

II/3A

Composite: Wide-Flange Beam Load Factor Design

Introduction

Chapter 3 illustrates the design of a composite, wide-flange beam bridge by the working stress method. This chapter illustrates load factor design for the same type of construction.

The example presented in this chapter deals with the design of a two-span, continuous beam (70 ft.—70 ft.), composite for positive and negative moments similar to Design IV of Chapter 3. The load factor design is in accordance with the 1973 Standard Specifications for Highway Bridges of the American Association of State Highway and Transportation Officials and their Interim Specifications dated 1974, 1975 and 1976. These specifications will be referred to for brevity as AASHTO followed by an article and section reference. USS COR-TEN B (ASTM A588, Grade A) is used for the steel portion of the composite beam. This is a high-strength low-alloy structural steel that is widely used in unpainted bridges where its enhanced atmospheric corrosion resistance is desired to help minimize maintenance costs.

The procedures for dead-load distribution, lateral distribution of live load, computation of reactions, shears, moments and deflections, determination of effective slab widths, section properties (except for plastic section modulus and related properties) and stresses in composite sections are the same for load factor and working stress designs. Descriptive text and illustrative calculations similar to those presented in Chapter 3 are not repeated but the similarity is pointed out.

General Design Considerations

Members designed by the Load Factor method are proportioned for multiples of the design loads. They are required to meet certain criteria for three theoretical load levels: 1) Maximum Design Load 2) Overload and 3) Service Load. The Maximum Design Load and Overload requirements are based on multiples of the service loads with certain other coefficients necessary to insure the required capabilities of the structure. **Service loads are defined as the same loads as used in working stress design.**

The Maximum Design Load criteria insures the structure's capability of withstanding a few passages of exceptionally heavy vehicles (simultaneously in more than one lane), in times of extreme emergency, that may induce significant permanent deformations.

The Overload criteria insures control of permanent deformations in a member, caused by occasional overweight vehicles equal to $5/3$ the design live and impact loads (simultaneously in more than one lane), that would be objectionable to riding quality of the structure.

The Service Load criteria insures that the live load deflection and fatigue life (for assumed fatigue loading) of a member are controlled within acceptable limits.

Moments, shears and other forces are determined by assuming elastic behavior of the structure except for a continuous beam of compact section where negative moments over supports, determined by elastic analysis, may be reduced by a maximum of 10%. This reduction, however, must be accompanied by an increase in the maximum positive moment equal to the average decrease of the negative moments in the span.

DESIGN LOADS

The moments, shears or forces to be sustained by a stress-carrying steel member are computed from the following formulas for the three loading levels.

$$\text{Service Load: } D + (L + I)$$

$$\text{Overload: } D + \frac{5}{3}(L + I)$$

$$\text{Maximum Design Load: } 1.30 \left[D + \frac{5}{3}(L + I) \right]$$

where D = dead load

L = live load

I = impact load

The factor 1.30 is included to compensate for uncertainties in strength, theory, loading, analysis, and material properties and dimensions. The factor 5/3 is incorporated to allow for overloads. Factors for other group loading combinations are given in AASHTO Art. 1.7.123.

COMPACT SECTIONS

Compact sections are able to form plastic hinges which rotate at near constant moment.

A steel section is considered compact when its geometry is such that the fully plastic bending-moment capacity can be reached without local buckling or lateral torsional buckling. Prevention of lateral buckling requires the presence of adequate bracing of the compression flange at suitable intervals. The following criteria define compactness:

1. Width-thickness ratio of compression flange projection should not exceed

$$\frac{b'}{t} = \frac{1,600}{\sqrt{F_y}}$$

where b' = width of projecting compression flange element

t = flange thickness

F_y = specified minimum yield point or yield strength, psi, of the type of steel being used

2. Depth-thickness ratio of the web should not exceed

$$\frac{d}{t_w} = \frac{13,300}{\sqrt{F_y}}$$

where d = beam depth

t_w = web thickness

3. The compression flange should be supported laterally by adequate bracing at intervals not exceeding either of the following:

$$L_b = \frac{7,000r_y}{\sqrt{F_y}} \text{ when } M_2 \geq 0.7M_1$$

$$L_b = \frac{12,000r_y}{\sqrt{F_y}} \text{ when } M_2 < 0.7M_1$$

where r_y = radius of gyration with respect to Y-Y axis

M_1 = larger of the bending moments at two adjacent braced points

M_2 = smaller of the bending moments at those braced points

The displacement or twisting of beams called lateral buckling may also be prevented by embedment of the top and sides of the compression flange in concrete.

4. Axial compression should not exceed

$$P = 0.15F_y A$$

where A = beam cross-sectional area

5. Shear should not exceed

$$V = 0.55F_y d t_w$$

DESIGN FOR MAXIMUM DESIGN LOADS

For a compact section, the maximum strength or maximum moment capacity is given by

$$M_u = F_y Z$$

where Z = plastic section modulus

M_u must be equal to or greater than moment induced in the beam by the maximum design load, that is,

$$F_y Z \geq 1.30 \left[D + \frac{5}{3}(L + I) \right]$$

Here, D , L and I represent moments induced by the service loads.

If a compact section is provided to carry negative moments at supports of a continuous beam, the negative moments determined by elastic theory may be reduced 10%. This reduction, however, must be accompanied by an increase in the maximum positive moment equal to the average decrease of the negative moments in the span. This redistribution of moments is the only exception in load-factor design to elastic theory for analysis of structures.

Adequately braced, noncompact sections may be used, but with lower moment capacity. With adequate bracing, the maximum strength of a symmetrical, non-compact section may be computed from

$$M_u = F_y S$$

where S = elastic section modulus

The section consequently must be proportioned so that

$$F_y S \geq 1.30 \left[D + \frac{5}{3}(L + I) \right]$$

For this relationship to be permitted, the following criteria must be satisfied:

1. Width-thickness ratio of the compression flange projection, when the bending

moment induced by the Maximum Design Load, M , equals the maximum strength, M_u , should not exceed

$$\frac{b'}{t} = \frac{2,200}{\sqrt{F_y}}$$

When $M < M_u$, b'/t may be increased in the ratio $\sqrt{M_u/M}$.

2. Depth-thickness ratio of the web should not exceed

$$\frac{D}{t_w} = 150$$

where D = clear unsupported distance between flange components

3. Spacing of lateral bracing of the compression flange should not exceed

$$L_b = \frac{20,000,000 A_f}{F_y d}$$

where A_f = cross-sectional area of compression flange

4. As for a compact section, axial compression should not exceed

$$P = 0.15 F_y A$$

5. Shear should not exceed either of the following values

$$V = \frac{3.5 E t_w^3}{D}$$

$$V = 0.58 F_y D t_w$$

where E = steel modulus of elasticity

For sections with geometric properties falling between the limits for compact sections and those for braced, noncompact sections, maximum strength may be calculated by straight-line interpolation between the moment capacities of the two types of sections. Web thickness, however, must satisfy Criterion 2 for compact sections.

When a member does not meet Criterion 3 for spacing of lateral bracing of braced, noncompact sections, it is called an unbraced section, and the AASHTO lateral buckling equation for maximum strength is

$$M_u = F_y S \left[1 - \frac{3 F_y}{4 \pi^2 E} \left(\frac{L_b}{b'} \right)^2 \right]$$

When $M_2 < 0.7 M_1$, this value of M_u may be increased 20% but may not exceed $F_y S$.

For sections unsymmetrical about the X-X axis but symmetrical about the Y-Y axis, maximum strength may be computed from the formula for M_u , given, except that when this formula is used, b' should be replaced by $0.9b'$.

The AASHTO lateral buckling equation for maximum strength, M_u , was developed for prismatic compression flanges. When a compression flange cover plate is terminated within an unbraced length, the compression flange section throughout this length is no longer prismatic and the AASHTO lateral buckling requirements are not directly applicable.

However, it can be shown that by a modification of application the AASHTO lateral buckling formula can be applied conservatively to cover-plated beams.* This can be done by rearranging the AASHTO formula and computing the critical buckling stress of the braced panel, in which the cover plate terminates, as that of the beam without cover plates. This stress may then be increased by 20% providing the ratio of compression flange axial forces at the ends of the braced panel are equal to or less than 0.7.

*United States Steel Research reviewed the basis for the AASHTO requirements and analyzed the buckling loads of stepped columns with various geometries. Based on these results a design procedure was developed which relates the strength of a stepped flange to that of a prismatic flange. For additional information on this procedure contact a USS Construction Representative through the nearest USS Sales Office.

The critical buckling stress, F_{cr} , is determined from the following rearrangement of the AASHTO buckling formula:

$$F_{cr} = \frac{M_u}{S} = F_y \left[1 - \frac{3F_y}{4\pi^2 E} \left(\frac{L_b}{b'} \right)^2 \right]$$

where b' = projecting compression flange width of the beam

S = section modulus of the steel section without the terminated cover plate

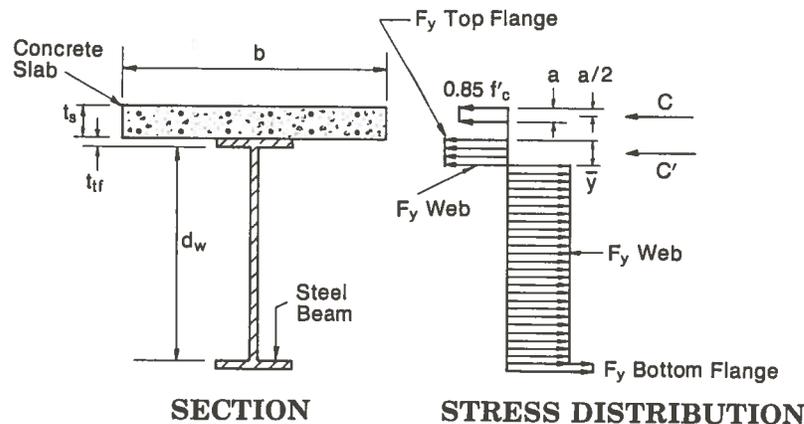
The maximum strength at any point in the panel is expressed as:

$$M_u = F_{cr} S_x$$

where S_x = section modulus at the point considered

For composite beams, those in which a concrete slab assists a steel section in resisting bending moments, the method for computing maximum strength depends on whether or not the steel section satisfies Criteria 2 and 5 previously given for compact sections and the stress-strain diagram for the steel exhibits a yield plateau followed by a strain-hardening range. If these criteria are satisfied, the beam is considered a compact, composite section.

Maximum strength in the positive-moment regions of a compact section with concrete slab on the top flange is computed for a fully plastic stress distribution on the section. This moment capacity equals the sum of the moments about the neutral axis of all compressive and tensile forces acting on the section.



COMPACT-COMPOSITE-BEAM STRESSES AT MAXIMUM DESIGN LOAD

The compressive force in the concrete slab is the smallest of the values of C computed from the following formulas.

1. Capacity of slab and its longitudinal steel reinforcement in the compression zone:

$$C = 0.85 f'_c b t_s + (A F_y)_c$$

where f'_c = specified 28-day compressive strength of concrete, psi

b = effective width of slab

t_s = slab thickness

$(A F_y)_c$ = product of area and yield point of that part of slab reinforcement parallel to the beam and lying in compression zone

2. Capacity of the steel section:

$$C = (A F_y)_{bf} + (A F_y)_{tf} + (A F_y)_w = \Sigma (A F_y)$$

where $(A F_y)_{bf}$ = product of area and yield point of bottom flange of steel section, including cover plate, if any

$(AF_y)_{t_f}$ = product of area and yield point of top flange of steel section

$(AF_y)_w$ = product of area and yield point of web of steel section

3. Capacity of shear connectors: $C = \Sigma Q_u$

where ΣQ_u = sum of ultimate strengths of shear connectors located between section under consideration and nearest section of zero moment

The depth of the assumed rectangular stress block (uniform stress distribution) for the slab is determined from the compressive force in the slab:

$$a = \frac{C - (AF_y)_c}{0.85f'_c}$$

When the compressive force in the slab is less than C computed for the capacity of the steel section, there will be a compressive force in the top portion of the steel section. This force is given by

$$C' = \frac{\Sigma(AF_y) - C}{2}$$

The distance \bar{y} of the neutral axis below the top of the steel section can be computed from one of the following formulas:

$$\bar{y} = \frac{C'}{(AF_y)_{t_f}} t_{t_f} \text{ when } C' < (AF_y)_{t_f}$$

$$\bar{y} = t_{t_f} + \frac{C' - (AF_y)_{t_f}}{(AF_y)_w} d_w \text{ when } C' \geq (AF_y)_{t_f}$$

where t_{t_f} = thickness of steel top flange

d_w = clear distance between flanges of steel section

The total tensile force acting on the section must equal the total compressive force for a beam subject only to bending and shear.

