

Note:

Designs presented in this chapter are in accordance with the Tenth Edition of the Standard Specifications for Highway Bridges of the American Association of State Highway and Transportation Officials (AASHTO) dated 1969. They should be reviewed for adequacy in conforming to the latest edition of the AASHTO Specifications and subsequent interims, especially with regard to fatigue provisions and welded stud shear connectors.

II/3

Composite: Wide-Flange Beam

Introduction

Composite and noncomposite wide-flange beam bridges are in common use. Non-composite beams generally are economical for simply supported spans shorter than about 40 ft and for continuous spans shorter than about 60 ft. Composite beams generally are economical for longer spans.

In this chapter, five design examples are given for wide-flange beams:

- I. A 40-ft simple-span noncomposite beam.
- II. A 40-ft simple-span, composite beam.
- III. A two-span, continuous beam (70 ft—70 ft), composite for positive moment only.
- IV. A two-span, continuous beam (70 ft—70 ft), composite for positive moment and negative moment.
- V. A four-span, continuous beam (70 ft—90 ft—90 ft—70 ft), composite for positive moment only.

The design example for a 40-ft noncomposite, simple-span bridge is given to permit comparison with a 40-ft composite, simple-span bridge. All design examples are for an interior stringer.

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The design examples are for ASTM A36 steel. Other structural steels such as ASTM A441, A572, and A588 have also proven economical for such structures. Design procedures for these high-strength low-alloy steels are similar to those for A36 steel except that the allowable stresses are generally higher.

Nomenclature for structural shapes adopted by the American Iron and Steel Institute in 1969 is used in this chapter; for example, the designation W33×130 replaces the former 33WF130.

General Design Considerations

In composite construction, shear connectors are provided between steel stringers and a concrete slab to make them act as a unit. Three elements, therefore, must be considered in design: (1) the reinforced concrete slab, (2) steel stringers, and (3) shear connectors.

Longitudinally, the reinforced concrete slab, located on the compression side of the steel stringer, acts as an effective cover plate. Transversely, the slab is designed to span between stringers as in noncomposite construction. Combined stresses due to composite action and due to slab action, which act at right angles to each other, are not critical and need not be investigated.

The steel stringers may be rolled beams, rolled beams with cover plates, or welded girders. Unsymmetrical sections, such as rolled beams with cover plates on the bottom flange or welded girders with bottom flange plates heavier than top flange plates, generally are economical for composite construction. Design of Composite, Welded Plate Girders is discussed in Chapter II/4.

Shear connectors provide mechanical connection between the slab and the steel stringers. (While there is a natural bond between the concrete slab and steel stringers, this bond is considered unreliable for providing the horizontal shearing resistance essential to composite action.) The connectors must be able to transfer horizontal shear between concrete slab and steel stringers so that the entire structure deforms as a unit for its full life.

Composite construction offers the following advantages over conventional steel-stringer-and-slab construction:

1. Greater economy.
2. Shallower construction.
3. Less deflection.
4. Greater factor of safety.
5. Better lateral bracing of the top flange.

Composite bridges may be built with or without temporary shoring. The design examples in this chapter are prepared for unshored construction, since most highway bridges are built without shores.

In unshored construction, the steel stringers must support their own weight plus the weight of the concrete slab. The composite section supports the weight of any additional dead load placed after the slab has hardened, plus all live load and impact. In shored construction, the steel stringers are temporarily supported during placing and hardening of the concrete slab and the composite section supports all loads after removal of the supports.

Sustained loads, such as dead loads, on concrete cause it to creep. In flexural members, creep reduces the intensity of the compressive stresses in the concrete. Thus under sustained loads, the concrete deck is less effective than for temporary loads.

The effect of creep is accounted for in composite construction by increasing the modular ratio n by a factor of 3. Stresses due to long-time dead loads on the composite section are computed with section properties based on the increased modular ratio $3n$.

Concrete is assumed ineffective in resisting tension. Thus, the slab is not considered part of the composite section in the negative-moment region of continuous, composite construction. Continuous designs are based upon one of the following assumptions:

1. Positive moments due to loads applied after the concrete slab has been placed and hardened are resisted by the composite section. Negative moments are resisted by the rolled beam plus cover plates. Shear connectors need be provided only in the positive-moment regions.
2. Positive moments due to loads applied after the concrete slab has been placed and hardened are resisted by the composite section. Negative moments also are resisted by a composite section that consists of the rolled beam and cover plates plus the longitudinal reinforcing bars in the deck. Shear connectors must be provided over the full length of the stringer.

A composite stringer bridge is designed as a series of T-beams. Each consists of one steel stringer and a portion of the concrete slab. The concrete is transformed into an equivalent area of steel by dividing the area of the slab by the modular ratio n (or $3n$ when creep is considered). The properties of the transformed section, and stresses at the top and bottom of the steel stringer and top of the concrete slab, are computed.

The assumed effective width of the slab as a T-beam flange must not exceed the following:

One-fourth the span of the stringer.

The spacing, center to center, of stringers.

Twelve times the least thickness of the slab.

For stringers having a flange on one side only, the effective flange width must not exceed one-twelfth the span of the stringer, six times the thickness of the slab, one-half the distance, center to center, to the adjacent stringer.

LATERAL DISTRIBUTION OF DEAD LOAD

Each interior stringer carries the weight of that portion of concrete slab extending a distance of one-half the stringer spacing on either side of the stringer. An outer stringer carries the weight of that portion of slab extending from the outer edge to a point midway between the outer stringer and the adjacent interior stringer.

The dead load of curbs, parapets, railings, and wearing surface, if placed after the slab has cured, may be considered equally distributed to all stringers.

If the overhang of the slab beyond the outer stringer is maintained at one-half the stringer spacing, total dead load on all stringers will be nearly equal.

LATERAL DISTRIBUTION OF LIVE LOAD

Live-load bending moments for an interior stringer are determined by applying to the stringer a fraction of the wheel loads, as prescribed in AASHO specifications. For a bridge consisting of a concrete slab on steel stringers and designed for two or more traffic lanes, this fraction is:

$$\text{Live-load distribution factor} = \frac{S}{5.5} \text{ wheels}$$

where S = average stringer spacing, ft, but not more than 14 ft.

Live-load bending moments for an outer stringer are determined by applying to the stringer the reactions due to wheel loads on the concrete slab, which is assumed to act as a simply supported beam between stringers. The fraction of wheel loads used, however, should not be less than:

$$\text{Live-load distribution factor} = \frac{S}{5.5} \text{ wheels}$$

when $S = 6$ ft or less.

$$\text{Live-load distribution factor} = \frac{S}{4 + 0.25S} \text{ wheels}$$

when S is more than 6 ft but less than 14 ft.

The live load applied to an outer stringer as determined by these formulas generally exceeds that obtained by assuming the slab to act as a simple span between stringers.

The live load applied to an outer stringer of a bridge designed for two or more lanes of traffic will be slightly less than that for an interior stringer. If stringers are positioned under the roadway to give equal dead loads to interior and outer stringers, often only the interior stringer need be designed and the same beam section may be used for the outer stringer.

In the calculation of stringer reactions and end shears, the live load of the wheel adjacent to the support should be distributed by assuming the concrete slab to act as a simple beam between stringers. For loads in other positions on the span, the same live-load distribution factors are used as for moment. (Many designs are made assuming the live-load distribution factor for moment applies for all shears and reactions. The resulting errors in reactions and shears are small.)

ALTERNATE INTERSTATE LOADING

Interstate loading, often called military loading, must be considered in the design of bridges on the interstate system. This loading governs moment in simple spans shorter than 38 ft and governs end shear in simple spans shorter than 30 ft.

Interstate loading does not govern any of the design examples in this chapter.

MAXIMUM MOMENTS IN CONTINUOUS STRINGERS

Curves of maximum moments in continuous spans may be computed or determined from tables based upon uniform moment of inertia of stringer. (See References 3 and 4 at end of chapter.) For the usual composite stringer with partial-length cover plates, these tables will yield a design well within reasonable accuracy. Moment curves for dead load and for live load based upon variable moment of inertia, when compared with moment curves based upon uniform moment of inertia, will show some variations. These, however, tend to be compensating; so the variations in total-moment curves are small. This is explained as follows:

To determine moments due to weight of concrete slab, stringers and framing details, the variable moment of inertia of the steel section alone is used. Since the moment of inertia is greater over the supports than in the positive-moment regions, negative moments will exceed and positive moments will be less than those based on uniform moment of inertia.

To determine moments due to the weight of curbs, parapets, railings and wearing surface, the moment of inertia of the composite section with an increased modular ratio of $3n$ is used in the positive-moment area and the moment of inertia of the steel section alone is used in the negative-moment area. Both positive and negative moments differ very little from those based upon uniform moment of inertia.

To determine moments due to live load plus impact, the moment of inertia of the composite section with a modular ratio of n is used in the positive-moment area and the moment of inertia of the steel section alone is used in the negative-moment area. The moment of inertia over the supports is less than in the positive-moment region. Hence, negative moments will be less than and positive moments will exceed those based on uniform moment of inertia.

The difference in negative moments for dead load tends to balance the difference in negative moments for live load and impact. The difference in positive moments for dead load tends to balance the difference in positive moments for live load and impact. Thus, the difference in total moments for uniform and variable moments of inertia depends on the relative magnitudes of dead-load and live-load moments. Stresses based on uniform moment of inertia generally will be within 3 or 4% of stresses based on variable moment of inertia. This is well within the limits of accuracy of the predicted loads on the stringer.

Curves of maximum moments based on uniform and variable moments of inertia are shown for Designs III and V, with stresses in the maximum-positive-moment section and the maximum-negative-moment section calculated under each assumption. The maximum increase in stress for variable moment of inertia was 0.3% and the maximum decrease was 4.3%.

COVER-PLATE CUTOFFS

The theoretical cutoff point of a cover plate is located where the stress in the rolled beam without the cover plate equals the allowable stress, exclusive of fatigue considerations. If the compression flange is unsupported, reduced allowable stresses, governed by lateral buckling requirements, should be used. From the theoretical cutoff point, the cover plate should be extended a terminal distance, as defined later. Stresses in the rolled beam at the actual end of the cover plate should not exceed allowable fatigue stresses.

The exact computation of theoretical cover-plate cutoff locations is very tedious when a portion of the load is carried by the steel section alone. For simply supported

spans, an approximate calculation of the theoretical length of the cover plate can be made on the assumption that the ratios of the section modulus without a cover plate to the section modulus with a cover plate are the same for the composite section without creep, for the composite section with creep, and for the steel section alone. The following approximate formula based on a parabolic moment diagram may be used:

$$L_{cp} = (L - 2a) \sqrt{1 - \frac{Z'_{bs}}{Z_{bs}}} + 2a$$

where L_{cp} = theoretical length of cover plate

L = span, ft

a = distance of maximum-moment section from midspan. For HS20 truck loading, $a = 0$ for spans less than 23.8 ft, 3.50 ft for spans between 23.8 and 33.8 ft, 2.33 ft for spans over 33.8 ft

Z'_{bs} = section modulus of rolled beam without a cover plate (steel section alone)

Z_{bs} = section modulus of rolled beam with a cover plate (steel section alone)

After the approximate theoretical cover-plate cutoff location has been found, stresses at that point in the section without the cover plate are determined. If the controlling stress (usually steel stress in the bottom flange) exceeds the allowable stress, the theoretical cover-plate cutoff point is moved slightly and stresses are recalculated. This procedure is repeated until the controlling stress equals the allowable stress.

For continuous spans when reinforcing bars are not considered part of the resisting section for negative moments, theoretical cover-plate cutoff locations in the non-composite negative-moment region can be determined by plotting the resisting moment of the steel section without a cover plate on the curve of maximum total moments and scaling the length of cover plate required. Theoretical cover-plate cutoff locations in the positive-moment regions are most easily determined by assuming locations, scaling moments at these locations from the curves of maximum moments, and calculating stresses. Assumed cutoff locations are adjusted until the controlling stress equals the allowable stress. When reinforcing bars are considered part of the resisting section for negative moments, theoretical cover-plate cutoff locations in the negative-moment regions are determined by the same procedure as described for positive moments.

All cover plates must extend beyond their theoretical ends by the terminal distance, or to a point where the stress in the beam flange is equal to the allowable fatigue stress in base metal adjacent to fillet welds, the greater length governing. The terminal distance is two times the nominal cover-plate width for cover plates not welded across their ends, and $1\frac{1}{2}$ times the nominal cover-plate width for cover plates welded across their ends. The weld connecting the cover plate to the flange beyond the theoretical cutoff must be continuous and of sufficient size to develop a total force not less than the computed force in the cover plate at its theoretical end.

FATIGUE

For rolled beams with cover plates, allowable fatigue stresses at the ends of the cover plates (rather than allowable stresses exclusive of fatigue considerations at the theoretical cutoffs of cover plates) usually control cover-plate cutoff locations. AASHTO specifications give the number of cycles of maximum stress to be assumed and corresponding formulas for allowable fatigue stress.

For freeways, expressways, and major highways and streets, the allowable fatigue stress in a rolled beam at the fillet-welded end of a cover plate is based on 500,000 cycles of stress and is given by

$$F_r = \frac{12,000k_1}{1-R}$$

where F_r = allowable fatigue stress, psi

k_1 = factor that depends on type of steel = 1.00 for A36 steel

R = ratio of minimum to maximum stress (tensile stress is positive; compressive stress, negative)

The allowable shear in a fillet weld for A36 steel is

$$F_r = \frac{10,800}{1 - 0.55R}$$

For minor highways and streets, the allowable fatigue stress in a rolled beam at the fillet-welded end of a cover plate is based on 100,000 cycles of repeated stress and is given by

$$F_r = \frac{18,000k_1}{1 - R}$$

The allowable fatigue stresses at cover-plate cutoffs in continuous spans near inflection points are low, particularly when the design is based on 500,000 cycles of stress. Consequently, the designer may economically specify a light rolled beam and extend top and bottom cover plates continuously from the negative-moment region to the positive-moment region, varying the thickness of the cover plates by butt welding plates of different thicknesses. Or the designer may specify a heavier rolled beam with thinner cover plates that can be cut off several feet each side of the inflection point.

Allowable fatigue stresses for base metal, for weld metal or base metal adjacent to butt welds, and for base metal adjacent to fasteners at field splices are given in AASHTO specifications. Fatigue of base metal (not adjacent to a butt weld) controls design only if reversal of stress occurs. Fatigue of weld metal or base metal adjacent to a butt weld controls design only if the ratio of minimum to maximum stress is less than 0.226 in a design for 500,000 cycles of stress, or if reversal occurs in a design for 100,000 cycles of stress.

If butt-welded splices conform to the following conditions, they may be designed in accordance with the allowable fatigue stress for base metal:

1. The parts joined are of equal thickness.
2. The parts joined are of equal width, or tapered as illustrated on p. II/4.2.
3. Weld soundness, established by radiographic inspection, meets specified requirements.
4. Welds are made smooth and flush by grinding in the direction of applied stress.

Allowable fatigue stresses for A36 base metal at 500,000 cycles are given by AASHTO specifications as follows:

$$\text{Tension: } F_r = \frac{20,500}{1 - 0.55R}$$

$$\text{Compression: } F_r = \frac{0.55F_y}{1 - \left(\frac{0.55F_y}{13,300} - 1\right)R}$$

where F_y = minimum yield strength of the material, psi.

Butt-welded splices that do not meet the above conditions must be designed for allowable fatigue stresses for weld metal or base metal adjacent to butt welds. For 500,000 cycles, these stresses are given by

$$\text{Tension: } F_r = \frac{17,200}{1 - 0.62R}$$

$$\text{Compression: } F_r = \frac{0.55F_y}{1 - \left(\frac{0.55F_y}{10.6} - 1 \right) R}$$

LATERAL BUCKLING

The bottom flanges of the stringers are laterally unsupported between diaphragms. Consequently, lateral buckling must be considered in determining the allowable compressive stresses in the negative-moment regions of continuous spans. The allowable compressive stress for A36 steel is $20,000 - 7.5(L/b)^2$ for $L/b \leq 36$, where L = length, in., of unsupported flange between diaphragms or other points of support, or the distance, in., from interior support to point of dead-load contraflexure, the shorter length governing, and b = flange width, in.

The allowable compressive stress at interior supports of continuous spans may be increased 20% over that permitted by the above formula, but may in no case exceed the allowable unit stress for a compression flange laterally supported its full length. The allowable stress at the theoretical cutoff points of cover plates in negative-moment regions is determined by the above formula, with no percentage increase.

WEB SHEARING STRESS

Web shearing stress may be determined on the basis that the web of the steel stringer carries the total external shear. This assumption neglects shear taken by the steel flanges and concrete slab. The shear is assumed to be uniformly distributed over the gross area of the web. Web shearing stress seldom is critical.

DESIGN OF SHEAR CONNECTORS

Many mechanical devices have been used to provide the necessary resistance to horizontal shear at the junction of concrete slabs and steel stringers. Steel studs automatically end-welded to the stringers and channels fillet welded to the stringers are the most common types used. Allowable loads for stud and channel shear connectors are given in the AASHTO specifications. Welded studs are used in all the following design examples.

To insure serviceability and durability of shear connectors, AASHTO specifications recommend design criteria based on fatigue under service loading. The number of connectors determined by these criteria are then checked to insure that they can develop the ultimate strength of the section.

The shear connectors required to resist fatigue are determined by an elastic analysis with the following formula:

$$S_r = \frac{V_r Q}{I}$$

where S_r = range of horizontal shear, pounds per linear inch, at junction of slab and stringer at point under consideration

V_r = range of shear, pounds, due to live load plus impact. At any section, the range of shear should be taken as the difference between minimum and maximum shear envelopes (excluding dead loads)

Q = statical moment, in.³, of transformed compressive concrete area, with modular ratio n , about neutral axis of composite section; or statical moment of area of concrete reinforcement for negative moment

I = moment of inertia, in.⁴, of transformed composite stringer, with modular ratio n in positive-moment regions; moment of inertia provided by steel beam and area of concrete reinforcement in negative-moment regions.

The allowable design range of load, pounds, on an individual shear connector is given for welded studs by

$$Z_r = \alpha d^2$$

where α = constant that depends on number of design cycles

d = diameter of stud, in.

The required pitch of shear connectors is determined by dividing the resistance of all connectors at a stringer cross section by S_r , the horizontal range of shear per linear inch. The maximum pitch should not exceed 24 in.

If slab reinforcement steel is not used in computing composite-section properties in negative-moment regions of the span, AASHO specifications require additional shear connectors to be placed at inflection points. The number of additional connectors equals

$$N_r = \frac{A_r f_r}{Z_r}$$

where A_r = area of reinforcement steel over interior support, sq in.

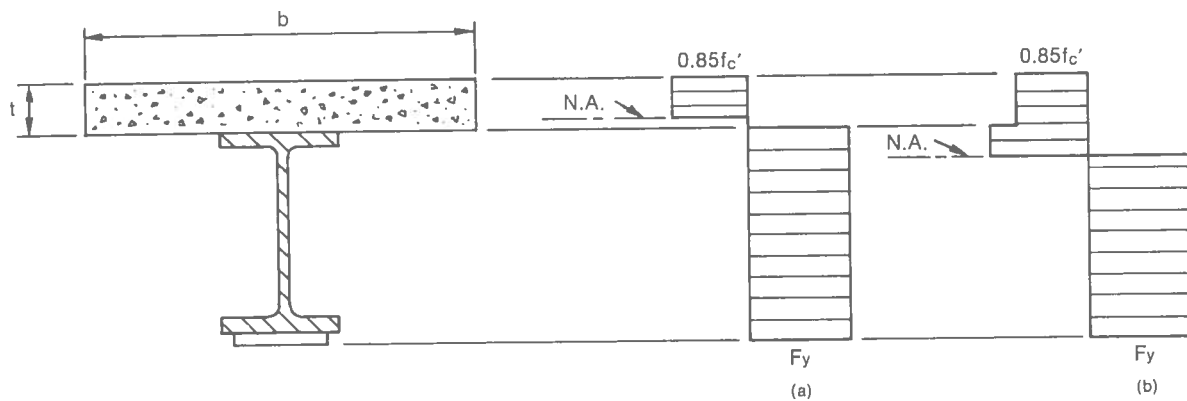
f_r = range of stress, psi, due to live load plus impact in slab reinforcement over support; 10,000 psi may be used in lieu of more accurate computations

These additional connectors must be placed within a distance of the inflection point equal to one-third the effective slab width.

The number of shear connectors established by fatigue considerations must be investigated for ultimate strength. This is relatively easy to do and assures the capability of developing the full plastic-stress distribution in the beam.

The procedure can be derived from an examination of the state of stress in a composite beam at ultimate moment.

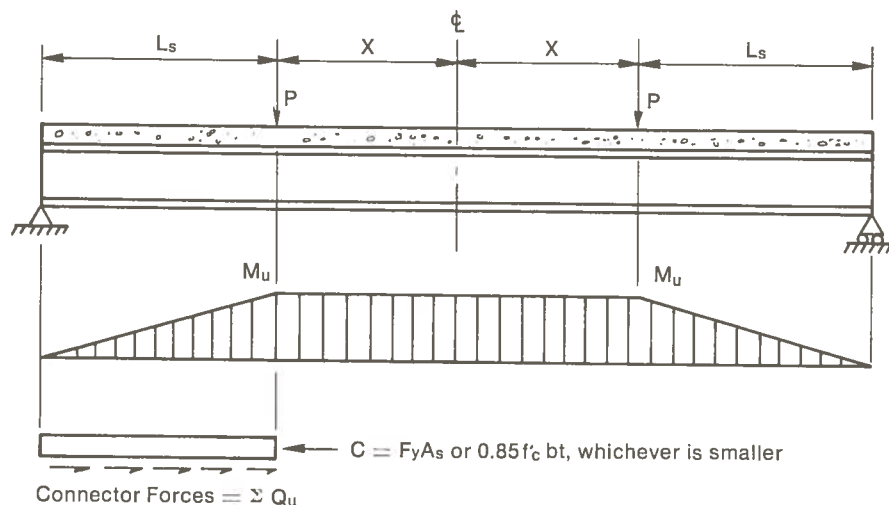
For most practical cross sections the neutral axis is located within the concrete slab. The plastic-stress distribution is as shown in (a) below. Occasionally, the neutral axis falls below the top of the stringer, with a resultant stress distribution as in (b).



STRESSES IN STRINGER

In case (a), the maximum compressive force in the concrete equals the plastic force in the entire steel area, $F_y A_s$, where F_y is the steel yield stress, psi, and A_s the area of steel section, sq in. In case (b) the maximum compressive force in the concrete equals $0.85f'_c bt$, where f'_c is the 28-day strength of the concrete, psi, b the effective width of slab, in., and t the slab thickness, in. The smaller of the two values, $F_y A_s$ and $0.85f'_c bt$, is the maximum possible compressive force that can occur in the concrete slab.

When a composite beam, such as shown on the next page, is loaded to ultimate moment, equilibrium with respect to the concrete slab must be satisfied over the length L_s , between the point of plastic moment M_u and the point of zero moment.



COMPOSITE BEAM LOADED TO ULTIMATE

Thus, the sum of the ultimate strengths of the shear connectors over the length L_s are required to balance the compressive slab force C . Tests have shown that the ultimate strength Q_u , pounds, of stud shear connector is proportional to the square root of the concrete strength:

$$Q_u = 930d^{3/2}\sqrt{f'_c}$$

where d is the stud diameter, in.

