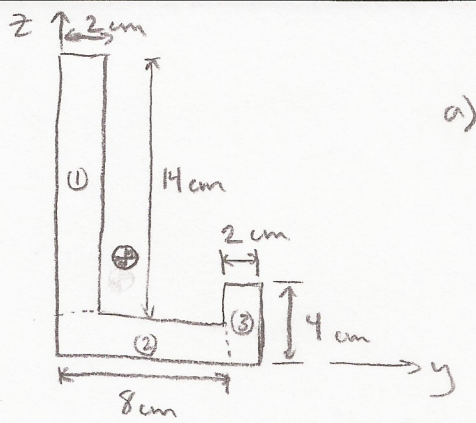


HW 6 - Problem 1



a)

i	A _i	\bar{y}_i	\bar{z}_i	I_{y_i}	I_{z_i}	dy_i	dz_i
1	28	1	9	457.33	9.333	-2.154	3.538
2	16	4	1	5.333	85.333	.846	-4.462
3	8	9	2	10.667	2.667	5.864	-3.462

$$I_{y_i} = \frac{bh^3}{12} \quad I_{z_i} = \frac{hb^3}{12}$$

a)

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} = \frac{28(1) + 16(4) + 8(9)}{28 + 16 + 8} = 3.154 \text{ cm}$$

$$\bar{z} = \frac{\sum A_i \bar{z}_i}{\sum A_i} = \frac{28(9) + 16(1) + 8(2)}{28 + 16 + 8} = 5.462 \text{ cm}$$

centroid

$$(\bar{y}, \bar{z}) = (3.154, 5.462)$$

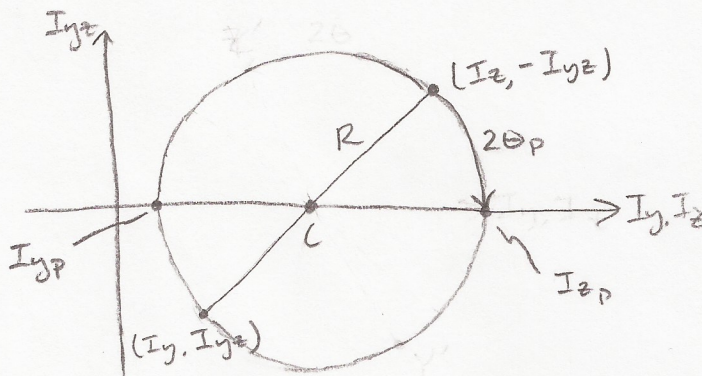
b)

$$I_y = \sum (I_{y_i} + A_i dz_i^2) = (457.33 + 28(3.538)^2) + (5.333 + 16(-4.462)^2) + (10.667 + 8(-3.462)^2) = 1238.25 \text{ cm}^4$$

$$I_z = \sum (I_{z_i} + A_i dy_i^2) = (9.333 + 28(-2.154)^2) + (85.333 + 16(.846)^2) + (2.667 + 8(5.864)^2) = 513.79 \text{ cm}^4$$

$$I_{yz} = \sum (\cancel{I_{yz_i}} + A_i dy_i dz_i) = 28(-2.154)(3.538) + 16(.846)(-4.462) + 8(5.864)(-3.462) = -436.19 \text{ cm}^4$$

c)



$$C = \frac{I_y + I_z}{2} = \frac{1238.25 + 513.79}{2} = 876.02$$

$$R = \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2} = \sqrt{\left(\frac{1238.25 - 513.79}{2}\right)^2 + (-436.19)^2}$$

$$I_{zp} = C + R = 876.02 + 566.99 = 1443.01 \text{ cm}^4 = I_{zp}$$

$$= 566.99$$

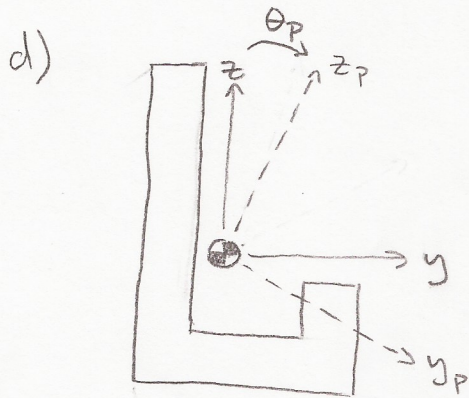
$$I_{yp} = C - R = 876.02 - 566.99 = 309.12 \text{ cm}^4 = I_{yp}$$



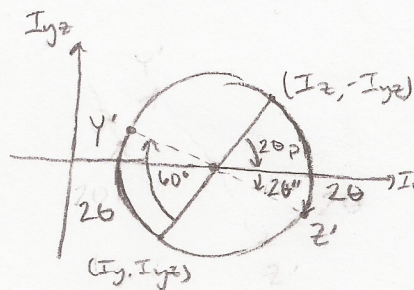
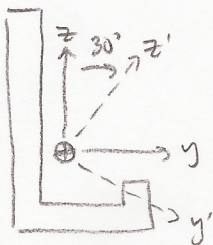
$$\sin(2\theta_p) = \frac{|I_{yz}|}{R}$$

$$2\theta_p = \sin^{-1}\left(\frac{|I_{yz}|}{R}\right)$$

$$\theta_p = \frac{1}{2} \sin^{-1}\left(\frac{|I_{yz}|}{R}\right) = \frac{1}{2} \sin^{-1}\left(\frac{436.19}{566.99}\right) = \boxed{25.15^\circ \text{ cw} = \theta_p}$$



e) $\theta = 30^\circ \text{ cw}$



$$2\theta_p + 2\theta'' = 60$$

$$2\theta'' = 60 - 2\theta_p$$

$$= 60 - 2(25.15) = 9.7^\circ$$

$$I_{y'} = C - R \cos(2\theta'') = 876.02 - 566.99 \cos(9.7^\circ) = \boxed{317.14 \text{ cm}^4}$$

$$I_{z'} = C + R \cos(2\theta'') = 876.02 + 566.99 \cos(9.7^\circ) = \boxed{1434.9 \text{ cm}^4}$$

$$I_{y'z'} = R \sin(2\theta'') = 566.99 \sin(9.7^\circ) = \boxed{95.53 \text{ cm}^4}$$