All quantities needed for computing the maximum bending strength M_u of the compact, composite section are thus determined. The section then must be so proportioned that

$$M_u \geq 1.30 \left[D + \frac{5}{3}(L + I) \right]$$

where again D , L and I are the moments induced in the member by the Service Loads.

When the steel section of a composite beam does not satisfy compactness requirements, maximum strength should be taken as the moment at first yielding. In other words, in a composite, noncompact section designed for Maximum Design Loads, the elastic stresses caused by multiples of the initial dead load, superimposed dead load, and live plus impact loads cannot exceed the yield stress. These stresses can be calculated by the usual elastic theory for composite beams, taking into account whether the construction is shored or unshored when the slab is cast.

DESIGN FOR OVERLOAD

To guard against objectionable deformation under occasional overloads, the following moment relationship must be observed for noncomposite sections,

$$0.8F_y S \geq \left[D + \frac{5}{3}(L + I) \right]$$

For the same reason, composite sections in positive bending must satisfy the relationship

$$0.95F_y S \geq \left[D + \frac{5}{3}(L + I) \right]$$

DESIGN FOR SERVICE LOADS

Fatigue is investigated in the same manner as in working stress design, using Service Loads and the provisions of AASHTO Art. 1.7.3. If the longitudinal reinforcing steel

in tension over the negative moment region is considered in computing section properties, the stress range in the reinforcing steel is limited to 20,000 psi.

SHEAR CONNECTORS

Requirements for shear connectors in load factor design are identical to requirements for working-stress design, which are applied in Chapters 3 and 4.

Design Example—Two-Span Continuous Beam (70-70 Ft) Composite for Positive and Negative Moment

To illustrate the load factor method, an interior stringer of a two-span bridge, similar to Design IV of Chapter 3, will be designed. The stringer consists of a rolled steel beam, with cover plates as required, that acts compositely with the concrete bridge deck in both positive-moment and negative-moment regions. For this purpose, in the negative-moment region, the section consists of the steel beam, cover plates, and longitudinal reinforcement in the slab. The following data apply to this design:

Roadway Section: The same as that shown for Design I in Chapter 3.

Specifications: 1973 AASHTO Standard Specifications for Highway Bridges, Interims 1974, 1975 and 1976

Loading: HS20-44

Structural Steel: ASTM A588, with $F_y = 50,000$ psi

Concrete: $f'_c = 4,000$ psi, modular ratio $n = 8$

Slab Reinforcing Steel: ASTM A615, Grade 40 with $F_y = 40,000$ psi

Loading Conditions:

Case 1—Weight of stringer and slab (DL_1) supported by the steel stringer alone.

Case 2—Superimposed dead load (DL_2) (curbs and railings) supported by the composite section, with the increased modular ratio $3n = 3 \times 8 = 24$.

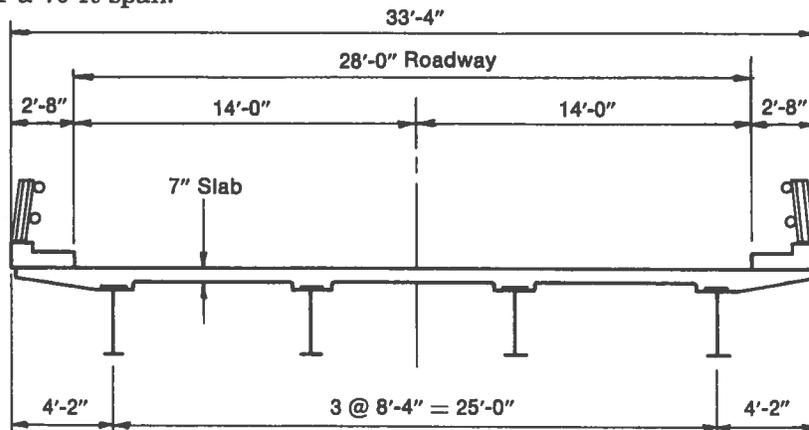
Case 3—Live load plus impact ($L+I$) supported by the composite section with the modular ratio $n = 8$.

Fatigue Considerations: 100,000 cycles of maximum stress for both truck and lane load as required for a secondary street or highway. (If the structure were considered located on a freeway, expressway, or major highway or street, it might be governed by a different number of stress cycles . . . see AASHTO Art. 1.7.3.)

LOADS, SHEARS AND MOMENTS

Moments and shears are determined by elastic theory with the initial assumption of uniform moment of inertia for both spans of the stringer.

The dead load DL_1 carried by the steel consists of the weight of the 7-in.-thick concrete slab and an assumed weight of 0.170 kips per ft for the stringer and framing details. The dead load DL_2 carried by the composite section comprises the proportional weight of the curbs and railings. Live load is HS20-44 truck loading, with impact for a 70-ft span.



TYPICAL SECTION

Dead Load Carried by Steel

$$\text{Slab} = 7/12 \times 8.33 \times 0.150 = 0.730$$

Steel beam, details, haunches, diaphragms = 0.170

$$DL_1 \text{ per stringer} = 0.900 \text{ k/ft}$$

Dead Load Carried by Composite Section *

Curbs and railings, $DL_2 = 0.660 \text{ k/ft}$

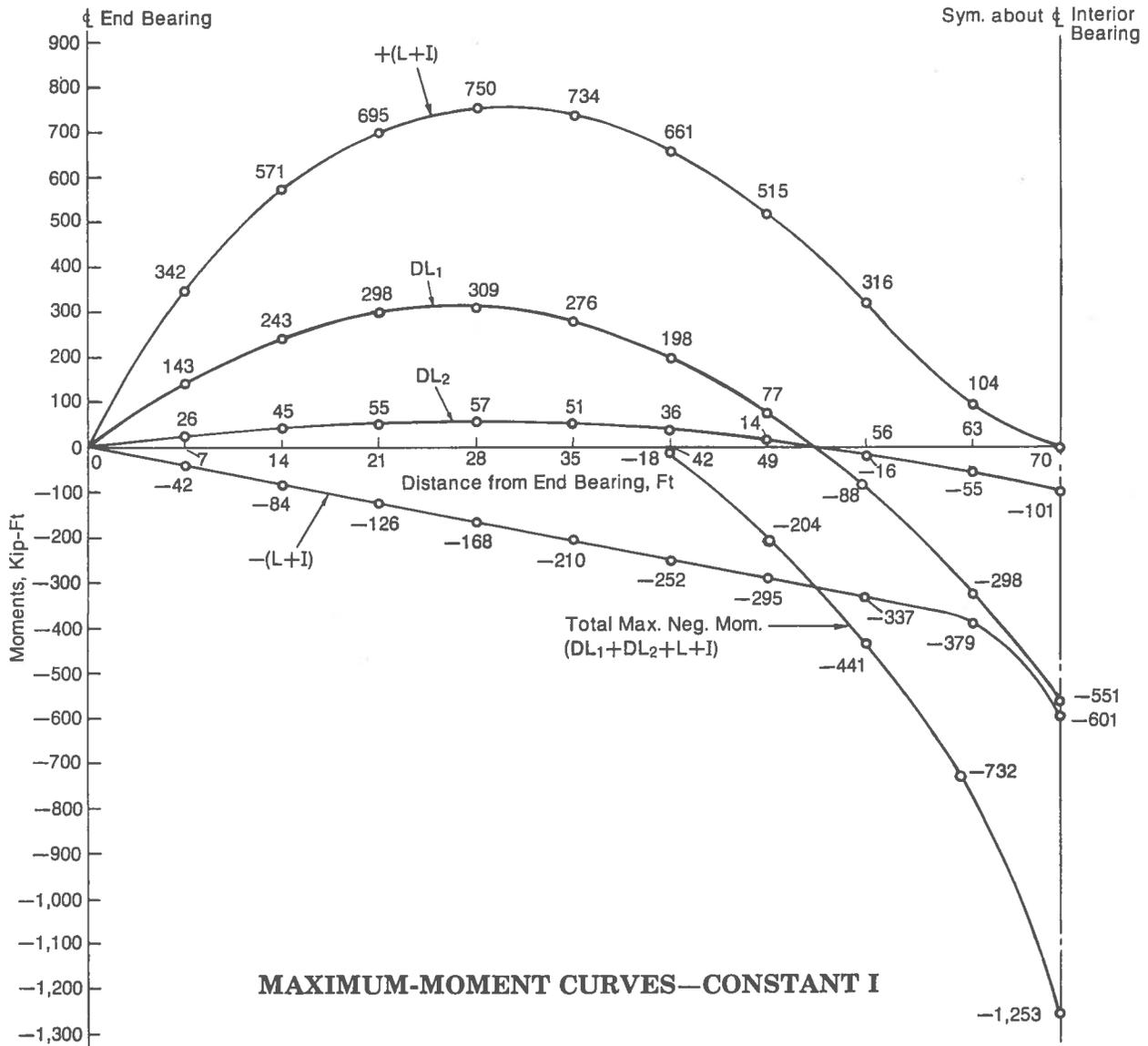
$$DL_2 \text{ per stringer} = 0.660/4 = 0.165 \text{ k/ft}$$

Live Load

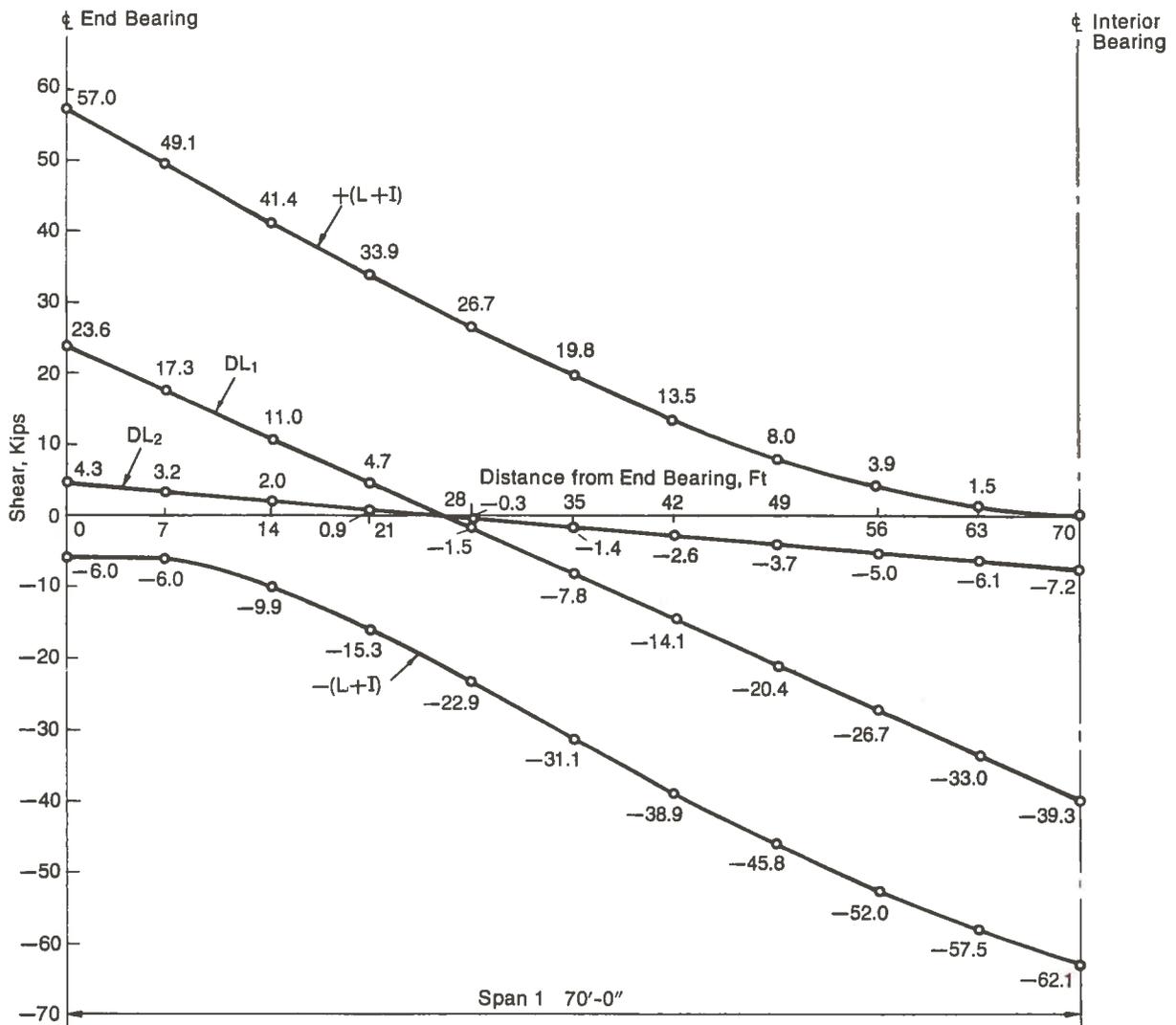
$$\text{Live-load distribution} = \frac{S}{5.5} = \frac{8.33}{5.5} = 1.51 \text{ wheels} = 0.755 \text{ axle}$$

$$\text{Impact} = \frac{50}{70+125} = 0.256$$

The curves shown for maximum moment and maximum shear may be calculated by any convenient method.