The required shear resistance to develop the compressive force C in the slab is furnished by N shear connectors between the point of maximum moment and the ends of span or inflection points, as determined by the following equation from AASHTO specifications:

$$N = \frac{C}{\phi Q_u}$$

where ϕ is a reduction factor equal to 0.85.

Since by concepts of ultimate-strength design, plastic moment is theoretically developed at every transition in section such as at ends of cover plates along the beam, N shear connectors should be provided between each transition location and the ends of span or inflection points. In this case, the C value used in computing N should be based on the weaker section at the transition when $F_y A_s < 0.85 f'_c b t$.

DEAD-LOAD DEFLECTIONS

The final elevations of the bridge deck under dead load should be in accordance with the finished elevations established in the plans. It is therefore necessary to establish the dead-load deflections of the beam to set the forms for the concrete slab and the screed guides for finishing the concrete slab at their proper elevations.

If esthetics are important for straight roadways, stringers may be cambered so that they will be straight under full dead load. For roadways on vertical curves, stringers may be cambered an amount equal to the dead-load deflection plus or minus sufficient additional camber so that the stringers will be a constant distance below the roadway surface under full dead load. For rolled-beam spans shorter than about 50 ft, however, dead-load deflections are small and cambering of the beam will be of little value.

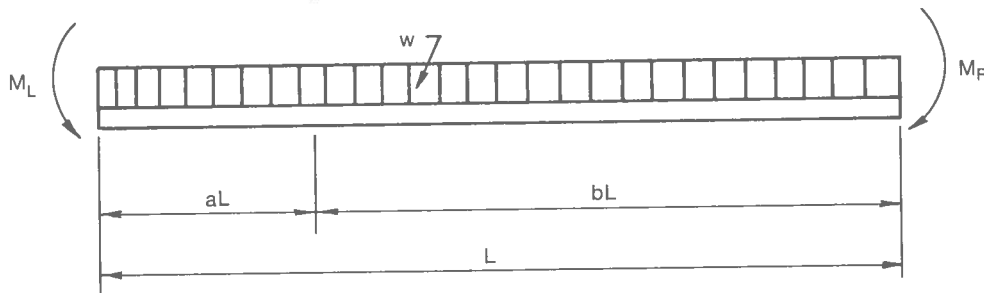
Rolled beams may have a slight mill camber when received from the rolling mill.

This mill camber should be turned upward when no other cambering is required.

For unshored construction, a composite stringer is subjected to both composite and noncomposite dead loads. Hence, dead-load deflections must be computed in two steps. Because the weight of the concrete slab, stringer and framing details act on the steel section alone, deflections due to these loads are calculated with the moment of inertia of the steel section alone. Because the weight of curbs, parapets, railing and wearing surface act on the composite section, deflections due to these loads are calculated with the moment of inertia of the composite section, with an increased modular ratio of $3n$ to allow for creep.

If exact deflections are required, calculations should include the variable moment of inertia of the stringer and composite section. The conjugate beam method or other methods may be used for this purpose. (See References 6, 7, and 8 at end of chapter.) In most cases, however, sufficient accuracy is obtained if deflection computations are based on the moment of inertia of the maximum-positive-moment section and variations in moment of inertia along the span are neglected. The following formula may be used:

$$\Delta = \frac{72wL^4}{E_s I} ab[1 + ab - 4(C_R - C_L)(1 + a) - 12C_L]$$



Uniformly Loaded Stringers

where Δ = deflection, in., at distance aL from left support

$$b = 1 - a$$

w = dead load, kips per ft

L = span, ft

E_s = modulus of elasticity of steel, ksi

I = moment of inertia, in.⁴, of steel section or of composite section with modular ratio $3n$, computed at point of maximum positive moment

$$C_R = M_R / wL^2$$

$$C_L = M_L / wL^2$$

M_R = bending moment at right support, kip-ft

M_L = bending moment at left support, kip-ft

For simply supported stringers, $M_R = M_L = 0$, reducing the above formula to:

$$\Delta = \frac{72wL^4}{E_s I} ab(1 + ab)$$

At midspan of a simple span:

$$\Delta = \frac{45wL^4}{2E_s I}$$

LIVE-LOAD DEFLECTIONS

AASHO specifications set minimum depth-to-span ratios and limit maximum deflections under live load plus impact.

For simply supported, composite stringers, the ratio of over-all depth of stringer (concrete slab plus steel stringer) to span should not be less than 1/25. The ratio of depth of steel stringer alone to span should not be less than 1/30. For continuous stringers, the span may be taken as the distance between dead-load inflection points.

Maximum deflection due to live load plus impact must not exceed 1/800 of the span. For bridges in urban areas used in part by pedestrians, the ratio preferably is 1/1000.

Live-load deflections are computed with the moment of inertia of the composite section with a modular ratio n . Deflections seldom control the design of composite stringers, because of the large moment of inertia of the composite section.

If more exact deflections are required, calculations should include the variable moment of inertia of the composite section and live load should be placed for maximum positive moment in the span.

The following formulas give the approximate maximum live-load deflection at midspan of a continuous stringer:

For HS truck loading:

$$\Delta = \frac{324}{E_s I_c} \left[P_T (L^3 - 555L + 4,780) - \frac{1}{3} (M_R + M_L) L^2 \right]$$

For lane loading:

$$\Delta = \frac{45L^2}{2E_s I_c} \left[L(w_L L + 1.6P_L) - 4.8(M_R + M_L) \right]$$

where Δ = deflection at midspan, in.

P_T = weight of one front truck wheel, kips, multiplied by the live-load distribution factor, plus impact.

I_c = moment of inertia, in.⁴, of the composite section with modular ratio n , computed at point of maximum positive moment

w_L = one-half weight of uniform lane load, kips per ft, multiplied by the distribution factor, plus impact

P_L = one-half weight of concentrated lane load for moment, kips, multiplied by the distribution factor, plus impact.

M_R = bending moment at right support, kip-ft

M_L = bending moment at left support, kip-ft

For simple beams, $M_R = M_L = 0$.

The following formulas give the live-load deflection at the 0.4 point in the end span of a continuous stringer, the approximate point of maximum deflection:

For HS truck loading:

$$\Delta_{0.4} = \frac{300}{E_s I_c} \left[P_T (L^3 + 3.89L^2 - 680L + 5,910) - 0.32ML^2 \right]$$

For lane loading:

$$\Delta_{0.4} = \frac{43L^2}{2E_s I_c} \left[L(w_L L + 1.5P_L) - 4.5M \right]$$

where $\Delta_{0.4}$ = deflection at distance of 0.4 L from simple support, in.

M = bending moment at continuous support, kip-ft

Truck loading will cause maximum positive moments and maximum live-load deflections in rolled-beam spans and in most welded girder spans.

DESIGN EXAMPLES

Five design examples as listed at the beginning of this chapter are presented to illustrate the design of wide-flange beams.

The following applies to all designs:

Roadway Section: See sketch in Design I.

Specifications: 1969 AASHTO Standard Specifications for Highway Bridges.

Loading: HS20-44

Structural Steel: A36

Allowable bending stress = 20,000 psi

Allowable shearing stress = 12,000 psi

Concrete: $f'_c = 4,000$ psi
 $f_c = 1,600$ psi
 $n = 8$

Loading Conditions:

Case 1—Weight of stringer and slab (DL_1) supported by the steel stringer alone.

Case 2—Superimposed dead load (DL_2) (curbs and railings) supported by the composite section with the modular ratio $n = 8$.

Case 3—Superimposed dead load (DL_2) (curbs and railings) supported by the composite section with the increased modular ratio $3n = 3 \times 8 = 24$.

Case 4—Live load plus impact ($LL + I$) supported by the composite section with the modular ratio $n = 8$.

Loading Combinations:

Combination A = Case 1 + 3 + 4.

Combination B = Case 2 + 4.

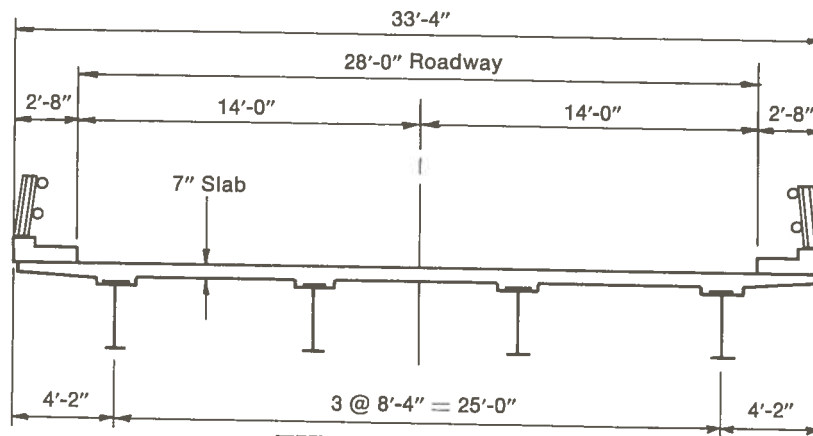
Combination C = Case 1 + 2 + 4.

Fatigue Considerations: 500,000 cycles of maximum stress for all designs except Alternate A of Design III. 100,000 cycles of maximum stress for Alternate A of Design III.

Design I—40-Ft Simple-Span Noncomposite Beam

LOADS, SHEARS AND MOMENTS

The dead load consists of the weights of the concrete slab, steel beam, framing details, concrete curbs and steel railings. The weight of curbs and railings may be assumed to be distributed equally to the four stringers by the deck slab and the diaphragms between stringers. Live load plus impact on a stringer are determined in accordance with the AASHTO wheel-load distribution for HS20-44 truck loading and impact factors. Since the span is greater than 38 ft, military loading does not govern. HS truck loading is used for design.



TYPICAL SECTION

Dead Loads

$$\text{Slab} = 7/12 \times 8.33 \times 0.150 = 0.730$$

$$\text{Steel beam, details, haunches, diaphragms} = 0.140$$

$$\text{Load per stringer} = 0.870 \text{ k/ft}$$

$$\text{Curbs} = 2.67 \times 0.75 \times 0.15 \times 2 = 0.600$$

$$\text{Railings} = 0.060$$

$$\text{Load on 4 stringers} = 0.660 \text{ k/ft}$$

$$\text{Curb and railing load per stringer} = 0.660/4 = 0.165 \text{ k/ft}$$

$$\text{Total dead load per stringer} = 0.870 + 0.165 = 1.035 \text{ k/ft}$$

Live Load

$$\text{Live load distribution} = \frac{S}{5.5} = \frac{8.33}{5.5} = 1.51 \text{ wheels} = 0.755 \text{ axle}$$

$$\text{Impact} = \frac{50}{40 + 125} = 0.303; \text{ maximum} = 30\%$$

Maximum bending moments occur near midspan. Dead-load moment is calculated for uniform dead load. Live-load moment may be obtained from the AASHO Table of Maximum Moments, Shears and Reactions—Simple Spans, One Lane for HS20-44 loading. (These tables do not cover military loading.) End shears are calculated similarly.

Maximum Moment

$$DL: M = \frac{wL^2}{8} = \frac{1.035 \times (40)^2}{8} = 207$$

$$LL: M = 0.755 \times 449.8 = 340$$

$$I: M = 0.30 \times 340 = \frac{102}{649} \text{ kip-ft}$$

Maximum Shear (End Reaction)

$$DL: V = 1.035 \times 20 = 20.7$$

$$LL: V = 0.755 \times 55.2 = 41.7$$

$$I: V = 0.30 \times 41.7 = \frac{12.5}{74.9} \text{ kips}$$

*AASHO Specifications, Section 1.3.1, states that for end shears and end reactions, no lateral or longitudinal distribution should be assumed for the wheel or axle adjacent to the end at which the stress is being calculated. Wheel loads in other positions are distributed for shear in the same manner as for moment. For simplicity in the above case, *all* wheel loads are distributed for shear the same as for moment. The resulting shear value is very little different from that which would follow from strict adherence to the Specifications.

SELECTION OF SECTION

For a short, simply supported noncomposite beam, it is usually economical to use a wide-flange beam without cover plates. Thus, the midspan moment controls the design, and it is unnecessary to determine an envelope of maximum moments for the span.

The required section modulus is calculated for an allowable bending stress of 20 ksi. A W33×130 beam is selected, and flexural and shearing stresses are checked. Shear in the webs of wide-flange beams is rarely critical.

Section Required

$$\text{Required } Z = \frac{649 \times 12}{20} = 389.4 \text{ in.}^3$$

Use W33×130 ($Z = 404.8 \text{ in.}^3$, $I = 6,699 \text{ in.}^4$). Check of bending stress shows

$$f_b = \frac{649 \times 12}{404.8} = 19.24 \text{ ksi} < 20$$

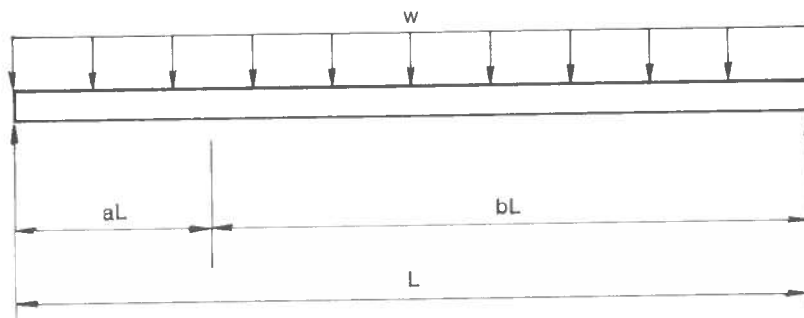
Check of shearing stress shows

$$f_v = \frac{74.9}{33.10 \times 0.580} = 3.9 \text{ ksi} < 12$$

DEFLECTIONS

For beam spans exceeding about 50 ft, stringers usually are cambered for dead-load deflection. A single camber ordinate at midspan is sufficient for cambering rolled beams to a parabolic curve in a fabrication shop. For all spans, the live-load deflection is limited to 1/800 of the span by AASHTO specifications.

Although mill camber is adequate for the stringers in this example, the dead-load deflection is calculated for illustrative purposes. The formulas used are those for deflection under uniform load and constant moment of inertia.



Deflections Due to Dead Load

$$DL: w = 1.035 \text{ kips per ft}$$

The dead-load deflection at midspan is

$$\Delta = \frac{45wL^4}{2E_s I}$$

where Δ = midspan deflection, in.

w = dead load, kips per ft

L = span, ft

E_s = modulus of elasticity of steel = $29(10)^3$ ksi

I = moment of inertia at midspan, in.⁴

$$\Delta = \frac{45 \times 1.035 (40)^4}{2 \times 29(10)^3 \times 6,669} = 0.307 \text{ in.}$$

Note: Turn mill camber upward. No other camber is required.

Deflection Due to Live Load+Impact

Maximum live-load deflections occur under two lanes of the HS20-44 truck loading, assumed to be distributed equally to the four stringers. The formula used is that for approximate maximum live-load deflection discussed under General Design Considerations. For simple beams, $M_R = M_L = 0$.

$$\Delta = \frac{324}{E_s I} P_T (L^3 - 555L + 4,780)$$

where Δ = midspan deflection, in.

P_T = concentrated load, kips, on four stringers = weight of front truck wheels \times distribution factor, plus impact, kips

I = moment of inertia at midspan, in.⁴

L = span, ft

E_s = modulus of elasticity of steel = $29(10)^3$ ksi

With two lanes of live load (four wheels abreast) plus 30% impact carried by four stringers,

$$P_T = 4 \times 4 \times 1.30 = 20.8 \text{ kips}$$

$$I_s = 4 \times 6,699 = 26,796 \text{ in.}^4$$

$$\Delta = \frac{324 \times 20.8 [(40)^3 - 555(40) + 4,780]}{29(10)^3 \times 26,796} = 0.402 \text{ in.}$$

The ratio of live-load deflection to span is

$$\frac{0.402}{40 \times 12} = \frac{1}{1,194} < \frac{1}{800}$$

Design II—40-Ft Simple-Span Composite Beam

Normally, the most economical section for a composite, simply supported, rolled beam is obtained when a cover plate is welded to the bottom flange over part of its length. Curves of maximum moments and maximum shears are required to determine the location of cover-plate cutoffs and shear-connector spacing.

LOADS, SHEARS AND MOMENTS

The dead load to be carried by the steel section alone, DL_1 , consists of the weight of the concrete slab plus the weight of the beam, cover plate and framing details. Weight of concrete curbs and steel railings makes up the additional dead load, DL_2 , to be carried by the composite section. This weight is assumed to be distributed equally to the four stringers by the deck slab and diaphragms between stringers. For live load plus impact, the AASHTO wheel load distribution for HS20-44 truck loading and impact factors are used.

Dead Load Carried by Steel

$$\text{Slab} = 7/12 \times 8.33 \times 0.150 = 0.730$$

Steel beam, details, haunches, diaphragms = 0.108

$$DL_1 \text{ per stringer} = 0.838 \text{ k/ft}$$

Dead Load Carried by Composite Section

Curbs and railings, $DL_2 = 0.660 \text{ k/ft}$

$$DL_2 \text{ per stringer} = 0.660/4 = 0.165 \text{ k/ft}$$

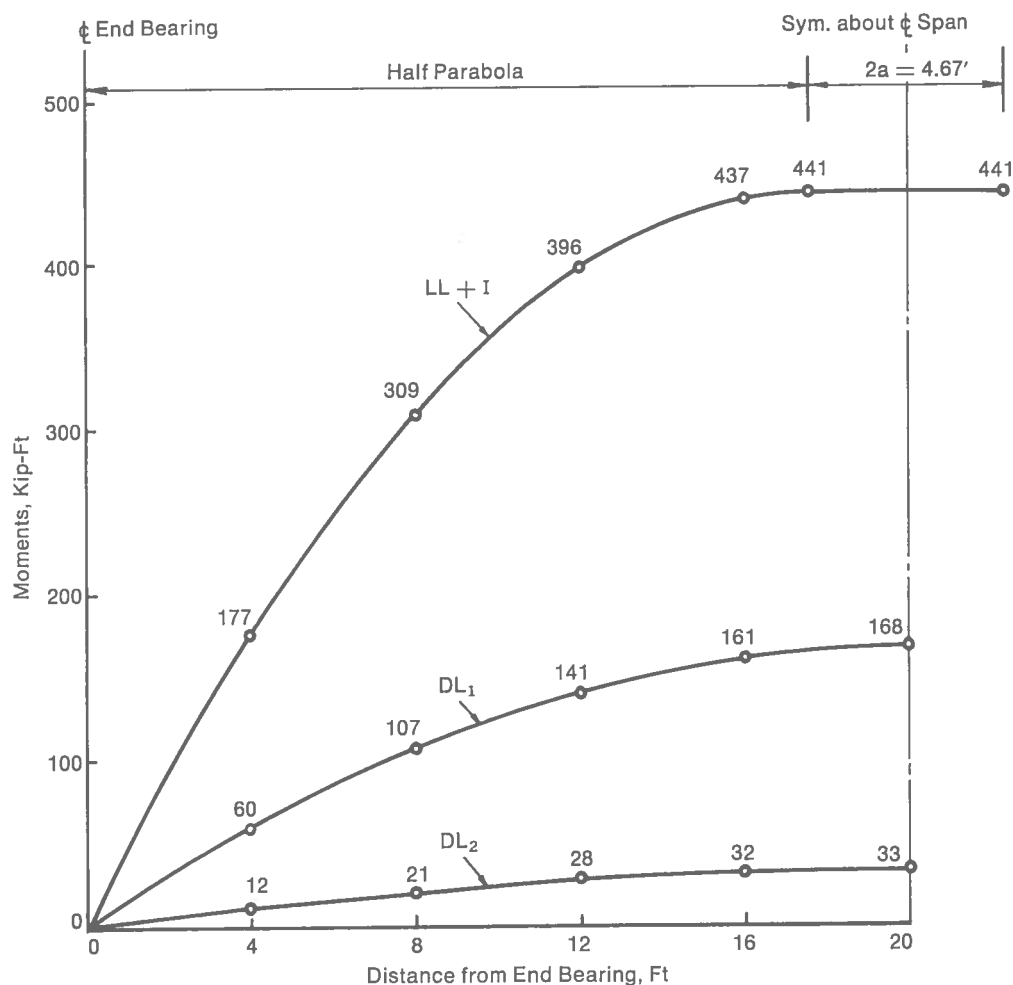
Live Load

$$\text{Live-load distribution} = \frac{S}{5.5} = \frac{8.33}{5.5} = 1.51 \text{ wheels} = 0.755 \text{ axle}$$

$$\text{Impact} = \frac{50}{40 + 125} = 0.303; \text{ maximum } 30\%$$

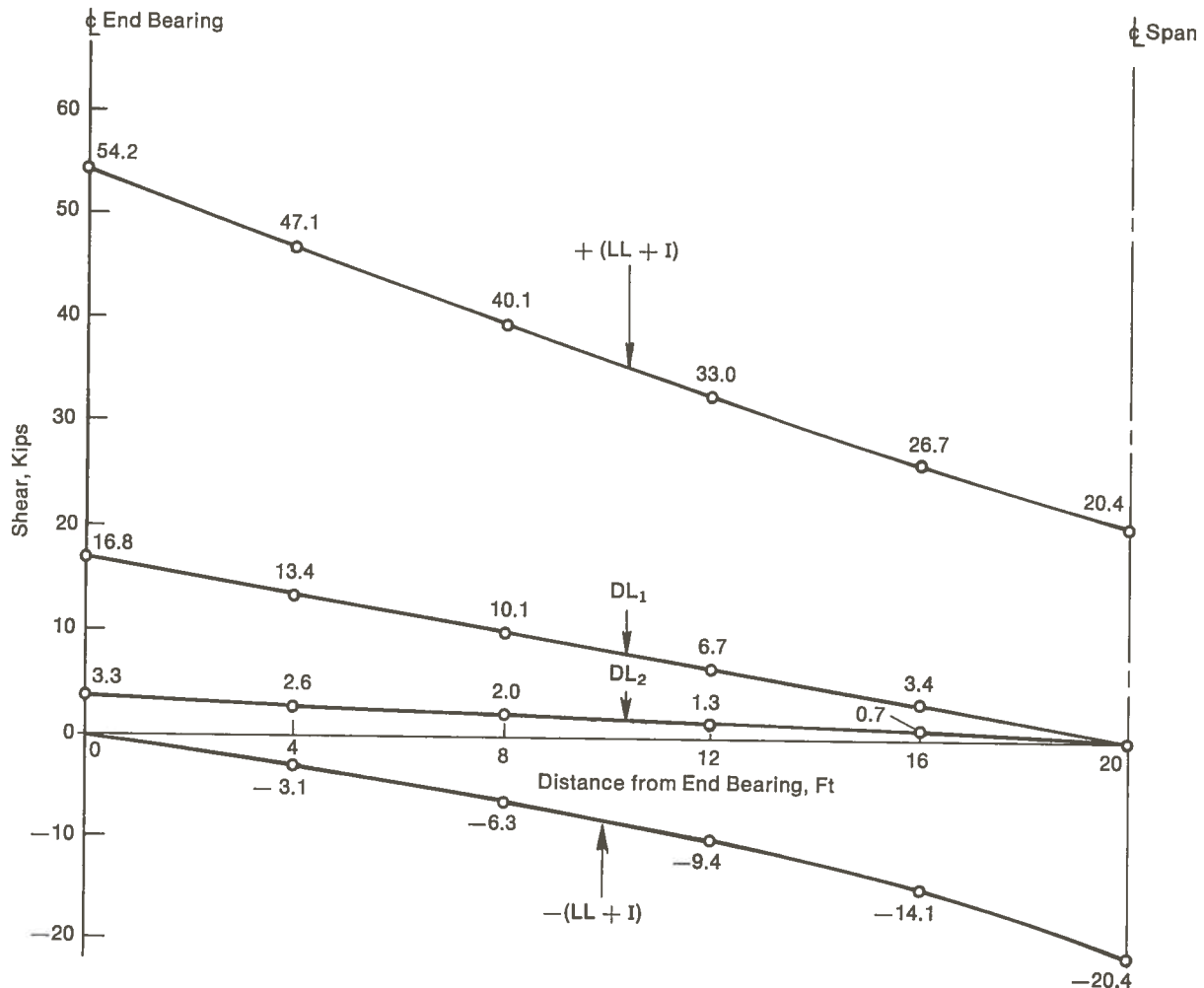
The curve of maximum moments for a system of two or three moving concentrated loads is closely approximated by two half-parabolas joined near midspan by a straight line of length $2a$, where a = distance of maximum-moment section from midspan (see Maximum Moment Curves). For *HS* truck loading on spans greater than 33.8 ft, an accurate value of $2a$ is 4.67 ft.

