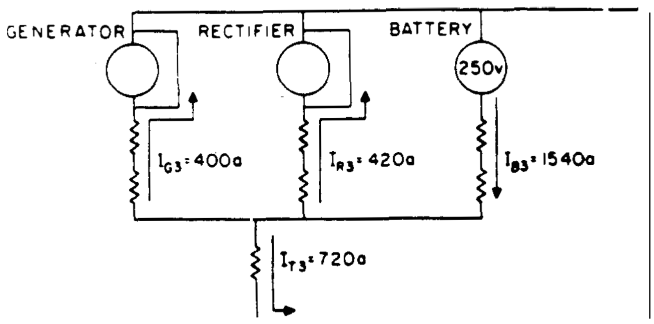

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Fig. 15(c). Step in suporpoiition mothod.

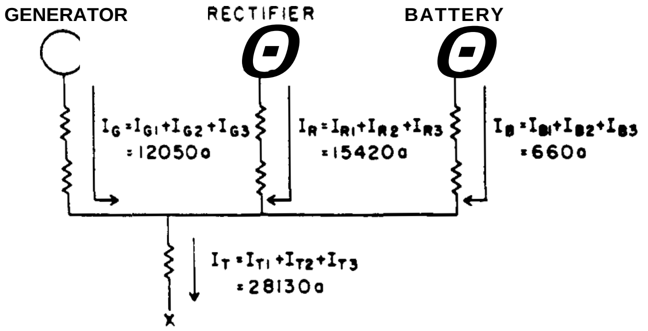

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Fig. 15(d). Stop in superposition mothod.

USING SUPERPOSITION METHOD

The resistance diagram for the system is shown in Fig. 14. Note that the circuit used for the rectifier is the circuit of Fig. 8(d). (It is assumed that the current from the rectifier will be less than 19,750 amperes, see Fig. 12, Section .172 for rectifier equivalent circuits.)

The rectifier equivalent circuit now contains a voltage source having a higher value than the other sources and therefore the system cannot be resolved into one equivalent value of resistance. The superposition method is used in this case.

The first step is to short out the rectifier and battery voltages and calculate the current flow from the generator through the branches of the system. The results are shown in Fig. 15(a).

In the next step, Fig. 15(b), the generator and battery voltages are shorted out and the current flow from the rectifier through the branches of the system is calculated. Note that the voltage forcing the current flow is 270 volts in this case whereas it is 250 volts in Fig. 15(a) and 15(c).

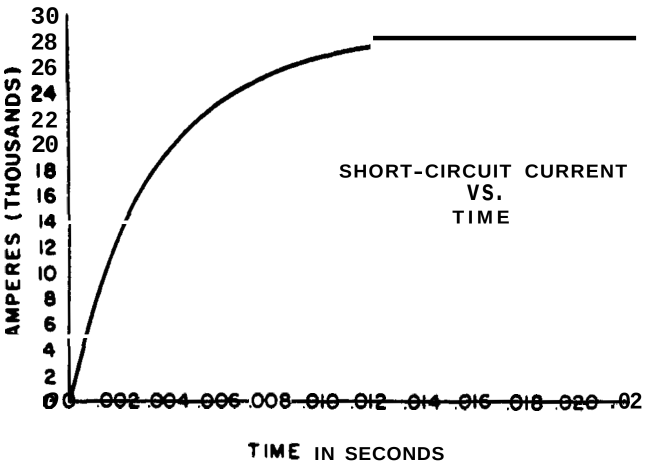

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Fig. 16. Current-time curve for total short-circuit current of exampl calculation.

The next step, Fig. 15(c), is to short out the voltages of the generator and rectifier sources and calculate the current flow from the battery.

The final step in the superposition method is to sum the currents in each branch with proper attention being given to direction of current flow. This is shown in Fig. 15(d).

The total current in the feeder as calculated by the superposition method is 28,130 amperes. This compares with the value of 26,900 amperes calculated by the method using an approximate value for the rectifier internal resistance.

Current-Time Curve

The initial rate of rise is 8.9 x 10^6 amperes per second and the total maximum current is 28,130 amperes. These values give a value of 0.028 millihenries for the single equivalent system inductance and a value of $\frac{250 \text{ volts}}{28{,}130 \text{ amperes}} = 0.0089$ ohms for the single equivalent system resistance.

The time constant for the short-circuit current is then

$$T_T = \frac{L_{eq}}{R_{eq}} = \frac{0.028 \times 10^{-3}}{0.0089} = 0.00315 \text{ seconds}$$

Thus at a time of 0.00315 seconds the current magnitude is 63.2% x 28,130 = 17,850 amperes and at a time of 0.0063 seconds the current magnitude is 87% x 28,130 = 24,400 amperes.

Based on this data the current-time curve for the short-circuit current is as shown in Fig. 16.

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D-c System Short-circuit Current Calculations

SUMMARY

The general procedure for calculating the system short-circuit current can be summarized by the following steps:

1. Obtain all system data, such as, the ratings of all equipment and the size, length, and arrangement of the conductors.
2. Draw a system one-line diagram.
3. Determine the equivalent circuits for all sources of short-circuit current.
4. Determine the inductance and resistance of all circuit conductors.
5. Construct the inductance diagram.
6. Resolve the inductance diagram to one equivalent value and calculate the initial rate of rise of the current.
7. Construct the resistance diagram.
8. Resolve the resistance diagram and calculate the maximum short-circuit current or use the superposition method to calculate the maximum current.
9. Construct the current-time curve.

It should be noted that circuit resistances and inductances are combined directly with source internal resistances and inductances in these system calculations. Also it is assumed that **any** system calculation will involve sufficient d-c system resistance and/or inductance to eliminate the ***peak*** which occurs at ½ cycle in a rectifier terminal short-circuit current.