*No future wearing surface is anticipated for this bridge. If a future wearing surface will be required, its weight must be included in the dead load carried by the composite section and distributed equally to all stringers.



MAXIMUM-SHEAR CURVES—CONSTANT I

DESIGN OF STRINGER SECTION

A W30×108 beam of COR-TEN B (A588, Grade A) steel is considered as the basic section. In positive moment regions, the top flange is in compression. Since the concrete slab provides local and lateral buckling restraint, only criterion 2 and 5 of the previously stated compact section criteria are applicable. In negative-moment regions, the bottom flange is in compression and without continuous buckling restraint. Therefore all compact section criteria must be investigated.

CHECK OF BEAM PROPERTIES

Actual width-thickness ratio of the projecting compression flange element (bottom flange in negative-moment regions) is

$$\frac{b'}{t} = \frac{(10.484 - 0.548)/2}{0.760} = 6.54$$

The allowable width-thickness ratio is

$$\frac{b'}{t} = \frac{1,600}{\sqrt{50,000}} = 7.16 > 6.54$$

Web depth-thickness ratio is

$$\frac{d}{t_w} = \frac{29.82}{0.548} = 54.4$$

The allowable depth-thickness ratio is

$$\frac{d}{t_w} = \frac{13,300}{\sqrt{50,000}} = 59.5 > 54.4$$

Diaphragms must be placed at the support and at a minimum of two points in each span, in accordance with AASHTO Art. 1.7.21, which limits spacing of diaphragms to a maximum of 25 ft. Assume two intermediate diaphragms in each span. For equal spacing, the distance between diaphragms would be

$$L_b = \frac{70}{3} = 23.3 \text{ ft}$$

For positive moments, the concrete slab braces the compression flange. In the negative-moment region, however, the bottom flange is in compression, and the compression is assumed to extend a distance of 18 ft from the pier to the dead-load inflection point. Consequently, the unsupported length of the compression flange for negative moments is $L_b = 18$ ft.

For the W30 × 108, $r_y = 2.15$. The moment at 18 ft from the support will be less than 0.7 the moment at the support. Hence, for a compact section, the maximum unsupported length of the compression flange in the negative-moment region may not exceed

$$L_b = \frac{12,000 \times 2.15}{\sqrt{50,000}} = 115 \text{ in.} = 9.62 \text{ ft} < 18 \text{ ft}$$

The maximum shear allowed for a compact section is

$$V = 0.55F_y d t_w = 0.55 \times 50 \times 29.82 \times 0.548 = 449 \text{ kips}$$

The actual shear for Maximum Design Load is

$$V = 1.30(39.3 + 7.2 + \frac{5}{3} \times 62.1) = 195 < 449 \text{ kips}$$

All requirements for compactness are satisfied, except the requirement for spacing of lateral bracing of the compression flange in the negative-moment region.

The section over the pier is checked next as a braced, noncompact section.

$$L_b = \frac{20,000,000 A_f}{F_y d} = \frac{20,000,000 \times 10.484 \times 0.760}{50,000 \times 29.82} = 106.9 \text{ in.} = 8.91 \text{ ft} < 18 \text{ ft}$$

The 18-ft unbraced length of the compression flange from the support to the inflection point exceeds the allowable for a braced noncompact section. Hence, the section must be considered an unbraced section.

STRENGTH OF UNBRACED SECTION IN THE NEGATIVE-MOMENT REGION

For the unbraced W30 × 108 section, the maximum strength M_u is reduced from $F_y S$ by the reduction formula previously given. Because the beam with reinforcing steel is unsymmetrical about the X-X axis, $0.9b'$ is used in the formula instead of b' ,

$$0.9b' = 0.9(10.484 - 0.548)/2 = 4.47 \text{ in.}$$

$$\begin{aligned} M_u &= F_y S \left[1 - \frac{3F_y}{4\pi^2 E} \left(\frac{L_b}{0.9b'} \right)^2 \right] = F_y S \left[1 - \frac{3 \times 50}{4\pi^2 \times 29,000} \left(\frac{18 \times 12}{4.47} \right)^2 \right] \\ &= F_y S (1 - 0.306) = 0.694 F_y S \end{aligned}$$

It is anticipated that the beam alone will suffice over part of the negative-moment region but that cover plates will be required in the immediate vicinity of the support.

The preceding investigation indicates that the design relationship for the negative-moment section compression flange is

$$1.30 \left[D + \frac{5}{3}(L+I) \right] \leq 0.833F_y S$$

and that the design relationship for the tension flange is

$$1.30 \left[D + \frac{5}{3}(L+I) \right] \leq F_y S$$

It can be seen that the relationship for Overload

$$D + \frac{5}{3}(L+I) \leq 0.95F_y S$$

does not govern.

Section properties are calculated for the cover-plated beam alone and for this beam plus the longitudinal reinforcing bars in the concrete slab. The allowable stresses in the cover-plated beam are

$$F_b = 0.833F_y = 0.833 \times 50 = 41.6 \text{ ksi for compression flange}$$

$$F_t = F_y = 50 \text{ ksi for tension flange}$$

Steel Section at Interior Support

Material	A	d	Ad	Ad ²	I _o	I
W30 × 108	31.8				4,470	4,470
Bottom Plate 1 × 12	12.00	15.41	-184.9	2,850		2,850
Top Plate 3/8 × 8	3.0	15.10	45.3	684		684

$$d_s = \frac{-139.6}{46.80} = -2.98 \text{ in.}$$

$$I_{NA} = \frac{8,004 - 2.98 \times 139.6}{7,588} \text{ in.}^4$$

$$d_{\text{Top}} = 14.91 + 0.38 + 2.98 = 18.27 \text{ in.}$$

$$d_{\text{Bot.}} = 14.91 + 1.00 - 2.98 = 12.93 \text{ in.}$$

$$S_{\text{Top}} = \frac{7,588}{18.27} = 415 \text{ in.}^3$$

$$S_{\text{Bot.}} = \frac{7,588}{12.93} = 587 \text{ in.}^3$$

Section with Reinforcing Steel at Interior Support

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	46.80		-139.6			8,004
Reinf. Steel 14 No. 6	6.16	19.58	120.6	2,362		2,362

$$d_s = \frac{-19.0}{52.96} = -0.36 \text{ in.}$$

$$I_{NA} = \frac{10,366 - 0.36 \times 19.0}{10,359} \text{ in.}^4$$

$$d_{\text{Reinf.}} = 15.65 + 1.0 + 3.30 = 19.95 \text{ in.}$$

$$S_{\text{Reinf.}} = \frac{10,359}{19.95} = 519 \text{ in.}^3$$

$$d_{\text{Top of steel}} = 14.91 + 0.38 + 0.36 = 15.65 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 14.91 + 1.00 - 0.36 = 15.55 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{10,359}{15.65} = 662 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{10,359}{15.55} = 666 \text{ in.}^3$$

Bending Moments (Constant I)

	DL ₁	DL ₂	LL+I
M, kip-ft	-551	-101	-601

**Steel Stresses for Maximum Negative Moment
Due to Maximum Design Load**

Top of Steel (Tension)		Bottom of Steel (Compression)	
For DL_1 :	$F_b = \frac{551 \times 12}{415} \times 1.30 = 20.7$	$F_b = \frac{551 \times 12}{587} \times 1.30 = 14.6$	
For DL_2 :	$F_b = \frac{101 \times 12}{662} \times 1.30 = 2.4$	$F_b = \frac{101 \times 12}{666} \times 1.30 = 2.4$	
For $L+I$:	$F_b = \frac{601 \times 12}{662} \times 1.30 \times \frac{5}{3} = 23.6$	$F_b = \frac{601 \times 12}{666} \times 1.30 \times \frac{5}{3} = 23.5$	
	$\overline{46.7} < 50.0 \text{ ksi}$	$\overline{40.5} < 41.6 \text{ ksi}$	

Reinforcing Steel Stress (Tension)

$$F_b = \frac{1.3 \times 12(101 + 1.667 \times 601)}{519} = 33.1 < 40 \text{ ksi}$$

The trial section is satisfactory for maximum negative moment.

Fatigue considerations limit the stress range in the reinforcement to 20,000 psi. The actual stress range is well within this value:

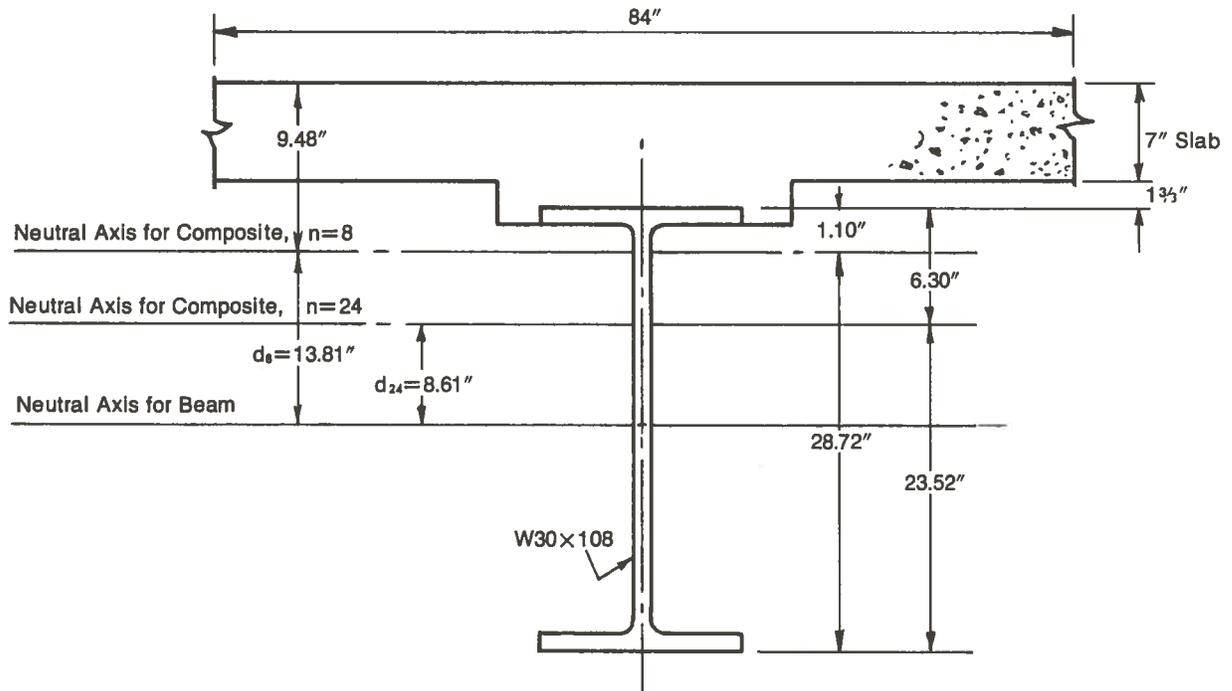
$$\text{Range} = \frac{601 \times 12}{519} = 13.9 < 20.0 \text{ ksi}$$

MAXIMUM POSITIVE MOMENT

The stringer is next investigated for maximum positive moment, which occurs 28 ft from the end support. There, the section consists of the W30×108 beam acting compositely with the slab and is compact. Properties are listed for the steel section alone and computed for the composite section with $n=8$ and $n=24$.