In this example, the midspan moment is 441 kip-ft, extending over the $2a$ distance. Moment ordinates then decrease parabolically from the maximum.



MAXIMUM-MOMENT CURVES

The curves of maximum and minimum live-load shears are calculated by moving a standard HS20-44 truck across the span to produce maximum and minimum shear at each tenth point.



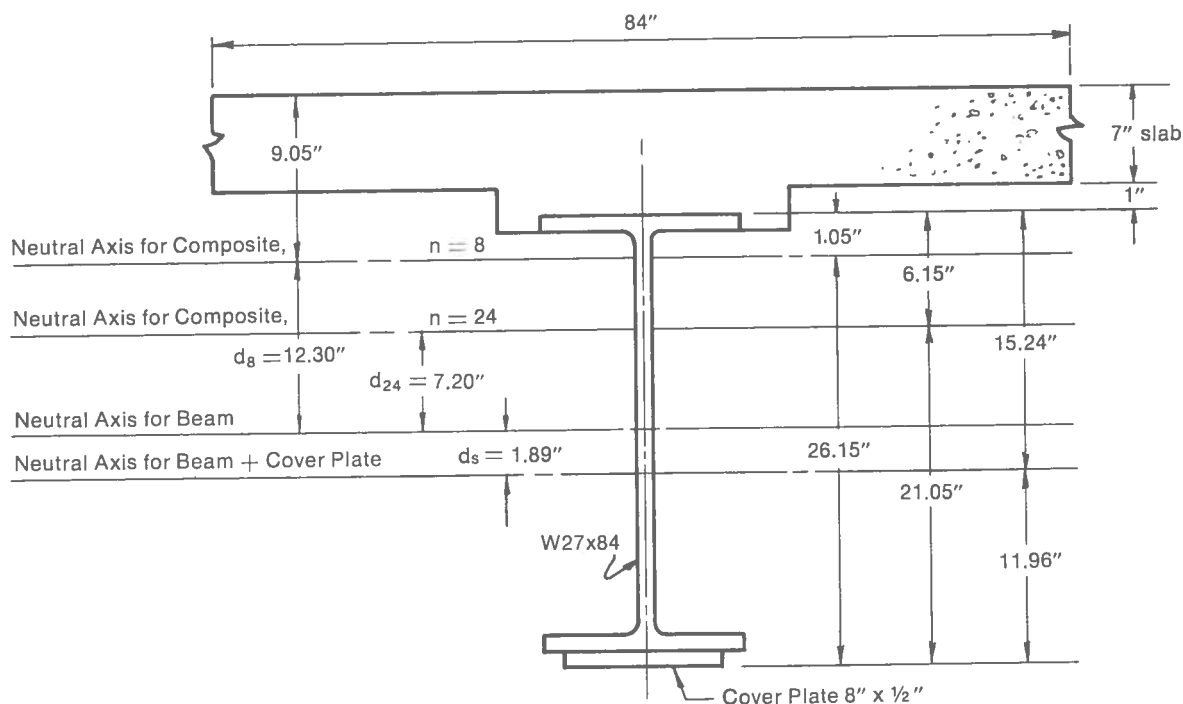
MAXIMUM-SHEAR CURVES

PROPERTIES OF COMPOSITE SECTION

Design of composite stringers is essentially a trial-and-error procedure involving selection of a steel section and investigation of steel and concrete stresses. By adjusting the steel section, the designer approaches an optimum stress condition.

Tables are available to facilitate selection of a trial steel section for certain conditions of slab thickness, stringer spacing, haunch depth and concrete strength. (See References 1 and 2 at end of chapter.) The tables list section properties for steel sections alone and composite sections. If tables are unavailable or out of the range of a given design problem, very little additional work is involved in a purely arbitrary selection of the initial trial section. The trial-and-error procedure converges rapidly to an acceptable design.

Maximum moment at midspan controls the design of the 40-foot stringer in this example. A W27 × 84 with an 8 × ½-in. bottom cover plate is investigated as a possible section. Section properties are computed for the steel beam and cover plate alone, the composite section with $n = 8$, and the composite section with $3n = 24$.



COMPOSITE SECTION

Effective Flange Width

$$\frac{1}{4} \text{ span} = \frac{1}{4} \times 40 \times 12 = 120 \text{ in.}$$

$$\text{Stringer spacing, c to c} = 8.33 \times 12 = 100 \text{ in.}$$

$$12 \times \text{slab thickness} = 12 \times 7 = 84 \text{ in. (governs)}$$

Steel Section for Maximum Positive Moment

Material	A	d	Ad	Ad ²	I _o	I
W27 × 84	24.71				2,825	2,825
Bot. Cover Plate 8 × ½	4.00	-13.60	-54.4	740		740

$$d_s = \frac{-54.4}{28.71} = -1.89 \text{ in.}$$

$$I_{NA} = \frac{3,565}{3,462 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 13.35 + 1.89 = 15.24 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 13.85 - 1.89 = 11.96 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{3462}{15.24} = 227 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{3462}{11.96} = 289 \text{ in.}^3$$

Composite Section, 3n = 24, for Maximum Positive Moment

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	28.71		- 54.4			3,565
Conc. 84 × 7/24	24.50	17.85	+437.3	7,806	100	7,906

$$d_{24} = \frac{382.9}{53.21} = 7.20 \text{ in.}$$

$$I_{NA} = \frac{11,471}{8,714 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 13.35 - 7.20 = 6.15 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 13.85 + 7.20 = 21.05 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{8,714}{6.15} = 1,418 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{8,714}{21.05} = 414 \text{ in.}^3$$

Composite Section, $n = 8$, for Maximum Positive Moment

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	28.71		-54.4			3,565
Conc. $84 \times \frac{7}{8}$	73.50	17.85	+1,312.0	23,419	300	23,719

$$d_8 = \frac{1,257.6}{102.21} = 12.30 \text{ in.}$$

$$+1,257.6 \text{ in.}^3$$

$$27,284$$

$$-12.30 \times 1,257.6 = -15,468$$

$$I_{NA} = \frac{11,816}{11,816} \text{ in.}^4$$

$$d_{\text{Top of steel}} = 13.35 - 12.30 = 1.05 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 13.85 + 12.30 = 26.15 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{11,816}{1.05} = 11,253 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{11,816}{26.15} = 452 \text{ in.}^3$$

$$d_{\text{Top of conc.}} = 1.05 + 8.00 = 9.05 \text{ in.}$$

$$Z_{\text{Top of conc.}} = \frac{11,816}{9.05} = 1,306 \text{ in.}^3$$

STRESSES IN COMPOSITE SECTION

The stresses at top and bottom of steel and top of concrete are computed for DL_1 , DL_2 and $LL+I$. With unshored construction, steel stresses are governed by Loading Combination A and concrete stresses by Loading Combination B. When rolled sections are used in composite design, the critical stress normally occurs in the bottom flange.

Midspan Bending Moments

	DL_1	DL_2	$LL+I$
M , kip-ft	168	33	441

Steel Stresses—Combination A

Top of Steel (Compression)

$$DL_1: f_b = \frac{168 \times 12}{227} = 8.88$$

$$DL_2: f_b = \frac{33 \times 12}{1,418} = 0.28$$

$$LL+I: f_b = \frac{441 \times 12}{11,253} = 0.47$$

$$9.63 \text{ ksi}$$

Bottom of Steel (Tension)

$$f_b = \frac{168 \times 12}{289} = 6.98$$

$$f_b = \frac{33 \times 12}{414} = 0.96$$

$$f_b = \frac{441 \times 12}{452} = 11.71$$

$$19.65 \text{ ksi}$$

Concrete Stresses—Combination B

Top of Concrete

$$DL_2: f_c = \frac{33 \times 12}{1,306 \times 8} = 0.038$$

$$LL+I: f_c = \frac{441 \times 12}{1,306 \times 8} = 0.507$$

$$0.545 \text{ ksi}$$

Maximum Shear Stress

Shear stress in the web of the beam is checked, although it is rarely critical in wide-flange beams that meet flexural requirements.

End Shears

	DL_1	DL_2	$LL+I$	Total
V , kips	16.8	3.3	54.2	74.3

$$f_v = \frac{74.3}{26.69 \times 0.463} = 6.0 \text{ ksi} < 12$$

LOCATION OF COVER-PLATE CUTOFFS

For locating cover-plate cutoffs, the section properties of the composite section without the bottom cover plate are computed. Properties of the steel section alone are obtained directly from Chapter I/4, **Hot Rolled Shapes and Plates** or the AISC Steel Construction Manual.

Composite Section, $3n = 24$, Near Supports

Material	A	d	Ad	Ad^2	I_o	I
W27 × 84	24.71				2,825	2,825
Conc. $84 \times 7/24$	24.50	17.85	+437.3	7,806	100	7,906

$$d_{24} = \frac{437.3}{49.21} = 8.89 \text{ in.}$$

$$49.21 \text{ in.}^2 \quad +437.3 \text{ in.}^3 \quad 10,731 \text{ in.}$$

$$-8.89 \times 437.3 = -3,888 \quad I_{NA} = \frac{-3,888}{6,843 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 13.35 - 8.89 = 4.46 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 13.35 + 8.89 = 22.24 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{6,843}{4.46} = 1,534 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{6,843}{22.24} = 308 \text{ in.}^3$$

Composite Section, $n = 8$, Near Supports

Material	A	d	Ad	Ad^2	I_o	I
W27 × 84	24.71				2,825	2,825
Conc. $84 \times 7/8$	73.50	17.85	+1,312	23,419	300	23,719

$$d_8 = \frac{1,312}{98.21} = 13.36 \text{ in.}$$

$$98.21 \text{ in.}^2 \quad +1,312 \text{ in.}^3 \quad 26,544$$

$$-13.36 \times 1,312 = -17,528 \quad I_{NA} = \frac{-17,528}{9,016 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 13.35 - 13.36 = 0$$

$$d_{\text{Bot. of steel}} = 13.35 + 13.36 = 26.71$$

$$Z_{\text{Top of steel}} = \frac{9,016}{0} = \infty$$

$$Z_{\text{Bot. of steel}} = \frac{9,016}{26.71} = 338 \text{ in.}^3$$

Change in Section

The approximate cover-plate cutoff location is determined, for a parabolic moment diagram, by the formula for cover-plate length given in General Design Considerations. Stresses are checked at the theoretical cutoff points 8.5 ft from the bearings, with moment values scaled from the Maximum Moment Curves. The stresses are compared with the allowable static tensile and compressive stresses and found satisfactory.

Approximate length L_{cp} , ft, for the 8 x ½-in. cover plate:

$$L_{cp} = (L - 2a) \sqrt{\frac{1 - Z'_{bs}}{Z_{bs}}} + 2a$$

where L = beam span = 40 ft

a = distance of maximum-moment section from midspan = 2.33 ft

Z'_{bs} = section modulus of W27 x 84 = 212 in.³

Z_{bs} = section modulus of beam with cover plate = 289 in.³

$$L_{cp} = \left[40 - (2 \times 2.33) \right] \sqrt{1 - \frac{212}{289}} + (2 \times 2.33) = 22.89, \text{ say } 23 \text{ ft}$$

Approximate cutoff point for the cover plate from center of bearing:

$$L/2 - L_{cp}/2 = 20.0 - 11.5 = 8.5 \text{ ft}$$

A location 8.5 ft from the bearing is investigated as the theoretical cutoff point of the cover plate. Stresses are checked there, in the beam without cover plate.

Bending Moments 8.5 ft from Support

	DL_1	DL_2	$LL+I$
M , kip-ft	112	22	320

Steel Stresses—Combination A

Top of Steel (Compression)

Bottom of Steel (Tension)

$$DL_1: f_b = \frac{112 \times 12}{212} = 6.34$$

$$f_b = \frac{112 \times 12}{212} = 6.34$$

$$DL_2: f_b = \frac{22 \times 12}{1,534} = 0.17$$

$$f_b = \frac{22 \times 12}{308} = 0.86$$

$$LL+I: f_b = \frac{320 \times 12}{\infty} = 0$$

$$f_b = \frac{320 \times 12}{338} = 11.36$$

6.51 ksi

18.56 ksi

A theoretical cutoff 8.5 ft from the support is satisfactory; but due to fatigue considerations, the plate must be extended. The terminal distance is computed as 1.5 times the cover-plate width: $1.5 \times 8 = 12$ in.

Try cutoffs at 7 ft 6 in. from the bearings.

Fatigue Check at Cover-Plate End

Fatigue stresses are checked in the beam adjacent to the fillet weld across the end of the cover plate. Dead- and live-load moments are scaled from the moment curve at this point, and stresses are calculated for the extreme fiber of the rolled section. Stresses alternate from full dead-load stress to dead- plus live-load stress, with no reversal. The allowable fatigue stress is given by the AASHTO specification formula for expressway structures at 500,000 cycles of loading.

Bending Moments 7.5 ft from Bearings

	DL_1	DL_2	$LL+I$
M , kip-ft	103	21	297

Steel Stresses—Combination A

Bottom of Steel (Tension)

$$DL_1: f_b = \frac{103 \times 12}{212} = 5.83$$

$$DL_2: f_b = \frac{21 \times 12}{308} = 0.82$$

$$LL + I: f_b = \frac{297 \times 12}{338} = \frac{10.54}{17.19} \text{ ksi}$$

Ratio of minimum to maximum stress in beam flange at cover-plate weld:

$$R = \frac{5.83 + 0.82}{17.19} = 0.387$$

The allowable fatigue stress is

$$F_r = \frac{12}{1 - R} = \frac{12}{1 - 0.387} = 19.58 \text{ ksi} > 17.19$$

Weld at Cover-Plate End

The fillet weld connecting the cover plate to the beam flange must develop the total force in the cover plate at its theoretical cutoff. This force is approximately equal to the extreme fiber stress in the cover plate at that point times the cross sectional area of the cover plate. A $\frac{3}{16}$ -in. fillet weld 32 in. long would be adequate based on the AASHTO specification of 12.4 ksi allowable shear stress. In this case, however, minimum requirements rather than strength considerations govern weld size. The $\frac{5}{8}$ -in. thickness of the beam flange requires at least a $\frac{1}{4}$ -in. fillet weld.

The allowable fatigue stress usually exceeds 12.4 ksi, except near inflection points of continuous beams. Fatigue should be checked where the designer believes it could govern.

Stresses 8.5 ft from End Support—Combination A

Cover Plate

$$DL_1: f_b = \frac{112 \times 12}{289} = 4.65$$

$$DL_2: f_b = \frac{22 \times 12}{414} = 0.64$$

$$LL + I: f_b = \frac{320 \times 12}{452} = \frac{8.50}{13.79} \text{ ksi}$$

Ratio of minimum to maximum stress in cover plates:

$$R = \frac{5.29}{13.79} = 0.384$$

The allowable weld fatigue stress in shear is

$$F_r = \frac{10.8}{1 - 0.55(0.384)} = 13.7 \text{ ksi} > 12.4$$

Allowable load on weld = $12.4 \times 0.707 = 8.76$ kips per in.

Force in cover plate = $8 \times \frac{1}{2} \times 13.79 = 55.16$ kips

Weld size required = $\frac{55.16}{8.76 \times 32} = 0.195$ in., say $\frac{3}{16}$ -in.

Use $\frac{1}{4}$ -in. fillet weld, required for flange thickness.

DESIGN OF SHEAR CONNECTORS

For shear connectors, $\frac{7}{8}$ -in.-dia. studs, 4-in. high, are used. The studs must satisfy the requirement: $H/d \geq 4.0$. The AASHTO specifications formula for ultimate strength of welded studs is used to determine the allowable load per stud. Strength requirements are satisfied with 27 shear connectors placed between the point of maximum moment at midspan and the end of the span. In addition, at least 24 shear connectors should be placed between the cover-plate cutoffs and nearest bearings.

$$\text{Concrete: } f'_c = 4,000 \text{ psi; } n = 8$$

$$\text{Studs: } \frac{7}{8}\text{-in.-dia, 4-in. high, } H/d = 4.0/0.875 = 4.6 > 4.0$$

The ultimate strength of a shear connector equals

$$Q_u = 0.93d^2\sqrt{f'_c} = 0.93(0.875)^2\sqrt{4,000} = 45.0 \text{ kips per stud}$$

With α given as 10.6 for 500,000 cycles of load in AASHTO specifications, the load range per shear connector is

$$Z_r = \alpha d^2 = 10.6(0.875)^2 = 8.11 \text{ kips per stud}$$

Shear Connectors—Strength Requirements

At midspan, the maximum compressive stress in the concrete is

$$H_1 = A_s F_y = 28.71 \times 36.0 = 1,033.6 \text{ kips (governs)}$$

$$H_2 = 0.85f'_c b t = 0.85 \times 4.0 \times 84.0 \times 7.0 = 1,999.2 \text{ kips}$$

The number of studs required between midspan and each support is

$$N = \frac{H_1}{\phi Q_u} = \frac{1,033.6}{0.85 \times 45.0} \approx 27$$

where ϕ = reduction factor = 0.85

At cover-plate cutoffs:

$$H_1 = A_s F_y = 24.71 \times 36.0 = 889.6 \text{ kips (governs)}$$

$$H_2 = 1,999.2 \text{ kips}$$

The number of studs required between the cover-plate cutoffs and nearest bearing is

$$N = \frac{H_1}{\phi Q_u} = \frac{889.6}{0.85 \times 45.0} = 23.3; \text{ use } 24$$

Shear Connector Spacing for Service Behavior (Fatigue)

Shear-connector-spacing requirements for service behavior under repeated loads are checked. Here, a variable pitch will result, in accordance with the variation in range of shear along the span. The allowable range of load per stud is 8.11 kips. Spacing is calculated at the end of the span first.

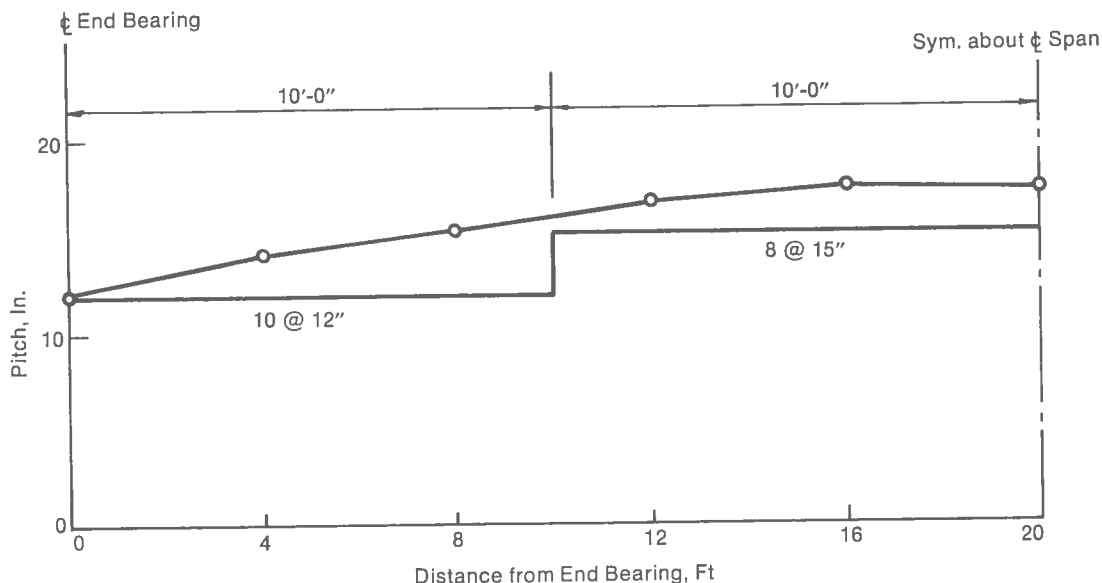
At supports, shear range for the live load = $V_r = 54.2 - 0 = 54.2$ kips. For $n = 8$, the horizontal shear per linear inch is

$$S_r = \frac{V_r Q}{I} = \frac{54.2 (73.5 \times 4.5)}{9,016} = 1.99 \text{ kips per in.}$$

$$\text{Spacing required (3 studs)} = \frac{3Z_r}{S_r} = \frac{3(8.11)}{1.99} = 12.23 \text{ in.}$$

Computation for shear-connector spacing under service behavior is repeated at each tenth point along the span, and a curve of required spacing is plotted. The curve provides an envelope within which the spacing diagram for fabrication is drawn. A single step in the spacing, from 12 to 15 in., is made at the quarter point of the span.

This spacing furnishes 54 studs from midspan to each bearing and 24 studs from the cover-plate cutoffs to nearest bearing. The number of studs supplied exceed the number required for strength and fatigue.



SHEAR-CONNECTOR SPACING

BEARING STIFFENERS

For rolled beams, end bearing stiffeners are not required if the shearing stress in the web does not exceed 75% of the allowable shearing stress for girder webs.

Reactions

	DL_1	DL_2	$LL+I$	Total
R , kips	16.8	3.3	54.2	74.3

$$f_v = \frac{74.3}{26.69 \times 0.463} = 6.01 \text{ ksi}$$

The allowable shear without stiffeners is

$$f_v = 12 \times 0.75 = 9.0 \text{ ksi} > 6.01$$

No end stiffeners are required.

DEFLECTIONS

Dead-load-deflection computations are carried out in two parts. DL_1 deflections are based on the moment of inertia of the steel section alone; DL_2 deflections are based on the moment of inertia of the composite section with $3n=24$. Calculations show that the total dead-load deflections are small. Ordinary mill camber turned upward is sufficient.