Steel Section W30×108

$$I_o = 4,470 \text{ in.}^4 \quad S_{\text{Top}} = S_{\text{Bot.}} = 300 \text{ in.}^3 \quad A = 31.8 \text{ in.}^2$$



POSITIVE-MOMENT SECTION

Composite Section, $3n=24$, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
W30×108	31.8				4,470	4,470
Conc. 84×7/24	24.5	19.79	484.9	9,595	100	9,695

$$d_s = \frac{484.9}{56.3} = 8.61 \text{ in.}$$

$$I_{NA} = \frac{14,165}{9,990} = 1.4175 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 14.91 - 8.61 = 6.30 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 14.91 + 8.61 = 23.52 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{9,990}{6.30} = 1,586 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{9,990}{23.52} = 425 \text{ in.}^3$$

Composite Section, $n=8$, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
W30×108	31.8				4,470	4,470
Conc. 84×7/8	73.5	19.79	1,454.6	28,786	300	29,086

$$d_s = \frac{1,454.6}{105.3} = 13.81 \text{ in.}$$

$$I_{NA} = \frac{33,556}{13,468} = 2.492 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 14.91 - 13.81 = 1.10 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 14.91 + 13.81 = 28.72 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{13,468}{1.10} = 12,244 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{13,468}{28.72} = 469 \text{ in.}^3$$

Check of Steel Stresses for Overload

Stresses are then calculated for Overload according to the relationship

$$D + \frac{5}{3}(L+I) \leq 0.95F_y S$$

Thus, the allowable stress for Overload is $F_b = 0.95 \times 50 = 47.5$ ksi. The results show that the relationship is satisfied by the composite section.

Bending Moments 28 Ft from End Support

	DL ₁	DL ₂	L+I
M, kip-ft	309	57	750

Steel Stresses for Overload—Combination A

Top of Steel (Compression)

Bottom of Steel (Tension)

For DL₁: $F_b = \frac{309 \times 12}{300} = 12.4$

$F_b = \frac{309 \times 12}{300} = 12.4$

For DL₂: $F_b = \frac{57 \times 12}{1,586} = 0.4$

$F_b = \frac{57 \times 12}{425} = 1.6$

For L+I: $F_b = \frac{750 \times 12}{12,244} \times \frac{5}{3} = 1.2$
14.0 ksi

$F_b = \frac{750 \times 12}{469} \times \frac{5}{3} = 32.0$
46.0 < 47.5 ksi

Check of Maximum Strength Under Maximum Design Loads

The strength of the composite section is checked for maximum positive moment with the relationship

$$1.30 \left[D + \frac{5}{3}(L+I) \right] \leq F_y Z$$

It has previously been established that the W30×108 beam and slab is a compact, composite section, since the compression flange is adequately braced by the slab.

Consequently, the maximum strength of the composite section is determined by the fully plastic moment capacity of the section. This is found by determining the compressive force in the slab, locating the neutral axis and calculating the total moment about the neutral axis of all forces acting on the section. The compressive force in the slab, if adequate shear connectors are provided, is the smaller value of either the fully plastic force developable by the steel section or the compressive strength of the slab. In this case, the plastic capacity of the steel section governs, as indicated below.

The capacity of the concrete slab, with concrete strength $f'_c = 4$ ksi and reinforcing steel with $F_y = 40$ ksi, is

$$C_1 = 0.85f'_c b t_s + (AF_y)_c = 0.85 \times 4 \times 84 \times 7 + 6.16 \times 40 = 2,246 \text{ kips}$$

The plastic force developable by the W30 is

$$C_2 = 31.8 \times 50 = 1,590 \text{ kips (controls)}$$

The neutral axis lies within the slab. The distance of the axis below the top of the slab, in this case, equals the depth a of the compressive stress block for the slab.

$$a = \frac{C - (AF_y)_c}{0.85f'_c b} = \frac{1,590 - 6.16 \times 40}{0.85 \times 4 \times 84} = 4.70 \text{ in.}$$

For this location of the neutral axis, the longitudinal reinforcement of the slab lies in the compression zone and therefore inclusion of the force in the bars in the calculation of a was warranted.

With the neutral axis located, the fully plastic moment M_u for the section can now be calculated.

$$M_u = \frac{0.85 \times 4 \times 84 (4.70)^2}{2} + 6.16 \times 40 (4.70 - 3.70) \\ + 1,590 [14.91 + 1.0 + 0.38 + (7.00 - 4.70)] = 32,960 \text{ kip-in.}$$

The maximum positive moment induced by the Maximum Design Load is

$$M = 1.30 \left[DL_1 + DL_2 + \frac{5}{3}(L+I) \right] = 1.30 \left(309 + 57 + \frac{5}{3} \times 750 \right) 12 \\ = 25,214 < 32,960 \text{ kip-in.}$$

Therefore, the composite section with the W30×108 is satisfactory for maximum strength in positive moment.

TOP-COVER-PLATE CUTOFF LOCATION

The next task in the design of the stringer is determining where the cover plates in the negative-moment region can be cut off. Since the top flange is in tension, the termination of the top cover plate is governed by the maximum strength relationship

$$1.30 \left[D + \frac{5}{3}(L+I) \right] \leq F_y S$$

Section properties are first calculated without the top cover plate but including the longitudinal reinforcing of the concrete slab. The cutoff location then is determined by trial.

Steel Section with Bottom Plate Only

Material	A	d	Ad	Ad ²	I _o	I
W30×108	31.8				4,470	4,470
Bottom Plate 1×12	12.00	15.41	-184.9	2,850		2,850

$$43.80 \text{ in.}^2 \quad -184.9 \text{ in.}^3 \quad 7,320$$

$$d_s = \frac{-184.9}{43.80} = -4.22 \text{ in.} \quad -4.22 \times 184.9 = -780$$

$$I_{NA} = \frac{6,540}{7,320} = 0.893 \text{ in.}^4$$

$$d_{\text{Top}} = 14.91 + 4.22 = 19.13 \text{ in.} \quad d_{\text{Bot.}} = 14.91 + 1.00 - 4.22 = 11.69 \text{ in.}$$

$$S_{\text{Top}} = \frac{6,540}{19.13} = 342 \text{ in.}^3 \quad S_{\text{Bot.}} = \frac{6,540}{11.69} = 559 \text{ in.}^3$$

Steel Section with Bottom Plate and Reinforcing Steel

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	43.80		-184.9			7,320
Reinf. Steel 14 No. 6	6.16	19.58	120.6	2,362		2,362

$$49.96 \text{ in.}^2 \quad -64.3 \text{ in.}^3 \quad 9,682$$

$$d_s = \frac{-64.3}{49.96} = -1.29 \text{ in.} \quad -1.29 \times 64.3 = -83$$

$$I_{NA} = \frac{9,599}{9,682} = 0.991 \text{ in.}^4$$

$$d_{\text{Top}} = 14.91 + 1.29 = 16.20 \text{ in.} \quad d_{\text{Bot.}} = 14.91 + 1.00 - 1.29 = 14.62 \text{ in.}$$

$$S_{\text{Top}} = \frac{9,599}{16.20} = 593 \text{ in.}^3 \quad S_{\text{Bot.}} = \frac{9,599}{14.62} = 657 \text{ in.}^3$$

$$d_{\text{Reinf.}} = 16.20 + 0.38 + 1.0 + 3.30 = 20.88 \text{ in.}$$

$$S_{\text{Reinf.}} = \frac{9,599}{20.88} = 460 \text{ in.}^3$$

Try a theoretical cutoff of the top plate 3.0 ft from the pier. The actual cutoff is 1.5 times the plate width away from the theoretical cutoff or 4.0 ft from the pier. The total length of plate satisfies the minimum cover plate length criteria of $(2D + 3.0)$ ft where D = depth of the beam in ft.

$$L_{\text{min}} = 2 \times 2.5 + 3.0 = 8.0 \text{ ft}$$

The allowable stress, from the strength relationship is

$$F_b = 50 \text{ ksi}$$

Bending Moments 3.0 Ft from Pier

	DL ₁	DL ₂	-(L+I)	+(L+I)
M, kip-ft	-438	-78	-470	35

Steel Stresses 3.0 Ft from Pier Due to Maximum Design Loads

Top of Steel (Tension)

$$\text{For } DL_1: F_b = \frac{438 \times 12}{342} \times 1.30 = 20.0$$

$$\text{For } DL_2: F_b = \frac{78 \times 12}{593} \times 1.30 = 2.1$$

$$\text{For } L+I: F_b = \frac{470 \times 12}{593} \times 1.30 \times \frac{5}{3} = 20.6$$

$$42.7 < 50.0 \text{ ksi}$$

The stress at the theoretical cutoff point is within the allowable.

Check the base metal stress adjacent to the fillet weld at the actual cutoff 4.0 ft from the pier.

Bending Moments 4.0 Ft from Pier

	DL_1	DL_2	$-(L+I)$	$+(L+I)$
M , kip-ft	-405	-72	-440	51

The allowable fatigue stress range for base metal at the end of cover plates falls into AASHTO fatigue category E. For 100,000 loading cycles, the allowable range of stress in the top flange of the W30 adjacent to the fillet weld at the end of the cover plate is

$$F_{sr} = 21 \text{ ksi}$$

The stress range induced by service loads at this point is

$$f_{sr} = \frac{(440+51) \times 12}{593} = 9.9 \text{ ksi} < 21 \text{ ksi}$$

Basic stress and fatigue are then checked in the longitudinal reinforcing steel at the actual cutoff 4.0 ft from the pier.

$$\text{For } DL_2: F_b = \frac{72 \times 12}{460} \times 1.30 = 2.4$$

$$\text{For } L+I: F_b = \frac{440 \times 12}{460} \times 1.30 \times \frac{5}{3} = 24.9$$

$$\underline{\underline{27.3}} < 40.0 \text{ ksi}$$

Stress range in the reinforcing steel is limited to 20 ksi. The actual stress range is determined to be

$$f_{sr} = \frac{(440+51) \times 12}{460} = 12.8 < 20 \text{ ksi}$$

Since all stresses are within the allowable and the fatigue stress range is within satisfactory limits, the top plate can be terminated 4 ft from the pier.

BOTTOM-PLATE CUTOFF LOCATION

The cutoff of the bottom cover plate is governed by the relationship for an unbraced section with the 20% increase in stress because the ratio of flange forces at the braced ends is less than 0.7:

$$1.30 \left[D + \frac{5}{3}(L+I) \right] \leq 1.2 \times 0.694 F_y S = 0.833 F_y S$$

Section properties are listed for the W30×108 and computed for the section composed of the W30 and the slab reinforcement.

Steel Section W30×108

$$I_o = 4,470 \text{ in.}^4$$

$$S_{\text{Top}} = S_{\text{Bot.}} = 300 \text{ in.}^3$$

$$A = 31.8 \text{ in.}^2$$

Section with W30 and Reinforcing Steel

Material	A	d	Ad	Ad ²	I _o	I
W30 × 108	31.8				4,470	4,470
Reinf. Steel 14 No. 6	6.16	19.58	120.6	2,362		2,362

$$d_s = \frac{120.6}{37.96} = 3.18 \text{ in.}$$

$$d_{\text{Top of steel}} = 14.91 - 3.18 = 11.73 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 14.91 + 3.18 = 18.09 \text{ in.}$$

$$d_{\text{Reinf.}} = 11.73 + 0.38 + 1.0 + 3.30 = 16.41 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{6,448}{11.73} = 550 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{6,448}{18.09} = 356 \text{ in.}^3$$

$$S_{\text{Reinf.}} = \frac{6,448}{16.41} = 393 \text{ in.}^3$$

$$I_{NA} = \frac{-3.18 \times 120.6}{6,448} = -\frac{384}{6,448} \text{ in.}^4$$

A theoretical cutoff of the bottom plate is tried 9.5 ft from the pier. Steel stresses at the bottom of the W30 govern and are checked first. The allowable stress for Maximum Design Load is

$$F_b = 0.833F_y = 0.833 \times 50 = 41.6 \text{ ksi}$$

Bending Moments 9.5 Ft from Pier

	DL ₁	DL ₂	-(L+I)	+(L+I)
M, kip-ft	-219	-41	-361	171

Steel Stresses 9.5 Ft from Pier Due to Maximum Design Loads

Bottom of Steel (Compression)

$$\text{For } DL_1: F_b = \frac{219 \times 12}{300} \times 1.30 = 11.4$$

$$\text{For } DL_2: F_b = \frac{41 \times 12}{356} \times 1.30 = 1.8$$

$$\text{For } L+I: F_b = \frac{361 \times 12}{356} \times 1.30 \times \frac{5}{3} = 26.4$$

$$39.6 < 41.6 \text{ ksi}$$

The stress at the theoretical cutoff point of the bottom plate is within the allowable.