Deflections Due to Dead Load

$$DL_1: w = 0.838 \text{ kips per ft}$$

$$DL_2: w = 0.165 \text{ kips per ft}$$

The dead-load deflection at midspan is

$$\Delta = \frac{45wL^4}{2E_sI}$$

where Δ = midspan deflections, in.

w = dead load, kips per ft

L = span, ft

E_s = modulus of elasticity of steel = $29(10)^3$ ksi

I = moment of inertia at midspan, in.⁴

Deflections Under DL_1

$$I_s = 3,462 \text{ in.}^4$$

$$\Delta = \frac{45 \times 0.838(40)^4}{2 \times 29(10)^3 \times 3,462} = 0.481 \text{ in.}$$

Deflections Under DL_2

$$I_{24} = 8,714 \text{ in.}^4$$

$$\Delta = \frac{45 \times 0.165(40)^4}{2 \times 29(10)^3 \times 8,714} = 0.039 \text{ in.}$$

Note: Turn mill camber upward. No other camber is required.

Deflection Due to Live Load + Impact

The deflection from live load plus impact is computed for two lanes of HS20-44 truck loading distributed equally to the four stringers. Computations show that the maximum live-load deflection is considerably below the allowable limit of $1/800$ of the span.

$$\Delta = \frac{324}{E_s I_c} P_T (L^3 - 555L + 4,780)$$

where Δ = midspan deflection, in.

P_T = concentrated load, kips, on four stringers = weight of front truck wheels \times distribution factor, plus impact, kips

I_c = moment of inertia of composite section at midspan, in.⁴

L = span, ft

$E_s = 29(10)^3$ ksi

Assume that two lanes of live load (four wheels abreast) are carried by four stringers. Then,

$$P_T = 4 \times 4 \times 1.30 = 20.8 \text{ kips}$$

$$I_s = 4 \times 11,816 = 47,264 \text{ in.}^4$$

$$\Delta = \frac{324 \times 20.8[(40)^3 - 555(40) + 4,780]}{29(10)^3 \times 47,264} = 0.229 \text{ in.}$$

The ratio of live-load deflection to span is

$$\frac{0.229}{40 \times 12} = \frac{1}{2,096} < \frac{1}{800}$$

FINAL DESIGN

Section and elevation views of the composite-beam design are shown on the next page. See also the detail drawing at the end of this chapter.

While live-load deflections are well within allowable limits for noncomposite and composite beams, the live-load deflection of the composite beam is less than 60% of that for the noncomposite beam. As span increases, live-load deflection may become an important consideration in the design of noncomposite beams. In composite design, however, live-load deflection seldom is critical.

Design III—Two-Span Continuous Beam (70-70 Ft), Composite For Positive Movement Only

This type of structure generally is applicable for spans of about 60 to 80 ft. For shorter continuous spans, noncomposite rolled beams usually will be more economical.

The design procedure for a two-span, continuous, composite, rolled-beam stringer in the positive-moment portion of the span is similar to the procedure for simple spans. Negative moments are assumed to be carried solely by the steel stringer in this example.

Economic design of a two-span, continuous composite beam requires a varying steel section. In negative-moment areas, cover plates are added to both flanges of the rolled beam, and in positive-moment areas, to the bottom flange only.

LOADS, SHEARS AND MOMENTS

An initial analysis of the two-span continuous stringer is made for moments and shears based on constant moment of inertia.

DL_1 is calculated as the weight of the 7-in.-thick concrete slab and an assumed weight of 0.170 kips per ft for the stringer and framing details. Weight of curbs and railings comprises DL_2 . Live load is the standard HS20-44 truck loading. Impact is computed for a 70-ft span.

Dead Load Carried by Steel

$$\text{Slab} = 7/12 \times 8.33 \times 0.150 = 0.730$$

$$\text{Steel beam, details, haunches, diaphragms} = 0.170$$

$$DL_1 \text{ per stringer} = \overline{0.900} \text{ k/ft}$$

Dead Load Carried by Composite Section

$$\text{Curbs and railings, } DL_2 = 0.660 \text{ k/ft}$$

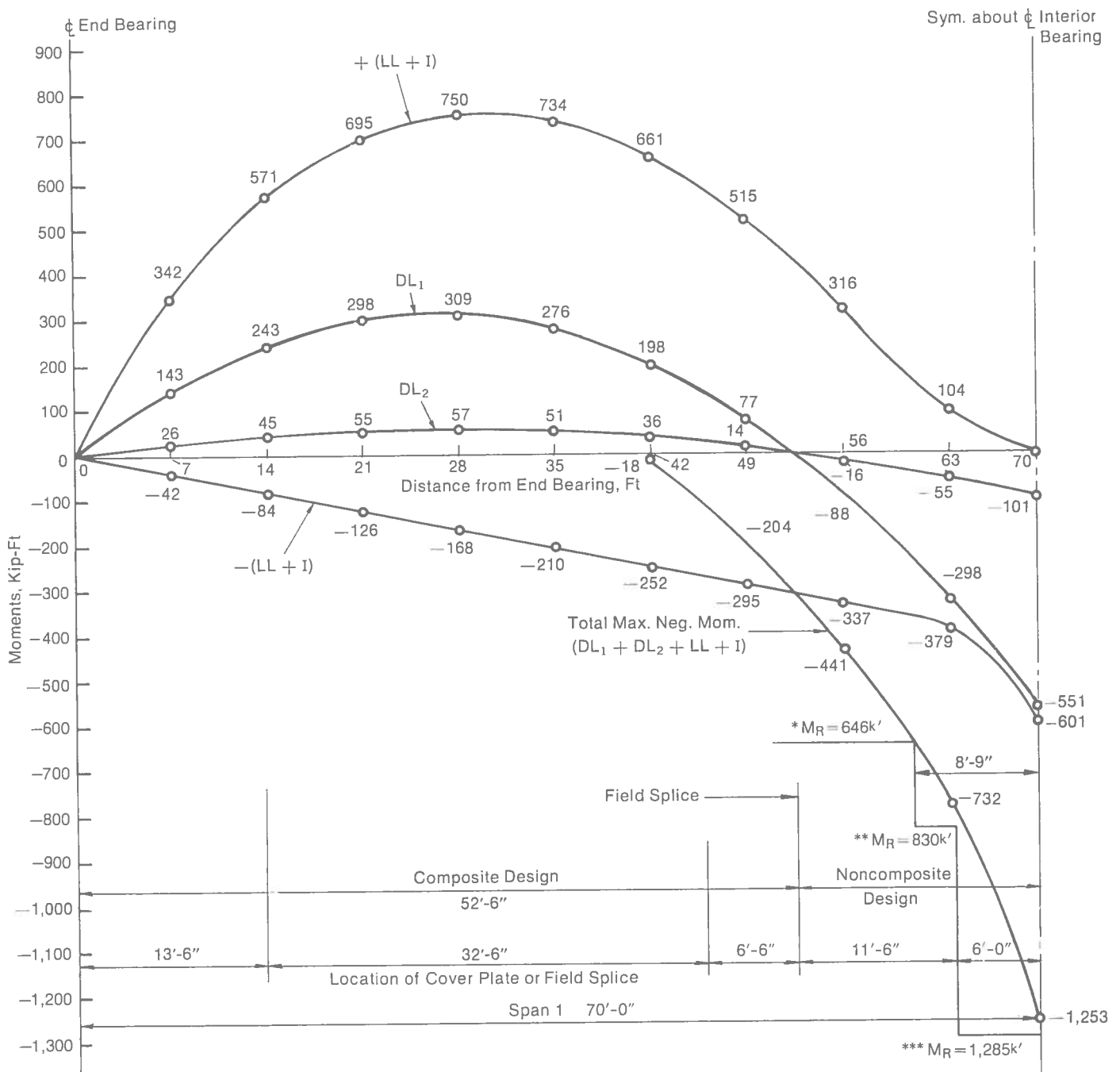
$$DL_2 \text{ per stringer} = 0.660/4 = 0.165 \text{ k/ft}$$

Live Load

$$\text{Live-load distribution} = \frac{S}{5.5} = \frac{8.33}{5.5} = 1.51 \text{ wheels} = 0.755 \text{ axle}$$

$$\text{Impact} = \frac{50}{70 + 125} = 0.256$$

Curves of maximum moments and maximum shears are calculated from tables of influence-line coefficients (see References 3 and 4 at end of chapter).

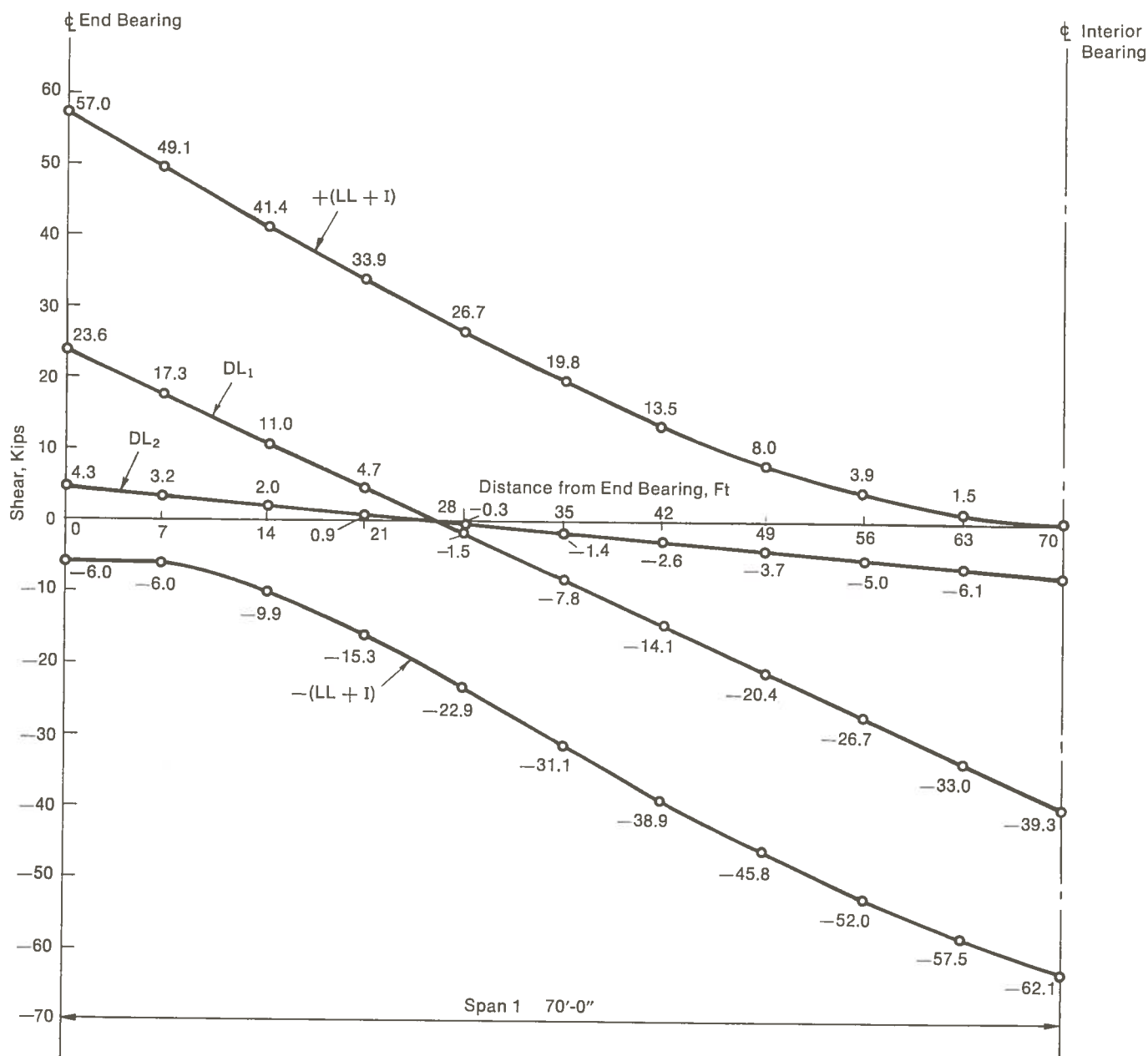


*Resisting Moment of W36×135 Alone

**Resisting Moment of W36×135 Plus 10" x 3/8" Cover Plates Top and Bottom

***Resisting Moment of W36×135 Plus 10" x 1" Cover Plates Top and Bottom

MAXIMUM-MOMENT CURVES—CONSTANT I



MAXIMUM-SHEAR CURVES—CONSTANT I

DESIGN OF STRINGER SECTION

As a first step in the design procedure, the control sections at the points of maximum negative moment and maximum positive moment are selected. A W36×135 beam with 10 x 1-in. cover plates top and bottom is investigated for maximum negative moment. Calculations show that the section satisfies stress limitations.

MAXIMUM-NEGATIVE MOMENT

From the maximum moment curves, the total moment at the interior support is -1,253 kip-ft. This moment must be taken by the W36×135 with top and bottom cover plates. For the W36×135, the moment of inertia $I_B = 7,796 \text{ in.}^4$ and the half-depth $c = 17.78 \text{ in.}$ The allowable tensile stress f in the plates is 20 ksi.

$$\text{Required area of cover plates} = \frac{12M}{fc} - \frac{I_B}{c^2} = \frac{12(1,253)}{20(18)} - \frac{7,796}{(18)^2} = 17.71 \text{ in.}^2$$

Try 2 cover plates 10 x 1 in., area = 20 in.²

Steel Section at Interior Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 135	39.70				7,796	7,796
2 Plates 10 × 1	20.00	18.28	365.6	6,683		6,683

$$59.70 \text{ in.}^2$$

$$I_{N.A.} = 14,479 \text{ in.}^4$$

$$Z = \frac{14,479}{18.78} = 771 \text{ in.}^3$$

$$\text{Maximum stress } f_b = \frac{1,253 \times 12}{771} = 19.50 \text{ ksi} < 20$$

$$\text{Resisting moment for 20 ksi allowable stress } M_R = \frac{20 \times 771}{12} = 1,285 \text{ kip-ft}$$

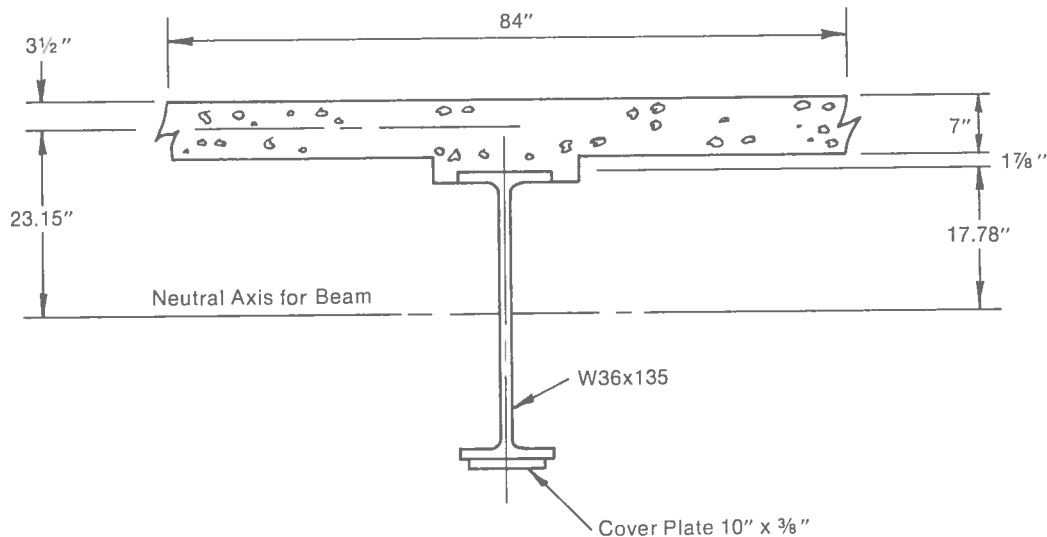
Allowable Compressive Stress near Support

Length L for lateral buckling is 17.5 ft, the distance from interior bearing to dead-load inflection point and b = flange width = 12 in.

$$F_b = 20,000 - 7.5 \left(\frac{L}{b} \right)^2 = 20,000 - 7.5 \left(\frac{17.5 \times 12}{12} \right)^2 = 17.70 \text{ ksi}$$

Because of continuity, AASHTO specifications permit a 20% increase in allowable stresses up to 20 ksi at the interior support.

$$F_b = 17.70 \times 1.20 = 21.2 \text{ ksi. Use 20 ksi.}$$



POSITIVE-MOMENT SECTION

MAXIMUM-POSITIVE MOMENT

A steel section consisting of a W36 × 135 with a bottom cover plate 10 x 3/8 in. is investigated for the region of maximum positive moment. Properties are computed for the steel section alone, the composite section with $n=8$, and the composite section with $3n=24$.

Steel Section, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 135	39.70				7,796	7,796
Bot. Cover Plate 10 × ¾	3.75	-17.96	-67.35	1,210		1,210

$$43.45 \text{ in.}^2$$

$$-67.35 \text{ in.}^3$$

$$9,006$$

$$d_s = \frac{-67.35}{43.45} = -1.55 \text{ in.}$$

$$-1.55 \times 67.35 = -104$$

$$I_{NA} = \frac{8,902}{8,902 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 17.78 + 1.55 = 19.33 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 17.78 + 0.38 - 1.55 = 16.61 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{8,902}{19.33} = 460 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{8,902}{16.61} = 536 \text{ in.}^3$$

Composite Section, 3n = 24, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
Steel section	43.45		-67.35			9,006
Conc. 84 × 7/24	24.50	23.15	567.18	13,130	100	13,330

$$67.95 \text{ in.}^2$$

$$499.83 \text{ in.}^3$$

$$22,236$$

$$d_{24} = \frac{499.83}{67.95} = 7.36 \text{ in.}$$

$$-7.36 \times 499.83 = -3,679$$

$$I_{NA} = \frac{18,557}{18,557 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 17.78 - 7.36 = 10.42 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 18.16 + 7.36 = 25.52 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{18,557}{10.42} = 1,781 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{18,557}{25.52} = 727 \text{ in.}^3$$

Composite Section, n = 8, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
Steel section	43.45		-67.35			9,006
Conc. 84 × ⅞	73.50	23.15	1,701.53	39,390	300	39,690

$$116.95 \text{ in.}^2$$

$$1,634.18 \text{ in.}^3$$

$$48,696$$

$$d_8 = \frac{1,634.18}{116.95} = 13.97 \text{ in.}$$

$$-13.97 \times 1,634.18 = -22,829$$

$$I_{NA} = \frac{25,867}{25,867 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 17.78 - 13.97 = 3.81 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 18.16 + 13.97 = 32.13 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{25,867}{3.81} = 6,789 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{25,867}{32.13} = 805 \text{ in.}^3$$

$$d_{\text{Top of conc.}} = 26.65 - 13.97 = 12.68 \text{ in.}$$

$$Z_{\text{Top of conc.}} = \frac{25,867}{12.68} = 2,040 \text{ in.}^3$$

Check of Steel and Concrete Stresses

Stresses are checked at the top and bottom of steel and at the top of concrete. Calculations show that the W36 × 135 with bottom cover plate satisfies stress limitations. The concrete stress is well within the allowable for compression.

Bending Moments 28 Ft from End Support

	DL_1	DL_2	$LL+I$
M , kip-ft	309	57	750

Steel Stresses—Combination A

Top of Steel (Compression)

$$DL_1: f_b = \frac{309 \times 12}{460} = 8.06$$

$$DL_2: f_b = \frac{57 \times 12}{1,781} = 0.38$$

$$LL+I: f_b = \frac{750 \times 12}{6,789} = \frac{1.33}{9.77} \text{ ksi}$$

Bottom of Steel (Tension)

$$f_b = \frac{309 \times 12}{536} = 6.92$$

$$f_b = \frac{57 \times 12}{727} = 0.94$$

$$f_b = \frac{750 \times 12}{805} = \frac{11.18}{19.04} \text{ ksi}$$

Concrete Stresses—Combination B

Top of Concrete (Compression)

$$DL_2: f_c = \frac{57 \times 12}{2,040 \times 8} = 0.042$$

$$LL+I: f_c = \frac{750 \times 12}{2,040 \times 8} = \frac{0.551}{0.593} \text{ ksi}$$

LOCATION OF COVER-PLATE CUTOFFS

The resisting moments of the rolled section alone, and the rolled section plus 10 x 1-in. cover plates, are plotted on the curve of maximum negative moments. The theoretical cutoff point, exclusive of fatigue considerations, for the plates is determined by scaling the location at which the resisting moment of the W36 x 135 alone equals the bending moment. Since the W36 x 135 has a section modulus $Z = 438.6\text{-in.}^3$ and the allowable compressive stress, determined by buckling is 17.70 ksi, this resisting moment equals

$$M_R = \frac{17.70 \times 438.6}{12} = 646 \text{ kip-ft}$$

The theoretical cutoff point is about 8.75 ft from the interior support. The minimum terminal distance required by AASHTO specifications sets the end of the 10 x 1-in. cover plates about 10 ft from that support.

Next, the allowable fatigue stress in the W36 x 135 adjacent to the fillet welds joining the ends of the cover plates to the flange is checked. Calculations show that the allowable fatigue stress 10 ft from the support is below the bending stress. Thus, fatigue governs the cutoff location.

Another check is made 12 ft from the support. Again, the allowable fatigue stress is lower than the bending stress. The procedure is repeated for a point 15 ft from the support and at the dead-load inflection point, 17.5 ft from the support. Fatigue requirements are not satisfied at these locations.

Allowable fatigue stresses are more highly restrictive in the vicinity of inflection points than elsewhere, because of stress reversals that occur where dead-load moments are small. A negative value of R in the formula for fatigue stress, $F_r = 12,000/(1-R)$, lowers the allowable fatigue stress to a greater extent than positive values of R . For the design under consideration, the ratio of bending stress to allowable fatigue stress increases with distance from the interior support, the inflection point. In this region, the W36 x 135 alone is unable to resist the fatigue stresses adjacent to fillet welds across the end of a cover plate.

Hence, the 10 x 3/8-in. bottom cover plate in the positive-moment portion of the span is extended through the inflection region and butt welded to the 10 x 1-in.

bottom cover plate. Also, a 10 x $\frac{3}{8}$ -in. cover plate is added to the top flange and butt welded to the 10 x 1-in. top cover plate.

Stresses 10 Ft from Interior Support

Bending stresses are computed 10 ft from the interior support. Also, the allowable fatigue stress is calculated in the W36×135 flanges and is found to govern.

Maximum Moments 10 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = -203$$

$$DL_2: M = -39$$

$$LL+I: M = \frac{+183}{-59} \text{ kip-ft}$$

With Negative Live-Load Moment

$$DL_1: M = -203$$

$$DL_2: M = -39$$

$$LL+I: M = \frac{-359}{-601} \text{ kip-ft}$$

Bending stress in top and bottom flange is

$$f_b = \frac{601 \times 12}{438.6} = 16.44 \text{ ksi}$$

Ratio of minimum to maximum stress at cover-plate ends is

$$R = \frac{-59}{-601} = 0.098$$

The allowable fatigue stress in the W36×135 flanges adjacent to the fillet welds for the cover-plate ends is

$$F_r = \frac{12.0}{1 - 0.098} = 13.30 < 16.44 \text{ ksi (overstressed)}$$

Since the W36×135 alone is inadequate in fatigue, try extending the cover plates 2 ft further from the support.

Stresses 12 Ft from Interior Support

Investigation of bending and fatigue stresses 12 ft from the interior support shows that fatigue governs.