The actual cutoff is 1.5 times the plate width beyond the theoretical cutoff, or 11 ft from the pier. Fatigue stress range is checked there for base metal adjacent to a fillet weld.

Bending Moments 11 Ft from Pier

	DL ₁	DL ₂	-(L+I)	+(L+I)
M, kip-ft	-174	-34	-354	215

Stresses at Bottom of Steel 11 Ft from Pier

With Negative Live-Load Moment

With Positive Live-Load Moment

$$\text{For } DL_1: f_b = \frac{-174 \times 12}{300} = -7.0$$

$$-7.0$$

$$\text{For } DL_2: f_b = \frac{-34 \times 12}{356} = -1.1$$

$$-1.1$$

$$\text{For } L+I: f_b = \frac{-354 \times 12}{356} = -11.9$$

$$-20.0 \text{ ksi}$$

$$f_b = \frac{215 \times 12}{356} = +7.2$$

$$-0.9 \text{ ksi}$$

Since there is no tension in the W30 section at the cutoff, there is no limitation on the stress range.

Reinforcing steel stresses are checked at the actual cutoff location 11.0 ft from the pier.

Reinforcing Steel Stresses 11.0 Ft from Pier (Tension)

$$\text{For } DL_2: F_b = \frac{34 \times 12}{393} \times 1.30 = 1.3$$

$$\text{For } L+I: F_b = \frac{354 \times 12}{393} \times 1.30 \times \frac{5}{3} = 23.4$$

$$\underline{24.7 < 40 \text{ ksi}}$$

Reinforcing Steel Fatigue Stress Range 11.0 Ft from Pier

Fatigue stress range in the reinforcing steel is limited to 20 ksi. The actual stress range is determined to be

$$f_{sr} = \frac{(354 + 215)12}{393} = 17.4 < 20 \text{ ksi}$$

All stresses are within the allowable, and the stress range is satisfactory for fatigue. Therefore, the bottom plate can be cut off 11 ft from the pier.

COVER-PLATE WELDS

The welds connecting the cover plates to the beam must be able to develop the computed forces in the cover plates at the theoretical cutoffs within the terminal length. The welds also must be able to resist the horizontal shear between the beam and the plate along the length of the plate. Usually, the former condition is more critical.

As with other components of the structure, both maximum strength under Maximum Design Loads and fatigue under service loads must be checked.

E70 XXX electrodes, with $F_u = 70$ ksi, will be used to make the welds. ASTM A588 steel also has a tensile strength $F_u = 70$ ksi.

$$\text{Weld strength} = 0.45F_u = 0.45 \times 70 = 31.5 \text{ ksi}$$

The permissible load on a fillet weld then is

$$P_u = 0.707 \times 31.5 = 22.3 \text{ kips per in. for a 1 in. fillet weld}$$

WELD AT END OF BOTTOM PLATE

In the following calculations, the bottom-cover-plate weld is investigated for maximum strength and fatigue under service loads at the theoretical cutoff.

Maximum Strength

Stresses in Bottom Plate Due to Maximum Design Loads (Compression)

$$\text{For } DL_1: F_b = \frac{219 \times 12}{559} \times 1.30 = 6.1$$

$$\text{For } DL_2: F_b = \frac{41 \times 12}{657} \times 1.30 = 1.0$$

$$\text{For } L+I: F_b = \frac{361 \times 12}{657} \times 1.30 \times \frac{5}{3} = 14.3$$

$$\underline{21.4 \text{ ksi}}$$

$$\text{Force in cover plate} = 21.4 \times 1 \times 12 = 257 \text{ kips}$$

The terminal development length, or distance along the plate edges available for making the terminal weld, is

$$L = 2 \times 1.5 \times 12 + 10.5 = 46.5 \text{ in.}$$

$$\text{Weld size required} = \frac{257}{46.5 \times 22.3} = 0.248 \text{ in.}$$

The fillet weld connecting the 1 in. cover plate to the 0.76 in. thick W30 flange is limited by AASHTO Art. 1.7.26 to a minimum size of $\frac{5}{16}$ in. Since the $\frac{5}{16}$ in. minimum exceeds the 0.248 in. weld size required for maximum strength, a $\frac{5}{16}$ in. weld will be used.

Service Loads—Fatigue

The range of stress in the bottom flange cover plate at the theoretical cutoff is

$$f_{sr} = \frac{(361 + 171)12}{657} = 9.72 \text{ ksi}$$

The range of force in the flange cover plate then becomes

$$P_{\text{range}} = 9.72(1 \times 12) = 117 \text{ kips}$$

Shear stress on the throat of fillet welds falls into AASHTO fatigue stress category F, and for 100,000 cycles, the allowable stress range cannot exceed 15 ksi. For the development length of 46.5 in. and a weld size of $\frac{5}{16}$ in., the actual weld shear stress range is

$$f_{sr} = \frac{117}{46.5(.707)(\frac{5}{16})} = 11.4 < 15 \text{ ksi}$$

WELD AT END OF TOP PLATE

The top-cover-plate weld is investigated for maximum strength under maximum design loads and fatigue under Service Loads in the same manner as the bottom-cover-plate weld.

Maximum Strength

Stress in Top Plate Due to Maximum Design Loads (Tension)

$$\text{For } DL_1: F_b = \frac{438 \times 12}{415} \times 1.3 = 16.5$$

$$\text{For } DL_2: F_b = \frac{78 \times 12}{662} \times 1.3 = 1.8$$

$$\text{For } L+I: F_b = \frac{470 \times 12}{662} \times 1.3 \times \frac{5}{3} = 18.5$$

36.8 ksi

Force in cover plate = $36.8 \times 8 \times .38 = 112$ kips.

The distance along the plate edges available for making the terminal weld is

$$L = 2 \times 1.5 \times 8 + 8 = 32 \text{ in.}$$

$$\text{Weld size required} = \frac{112}{32 \times 22.3} = 0.16 \text{ in.}$$

The 0.76 in. thick W30 flange requires a $\frac{5}{16}$ in. minimum fillet weld. Therefore, investigate fatigue on the $\frac{5}{16}$ in. weld.

The range of stress in the top flange cover plate at the theoretical cutoff is

$$f_{sr} = \frac{(470 + 35)}{662} \times 12 = 9.15 \text{ ksi}$$

The range of force in the flange cover plate then becomes

$$P_{\text{range}} = 9.15(.38 \times 8) = 27.5 \text{ kips}$$

The actual weld shear stress range is

$$f_{sr} = \frac{27.5}{32(.707)(\frac{5}{16})} = 3.89 \text{ ksi} < 15 \text{ ksi}$$

FATIGUE AT STUD WELDS

Fatigue must be investigated for base metal adjacent to stud shear connectors on the tension flange. The following calculations show that the fatigue stress range is not critical at either the maximum negative-moment section over the support or at cover-plate cutoffs.

Stud-weld Fatigue Stress Range at Pier Location

Tensile stress in the top cover plate over the support falls into AASHTO fatigue stress category C. For 100,000 cycles, the allowable stress range is 32 ksi. The live-load stress range in the top cover plate at the pier is determined to be

$$f_{sr} = \frac{(601 - 0) \times 12}{662} = 10.9 < 32 \text{ ksi}$$

Studs therefore may be welded to the top cover plate over the support without any reduction in strength of the section.

Stud-Weld Fatigue Stress Range at Top-Plate Cutoff

The maximum stress range in the top flange of the W30 for the section 4 ft from the pier, where studs may be welded to the flange near the end of the top cover plate, was previously computed to be 9.9 ksi since this stress range is less than the 32 ksi allowable, studs may be welded to the top flange of the W30 near the top-plate cutoff without a reduction in the strength of the section.

Stud-Weld Fatigue Stress Range at Bottom-Plate Cutoff

The maximum stress range in the top flange of the W30 for the section 11.0 ft from the pier where the bottom plate is terminated is determined to be

$$f_{sr} = \frac{(354 + 215) \times 12}{550} = 12.4 < 32 \text{ ksi}$$

Studs, therefore, may be welded to the top flange of the W30 at the bottom-plate cutoff without a reduction in strength of the section.

SHEAR-CONNECTOR SPACING

Studs $\frac{3}{8}$ in. diameter and 4 in. long are selected for shear connectors. Three studs are placed per row across the top flange or cover plate, for embedment in the concrete slab. Spacing of the rows is determined with the same criteria used for working-stress design and illustrated in Chapter 3. To avoid repetition, detailed calculations are not given here.

The spacing, shown in a diagram, satisfies the requirements for fatigue under Service Loads in the positive-moment region, and maximum spacing of 24 in. in the negative-moment region. However, the number of connectors must also be checked to insure that ultimate strength of the composite section can be achieved. The ultimate strength of welded studs is determined by the formula

$$S_u = 0.4d^2(f'_c E_c)^{1/2} \quad \text{where } E_c = 57,000 \sqrt{f'_c}$$

Therefore

$$S_u = 0.4d^2 \sqrt{57,000(f'_c)}^{1/2} = 0.4(4)^2 \sqrt{57,000(.875)}^{1/2} = 36,800 \text{ psi}$$

The number of connectors required between the point of maximum positive moment and the end support is determined by the formula

$$N_1 = P / (0.85 \times S_u)$$

where P is the smaller of the two values:

$$P_1 = A_s F_y = 31.8(50) = 1590^k \text{ (controls)}$$

$$P_2 = 0.85 f'_c b_c = 0.85(4)(84)(7) = 1999^k$$

Thus the number of connectors required is

$$N_1 = \frac{1590}{0.85(36.8)} = 50.8 \text{ or } \frac{50.8}{3} = 17 \text{ rows}$$

Service load design provides 22 rows of shear connectors between the end support and the maximum positive-moment location. The ultimate strength requirement for this region is therefore satisfied.

The number of connectors required between the point of maximum positive moment and the interior support is determined by the formula

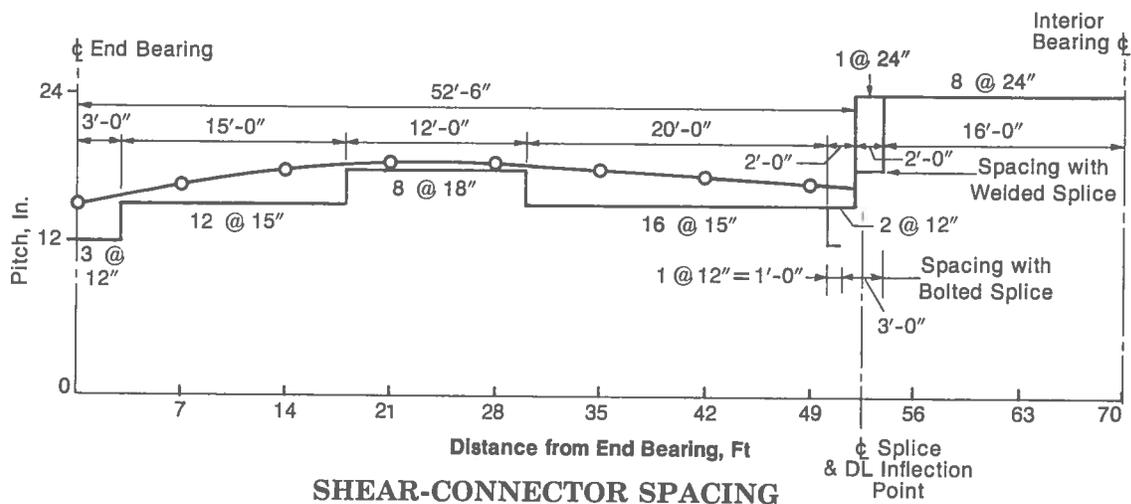
$$N_2 = (P + P_3) / (.85 S_u)$$

where $P_3 = A_s F_y = 6.16 \times 40 = 246^k$

Therefore the number of connectors required is

$$N_2 = (1590 + 246) / (.85 \times 36.8) = 58.7 \text{ or } \frac{58.7}{3} = 20 \text{ rows}$$

The number of rows furnished by service load design between the maximum positive-moment location and the interior support is determined to be 29. This number satisfies the ultimate strength requirement.