Maximum Moments 12 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = -145$$

$$DL_2: M = -28$$

$$LL+I: M = \frac{+247}{74} \text{ kip-ft}$$

With Negative Live-Load Moment

$$DL_1: M = -145$$

$$DL_2: M = -28$$

$$LL+I: M = \frac{-350}{-523} \text{ kip-ft}$$

Bending stress in top and bottom flange is

$$f_b = \frac{523 \times 12}{438.6} = 14.31 \text{ ksi}$$

Ratio of minimum to maximum stress at cover-plate ends is

$$R = \frac{74}{-523} = -0.141$$

The allowable fatigue stress in the W36×135 is

$$F_r = \frac{12.0}{1 - (-0.141)} = 10.52 \text{ ksi} < 14.31 \text{ (overstressed)}$$

Since the W36×135 again is inadequate in fatigue, try extending the cover plates 3 ft further from the support.

Stresses 15 Ft from Interior Support

Investigation of stresses 15 ft from the interior support shows that fatigue still governs.

Maximum Moments 15 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = -60$$

$$DL_2: M = -11$$

$$LL+I: M = \frac{+345}{274} \text{ kip-ft}$$

With Negative Live-Load Moment

$$DL_1: M = -60$$

$$DL_2: M = -11$$

$$LL+I: M = \frac{-325}{-396} \text{ kip-ft}$$

Bending stress in top and bottom flange is

$$f_b = \frac{396 \times 12}{438.6} = 10.83 \text{ ksi}$$

Ratio of minimum to maximum stress at cover-plate ends is

$$R = \frac{274}{-396} = -0.692$$

The allowable fatigue stress in the W36×135 is

$$F_r = \frac{12.0}{1 - (-0.692)} = 7.09 \text{ ksi} < 10.83 \text{ (overstressed)}$$

Since the W36×135 still is inadequate, try extending the cover plates another 2.5 ft, to the dead-load inflection point.

Stresses at Inflection Point

Because of stress reversal, allowable fatigue stress is even smaller at the inflection point than nearer the support and continues to govern.

Maximum Moments 17.5 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = 0$$

$$DL_2: M = 0$$

$$LL+I: M = 420 \text{ kip-ft}$$

With Negative Live-Load Moment

$$DL_1: M = 0$$

$$DL_2: M = 0$$

$$LL+I: M = -310 \text{ kip-ft}$$

Bending stress in top and bottom flange is

$$f_b = \frac{420 \times 12}{438.6} = 11.49 \text{ ksi}$$

Ratio of minimum to maximum stress is

$$R = \frac{-310}{420} = -0.738$$

The allowable fatigue stress in the W36×135 is

$$F_r = \frac{12.0}{1 - (-0.738)} = 6.90 \text{ ksi} < 11.49 \text{ (overstressed)}$$

The W36×135 still is inadequate in fatigue, so extend the 10 x 3/8-in. bottom cover plate in the positive-moment region and butt weld it to the bottom 10 x 1-in. cover plate. Also, add a top cover plate 10 x 3/8-in. in the inflection-point region and butt weld it to the 10 x 1-in. top cover plate.

NEGATIVE-MOMENT TRANSITION SECTION

Fatigue limitations for weld metal or base metal adjacent to butt welds are not so restrictive as the fatigue limitations for metal adjacent to fillet welds. Trials indicate that the butt-welded joints between the 10 x 1-in. cover plates and the 10 x 3/8-in. cover plates may be located as close as 6 ft to the interior support.

Steel Section 6 Ft from Interior Support

Material	A	d	Ad ²	I _o	I
W36×135	39.70			7,796	7,796
2 Plates 10×3/8	7.50	17.97	2,422		2,422

$$47.20 \text{ in.}^2$$

$$I_{NA} = 10,218 \text{ in.}^4$$

$$Z = \frac{10,218}{18.15} = 563 \text{ in.}^3$$

As computed previously, the allowable compressive stress, determined by lateral buckling, is 17.70 ksi. Hence, the resisting moment is

$$M_R = \frac{17.70 \times 563}{12} = 830 \text{ kip-ft}$$

Maximum Moments 6 Ft from Interior Support

With Positive Live-Load Moment

With Negative Live-Load Moment

$$DL_1: M = -330$$

$$DL_1: M = -330$$

$$DL_2: M = -61$$

$$DL_2: M = -61$$

$$LL+I: M = + \frac{85}{-306} \text{ kip-ft}$$

$$LL+I: M = \frac{-395}{-786} \text{ kip-ft}$$

Bending stress in top and bottom cover plates at the butt-welded splice is

$$f_b = \frac{786 \times 12}{563} = 16.75 \text{ ksi} < 17.70 \text{ ksi}$$

Ratio of minimum to maximum stress in the butt weld is

$$R = \frac{-306}{-786} = 0.389$$

The allowable butt-weld fatigue stress in tension is

$$F_r = \frac{17.2}{1 - 0.62(0.389)} = 22.66 \text{ ksi} > 16.75$$

The allowable butt-weld fatigue stress in compression is

$$F_r = \frac{0.55 \times 36}{1 - \left(\frac{0.55 \times 36}{10.6} - 1 \right) 0.389} = 29.91 \text{ ksi} > 16.75$$

Lateral buckling governs, $F_b = 17.70 \text{ ksi}$. Since the bending stress is smaller, the W36×135 with 10 x 3/8-in. cover plates is satisfactory 6 ft from the interior support.

CUTOFF OF TOP COVER PLATE

To satisfy fatigue stresses, the 10 x $\frac{3}{8}$ -in. top cover plate must extend sufficiently into the composite area that live-load positive moments at the end of the plate are carried by the composite section. Preliminary calculations indicate that shear connectors over a length of about 6 ft are required to develop the compressive stress that occurs in the concrete slab under composite action. Since shear connectors start about 17.5 ft from the interior support, a trial cutoff point 24 ft from that support is investigated.

Maximum Moments 24 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = 130$$

$$DL_2: M = 25$$

$$LL+I: M = 580$$

With Negative Live-Load Moment

$$DL_1: M = +130$$

$$DL_2: M = +25$$

$$LL+I: M = \frac{-275}{-120 \text{ kip-ft}}$$

Stresses at Top of Steel—Combination A

Compression (Composite Section)

$$DL_1: f_b = \frac{130 \times 12}{460} = 3.39$$

$$DL_2: f_b = \frac{25 \times 12}{1,781} = 0.17$$

$$LL+I: f_b = \frac{580 \times 12}{6,789} = 1.03$$

4.59 ksi

Tension (Steel Only)

$$f_b = \frac{120 \times 12}{460} = 3.13 \text{ ksi}$$

Ratio of minimum to maximum stress at top of W36 x 135 is

$$R = \frac{3.13}{-4.59} = -0.682$$

The allowable fatigue stress at the fillet weld is

$$F_r = \frac{12.0}{1 - (-0.682)} = 7.13 \text{ ksi} > 4.59$$

Thus, fatigue requirements are satisfied if the top cover plate ends 24 ft from the interior support.

Weld at End of Top Cover Plate

The fillet weld connecting the top cover plate to the W36 x 135 flange within the terminal distance of the end must develop the force in the plate. The terminal distance is 1.5 times the plate width, or 15 in. for the 10-in.-wide cover plate. The weld size is determined at the theoretical cutoff point, 15 in. from the plate end, 22.75 ft from the interior support. A $\frac{1}{16}$ -in. fillet weld 40 in. long would be adequate. Minimum requirements, rather than strength considerations govern, however. The $\frac{13}{16}$ -in. thickness of the W36 x 135 flange requires at least a $\frac{5}{16}$ -in. weld.

Maximum Moments 22.75 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = 94$$

$$DL_2: M = 20$$

$$LL+I: M = 540$$

With Negative Live-Load Moment

$$DL_1: M = +94$$

$$DL_2: M = +20$$

$$LL+I: M = \frac{-287}{-173 \text{ kip-ft}}$$

The steel section consists of the W36 x 135 and top and bottom cover plates 10 x $\frac{3}{8}$ -in. As computed for the transition section 6 ft from the interior support, the section modulus $Z = 563 \text{ in.}^3$

Composite Section, 3n = 24, 22.75 Ft from Interior Support

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	47.20				10,218	10,218
Conc. 84 × 7/24	24.50	23.15	567.18	13,130	100	13,230

$$d_s = \frac{567.18}{71.70} = 7.91 \text{ in.} \quad \begin{array}{l} 71.70 \text{ in.}^2 \\ 567.18 \text{ in.}^3 \\ 23,448 \\ - 7.91 \times 567.18 = - 4,486 \\ \hline I_{NA} = 18,962 \text{ in.}^4 \end{array}$$

$$d_{\text{Top of steel}} = 18.15 - 7.91 = 10.24 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 18.15 + 7.91 = 26.06 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{18,962}{10.24} = 1,852 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{18,962}{26.06} = 728 \text{ in.}^3$$

Composite Section, n = 8, 22.75 Ft from Interior Support

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	47.20				10,218	10,218
Conc. 84 × 3/8	73.50	23.15	1,701.53	39,390	300	39,690

$$d_s = \frac{1,701.53}{120.70} = 14.10 \text{ in.} \quad \begin{array}{l} 120.70 \text{ in.}^2 \\ 1,701.53 \text{ in.}^3 \\ 49,908 \\ - 14.10 \times 1,701.53 = - 23,992 \\ \hline I_{NA} = 25,916 \text{ in.}^4 \end{array}$$

$$d_{\text{Top of steel}} = 18.15 - 14.10 = 4.05 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 18.15 + 14.10 = 32.25 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{25,916}{4.05} = 6,399 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{25,916}{32.25} = 804 \text{ in.}^3$$

Stresses at Top of Steel—Combination A

Compression (Composite Section)

Tension (Steel Only)

$$DL_1: f_b = \frac{94 \times 12}{563} = 2.00$$

$$f_b = \frac{173 \times 12}{563} = 3.69 \text{ ksi}$$

$$DL_2: f_b = \frac{20 \times 12}{1,852} = 0.13$$

$$LL + I: f_b = \frac{540 \times 12}{6,399} = \frac{1.01}{3.14} \text{ ksi}$$

Ratio of minimum to maximum stress in top cover plate is

$$R = \frac{-3.14}{3.69} = -0.851$$

The allowable weld fatigue stress in shear is

$$F_r = \frac{10.8}{1 - 0.55(-0.851)} = 7.36 \text{ ksi}$$

$$\text{Allowable load on weld} = 7.36 \times 0.707 = 5.20 \text{ kips per in.}$$

$$\text{Force in cover plate} = 10 \times \frac{3}{8} \times 3.69 = 13.84 \text{ kips}$$

$$\text{Weld size required} = \frac{13.84}{5.20 \times 40} = 0.066 \text{ in., say } \frac{1}{16} \text{ in.}$$

Use 5/16-in. fillet weld required for flange thickness.

CUTOFF OF BOTTOM COVER PLATE

The theoretical cutoff point for the 10 x 3/8-in. bottom cover plate is located by trial at 17 ft from the end support. Fatigue limitations set the plate end at 13.5 ft from the support. There is no reversal of stress in this region of the span. The maximum stress range occurs with cycling between maximum positive live-load moment and maximum negative live-load moment.

Weld size is governed by material thickness. A check on strength is included for illustrative purposes.

Stresses 17 Ft from End Support

Between the end support and the theoretical cutoff point 17 ft away, the steel section consists of the W36 x 135, with section modulus $Z = 438.6 \text{ in.}^3$

Composite Section, 3n = 24, near End Support

Material	A	d	Ad	Ad ²	I _o	I
W36 x 135	39.70				7,796	7,796
Conc. 84 x 7/24	24.50	23.15	567.18	13,130	100	13,230

$$d_{NA} = \frac{567.18}{64.20} = 8.83 \text{ in.} \quad \begin{array}{l} 64.20 \text{ in.}^2 \\ 567.18 \text{ in.}^3 \\ 21,026 \\ - 8.83 \times 567.18 = - 5,008 \\ I_{NA} = 16,018 \text{ in.}^4 \end{array}$$

$$d_{\text{Top of steel}} = 17.18 - 8.83 = 8.95 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 17.78 + 8.95 = 26.73 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{16,018}{8.95} = 1,790 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{16,018}{26.73} = 599 \text{ in.}^3$$

Composite Section, n = 8, near End Support

Material	A	d	Ad	Ad ²	I _o	I
W36 x 135	39.70				7,796	7,796
Conc. 84 x 7/8	73.50	23.15	1,701.53	39,390	300	39,690

$$d_s = \frac{1,701.53}{113.20} = 15.03 \text{ in.} \quad \begin{array}{l} 113.20 \text{ in.}^2 \\ 1,701.53 \text{ in.}^3 \\ 47,486 \\ - 15.03 \times 1,701.53 = - 25,574 \\ I_{NA} = 21,912 \text{ in.}^4 \end{array}$$

$$d_{\text{Top of steel}} = 17.78 - 15.03 = 2.75 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 17.78 + 15.03 = 32.81 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{21,912}{2.75} = 7,968 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{21,912}{32.81} = 668 \text{ in.}^3$$

Bending Moments 17 Ft from End Support

	DL ₁	DL ₂	LL + I
M, kip-ft	274	50	640

Steel Stresses—Combination A

Top of Steel (Compression)

$$DL_1: f_b = \frac{274 \times 12}{438.6} = 7.50$$

$$DL_2: f_b = \frac{50 \times 12}{1,790} = 0.34$$

$$LL + I: f_b = \frac{640 \times 12}{7,968} = 0.96$$

8.80 ksi

Bottom of Steel (Tension)

$$f_b = \frac{274 \times 12}{438.6} = 7.50$$

$$f_b = \frac{50 \times 12}{599} = 1.00$$

$$f_b = \frac{640 \times 12}{668} = 11.50$$

20.00 ksi

Fatigue Check at Cover-Plate End

The composite section with W36 × 135 is adequate for bending stresses. Fatigue stresses, however, will govern. The 10 × ¾-in. bottom cover plate must be extended beyond the 15-in. minimum terminal distance. Try ending the plate 13.5 ft from the end support.

Maximum Moments 13.5 Ft from End Support

With Positive Live-Load Moment

$$\begin{aligned} DL_1: M &= 232 \\ DL_2: M &= 41 \\ LL+I: M &= 560 \end{aligned}$$

With Negative Live-Load Moment

$$\begin{aligned} DL_1: M &= +232 \\ DL_2: M &= +41 \\ LL+I: M &= -\frac{78}{195} \text{ kip-ft} \end{aligned}$$

Stresses at Bottom of Steel—Combination A

Tension (Composite Section)

$$\begin{aligned} DL_1: f_b &= \frac{232 \times 12}{438.6} = 6.35 \\ DL_2: f_b &= \frac{41 \times 12}{599} = 0.82 \\ LL+I: f_b &= \frac{560 \times 12}{668} = \frac{10.06}{17.23} \text{ ksi} < 20 \end{aligned}$$

Tension (Steel Only)

$$f_b = \frac{195 \times 12}{438.6} = 5.34 \text{ ksi}$$

Ratio of minimum to maximum stress in beam flange at cover-plate weld:

$$R = \frac{5.34}{17.23} = 0.310$$

The allowable fatigue stress is

$$F_r = \frac{12.0}{1 - 0.310} = 17.39 \text{ ksi} > 17.23$$

Since allowable stresses are satisfied, the plate can be terminated 13.5 ft from the end support.

Weld at End of Bottom Cover Plate

The fillet weld connecting the bottom cover plate to the W36 × 135 flange within 15 in. of the end must develop the force in the plate. The weld size is determined at the theoretical cutoff point 14.75 ft from the end support. A ¾-in. fillet weld 40 in. long would be adequate, but the 13/16-in. flange of the W36 × 135 requires at least a 5/16-in. weld.

Maximum Moments 14.75 Ft from End Support

With Positive Live-Load Moment

$$\begin{aligned} DL_1: M &= 247 \\ DL_2: M &= 45 \\ LL+I: M &= 590 \end{aligned}$$

With Negative Live-Load Moment

$$\begin{aligned} DL_1: M &= +247 \\ DL_2: M &= +45 \\ LL+I: M &= -\frac{88}{204} \text{ kip-ft} \end{aligned}$$

Stresses at Bottom of Steel—Combination A

Tension (Composite Section)

$$\begin{aligned} DL_1: f_b &= \frac{247 \times 12}{536} = 5.53 \\ DL_2: f_b &= \frac{45 \times 12}{727} = 0.74 \\ LL+I: f_b &= \frac{590 \times 12}{805} = \frac{8.80}{15.07} \text{ ksi} \end{aligned}$$

Tension (Steel Only)

$$f_b = \frac{204 \times 12}{536} = 4.57 \text{ ksi}$$

Ratio of minimum to maximum stress in bottom cover plate is

$$R = \frac{4.57}{15.07} = 0.303$$

The allowable weld fatigue stress in shear is

$$F_r = \frac{10.8}{1 - 0.55(0.303)} = 12.96 \text{ ksi} > 12.4$$

Allowable load on weld = $12.4 \times 0.707 = 8.76$ kips per in.

Force in cover plate = $10 \times \frac{3}{8} \times 15.07 = 56.5$ kips

$$\text{Weld size required} = \frac{56.5}{8.76 \times 40.0} = 0.161 \text{ in., say } \frac{3}{16} \text{ in.}$$

Use $\frac{5}{16}$ in. fillet weld, required for flange thickness.

From the preceding calculations, it is evident that the stringer design is governed to a great extent by fatigue at the cover-plate cutoff points. Two alternate designs have been made, aimed at minimizing the fatigue limitations by partially or completely eliminating cover plates. These alternate designs are discussed in detail at the end of this example.

COMPARISON OF FATIGUE FORMULAS FOR 100,000 AND 500,000 CYCLES

The preceding designs have been based on 500,000 cycles of loading, corresponding to fatigue conditions for heavily travelled expressways. Rolled-beam composite design, however, finds frequent application for structures carrying secondary roadways over expressways, for which fatigue limitations may be based on 100,000 applications of load. For comparison, tensile-fatigue formulas are given below for A36 steel, for butt welds and base metal adjacent to fillet welds at 100,000 and 500,000 cycles.

Cycles	Butt Welds	Base Metal Adjacent to Fillet Welds
100,000	$F_r = \frac{20.5}{1 - 0.55R}$	$F_r = \frac{18.0}{1 - R}$
500,000	$F_r = \frac{17.2}{1 - 0.62R}$	$F_r = \frac{12.0}{1 - R}$

The allowable fatigue stresses for 100,000 cycles are about 20 to 50% higher than those for 500,000 cycles. Thus, the extent to which fatigue controls cover-plate cutoff locations is proportionately reduced in structures carrying secondary roadways. This is illustrated in a third alternate design based on 100,000 cycles of fatigue loading, appearing at the end of the present example.

RE-ANALYSIS BASED ON VARIABLE SECTION

After material sizes and cover-plate cutoff locations have been determined, moments based on variable moment of inertia can be obtained. Curves of maximum moments are plotted from the data. For comparison, the moment curves based on constant moment of inertia are shown by dashed lines. The diagram shows that there is little difference between the curves for variable and constant moment of inertia.

Stresses based on the variable section are first checked at the points of maximum-negative and maximum-positive moment.

Stresses for Maximum-Negative Moment

The effect of variable moment of inertia at the negative-moment section is a decrease in stress from 19.50 to 18.72 ksi.

	DL_1	DL_2	$LL+I$	Total
M , kip-ft	-609	-92	-502	-1,203

Figure 10 is a graph showing Maximum-Moment Curves—Variable I for a single-span bridge (Span 1: 70'-0"). The vertical axis represents Moments, Kip-Ft, ranging from -1,300 to 800. The horizontal axis represents Distance from End Bearing, Ft, ranging from 0 to 70. The graph includes curves for Dead Load (DL₁, DL₂), Live Load (LL), and their combinations (+LL + I, -(LL + I)). The legend indicates that solid lines represent Variable I and dashed lines represent Constant I. The total maximum negative moment (DL₁ + DL₂ + LL + I) is shown as a thick solid line at the bottom right, reaching -1,203 Kip-Ft at the interior bearing. Other key values include a maximum positive moment of 783 Kip-Ft for DL₁ and a total maximum negative moment of -713 Kip-Ft for the combination of DL₁, DL₂, LL, and I.

The effect of variable moment of inertia at the positive-moment section is an increase in steel stress from 19.04 to 19.07 ksi and in concrete stress from 0.593 to 0.620 ksi.

	DL_1	DL_2	$LL+I$
M , kip-ft	286	60	783

Steel Stresses—Combination A

Top of Steel (Compression)

$$DL_1: f_b = \frac{286 \times 12}{460} = 7.46$$

$$DL_2: f_b = \frac{60 \times 12}{1,781} = 0.40$$

$$LL+I: f_b = \frac{783 \times 12}{6,789} = 1.38$$

9.24 ksi

Bottom of Steel (Tension)

$$f_b = \frac{286 \times 12}{536} = 6.40$$

$$f_b = \frac{60 \times 12}{727} = 0.98$$

$$f_b = \frac{783 \times 12}{805} = 11.67$$

19.05 ksi

Concrete Stresses—Combination B

$$DL_2: f_c = \frac{60 \times 12}{2,040 \times 8} = 0.044$$

$$LL+I: f_c = \frac{783 \times 12}{2,040 \times 8} = 0.576$$

0.620 ksi

Check of Fatigue Stresses

Stresses at the ends of the top and bottom cover plates, as well as at the butt weld between cover plates, are checked and exhibit only slight changes from the values based on constant moment of inertia.

Maximum Moments 24 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = 95$$

$$DL_2: M = 30$$

$$LL+I: M = 630$$

With Negative Live-Load Moment

$$DL_1: M = + 95$$

$$DL_2: M = + 30$$

$$LL+I: M = -230$$

-105 kip-ft

Stresses at Top of Steel—Combination A

Compression (Composite Section)

$$DL_1: f_b = \frac{95 \times 12}{460} = 2.48$$

$$DL_2: f_b = \frac{30 \times 12}{1,781} = 0.20$$

$$LL+I: f_b = \frac{630 \times 12}{6,789} = 1.11$$

3.79 ksi

Tension (Steel Only)

$$f_b = \frac{105 \times 12}{460} = 2.74 \text{ ksi}$$

Ratio of minimum to maximum stress at top of W36×135 is

$$R = \frac{2.74}{-3.79} = -0.723$$

The allowable fatigue stress at the fillet weld is

$$F_r = \frac{12.0}{1 - (-0.723)} = 6.96 \text{ ksi} > 3.79$$

Thus, at the cover-plate end 24 ft from the interior support, stresses are slightly lower for variable moment of inertia than for constant moment of inertia.

Maximum Moments 13.5 Ft from End Support

With Positive Live-Load Moment

$$DL_1: M = 221$$

$$DL_2: M = 44$$

$$LL+I: M = 571$$

With Negative Live-Load Moment

$$DL_1: M = +221$$

$$DL_2: M = + 44$$

$$LL+I: M = - 68$$

197 kip-ft

Stresses at Bottom of Steel—Combination A

Tension (Composite Section)

$$DL_1: f_b = \frac{221 \times 12}{438.6} = 6.05$$

$$DL_2: f_b = \frac{44 \times 12}{599} = 0.88$$

$$LL+I: f_b = \frac{571 \times 12}{668} = 10.26$$

17.19 ksi

Tension (Steel Only)

$$f_b = \frac{197 \times 12}{438.6} = 5.39 \text{ ksi}$$

Ratio of minimum to maximum stress in beam flange at cover-plate weld:

$$R = \frac{5.39}{17.19} = 0.314$$

The allowable fatigue stress is

$$F_r = \frac{12.0}{1 - 0.314} = 17.49 \text{ ksi} > 17.19$$

At 13.5 ft from the end support, stresses at the cover-plate end for constant and variable moment of inertia differ by less than 1%.