WELDED FIELD SPLICE

As in Chapter 3, the stringer is spliced near the dead-load inflection point, 17.5 ft from the pier, in one span. A full-strength welded splice is to be used, with all welds ground smooth. Therefore, only fatigue stresses need be investigated.

Fatigue in the butt-welded splice falls into AASHTO fatigue stress category B because the parts joined have the same width and thickness and the welds will be finished smooth and flush. For 100,000 cycles, the allowable range of stress in the base metal is 45 ksi.

Because of the splice location, the dead-load moments DL_1 and DL_2 are both zero. The live-load maximum positive moment is 420 kip-ft and the maximum negative moment is -310 kip-ft. The service-load stress range is calculated to be

$$\text{For positive moment } f_b = \frac{420(12)}{469} = +10.7$$

$$\text{For negative moment } f_b = \frac{310(12)}{356} = \frac{-10.4}{21.1} \text{ ksi range} \\ < 45 \text{ ksi}$$

BOLTED FIELD SPLICE

As in Chapter 3, a bolted field splice is designed as an alternative to the welded splice. The splice is to be a friction-type connection made with $\frac{7}{8}$ -in.-dia A325 bolts.

Shears 17.5 Ft from Pier, Kips

	For Service Loads	Factor	For Overload	Factor	Max Design Loads
DL_1 :	23.0	1	23.0	1.30	29.9
DL_2 :	4.5	1	4.5	1.30	5.8
$L+I$:	<u>48.0</u>	5/3	<u>80.0</u>	1.30	<u>104.1</u>
	75.5		107.5		139.8

Moments 17.5 Ft from Pier, Kip-Ft

	For Service Loads	Factor	For Overload	Factor	Max Design Loads
DL_1+DL_2 :	0	1	0	1.30	0
$-(L+I)$:	-310				
$+(L+I)$:	<u>420</u>	5/3	<u>700</u>	1.30	<u>910</u>
Maximum:	420		700		910

For load factor design of a bolted field splice, AASHTO Specifications require that splice material be proportioned for Maximum Design Loads and resistance to fatigue under service loads. Friction connections must resist slip deformation under Overload, therefore the fasteners must be designed for an allowable stress $F_v = 21$ ksi for Overload $D + (5/3)(L+I)$. The allowable bolt load in double shear is

$$P = 2 \times 0.6013 \times 21 = 25.3 \text{ kips per bolt}$$

For design of the splice material for Maximum Design Loads, the design moment is calculated as the greater of:

Average of the calculated moment on the section and maximum moment capacity of the section.

75% of maximum capacity of the section.

The calculated moment is that induced by the Maximum Design Load $1.30[D + (5/3)(L+I)]$. Splice material should have a capacity equal at least to the design moment. The section capacity is based on the gross section minus any flange-area loss due to bolt holes in excess of 15% of each flange area.

Base metal fatigue should be investigated at the gross beam section near friction type fasteners.

Section Properties at Splice

Design of the splice begins with calculation of the section properties, section capacity, and design moment and shear.

Properties of W30 × 108

$$I = 4,470 \text{ in.}^4$$

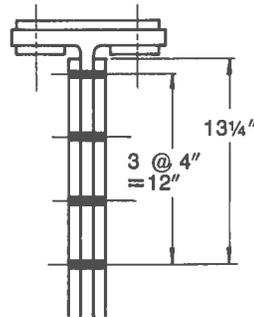
$$t_f = 0.76 \text{ in.}$$

$$b_f = 10.484 \text{ in.}$$

$$A = 31.80 \text{ in.}^2$$

$$t_w = 0.548 \text{ in.}$$

$$D = 29.82 \text{ in.}$$



DETAIL OF WEB SPLICE

Area of Flange Holes

$$A_H = 2 \times 1.0 \times 0.76 = 1.52 \text{ in.}^2$$

Area of Flange

$$A_f = 10.484 \times 0.76 = 7.97 \text{ in.}^2$$

The bolt holes remove

$$\% \text{ of flange} = \frac{1.52}{7.97} \times 100 = 19.07 \%$$

Therefore, $19.07\% - 15\% = 4.07\%$ of the flange area must be deducted for determination of the net section. With this deduction, the net moment of inertia is

$$I_{\text{net}} = 4,470 - 2 \times 0.0407 \times 7.97(14.53)^2 = 4,333 \text{ in.}^4$$

Design Moments and Shears

For $F_y = 50$ ksi, the net-section moment capacity is

$$M_{\text{net}} = \frac{50 \times 4,333}{14.91 \times 12} = 1,211 \text{ kip-ft}$$

$$75\% M_{\text{net}} = 0.75 \times 1,211 = 908 \text{ kip-ft}$$

The average of the calculated moment for the Maximum Design Loads and the net capacity of the section is

$$M_{\text{av}} = \frac{910 + 1,211}{2} = 1,061 > 908 \text{ kip-ft}$$

The design moment therefore is 1,061 kip-ft.

The design shear is obtained by multiplying the calculated shear for the Maximum Design Loads by the ratio of the design moment to the calculated moment on the section.

$$\text{Design shear} = 139.8 \times \frac{1,061}{910} = 163.0 \text{ kips}$$

Web-Splice Design

The web splice plates are proportioned to carry the design shear, a moment M_s due to the eccentricity of this shear and a portion M_w of the design moment on the section. The share of the design moment to be resisted by the web is obtained by multiplying the design moment by the ratio of the moment of inertia of the web $I_w = 1,035 \text{ in.}^4$ to the net moment of inertia of the entire section $I_{\text{net}} = 4,333 \text{ in.}^4$

Web Moments for Design Loads

$$M_v = 163.0 \times \frac{5}{12} = 68$$

$$M_w = 1,061 \times \frac{1,035}{4,333} = 253$$

321 kip-ft

Try two $\frac{3}{8} \times 26\frac{1}{2}$ -in. web splice plates with a gross area of 19.88 sq in. Their gross moment of inertia is

$$I_g = \frac{2 \times 0.375 (26.5)^3}{12} = 1,163 \text{ in.}^4$$

Try three rows of bolts with seven bolts per row on each side of the joint. The area of one hole is $0.375 \times 1 = 0.375$ sq in. The holes remove

$$\% \text{ of plate} = \frac{7 \times 0.375}{0.375 \times 26.5} \times 100 = 26.4 \%$$

Consequently, the fraction of the hole area that must be deducted in determination of the net section is

$$\frac{26.4 - 15}{26.4} = 0.432$$

d² for Holes

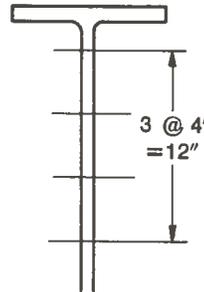
$$4^2 = 16$$

$$8^2 = 64$$

$$12^2 = 144$$

$$\Sigma d^2 = 224$$

**BOLT
SPACING
IN WEB**



$$Ad^2 \text{ web holes} = 0.432 \times 4 \times 0.375 \times 224 = 145 \text{ in.}^4$$

The net moment of inertia of the web splice plates then is

$$I_{\text{net}} = 1,163 - 145 = 1,018 \text{ in.}^4$$

Hence, the maximum bending stress in the plates for design loads is

$$f_b = \frac{321 \times 12 \times 13.25}{1,018} = 50.1 \approx 50 \text{ ksi}$$

The plates are satisfactory for bending. The allowable shear stress is

$$F_v = 0.55F_y = 0.55 \times 50 = 27.5 \text{ ksi}$$

The shear stress for the design shear is

$$f_v = \frac{163.0}{19.88} = 8.20 < 27.5 \text{ ksi}$$

The $\frac{3}{8} \times 26.5$ -in. web splice plates are satisfactory for strength requirements. The plates are next checked for fatigue under service loads.

Web Bending Stress Range for Service Loads

The range of moment carried by the web equals

$$M_w = (420 + 310) \times \frac{1,035}{4,333} = 174.4 \text{ kip-ft}$$

The maximum bending stress range in the gross section of the web splice plate is

$$f_b = \frac{174.4 \times 12 \times 13.25}{1,163} = 23.8 \text{ ksi}$$

Allowable Fatigue Stress Range for Splice Material

Fatigue in base metal adjacent to friction-type fasteners is classified by AASHTO as category B. For 100,000 cycles, the associated allowable stress range is 45 ksi. The actual fatigue stress range is within the allowable and the web splice plates are satisfactory.

Use two $\frac{3}{8} \times 26\frac{1}{2}$ -in. web splice plates.

Web Bolts

The 21 bolts in the web splice must carry the vertical shear, the moment due to the eccentricity of this shear about the centroid of the bolt group, and the portion of the beam moment taken by the web. These forces are induced by the Overload $D + (5/3)(L + I)$. The allowable load in double shear was previously computed to be $P = 25.3$ kips per bolt.

The polar moment of inertia of the bolt group about its centroid is

$$I = 6 \times 224 + 14(3)^2 = 1,470 \text{ in.}^4$$

The distance from the centroid to the outermost bolt is

$$d = \sqrt{12^2 + 3^2} = 12.38 \text{ in.}$$

Web Moments for Overload

$$M_v = 107.5 \times \frac{5}{12} = 44.8$$

$$M_w = 700 \times \frac{1,035}{4,333} = 167.2$$

212.0 kip-ft

The load per bolt due to shear is

$$P_s = \frac{107.5}{21} = 5.1 \text{ kips}$$

The load on the outermost bolt due to moment is

$$P_m = \frac{212 \times 12 \times 12.38}{1,470} = 21.4 \text{ kips}$$

The vertical component of this load is

$$P_v = 21.4 \times \frac{3}{12.38} = 5.2 \text{ kips}$$

The horizontal component is

$$P_h = 21.4 \times \frac{12}{12.38} = 20.8 \text{ kips}$$

Hence, the total load on the outermost bolt is the resultant

$$P = \sqrt{20.8^2 + (5.2 + 5.1)^2} = 23.2 < 25.3 \text{ kips}$$

Use 21 $\frac{3}{8}$ -in.-dia A325 bolts in three columns.

Flange-Splice Design

The flange splice plates are proportioned for design loads and checked for fatigue in a similar manner to that for the web plates. The flange splice carries that portion of the design moment not carried by the web. Deducting the moment taken by the web from the design moment on the section, the flanges must be designed for

$$M_f = 1,061 - 253 = 808 \text{ kip-ft}$$

Compressive and tensile forces in the flanges form a couple that supply this capacity. Each force equals

$$P_f = \frac{808 \times 12}{29.82 - 0.76} = 334 \text{ kips}$$

To carry this force, the splice plate on each flange must have an area of at least

$$A_f = \frac{334}{50} = 6.68 \text{ in.}^2$$

Try a $\frac{3}{8} \times 10$ -in. plate on the outer surface of each flange and two $\frac{7}{16} \times 4$ -in. plates on the inner surface of each flange. The net area of these plates after deduction of holes for two bolts is

$$A_f = (\frac{3}{8} \times 10) + (2 \times \frac{7}{16} \times 4) - [(2 \times 1 \times \frac{3}{8}) + (2 \times 1 \times \frac{7}{16}) - 0.15 \{ (\frac{3}{8} \times 10) + (2 \times \frac{7}{16} \times 4) \}] = 6.71 > 6.68 \text{ in.}^2$$

Use one $\frac{3}{8} \times 10$ -in. plate and two $\frac{7}{16} \times 4$ -in. plates. The plates are then checked for fatigue under service load.