Maximum Moments 6 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = -390$$

$$DL_2: M = -50$$

$$LL+I: M = +80$$

-360 kip-ft

With Negative Live-Load Moment

$$DL_1: M = -390$$

$$DL_2: M = -50$$

$$LL+I: M = -330$$

-770 kip-ft

Bending stress in top and bottom cover plates at the butt-welded splice is

$$f_b = \frac{770 \times 12}{563} = 16.42 \text{ ksi} < 17.70$$

Ratio of minimum to maximum stress in the butt weld is

$$R = \frac{-360}{-770} = 0.467$$

The allowable butt-weld fatigue stress in tension is

$$F_r = \frac{17.2}{1 - 0.62(0.467)} = 24.19 \text{ ksi} > 16.42$$

The allowable butt-weld fatigue stress in compression is

$$F_r = \frac{0.55 \times 36}{1 - \left(\frac{0.55 \times 36}{10.6} - 1 \right) 0.467} = 33.28 \text{ ksi} > 16.42$$

At 6 ft from the interior support, bending stress is smaller for variable moment of inertia, and lateral buckling still governs.

The computations illustrate that a uniform moment of inertia may be assumed with sufficient accuracy in analysis of two-span, continuous, composite stringers.

BEARING STIFFENERS

Bearing stiffeners are not required at the supports of rolled beams if the web shearing stress does not exceed 75% of the allowable shearing stress. Calculations indicate that shearing stresses are well below this value. Hence, stiffeners are not necessary.

Reactions at End Support

	DL_1	DL_2	$LL+I$	Total
R , kips	23.6	4.3	57.0	84.9

$$f_v = \frac{84.9}{0.598 \times 35.55} = 4.0 \text{ ksi}$$

The allowable shear without stiffeners is

$$f_v = 12 \times 0.75 = 9 \text{ ksi} > 4.0$$

No end stiffeners are required.

Shears at Interior Support

	DL_1	DL_2	$LL+I$	Total
V , kips	39.3	7.2	62.1	108.6

$$f_v = \frac{108.6}{0.598 \times 35.55} = 5.1 \text{ ksi} < 9 \text{ ksi}$$

No stiffeners are required at the interior support.

DESIGN OF SHEAR CONNECTORS

Welded stud shear connectors, $\frac{7}{8}$ in. in diameter and 4 in. high, are provided in the composite region of the span and in the negative-moment region adjacent to the dead-load inflection point. The allowable load on the shear connectors is calculated according to AASHTO Specifications for an H/d ratio greater than 4.0. Service behavior under repeated load governs the spacing. The required spacing is calculated for points along the span, and the theoretical spacing curve is plotted. The actual, stepped spacing diagram is enclosed within this theoretical spacing curve. The shear connector pitch is changed in 3-in. increments.

$$\text{Concrete: } f'_c = 4,000 \text{ psi; } n = 8$$

$$\text{Studs: } \frac{7}{8}\text{-in.-dia, 4-in.-high, } H/d = 4.0/0.875 = 4.6 > 4.0$$

The ultimate strength of a shear connector equals

$$Q_u = 0.93d^2\sqrt{f'_c} = 0.93(0.875)^2\sqrt{4,000} = 45.0 \text{ kips per stud}$$

With α given as 10.6 for 500,000 cycles of load in AASHTO specifications, the load range per shear connector is

$$Z_r = \alpha d^2 = 10.6(0.875)^2 = 8.11 \text{ kips per stud}$$

Shear Connectors—Strength Requirements

At midspan, the maximum compressive stress in the concrete is

$$H_1 = A_s F_y = [39.70 + (10.0 \times 0.375)]36.0 = 1,564 \text{ kips (governs)}$$

$$H_2 = 0.85f'_c bt = 0.85 \times 4.0 \times 84.0 \times 7.0 = 1,999.2 \text{ kips}$$

The number of studs required from point of maximum moment ($0.4L$) to end support and to dead-load inflection point is

$$N = \frac{H_1}{\phi Q_u} = \frac{1,564}{0.85 \times 45.0} = 40.8$$

where ϕ = reduction factor = 0.85

At the cover-plate cutoffs, 13.5 ft from the end support:

$$H_1 = A_s F_y = 39.70 \times 36.0 = 1,429 \text{ kips (governs)}$$

$$H_2 = 1,999.2 \text{ kips}$$

The number of studs required between the cover-plate cutoff and the end bearing is

$$N = \frac{H_1}{\phi Q_n} = \frac{1,429}{0.85 \times 45.0} = 37.4$$

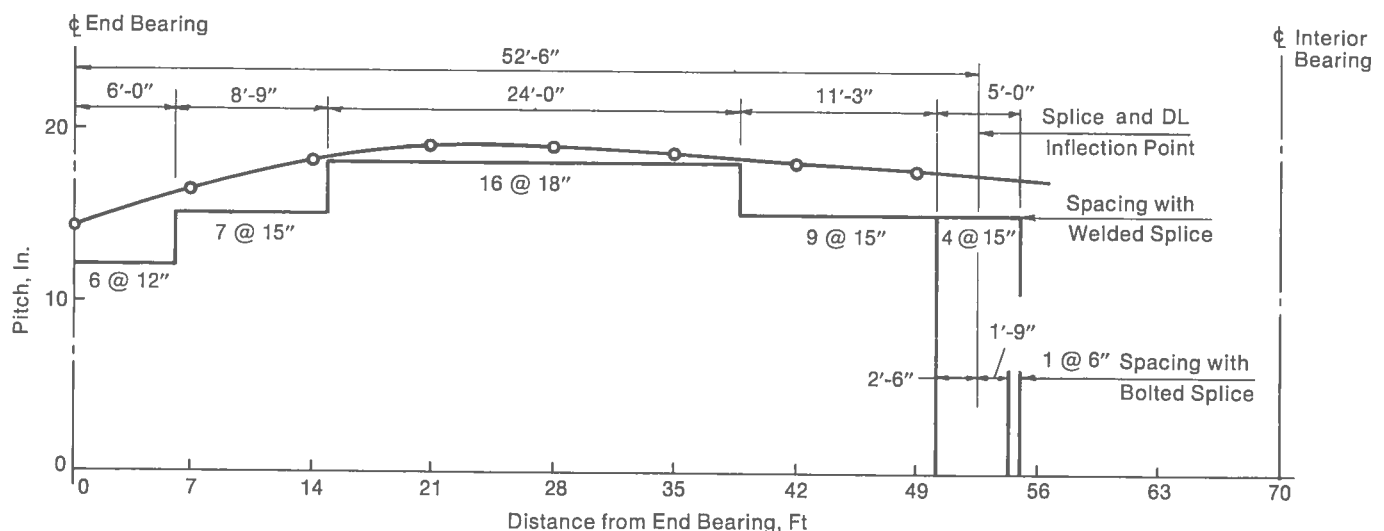
Shear-Connector Spacing for Service Behavior (Fatigue)

The required spacing of shear connectors under repeated loads is calculated first for the end support. There, shear range for the live load = $V_r = 57.0 - (-6.0) = 63.0$ kips. For $n = 8$, the horizontal shear per linear inch is

$$S_r = \frac{V_r Q}{I} = \frac{63.0 (73.5 \times 8.12)}{21,912} = 1.72 \text{ kips per in.}$$

$$\text{Spacing required (3 studs)} = \frac{3 Z_r}{S_r} = \frac{3 \times 8.11}{1.72} = 14.1 \text{ in.}$$

Next, the shear-connector spacing required at each tenth point is computed. Then, the theoretical spacing envelope is plotted. The stepped spacing diagram for fabrication is drawn within the envelope and provides 66 studs from the end support to the $0.4L$ point and 57 studs from there to the dead-load inflection point. From the end support to the cover-plate end, 39 studs are provided. The number of studs supplied exceed the number required for strength and fatigue.



SHEAR-CONNECTOR SPACING

Shear Connectors Required for Slab Reinforcement

To develop the slab reinforcement as recommended by AASHTO specifications, 6 extra studs are required in the negative-moment region adjacent to the inflection point.

For longitudinal reinforcement within the effective flange width in the negative-moment region, 14 No. 5 bars are specified. These provide an area

$$A_r = 14 \times 0.31 = 4.34 \text{ in.}^2$$

and have a yield strength of

$$H_3 = A_r F_y = 4.34 \times 40.0 = 173.6 \text{ kips}$$

This is equivalent to

$$N = \frac{H_3}{\phi Q_u} = \frac{173.6}{0.85 \times 45.0} = 4.5 \text{ studs}$$

Live load plus impact, however, requires

$$N = \frac{A_r f_r}{Z_r} = \frac{4.34 \times 10.0}{8.11} = 5.3 \text{ studs}$$

Use 6 extra studs adjacent to the dead-load inflection point to develop the slab reinforcement in the negative-moment region.

Shear Connectors Required Beyond End of Top Cover Plate

At the end of the 10 x 3/8-in. top cover plate, 24 ft from the interior support, full composite action must be insured. Thus, enough shear connectors must be provided between plate end and the inflection point to develop the maximum force in the concrete slab at the plate end. A 15 in. pitch is used over this distance.

Moments 24 Ft from Interior Support

	DL ₂	LL+I	Total
M, kips	25	580	605

Concrete Stresses—Combination B

Top of Slab (Compression)

$$DL_2: f_c = \frac{25 \times 12}{2,040 \times 8} = 0.018$$

$$LL+I: f_c = \frac{580 \times 12}{2,040 \times 8} = 0.426$$

0.444 ksi

Bottom of Slab (Compression)

$$f_c = \frac{25 \times 12 \times 5.68}{25,867 \times 8} = 0.008$$

$$f_c = \frac{580 \times 12 \times 5.68}{25,867 \times 8} = 0.191$$

0.199 ksi

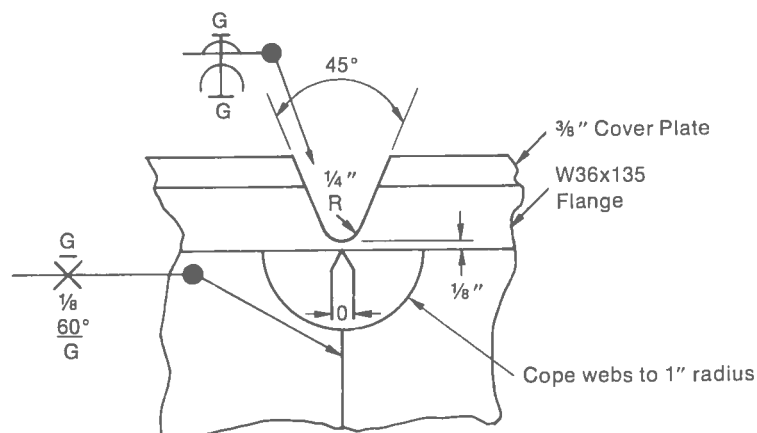
The average stress in the concrete slab is 1/2 (0.444 + 0.199) = 0.321 ksi and the total force in the slab is 84 × 7 × 0.321 = 189 kips. With a factor of safety of 3.0 for stud strength, the number of studs required to develop 189 kips is

$$N = \frac{189}{45.0/3} = 12.6$$

Use 5 rows of studs at 15-in. pitch = 15 studs over a length of 5 ft.

WELDED FIELD SPLICE

A full-penetration butt weld is used to make a field splice for the two-span stringer at one dead-load inflection point, 17.5 ft from the interior support. The field splice is designed as a full-strength welded splice, as shown below. All welds are to be ground smooth. Hence, there are no fatigue restrictions.



WELDED FIELD SPLICE

BOLTED FIELD SPLICE

A bolted field splice is investigated as an alternate design. The splice, made with $\frac{3}{8}$ -in.-dia, high-strength friction bolts, must meet static strength requirements. It also must satisfy requirements for fatigue in base metal adjacent to friction-type fasteners.

For static strength, the splice material is proportioned to carry the greater of:

- (1) 75% of the moment capacity of the net section.
- (2) The average of the actual maximum moment and the moment capacity of the net section.

The net section is the gross section on the weaker side of a splice less bolt holes.

Fatigue need not be considered when calculating bolt stresses, since no reduction in allowable bolt stress is required by AASHTO specifications, regardless of the stress range or number of stress repetitions. Fatigue in base metal adjacent to friction-type connectors, however, should be considered in the design of splice plates. A fatigue design moment, defined as follows, assures that splice-plate stresses will be less than the allowable fatigue stress.

$$\text{Fatigue design moment} = \text{actual maximum moment} \times \frac{\text{allowable tensile stress}}{\text{allowable fatigue stress}}$$

If greater than moments (1) and (2), the fatigue design moment controls the splice plate design. In this example, (2) governs, because it is greater than (1) and the fatigue design moment. In other designs, the fatigue design moment may govern.

The shear to be used in design of a splice is not as well standardized as is the moment. Some designs are made for 75% of the shear capacity of the web or the average of the actual shear and the shear capacity of the web, whichever is greater. Such a design is usually quite conservative, particularly for a rolled beam with a heavy web and high shear capacity. Since both shear and moment are directly related to applied load, it would appear reasonable to use a design shear increased by the same proportion as the design moment. Accordingly, the field splice is designed for shear determined as follows:

$$\text{Design shear} = \text{actual maximum shear} \times \frac{\text{design moment}}{\text{actual maximum moment}}$$

Design calculations for the splice begin with tabulation of maximum shear and moment at a point 17.5 ft from the interior support, and computation of net-section properties.

Maximum Shear

$$DL_1: V = 23.0$$

$$DL_2: V = 4.5$$

$$LL + I: V = \frac{48.0}{75.5} \text{ kips}$$

Maximum Moments

$$\text{Positive } M = 420 \text{ kip-ft}$$

$$\text{Negative } M = -310 \text{ kip-ft}$$

Net Section Properties

Material	A	d	Ad ²	I _o	I
W36 × 135	39.70			7,796	7,796
2 Plates 10 × $\frac{3}{8}$	7.50	17.97	2,422		2,422

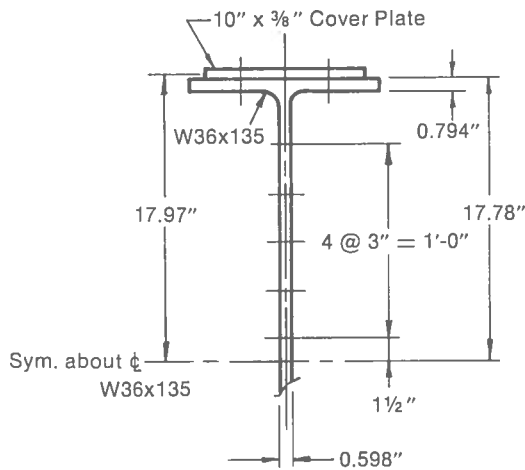
$$47.20 \text{ in.}^2$$

$$I = 10,218 \text{ in.}^4$$

Use $\frac{3}{8}$ -in.-dia, high-strength bolts (ASTM A325), with 1-in. hole diameter.

$$\text{Flange hole area} = 1 \times 1.169 = 1.169 \text{ in.}^2$$

$$\text{Web hole area} = 1 \times 0.598 = 0.598 \text{ in.}^2$$



SECTION AT SPLICE

d^2 for Holes

$$\begin{aligned}(1.5)^2 &= 2.25 \\ (4.5)^2 &= 20.25 \\ (7.5)^2 &= 56.25 \\ (10.5)^2 &= 110.25 \\ (13.5)^2 &= 182.25 \\ \Sigma d^2 &= 371.25 \text{ in.}^2\end{aligned}$$

Net Moment of Inertia

$$\begin{aligned}I \text{ of beam} &= 10,218 \\ Ad^2 \text{ flg. holes} &= -1.169 \times 2 \times 2(17.57)^2 = -1,443 \\ Ad^2 \text{ web holes} &= 0.598 \times 2 \times 371.25 = \frac{444}{8,331 \text{ in.}^4} \\ I_N &= \frac{444}{8,331 \text{ in.}^4}\end{aligned}$$

Required Moment and Shear Capacity of Splice

Fatigue is investigated but is found not to govern design of splice material. Moments for design are determined by the average of the maximum moment and the moment capacity of the net section.

Ratio of minimum to maximum stress in the cover plate is

$$R = \frac{-310}{420} = -0.738$$

The allowable fatigue stress in tension is

$$F_r = \frac{20.5k_1}{1 - 0.55R} = \frac{20.5 \times 1}{1 - 0.55(-0.738)} = 14.58 \text{ ksi}$$

The allowable fatigue stress in compression is

$$F_r = \frac{0.55F_y}{1 - \left(\frac{0.55F_y}{13.3} - 1\right)R} = \frac{0.55 \times 36}{1 - \left(\frac{0.55 \times 36}{13.3} - 1\right)(-0.738)} = 14.55 \text{ ksi (governs)}$$

For the splice plates, then, the fatigue design moment is

$$M_{\text{fat}} = \frac{420 \times 20}{14.55} = 578 \text{ kip-ft}$$

The moment capacity of the net section is

$$M_{\text{net}} = \frac{20 \times 8,331}{18.15 \times 12} = 765 \text{ kip-ft}$$

$$75\% M_{\text{net}} = 0.75 \times 765 = 574 \text{ kip-ft}$$

The average moment capacity is

$$M_{\text{av}} = \frac{420 + 765}{2} = 593 \text{ kip-ft} > 578$$

Because M_{av} is larger than the fatigue design moment and 75% of M_{net} , the splice is designed for 593 kip-ft.

The shear capacity corresponding to the average-moment capacity is

$$V = 75.5 \times \frac{593}{420} = 107 \text{ kips}$$

Web-Splice Design

The web splice is designed to carry the total shear at the section, the moment due to the eccentricity of this shear, and a portion of the total moment on the section. The share of total moment to be resisted by the web is obtained by multiplying the total moment by the ratio of the net moment of inertia of the web to the net moment of inertia of the entire section.

A check is made of the shear in the extreme bolt and extreme fiber stress of the web splice plate. Computations show that both values are within allowable limits.

Deduction of twice the flange thickness from the depth of the W36×135 yields the web depth: $35.55 - 2 \times 0.795 = 33.96$ in. The net moment of inertia of the web then is

$$I_w = \frac{0.598(33.96)^3}{12} - 444 = 1,508 \text{ in.}^4$$

Web Moment

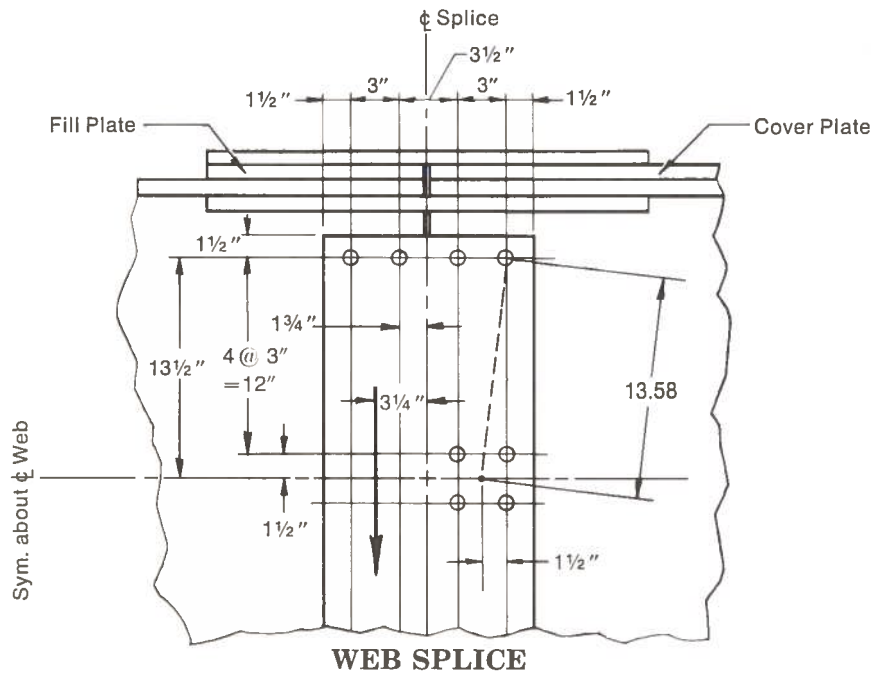
$$M_v = \frac{107 \times 3.25}{12} = 29$$

$$M_w = \frac{1,508}{8,331} \times 593 = \frac{108}{137} \text{ kip-ft}$$

Moment of Inertia of Bolts

$$I_{x-x} = 2 \times 2 \times 371.25 = 1,485$$

$$I_{y-y} = 20(1.5)^2 = \frac{45}{1,530 \text{ in.}^4}$$



Shear on Bolts

Load per bolt due to shear is

$$P_s = \frac{107}{20} = 5.35 \text{ kips}$$

Load on the outermost bolt due to moment is

$$P_m = \frac{137 \times 12 \times 13.58}{1,530} = 14.59 \text{ kips}$$

The vertical component of this load is

$$P_v = \frac{14.59 \times 1.5}{13.58} = 1.61 \text{ kips}$$

And the horizontal component is

$$P_h = \frac{14.59 \times 13.5}{13.58} = 14.50 \text{ kips}$$

Hence, the total load on the outermost bolt is the resultant

$$P = \sqrt{(5.35 + 1.61)^2 + (14.50)^2} = 16.08 \text{ kips}$$

Allowable double shear on $\frac{7}{8}$ -in.-dia, *HS* bolt = 16.2 kips > 16.08. The bolts and bolt arrangement are satisfactory.

Web Splice Plates—Design for Average Moment

For the web splice, assume two plates $12\frac{1}{2} \times \frac{3}{8}$ -in. by 2 ft 6 in. long.

$$I = 2 \times 0.375 \left(\frac{(30)^3}{12} - 742.5 \right) = 1,131 \text{ in.}^4$$

These plates resist the portion of the bending moment carried by the web, 108 kip-ft. This produces a maximum stress

$$f = \frac{108 \times 12 \times 15}{1,131} = 17.19 \text{ ksi} < 20 \text{ (allowable)}$$

The assumed plates are satisfactory.

Flange-Splice Design

The flange splice carries that portion of the total moment not carried by the web. The splice plates transmit this moment across the splice as a couple in axial tension and compression, and into the cover plates and W36 \times 135 flanges by double shear on $\frac{7}{8}$ -in.-dia bolts. Two rows of bolts are used, with $\frac{9}{16}$ and $\frac{5}{8}$ -in.-thick splice plates.