The range of moment carried by the flanges equals

$$M_f = (420 + 310) \left(1 - \frac{10.35}{43.33} \right) = 555 \text{ kip-ft}$$

The range of force in each flange is computed to be

$$P_f = \frac{555 \times 12}{29.82 - 0.76} = 229 \text{ kips}$$

The maximum stress range in the gross section of the flange splice plate then is

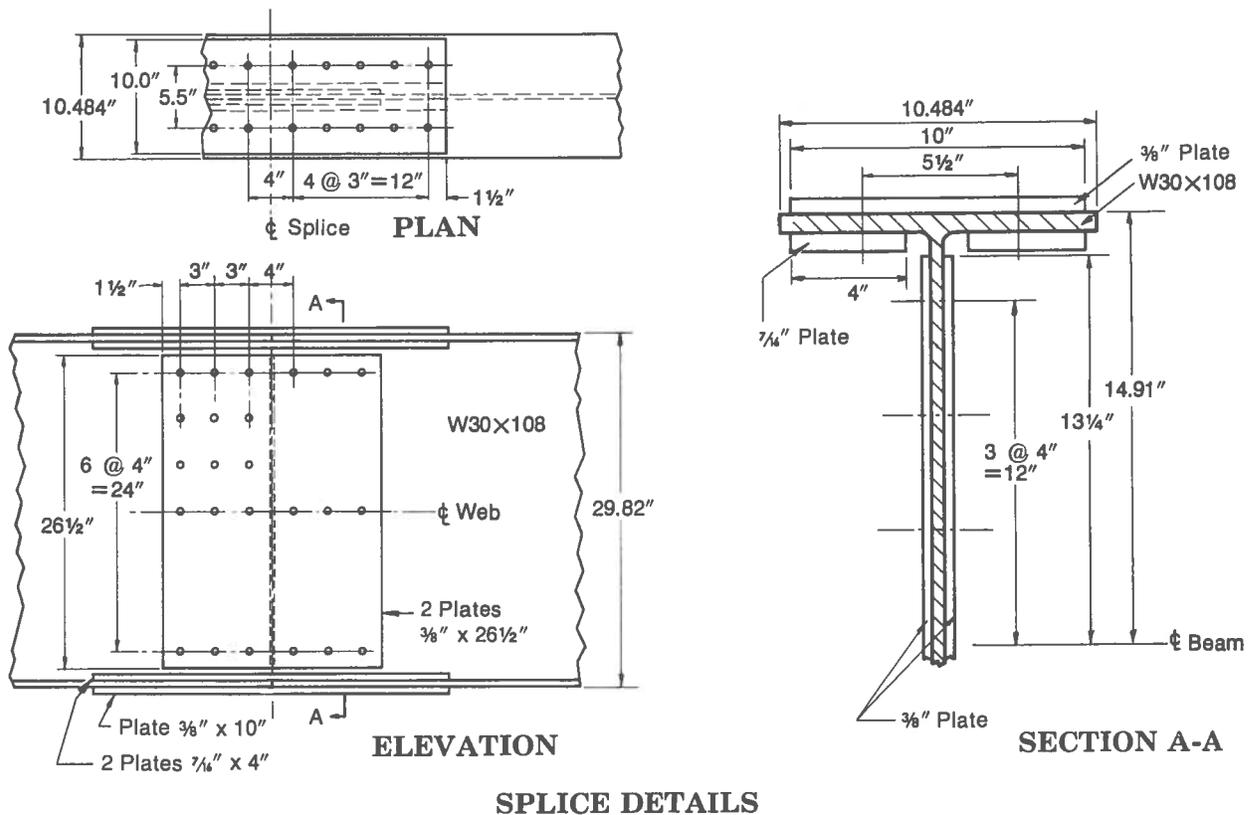
$$f_f = \frac{229}{(\frac{3}{8} \times 10 + 2 \times \frac{7}{16} \times 4)} = 31.6 \text{ ksi} < 45 \text{ ksi}$$

Flange Bolts

The number of bolts required in the flange splice is determined by the capacity needed for transmitting the flange force under the Overload $D + (5/3)(L + I)$. The flange moment is the moment on the section less the moment carried by the web:

$$M_f = 700 - 167 = 533 \text{ kip-ft}$$

Use 10 bolts in two rows. Details of the splice are shown below.



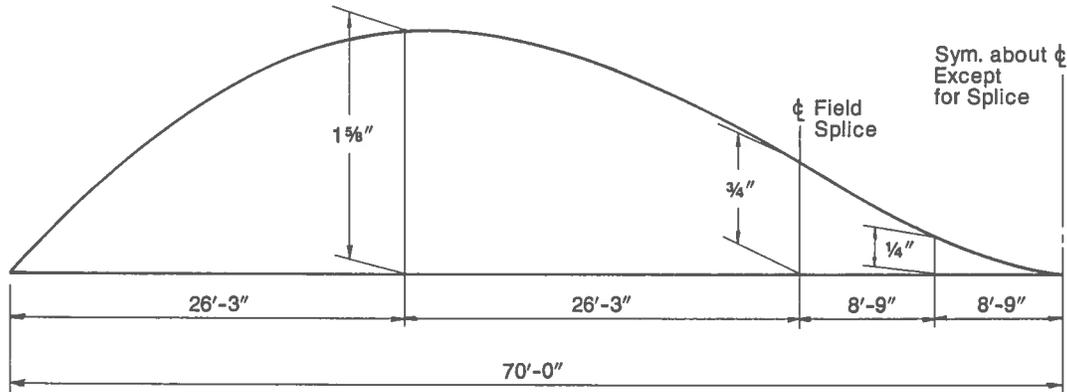
For this moment, the force in the flange is

$$P_f = \frac{533 \times 12}{29.82 - 0.76} = 220 \text{ kips}$$

$$\text{Bolts required} = \frac{220}{25.3} = 8.7$$

DEFLECTIONS

Dead-load and live-load deflections are computed for Service Loads, in the same way as for working-stress design.

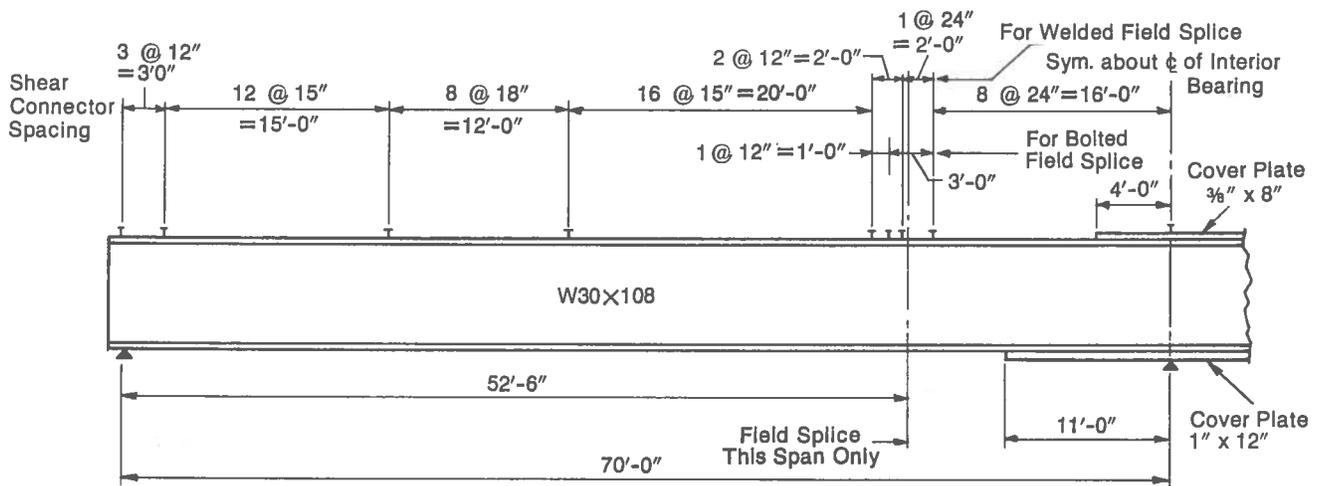


CAMBER DIAGRAM

Live-load deflection is 0.902 in. The deflection-span ratio is 1/930, well within the allowable value of 1/800.

FINAL DESIGN

An elevation of the two-span, continuous, composite stringer is shown below. See also the detail drawing at the end of this chapter. An alternate design is examined next.



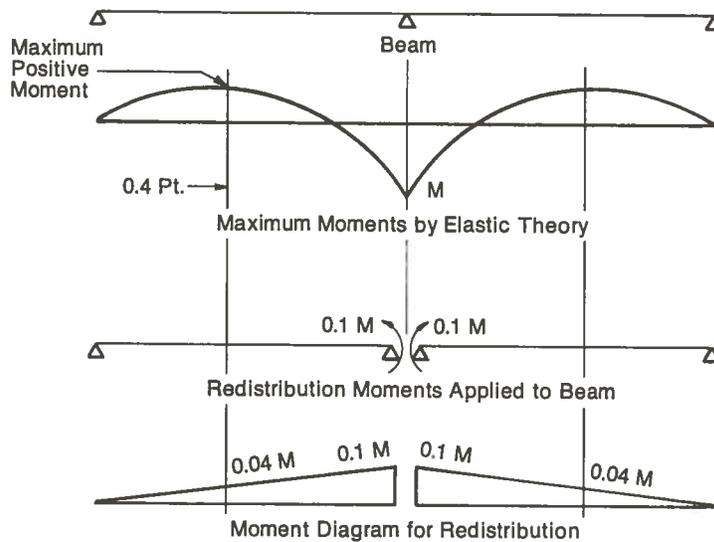
BEAM ELEVATION

Alternate Design

In the negative-moment region, the stringer as designed in the first part of this chapter does not qualify as a compact section. Although satisfying every other compactness requirement, it does not meet the restrictions on unbraced length of compression flange with only two equally spaced lines of diaphragms per span. If the

unbraced length is decreased, by adding another line of diaphragms in each span, to make the negative-moment section compact, a lighter section results. If the savings in beam material thus achieved is greater than the cost of the additional diaphragms, the structure with the compact section would be more economical. This possibility is investigated for an alternate design in which compactness is obtained in the negative-moment region, with the W30×108 plus cover plates, by adding a line of diaphragms 9 ft from the center support. Since it was shown on page 3A.9 that the W30×108 alone satisfies compactness requirements for an unbraced length of up to 9.62 ft, it is clear that a diaphragm line 9 ft from the support will insure compactness for the W30×108 plus cover plates over a part of the 9 ft length.

For the compact, negative-moment section, the moments from the original analysis by elastic theory may be redistributed. They may be reduced by 10% in the negative-moment region and increased by 4% in the positive-moment region, as permitted for compact sections. This redistribution is equivalent to applying at the ends of each span at the center support, a positive moment equal to 10% of the maximum negative moment by elastic theory (see diagram).



MOMENT REDISTRIBUTION FOR COMPACT SECTION

If the bending moments in each span are then superimposed on the original moments, a 10% reduction in the maximum negative moment and a 4% increase in the maximum positive moment results.

MAXIMUM POSITIVE MOMENT

Because of the moment redistribution, the positive-moment section of the stringer must be investigated to determine whether the W30×108 alone is sufficient to carry the increase in maximum positive moment.

Redistributed Positive Moments, Kip-Ft

$$DL_1: M = 309 + 0.04 \times 551 = 331$$

$$DL_2: M = 57 + 0.04 \times 101 = 61$$

$$L+I: M = 750 + 0.04 \times 601 = 774$$

Stresses induced by the redistributed positive moments are determined with the section moduli previously computed. As before, the allowable stress for the Overload $D + (5/3)(L+I)$ is 47.5 ksi.

Steel Stresses for Overload—Combination A

Bottom of Steel (Tension)

$$\text{For } DL_1: F_b = \frac{331 \times 12}{300} = 13.2$$

$$\text{For } DL_2: F_b = \frac{61 \times 12}{425} = 1.7$$

$$\text{For } L+I: F_b = \frac{774 \times 12}{469} \times \frac{5}{3} = 33.0$$

$$\underline{47.9 \approx 47.5 \text{ ksi}}$$

The section is overstressed 1% but is considered satisfactory for Overload.

Maximum strength is checked next, to determine if the fully plastic moment capacity of the section $M_u = 32,990$ kip-in. is adequate. The Maximum Design Loads induce a maximum positive moment after redistribution equal to

$$M_u = 1.30 \left[DL_1 + DL_2 + \frac{5}{3}(L+I) \right] = 1.30 \left(331 + 61 + \frac{5}{3} \times 774 \right)$$

$$= 2,187 \text{ kip-ft} = 26,244 \text{ kip-in.} < 32,990$$

Consequently, the section has adequate strength for maximum positive moment.

MAXIMUM NEGATIVE MOMENT

Next, the section at the pier is proportioned for the redistributed maximum negative moment. A trial section consisting of the W30 × 108 with a $\frac{5}{16} \times 7$ -in. top cover plate and a $\frac{1}{2} \times 9\frac{1}{2}$ -in. bottom cover plate is chosen.

Redistributed Negative Moments at Pier, Kip-Ft

$$DL_1: M = 0.9 \times 551 = 496$$

$$DL_2: M = 0.9 \times 101 = 91$$

$$L+I: M = 0.9 \times 601 = 541$$

Properties are calculated for the cover-plated beam alone and for this beam plus the longitudinal reinforcing in the concrete slab.

Steel Section at Interior Support

Material	<i>A</i>	<i>d</i>	<i>Ad</i>	<i>Ad</i> ²	<i>I</i> _o	<i>I</i>
W30 × 108	31.8				4,470	4,470
Top Plate $\frac{5}{16} \times 7$	2.19	15.07	33.0	497		497
Bottom Plate $\frac{1}{2} \times 9\frac{1}{2}$	4.75	-15.16	-72.0	1,092		1,092

$$d_s = \frac{39.0}{38.74} = 1.007 \text{ in.}$$

$$I_{NA} = \frac{6,059}{6,020} \text{ in.}^4$$

$$d_{\text{Top}} = 14.91 + 0.31 + 1.01 = 16.23 \text{ in.}$$

$$d_{\text{Bot.}} = 14.91 + 0.50 - 1.01 = 14.40 \text{ in.}$$

$$S_{\text{Top}} = \frac{6,020}{16.23} = 371 \text{ in.}^3$$

$$S_{\text{Bot.}} = \frac{6,020}{14.40} = 418 \text{ in.}^3$$

Steel Section at Interior Support with Reinforcing Steel

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	38.74		- 39.0			6,059
Reinf. Steel 14 No. 6	6.16	19.52	120.2	2,347		2,347

$$44.90 \text{ in.}^2 \qquad 81.2 \text{ in.}^3 \qquad 8,406$$

$$d_s = \frac{81.2}{44.90} = 1.81 \text{ in.}$$

$$-1.81 \times 81.2 = -147$$

$$I_{NA} = \frac{8,259}{8,259} \text{ in.}^4$$

$$d_{\text{Top of steel}} = 14.91 + 0.31 - 1.81 = 13.41 \text{ in.} \qquad d_{\text{Bot. of steel}} = 14.91 + 0.50 + 1.81 = 17.22 \text{ in.}$$

$$S_{\text{Top of steel}} = \frac{8,259}{13.41} = 616 \text{ in.}^3$$

$$S_{\text{Bot. of steel}} = \frac{8,259}{17.22} = 480 \text{ in.}^3$$

$$d_{\text{Reinf.}} = 19.52 - 1.81 = 17.71 \text{ in.}$$

$$S_{\text{Reinf.}} = \frac{8,259}{17.71} = 466 \text{ in.}^3$$

Check at Overload

Because the section at the support is made compact by introducing an additional line of diaphragms to brace the compression flange, the Overload condition governs the design. The moment relationship is

$$\left[D + \frac{5}{3}(L + I) \right] \leq 0.80 F_y S$$

The allowable stress is thus $(0.80)(50) = 40$ ksi for the beam and $(0.80)(40) = 32$ ksi for the reinforcing steel.

Steel Stresses for Overload—Combination A

Top of Steel (Tension)

Bottom of Steel (Compression)

$$\text{For } DL_1: F_b = \frac{496 \times 12}{371} = 16.0$$

$$F_b = \frac{496 \times 12}{418} = 14.2$$

$$\text{For } DL_2: F_b = \frac{91 \times 12}{616} = 1.8$$

$$F_b = \frac{91 \times 12}{480} = 2.3$$

$$\text{For } L + I: F_b = \frac{541 \times 12}{616} \times \frac{5}{3} = 17.6$$

$$F_b = \frac{541 \times 12}{480} \times \frac{5}{3} = 22.5$$

$$35.4 < 40 \text{ ksi}$$

$$39.0 < 40 \text{ ksi}$$

Reinforcing Steel Stress (Tension)

$$DL_2: F_b = \frac{91 \times 12}{466} = 2.3$$

$$L + I: F_b = \frac{541 \times 12}{466} \times \frac{5}{3} = 23.2$$

$$25.5 < 32 \text{ ksi}$$

The trial section is satisfactory for maximum negative moment for Overload.

Fatigue Check

Fatigue in the reinforcing steel is checked next for a live-load moment range from 0 to 601 kip-ft, without the 10% reduction.

$$f_{sr} = \frac{601(12)}{466} = 15.5 < 20 \text{ ksi}$$

The trial section therefore is satisfactory for Overload.

COVER-PLATE CUTOFF

The cover plates are cut off 5 ft from the pier. Stresses for Overload are calculated at the theoretical cutoff location 1.5 times the plate width closer to the pier ($5.00 - 1.5 \times 9.5/12 = 3.81$ ft = 3 ft-9¾ in. from the pier). Fatigue is investigated at the actual termination.

Section with W30 and Reinforcing Steel

Material	A	d	Ad	Ad ²	I _o	I
W30×108	31.8				4,470	4,470
Reinf. Steel 14 No. 6	6.16	19.52	120.2	2,347		2,347

$$37.96 \text{ in.}^2$$

$$120.2 \text{ in.}^3$$

$$6,817$$

$$d_s = \frac{120.2}{37.96} = 3.17 \text{ in.}$$

$$-3.17 \times 120.2 = -\frac{381}{I_{NA} = 6,436 \text{ in.}^4}$$

$$d_{\text{Top}} = 14.91 - 3.17 = 11.74 \text{ in.}$$

$$d_{\text{Bot.}} = 14.91 + 3.17 = 18.08 \text{ in.}$$

$$S_{\text{Top}} = \frac{6,436}{11.74} = 548 \text{ in.}^3$$

$$S_{\text{Bot.}} = \frac{6,436}{18.08} = 356 \text{ in.}^3$$

$$d_{\text{Reinf.}} = 19.52 - 3.17 = 16.35 \text{ in.}$$

$$S_{\text{Reinf.}} = \frac{6,436}{16.35} = 394 \text{ in.}^3$$

Bending Moments 3.81 Ft from Pier

	DL ₁	DL ₂	(L+I)
M, kip-ft	-410	-76	-449

Steel Stresses for Overload 3.81 Ft from Pier

Bottom of Steel (Compression)

$$\text{For } DL_1: F_b = \frac{(410 - 0.095 \times 551)12}{300} = 14.3$$

$$\text{For } DL_2: F_b = \frac{(76 - 0.095 \times 101)12}{356} = 2.2$$

$$\text{For } L+I: F_b = \frac{(449 - 0.095 \times 601)12}{356} \times \frac{5}{3} = 22.0$$

$$35.5 < 40.0 \text{ ksi}$$

Stresses at the theoretical cutoff of the cover plates are within the allowable. Next, fatigue is investigated at the actual ends of the plates adjacent to the fillet weld.

Bending Moments 5 Ft from Pier

	DL ₁	DL ₂	-(LL+I)	+(L+I)
M, kip-ft	-361	-69	-412	64

It is obvious from the above moments that no tension will exist in the bottom flange. Therefore, no restrictions are placed on the fatigue stress range there.

Tensile stress range in the top flange is computed to be

$$f_{sr} = \frac{(412+64)(12)}{548} = 10.4 < 21 \text{ ksi}$$

Finally, the slab reinforcement is checked for Overload and for fatigue under Service Load at the theoretical cut-off location.

Reinforcing Steel Stresses 3.81 Ft from Pier

$$\text{For } DL_2: F_b = \frac{(76 - 0.095 \times 101)12}{394} = 2.0$$

$$\text{For } L+I: F_b = \frac{(449 - 0.095 \times 601)12}{394} \times \frac{5}{3} = 19.9$$

$$21.9 < 32 \text{ ksi}$$

Reinforcing Steel Fatigue Stress Range 3.81 Ft from Pier

$$\text{Range} = \frac{(449+42)12}{394} = 15.0 < 20 \text{ ksi}$$

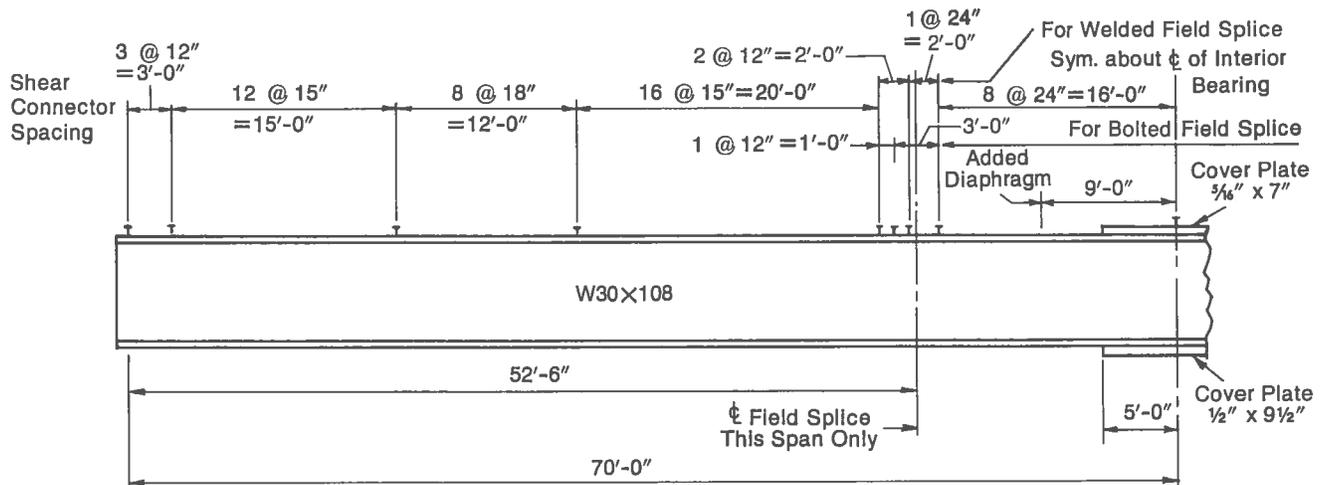
All stresses are within the allowable, and the stress range is satisfactory for fatigue. Therefore, the cover plates can be cut off 5 ft from the center support.

COVER-PLATE WELDS

Use the minimum weld size of $\frac{5}{16}$ in. for the cover plates.

FINAL ALTERNATE DESIGN

An elevation view of the alternate design is shown below.



BEAM ELEVATION—ALTERNATE DESIGN

COMPARISON OF ALTERNATE AND ORIGINAL DESIGN

As a measure of the relative economy of the alternate design with a compact negative-

moment section and the original design with a noncompact negative-moment section, the difference in weights of steel required for the two designs is calculated.

**Weight Comparison for Beam Only
Noncompact Section**

	Material	Weight, lb per ft	Weight, lb
Beam	W30 × 140 ft	108	15,120
Top cover plate	$\frac{3}{8} \times 8 \times 8$ ft	10.2	82
Bottom cover plate	$1 \times 12 \times 22$ ft	40.8	898
Total			<u>16,100</u>

Compact Section

Beam	W30 × 140 ft	108	15,120
Top cover plate	$\frac{5}{16} \times 7 \times 10$ ft	7.4	74
Bottom cover plate	$\frac{1}{2} \times 9\frac{1}{2} \times 10$ ft	16.2	<u>162</u>
Total			15,356
Beam material savings = 16,100 – 15,356 = 744 lb per beam			

Weight of Two Lines of Diaphragms

$$2 \text{ C}10 \times 15.3 \times 25 \text{ ft}$$

$$\text{Weight} = 2 \times 25 \times 15.3 = 765 \text{ lb}$$

$$\text{Weight per beam} = \frac{765}{4} = 191 \text{ lb}$$

Net savings = 744 – 191 = 553 lb per beam, or 2,212 lb per bridge

With a weight savings of only about 3%, it is difficult to say whether the alternate design is more economical than the original design when added fabrication and erection costs for the additional line of diaphragms is taken into account.

A detail drawing of the original design is shown on the following page.