Flange Bolts Required

The required average-moment capacity of the flange splice is

$$M_f = 593 - 108 = 485 \text{ kip-ft}$$

Compressive and tensile forces in the flanges form a couple that supply this capacity.

$$P_f = \frac{485 \times 12}{36.30 - 1.17} = 166 \text{ kips}$$

$$\text{Bolts required} = \frac{166}{16.2} = 11. \text{ Use 12.}$$

Flange Splice Plates—Design for Average Moment

To carry the 166-kip force on the flange, the flange splice plates must have an area of at least

$$A = \frac{166}{20} = 8.30 \text{ in.}^2$$

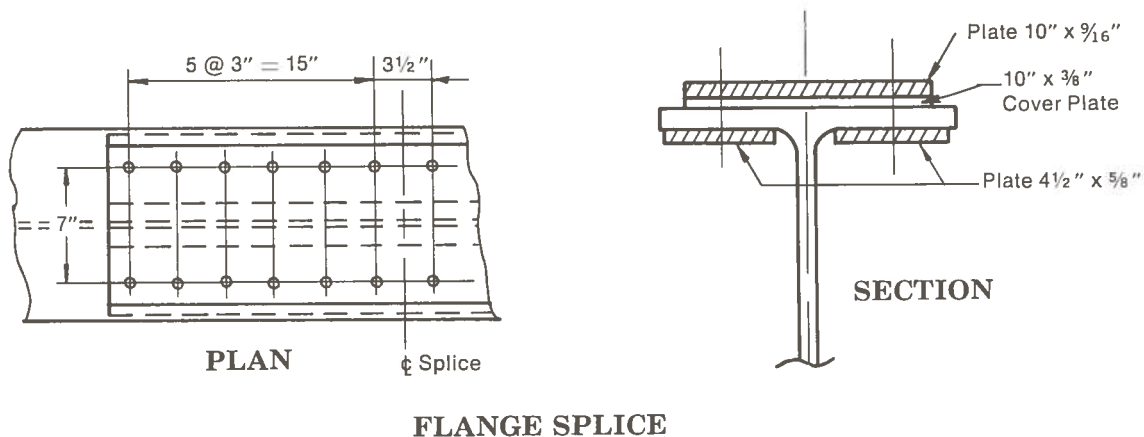
Use on each flange one plate $10 \times \frac{9}{16}$ in. and two plates $4\frac{1}{2} \times \frac{5}{8}$ in.

The net area supplied is

$$(10 - 2)\frac{9}{16} = 4.50$$

$$(9 - 2)\frac{5}{8} = 4.38$$

$$\text{Net } A = 8.88 \text{ in.}^2 > 8.30$$



SECTION

When the field splice is welded, the 10 x $\frac{3}{8}$ -in. top cover plate extends past the splice to the cutoff point 24 ft from the interior support. This cutoff location is governed by allowable fatigue stress in the metal adjacent to the fillet weld across the end of the cover plate. When the field splice is bolted, the top cover plate may be cut off at the splice, since, between the first line of bolts on each side of the splice, no stress exists either in the cover plate or rolled beam. The stress is carried solely by the splice plates across this region and transferred gradually back into the stringer flange through the bolts. Thus, no welding across the end of the cover plate is required, nor is there any notch effect from the abrupt ending of the plate.

A 10 x $\frac{3}{8}$ -in. fill plate is used beneath the top splice plate beyond the cover-plate cutoff. Elimination of the fillet weld there also renders it unnecessary to fully develop the slab at 24 ft from the interior support. Adjacent to the splice, 15-in. shear-connector pitch is used and is terminated short of the flange splice plates. Two extra rows of shear connectors are still required to develop the slab reinforcement in the negative-moment region and are placed on the corresponding side of the splice at 6-in. pitch.

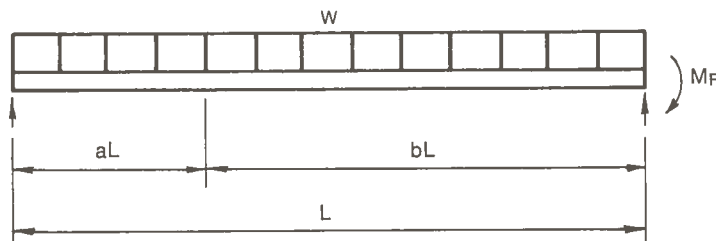
DEFLECTIONS

The final step in this design is calculation of dead-load camber and live-load deflection. Deflections due to dead load are computed, by the formula given earlier, at the splice, midway between the end support and splice, and midway between splice and interior support. These ordinates provide sufficient data for the fabricator to camber the beams. The moment of inertia at midspan is assumed constant throughout the span in these calculations.

Deflections Due to Dead Loads

$$DL_1: w = 0.900 \text{ kips per ft} \quad M_R = 551 \text{ kip-ft}$$

$$DL_2: w = 0.165 \text{ kips per ft} \quad M_R = 101 \text{ kip-ft}$$



Uniformly Loaded Span

$$\Delta = \frac{72wL^4}{E_s I} ab[1 + ab - 4C_R(1 + a)]$$

where Δ = deflection, in., at distance aL from end support

$$b = 1 - a$$

w = dead load, kips per ft

L = span, ft

E_s = modulus of elasticity of the steel = $29(10)^3$ ksi

I = moment of inertia at midspan, in.⁴

$$C_R = M_R / wL^2$$

M_R = bending moment at interior support, kip-ft

Deflections Under DL₁

$$I_s = 8,902 \text{ in.}^4 \quad C_R = \frac{551}{0.900(70)^2} = 0.125$$

$$\Delta = \frac{72 \times 0.900(70)^4}{29(10)^3 \times 8,902} ab[1 + ab - 4 \times 0.125(1 + a)] = 6.03ab[1 + ab - 0.5(1 + a)]$$

At $a = \frac{3}{8}$,

$$\Delta = 6.03(0.375)(0.625)[1 + 0.375(0.625) - 0.5(1.375)] = 0.772 \text{ in.}$$

At $a = \frac{3}{4}$,

$$\Delta = 6.03(0.750)(0.250)[1 + 0.75(0.25) - 0.5(1.750)] = 0.353 \text{ in.}$$

At $a = \frac{7}{8}$,

$$\Delta = 6.03(0.875)(0.125)[1 + 0.875(0.125) - 0.5(1.875)] = 0.113 \text{ in.}$$

Deflections Under DL₂

$$I_{24} = 18,557 \text{ in.}^4 \quad C_R = \frac{101}{0.165(70)^2} = 0.125$$

$$\Delta = \frac{72 \times 0.165(70)^4}{29(10)^3 \times 18,557} ab[1 + ab - 4 \times 0.125(1 + a)] = 0.530ab[1 + ab - 0.5(1 + a)]$$

At $a = \frac{3}{8}$,

$$\Delta = 0.530(0.375)(0.625)[1 + 0.375(0.625) - 0.5(1.375)] = 0.068 \text{ in.}$$

At $a = \frac{3}{4}$,

$$\Delta = 0.530(0.75)(0.25)[1 + 0.75(0.25) - 0.5(1.75)] = 0.031 \text{ in.}$$

At $a = \frac{7}{8}$,

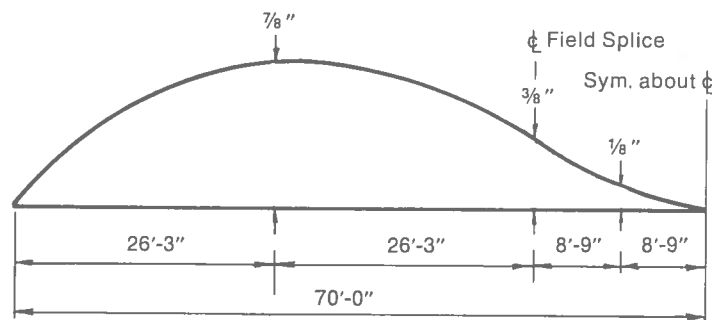
$$\Delta = 0.530(0.875)(0.125)[1 + 0.875(0.125) - 0.5(1.875)] = 0.010 \text{ in.}$$

Total DL Deflections

$$\text{At } a = \frac{3}{8}, \Delta = 0.772 + 0.068 = 0.840 \text{ in., say } \frac{7}{8} \text{ in.}$$

$$\text{At } a = \frac{3}{4}, \Delta = 0.353 + 0.031 = 0.384 \text{ in., say } \frac{3}{8} \text{ in.}$$

$$\text{At } a = \frac{7}{8}, \Delta = 0.113 + 0.010 = 0.123 \text{ in., say } \frac{1}{8} \text{ in.}$$



CAMBER DIAGRAM

Deflection Due To Live Load + Impact

Live load for deflection consists of two lanes of truck loading, which are distributed equally to the four stringers. The approximate formula given earlier for deflection at the 0.4 point of end spans of continuous beams due to HS truck loading is used. The moment of inertia at midspan is assumed constant throughout the span. Calculations show that live-load deflections are less than half the allowable value.

The live-load deflection, in., 28 ft from the end support is given by

$$\Delta = \frac{300}{E_s I} [P_T (L^3 + 3.89L^2 - 680L + 5,910) - 0.32M_R L^2]$$

where P_T = weight of front truck wheel \times distribution factor, plus impact, kips

I = moment of inertia at midspan, in.⁴

L = span, ft

$E_s = 29(10)^3$ ksi

M_R = bending moment due to live load plus impact at the interior support, kip-ft

Assume that two lanes of live load (four wheels abreast) plus 25.6% impact are equally distributed over four stringers.

$$P_T = 4 \times 4 \times 1.256 = 20.1 \text{ kips}$$

$$I_s = 4 \times 25,867 = 103,468 \text{ in.}^4$$

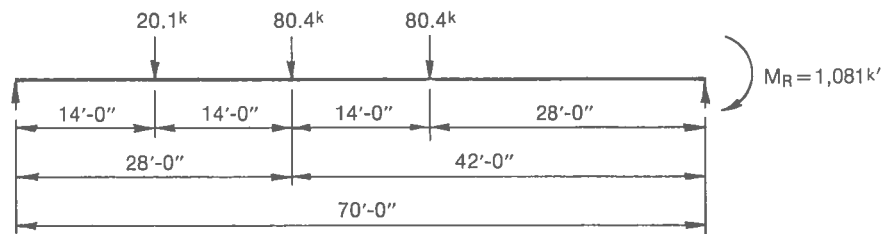
The moment M_R at the interior support can be computed in any of several ways; for example, by influence coefficients, as shown below. (See References 3 and 4 at the end of the chapter.)

$$20.1 \times 70 \times 0.0480 = 68$$

$$80.4 \times 70 \times 0.0840 = 473$$

$$80.4 \times 70 \times 0.0960 = 540$$

$$M_R = 1,081 \text{ kip-ft}$$



Girder Loaded For Maximum Deflection

The maximum live-load deflection therefore is

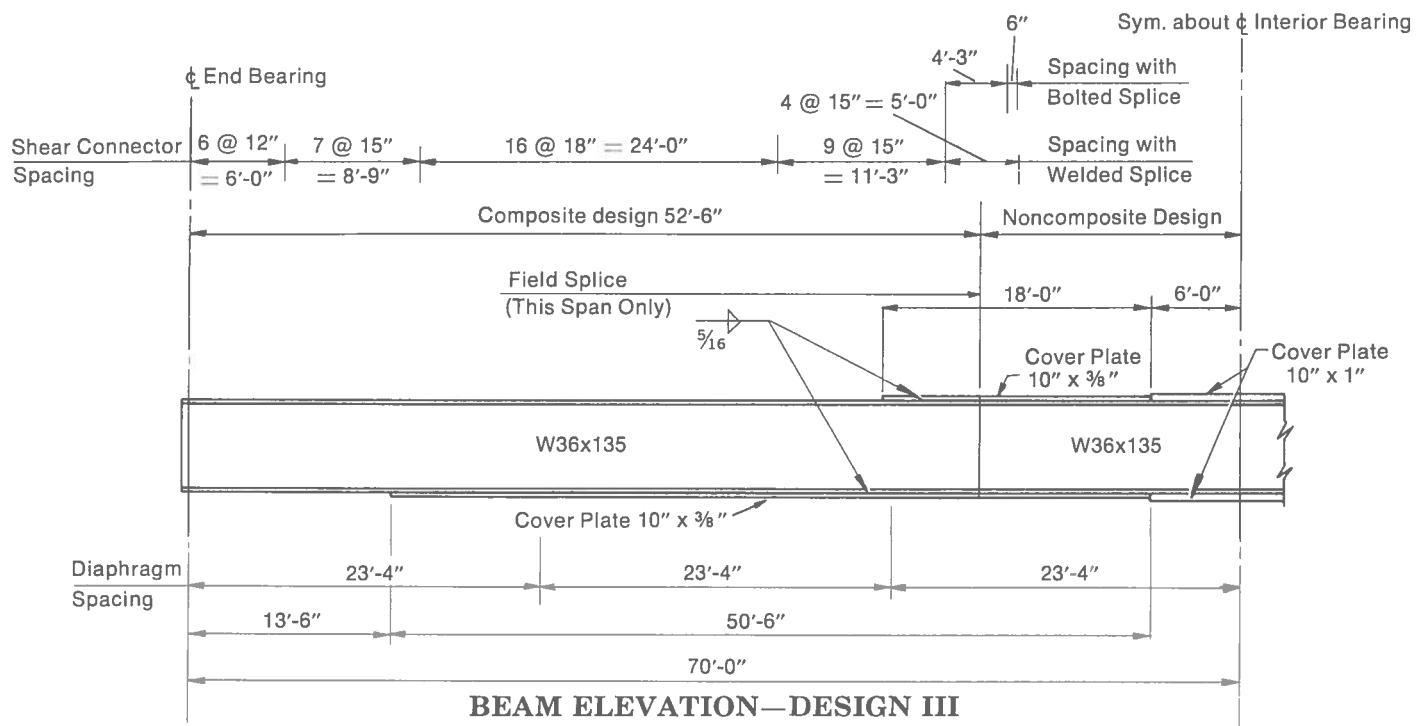
$$\begin{aligned} \Delta &= \frac{300}{29(10)^3 \times 103,468} \{ 20.1[(70)^3 + 3.89(70)^2 - 680(70) + 5,910] - 0.32(1,081)(70)^2 \} \\ &= \frac{300 \times 4,744,449}{29(10)^3 \times 103,468} = 0.472 \text{ in.} \end{aligned}$$

The ratio of live-load deflection to span is

$$\frac{0.472}{70 \times 12} = \frac{1}{1,780} < \frac{1}{800}$$

FINAL DESIGN

An elevation of the two-span, continuous, composite stringer is shown on the next page. See also the detail drawing at the end of this chapter. Alternate designs will be examined next.



Design III—Alternate A

An alternate design for the two span, continuous, composite stringer employs a $W36 \times 160$ beam with $10\frac{1}{2} \times \frac{5}{8}$ -in. cover plates added to the top and bottom flanges at the interior support. This rolled beam was selected because it is the lightest section that requires no bottom cover plate in the positive-moment region. It also permits the cover plates to end between the interior support and the inflection point without exceeding the allowable fatigue stress.

Properties are computed for the steel section alone in the maximum-negative-moment region, and for the composite section with $n=8$ and $3n$ equal to 24 in the maximum-positive-moment region. Bending moments and shears are computed for constant moment of inertia in both spans. Stresses are calculated at the top and bottom of steel and at the top of concrete.

MAXIMUM NEGATIVE MOMENT

From the maximum-moment curves, the total moment at the interior support is $-1,253$ kip-ft. This moment must be taken by the $W36 \times 160$ with top and bottom cover plates. For the $W36 \times 160$, the moment of inertia is $9,739$ in.⁴, the half-depth $c=18$ in., and the section modulus $Z=541$.

Steel Section at Interior Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 160	47.09				9,739	9,739
2 Plates $10\frac{1}{2} \times \frac{5}{8}$	13.13	18.31	240.35	4,401		4,401

$$60.22 \text{ in.}^2$$

$$I_{NA} = 14,140 \text{ in.}^4$$

$$Z = \frac{14,140}{18.63} = 759 \text{ in.}^3$$

$$\text{Maximum stress } f_b = \frac{1,253 \times 12}{759} = 19.80 \text{ ksi} < 20$$

Allowable Compressive Stress Near Interior Support

$$F_b = 20,000 - 7.5 \left(\frac{L}{b} \right)^2 = 20,000 - 7.5 \left(\frac{17.5 \times 12}{12} \right)^2 = 17.70 \text{ ksi}$$

Because of continuity, AASHTO specifications permit a 20% increase in allowable stresses up to 20 ksi at the interior support.

$$F'_b = 17.73 \times 1.20 = 21.28 \text{ ksi. Use 20 ksi.}$$

MAXIMUM POSITIVE MOMENT

The W36 × 160 is investigated for the region of maximum positive moment. Properties are computed for the composite section with $n = 8$ and $3n = 24$.

Composite Section, $3n = 24$, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 160	47.09				9,739	9,739
Conc. 84 × 7/24	24.50	23.10	565.95	13,073	100	13,173

$$d_{24} = \frac{565.95}{71.59} = 7.91 \text{ in.}$$

$$I_{NA} = \frac{22,912}{18,435} \text{ in.}^4$$

$$d_{\text{Top of steel}} = 18.00 - 7.91 = 10.09 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 18.00 + 7.91 = 25.91 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{18,435}{10.09} = 1,827 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{18,435}{25.91} = 712 \text{ in.}^3$$

Composite Section, $n = 8$, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 160	47.09				9,739	9,739
Conc. 84 × 7/8	73.50	23.10	1,698	39,224	300	39,524

$$d_8 = \frac{1,698}{120.59} = 14.08 \text{ in.}$$

$$I_{NA} = \frac{49,263}{25,355} \text{ in.}^4$$

$$d_{\text{Top of steel}} = 18.00 - 14.08 = 3.92 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 18.00 + 14.08 = 32.08 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{25,355}{3.92} = 6,468 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{25,355}{32.08} = 790 \text{ in.}^3$$

$$d_{\text{Top of conc.}} = 26.38 - 14.08 = 12.30 \text{ in.}$$

$$Z_{\text{Top of conc.}} = \frac{25,355}{12.30} = 2,061 \text{ in.}^3$$

Check of Steel and Concrete Stresses

Stresses are checked at top and bottom of steel and at the top of concrete. Calculations show that the W36 × 160 satisfies stress limitations. The concrete stress is well within the allowable for compression.

Bending Moments 28 Ft from End Support

	DL_1	DL_2	$LL + I$
M , kip-ft	309	57	750

Steel Stresses—Combination A

Top of Steel (Compression)

$$DL_1: f_b = \frac{309 \times 12}{541} = 6.85$$

$$DL_2: f_b = \frac{57 \times 12}{1,827} = 0.37$$

$$LL+I: f_b = \frac{750 \times 12}{6,468} = 1.39$$

8.61 ksi

Bottom of Steel (Tension)

$$f_b = \frac{309 \times 12}{541} = 6.85$$

$$f_b = \frac{57 \times 12}{712} = 0.96$$

$$f_b = \frac{750 \times 12}{790} = 11.39$$

19.20 ksi

Concrete Stresses—Combination B

$$DL_2: f_c = \frac{57 \times 12}{2,061 \times 8} = 0.042$$

$$LL+I: f_c = \frac{750 \times 12}{2,061 \times 8} = 0.546$$

0.588 ksi

LOCATION OF COVER-PLATE CUTOFFS

Stresses are checked at theoretical cutoff points for the $10\frac{1}{2} \times \frac{5}{8}$ -in. cover plates 6.0 ft from the interior support and found to be less than allowable values. The actual ends, 7 ft 4 in. from the support are then investigated for fatigue strength at the fillet welds. Finally, the size of weld connecting the ends of the cover plates to the flanges is determined.

Maximum Moments 6 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = -330$$

$$DL_2: M = -60$$

$$LL+I: M = +85$$

-305 kip-ft

With Negative Live-Load Moment

$$DL_1: M = -330$$

$$DL_2: M = -60$$

$$LL+I: M = -400$$

-790 kip-ft

For the $W36 \times 160$ alone, with $Z = 541 \text{ in.}^3$, the maximum stress is

$$f_b = \frac{790 \times 12}{541} = 17.52 \text{ ksi}$$

Since this is less than 17.70 ksi, the allowable compressive stress in the bottom flange for lateral buckling, the theoretical cutoff 6 ft from the interior support is satisfactory.

The required terminal distance is at least $1.5 \times 10\frac{1}{2} = 15.75 \text{ in.}$

Try cutoff of the cover plates 6 ft + 1 ft 4 in. = 7 ft 4 in. from the interior support.

Fatigue Check at Cover-Plate End

Bending stresses are computed 7 ft 4 in. from the interior support and are found to be less than the allowable fatigue stress in the $W36 \times 160$ flanges.

Maximum Moments 7 Ft 4 In. from Interior Support

With Positive Live-Load Moment

$$DL_1: M = -285$$

$$DL_2: M = -50$$

$$LL+I: M = +115$$

-220 kip-ft

With Negative Live-Load Moment

$$DL_1: M = -285$$

$$DL_2: M = -50$$

$$LL+I: M = -380$$

-715 kip-ft

Bending stress in top and bottom flanges is

$$f_b = \frac{715 \times 12}{541} = 15.86 \text{ ksi}$$

Ratio of minimum to maximum stress at cover-plate ends is

$$R = \frac{-220}{-715} = 0.308$$

The allowable fatigue stress in the W36×160 flanges adjacent to the fillet welds for the plate ends is

$$F_r = \frac{12.0}{1 - 0.308} = 17.34 \text{ ksi} > 15.86$$

Welds at Cover-Plate Ends

The weld size is determined by the force in each cover plate at the theoretical cutoff point, 6 ft from the interior support. Computations show that a $\frac{1}{4}$ -in. fillet weld 42.5 in. long in the terminal region of each plate would be satisfactory. For the 1-in.-thick flanges of the W36×160, however, a $\frac{5}{16}$ -in. weld is required.

The stress in the cover plates at theoretical cutoff is

$$f_b = \frac{790 \times 12}{759} = 12.49 \text{ ksi}$$

Ratio of minimum to maximum stress in the cover plates 6 ft from the interior support is

$$R = \frac{-305}{-790} = 0.386$$

The allowable weld fatigue stress in shear is

$$F_r = \frac{10.8}{1 - 0.55(0.386)} = 13.7 \text{ ksi} > 12.4 \text{ ksi}$$

Allowable load on weld = $12.4 \times 0.707 = 8.76$ kips per in.

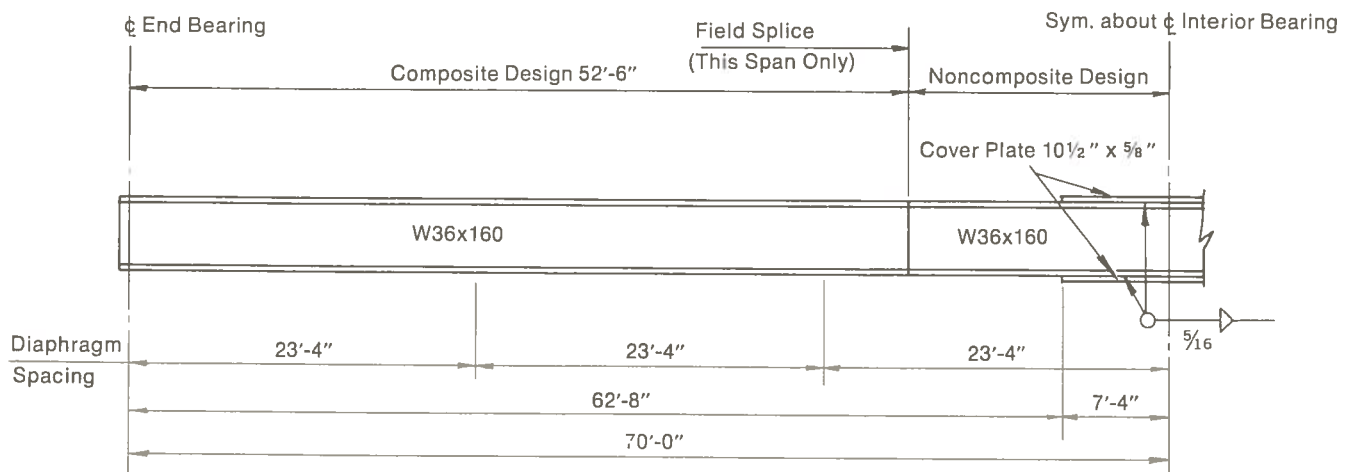
Force in cover plate = $10.5 \times \frac{5}{8} \times 12.49 = 81.9$ kips

Weld size required = $\frac{81.9}{8.76 \times 42.5} = 0.22$ in., say $\frac{1}{4}$ in.

Use $\frac{5}{16}$ -in. fillet weld, required for flange thickness.

FINAL DESIGN—ALTERNATE A

An elevation of the two-span, composite Alternate A is shown below. The alternate designs will be discussed later.



BEAM ELEVATION DESIGN III—ALTERNATE A

Design III—Alternate B

A second alternate design utilizes a W36 × 160 butt welded to a W36 × 230. This arrangement eliminates cover plates and the low allowable fatigue stresses at the ends of cover plates.

Section properties and stresses are computed for the maximum-negative and maximum-positive-moment regions. Again, bending moments and shears are calculated for constant moment of inertia in both spans. The stress is also checked at the butt-welded field splice and found to be less than the allowable fatigue stress.

MAXIMUM NEGATIVE MOMENT

From the maximum-moment curves, the total moment at the interior support is −1,253 kip-ft. This moment must be taken by the W36 × 230, with section modulus $Z = 835.5 \text{ in.}^3$. The maximum stress is

$$f_b = \frac{1,253 \times 12}{835.5} = 18.00 \text{ ksi} < 20$$

Allowable Compressive Stress Near Interior Support

The length L for lateral buckling is 17.5 ft, the distance from interior bearing to dead-load inflection point.

$$F_b = 20,000 - 7.5 \left(\frac{L}{b} \right)^2 = 20,000 - 7.5 \left(\frac{17.5 \times 12}{16.47} \right)^2 = 18.78 \text{ ksi}$$

Because of continuity, AASHTO specifications permit a 20% increase in allowable stress up to 20 ksi at the interior support.

$$F'_b = 1.20 \times 18.78 = 22.54 \text{ ksi. Use 20 ksi.}$$

MAXIMUM POSITIVE MOMENT

The W36 × 160 is investigated for the region of maximum positive moment. Properties are computed for the composite section with $n = 8$ and $3n = 24$. For the W36 × 160, the section modulus $Z = 541$.

Composite Section, $3n = 24$, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 160	47.09				9,739	9,739
Conc. 84 × 7 24	24.50	22.44	549.7	12,335	100	12,435

$$d_{NA} = \frac{549.7}{71.59} = 7.68 \text{ in.} \quad \begin{array}{l} 71.59 \text{ in.}^2 \quad 549.7 \text{ in.}^3 \quad 22,174 \\ -7.68 \times 549.7 = -4,222 \\ I_{NA} = \frac{17,952}{17,952} \text{ in.}^4 \end{array}$$

$$d_{\text{Top of steel}} = 18.00 - 7.68 = 10.32 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 18.00 + 7.68 = 25.68 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{17,952}{10.32} = 1,740 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{17,952}{25.68} = 699 \text{ in.}^3$$

Composite Section, n=8, 28 Ft from End Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 160	47.09				9,739	9,739
Conc. 84 × 7/8	73.50	22.44	1,649.3	37,010	300	37,310

$$d_8 = \frac{1,649.3}{120.59} = 13.68 \text{ in.}$$

$$I_{NA} = \frac{47,049}{-13.68 \times 1649.3} = -22,562 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 18.00 - 13.68 = 4.32 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 18.00 + 13.68 = 31.68 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{24,487}{4.32} = 5,668 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{24,487}{31.68} = 773 \text{ in.}^3$$

$$d_{\text{Top of conc.}} = 25.94 - 13.68 = 12.26 \text{ in.}$$

$$Z_{\text{Top of conc.}} = \frac{24,487}{12.26} = 1,997 \text{ in.}^3$$

Check of Steel and Concrete Stresses

Stresses are checked at top and bottom of steel and at the top of concrete. Calculations show that the W36 × 160 satisfies stress limitations. The concrete stress is well within the allowable for compression.

Bending Moments 28 Ft from End Support

	DL ₁	DL ₂	LL + I
M, kip-ft	309	57	750

Steel Stresses—Combination A

Top of Steel (Compression)

$$DL_1: f_b = \frac{309 \times 12}{541} = 6.85$$

$$DL_2: f_b = \frac{57 \times 12}{1,740} = 0.39$$

$$LL + I: f_b = \frac{750 \times 12}{5,668} = \frac{1.59}{8.83} \text{ ksi}$$

Bottom of Steel (Tension)

$$f_b = \frac{309 \times 12}{541} = 6.85$$

$$f_b = \frac{57 \times 12}{699} = 0.98$$

$$f_b = \frac{750 \times 12}{773} = \frac{11.64}{19.47} \text{ ksi}$$

Concrete Stresses—Combination B

Top of Concrete (Compression)

$$DL_2: f_c = \frac{57 \times 12}{1,997 \times 8} = 0.043$$

$$LL + I: f_c = \frac{750 \times 12}{1,997 \times 8} = \frac{0.563}{0.606} \text{ ksi}$$

Fatigue Check of Butt-Welded Field Splice

The W36 × 230 and W36 × 160 are spliced at the dead-load inflection point, 17.5 ft from the interior support. Stresses in the W36 × 160 are computed at this location and found to be less than the allowable fatigue stress for the butt-welded splice.

$$DL_1 + DL_2: M = 0 \text{ kip-ft}$$

$$DL_1 + DL_2: M = 0 \text{ kip-ft}$$

$$LL + I: M = 420 \text{ kip-ft}$$

$$LL + I: M = -310 \text{ kip-ft}$$

Actual stress in top and bottom flanges is

$$f_b = \frac{420 \times 12}{541} = 9.32 \text{ ksi}$$

Ratio of minimum to maximum stress in butt-welded flange splice is

$$R = \frac{-310}{420} = -0.738$$

The allowable fatigue stress in tension is

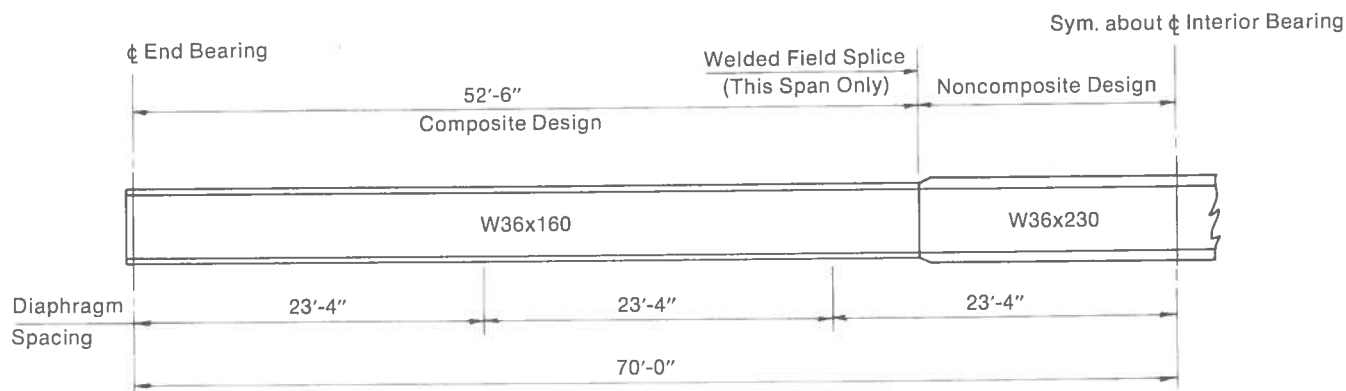
$$F_r = \frac{17.2}{1 - 0.62(-0.738)} = 11.80 \text{ ksi} > 9.32$$

The allowable fatigue stress in compression is

$$F_r = \frac{0.55 \times 36}{1 - \left[\frac{0.55 \times 36}{10.6} - 1 \right] (-0.738)} = 12.06 \text{ ksi} > 9.32$$

FINAL DESIGN—ALTERNATE B

An elevation of the two-span, composite Alternate B is shown below. The alternate designs will be discussed next.



BEAM ELEVATION DESIGN III—ALTERNATE B

Design III—Alternates A and B: Economic Considerations

Design III and its Alternates A and B are different solutions to the same structural problem. Design III is presented in considerable detail as an example that illustrates the most likely conditions to be encountered by a designer. It may not necessarily be the most advantageous of the three designs from an economic standpoint. The relative economies achieved by Design III, Alternate A, and Alternate B are dependent on the quantity of steel required for each design and amount of fabrication involved.

Alternate A requires about 7½% more steel, and Alternate B about 16½% more steel than Design III. But Alternate A has less than a third as much fillet weld as Design III and only about a fifth as much length of plate shearing. Alternate B eliminates fillet welds and shearing entirely, except for minor details, but requires a major butt weld of the two rolled sections.

In a comparison of the two alternate designs with Design III, the smaller amount of fabrication in Alternate A tends to offset its greater weight. The even smaller amount of fabrication of Alternate B tends to offset its still greater weight. The relative importance of steel quantity versus amount of fabrication in deciding the economy

of a structure varies from one part of the country to another, from one time to another, and with local conditions. Furthermore, the most economic design arrangements may change with span. For slightly longer spans, lighter rolled sections with cover plates may prove superior to heavier rolled sections with a minimum of cover-plate material.

Design III—Alternate C

Many bridges carrying minor highways and streets are designed for 100,000 cycles of maximum stress. The design procedure is similar to that used for 500,000 cycles of maximum stress, and the same rolled beam and cover plates may be used at the points of maximum moments. However, allowable fatigue stresses in base metal adjacent to fillet welds are considerably higher for 100,000 cycles of maximum stress. When a light rolled beam is used, the heavy cover plates required for maximum negative moment may be cut off a few feet each side of the interior support. Also, the bottom cover plate required for maximum positive moment may be cut off between the point of maximum positive moment and the inflection point.

For 100,000 stress cycles, Design III—Alternate C uses the same W36×135 beam and cover plates at the points of maximum moment as were used for Design III. For Alternate C, however, the 10 x 1-in., negative-moment cover plates can be cut off near the interior support and fillet welded across their ends. As for Design III, the theoretical cutoff point is 8.75 ft from the interior support, but, for Alternate C, the allowable fatigue stress at the fillet weld is satisfied at only 10 ft from the support. Thus, the 10 x 1-in. top and bottom cover plates are ended at this point, rather than butt welded to 10 x 3/8-in. plates as in Design III.

Maximum Moments 8.75 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = -240$$

$$DL_2: M = -40$$

$$LL+I: M = \frac{+155}{-125} \text{ kip-ft}$$

With Negative Live-Load Moment

$$DL_1: M = -240$$

$$DL_2: M = -40$$

$$LL+I: M = \frac{-365}{-645} \text{ kip-ft}$$

For the W36×135 alone, with $Z=438.6$, the maximum stress is

$$f_b = \frac{645 \times 12}{438.6} = 17.65 \text{ ksi}$$

Since this is less than 17.70 ksi, the allowable compressive stress in the bottom flange for lateral buckling, the theoretical cutoff 8.75 ft from the interior support is satisfactory. The required terminal distance is at least $1.5 \times 10 = 15$ in.

Try cutoff of the cover plates $8.75 + 1.25 = 10$ ft from the interior support.

Fatigue Check for 100,000 Cycles

Bending stresses are computed 10 ft from the interior support and found to be less than the allowable fatigue stress in the W36×135 flanges.

Maximum Moments 10 Ft from Interior Support

With Positive Live-Load Moment

$$DL_1: M = -203$$

$$DL_2: M = -39$$

$$LL+I: M = \frac{+183}{-59} \text{ kip-ft}$$

With Negative Live-Load Moment

$$DL_1: M = -203$$

$$DL_2: M = -39$$

$$LL+I: M = \frac{-359}{-601} \text{ kip-ft}$$

Bending stress in top and bottom flanges is

$$f_b = \frac{601 \times 12}{438.6} = 16.44 \text{ ksi}$$

Ratio of minimum to maximum stress in cover-plate ends is

$$R = \frac{-59}{-601} = 0.098$$

The allowable fatigue stress in the W36 × 135 flanges adjacent to the fillet welds at the plate ends is

$$F_r = \frac{18.0}{1 - 0.098} = 19.96 \text{ ksi} > 16.44$$

Welds at Cover-Plate Ends

The weld size is determined by the force in each cover plate at the theoretical cutoff point, 8 ft 9 in. from the interior support. Computations show that the minimum permissible size for the $\frac{13}{16}$ -in.-thick flanges of the W36 × 135, a $\frac{5}{16}$ -in. fillet weld, is adequate. Length of weld is 40 in.

The stress in the cover plates at theoretical cutoff is

$$f_b = \frac{645 \times 12}{771} = 10.04 \text{ ksi}$$

Ratio of minimum to maximum stress in the cover plates 8 ft 9 in. from the interior support is

$$R = \frac{-125}{-645} = 0.194$$

The allowable weld fatigue stress in shear is

$$F_r = \frac{12.0}{1 - 0.5(0.194)} = 13.29 \text{ ksi} > 12.4$$

Allowable load on weld = $12.4 \times 0.707 = 8.76$ kips per in.

Force in cover plate = $10 \times 1 \times 10.04 = 100.4$ kips

Weld size required = $\frac{100.4}{8.76 \times 40} = 0.29$ in.

Use minimum $\frac{5}{16}$ -in. fillet weld.

CUTOFFS OF POSITIVE-MOMENT COVER PLATE

A 10 × $\frac{3}{8}$ -in. cover plate is used along the bottom flange in the positive-moment region. A location 40 ft from the end support is investigated as the theoretical cutoff point nearest the interior support.

Stresses 40 Ft from End Support

The composite section between the end of the 10 × $\frac{3}{8}$ -in. cover plate and the dead-load inflection point consists of the W36 × 135 and the 7-in. concrete slab. As computed in Design III, $Z = 1,790 \text{ in.}^3$ at top of steel and 599 in.^3 at bottom of steel for $3n = 24$, and $Z = 7,968 \text{ in.}^3$ at top of steel and 668 in.^3 at bottom of steel for $n = 8$.

Bending Moments 40 Ft from End Support

	DL_1	DL_1	$LL+I$
M , kip-ft	225	45	690

Steel Stresses—Combination A

Top of Steel (Compression)

$$DL_1: f_b = \frac{225 \times 12}{438.6} = 6.16$$

$$DL_2: f_b = \frac{45 \times 12}{1,790} = 0.30$$

$$LL + I: f_b = \frac{690 \times 12}{7,968} = \frac{1.04}{7.50 \text{ ksi}}$$

Bottom of Steel (Tension)

$$f_b = \frac{225 \times 12}{438.6} = 6.16$$

$$f_b = \frac{45 \times 12}{599} = 0.90$$

$$f_b = \frac{690 \times 12}{668} = \frac{12.40}{19.46 \text{ ksi} < 20}$$

Fatigue Check 45 Ft from End Support—100,000 Cycles

Fatigue requirements are fulfilled if the end of the cover plate is set at 45 ft from the end support. Stresses are computed for both maximum-positive and maximum-negative moments and found to be less than the allowable fatigue stress. Weld size is governed by material thickness (computations are not shown).

Maximum Moments 45 Ft from End Support

With Positive Live-Load Moment

$$DL_1: M = 145$$

$$DL_2: M = 28$$

$$LL + I: M = \frac{600}{773 \text{ kip-ft}}$$

With Negative Live-Load Moment

$$DL_1: M = 145$$

$$DL_2: M = 28$$

$$LL + I: M = \frac{-270}{-97 \text{ kip-ft}}$$

Stresses at Bottom of Steel—Combination A

Tension (Composite Section)

$$DL_1: f_b = \frac{145 \times 12}{438.6} = 3.97$$

$$DL_2: f_b = \frac{28 \times 12}{599} = 0.56$$

$$LL + I: f_b = \frac{600 \times 12}{668} = \frac{10.78}{15.31 \text{ ksi}}$$

Tension (Steel Only)

$$f_b = \frac{97 \times 12}{438.6} = 2.65 \text{ ksi}$$

Negative Moment

Ratio of minimum to maximum stress in beam flange at cover-plate weld:

$$R = \frac{-2.65}{15.31} = -0.173$$

The allowable fatigue stress is

$$F_r = \frac{18.0}{1 - (-0.173)} = 15.34 \text{ ksi} > 15.31$$

Since allowable stresses are satisfied, the plate can be terminated 45 ft from the end support.

Fatigue Check at Cutoff 15 Ft 9 In. from End Support—100,000 Cycles

The theoretical cutoff point nearest the end support for the bottom cover plate is 17 ft from that support, as established in Design III. Fatigue requirements based on 100,000 cycles are satisfied if the cover plate ends 15.75 ft from the end support.

Maximum Moments 15.75 Ft from End Support
With Positive Live-Load Moment With Negative Live-Load Moment

$$DL_1: M = 260$$

$$DL_1: M = 260$$

$$DL_2: M = 46$$

$$DL_2: M = 46$$

$$LL+I: M = \frac{615}{921} \text{ kip-ft}$$

$$LL+I: M = -\frac{93}{213} \text{ kip-ft}$$

Stresses at Bottom of Steel—Combination A
Tension (Composite Section) Tension (Steel Only)

$$DL_1: f_b = \frac{260 \times 12}{438.6} = 7.11$$

$$f_b = \frac{213 \times 12}{438.6} = 5.83 \text{ ksi}$$

$$DL_2: f_b = \frac{46 \times 12}{599} = 0.92$$

$$LL+I: f_b = \frac{615 \times 12}{668} = \frac{11.05}{19.08} \text{ ksi}$$

Ratio of minimum to maximum stress in beam flange at cover-plate weld:

$$R = \frac{5.83}{19.08} = 0.305$$

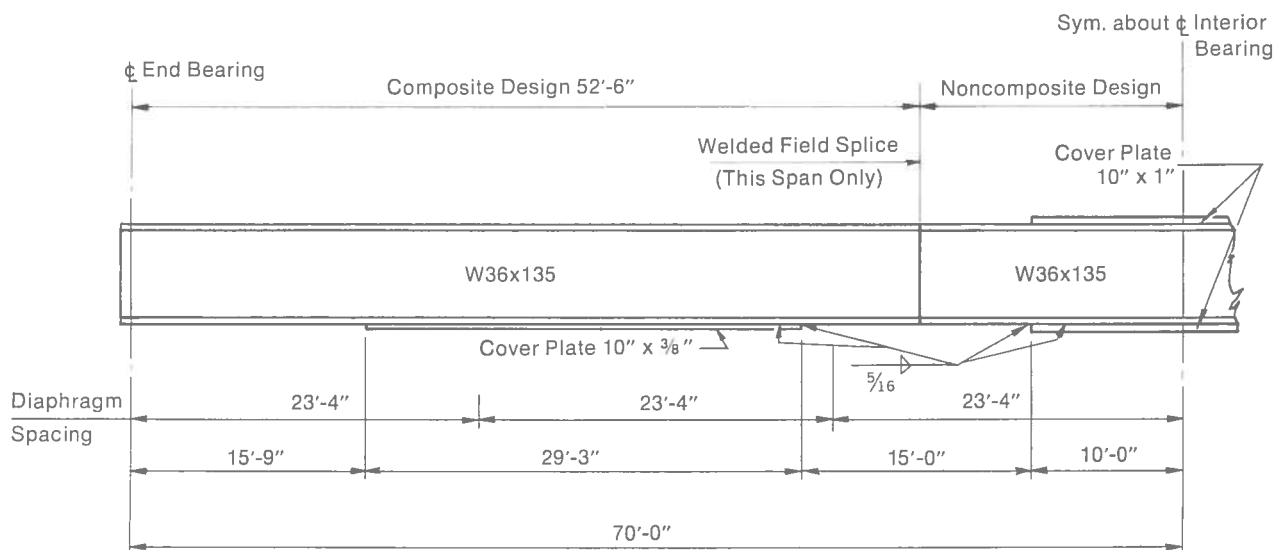
The allowable fatigue stress is

$$F_r = \frac{18.0}{1 - 0.305} = 25.90 \text{ ksi} > 19.08$$

Since allowable stresses are satisfied, the plate can be ended 15.75 ft from the end support.

FINAL DESIGN—ALTERNATE C

An elevation of the two span, continuous, composite Alternate C is shown below.



BEAM ELEVATION DESIGN III—ALTERNATE C

In Design III, composite action is developed only for positive bending moments. The design procedure for this example is similar to that for Design III, except that longitudinal reinforcing bars in the concrete slab are considered to act compositely with the beam in the negative-moment region. Concrete is assumed to be cracked under tensile stress and not acting as part of the composite section, except to transfer shear from the reinforcing bars to the beam. Reinforcing bars in the direction of the stringers are assumed to be effective if embedded in a width of slab not exceeding the following:

- ### MAXIMUM NEGATIVE MOMENT

The diagram illustrates the cross-section of a composite beam with the following dimensions and details:

- Top Flange:** Total width is 84" ($12 \times 7"$). It contains 14 #5 bars spaced at 6" ($14 \times 6" = 78"$). The top slab thickness is 7".
- Reinforcement:** Top bars are 1" from the top surface. Bottom bars are 1.25" from the bottom surface. A 1" haunch is provided at the top of the web.
- Web:** The web is a W36x135 section. The distance from the top flange to the neutral axis for the steel section is 17.78". The distance from the bottom flange to the neutral axis for the steel section is 19.30".
- Neutral Axes:**
 - Neutral Axis for Composite: Located 17.88" from the top flange and 19.65" from the bottom flange.
 - Neutral Axis for W36x135: Located 17.53" from the bottom flange.
 - Neutral Axis for Steel Section: Located 19.30" from the bottom flange.
- Cover Plates:** Top cover plate is 10" x 5/8". Bottom cover plate is 10" x 1".
- Other Dimensions:**
 - Distance from top flange to top reinforcement: 22.82".
 - Distance from top reinforcement to neutral axis for composite: 3.06".
 - Distance from neutral axis for composite to bottom reinforcement: 3.94".
 - Distance from bottom reinforcement to bottom flange: 19.30".
 - Distance from bottom flange to neutral axis for steel section: 17.53".

II/3.65

Steel Section at Interior Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 135	39.70				7,796	7,796
Bottom Plate 10 × 1	10.00	-18.28	-182.8	3,342		3,342
Top Plate 10 × 5/8	6.25	18.09	113.1	2,045		2,045

$$d_s = \frac{-69.7}{55.95} = -1.25 \text{ in.}$$

$$55.95 \text{ in.}^2$$

$$-69.7 \text{ in.}^3$$

$$-1.25 \times 69.7 = -87$$

$$I_{N.A.} = \frac{13,183}{13,096 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 17.78 + 0.62 + 1.25 = 19.65 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 17.78 + 1.00 - 1.25 = 17.53 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{13,096}{19.65} = 666 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{13,096}{17.53} = 747 \text{ in.}^3$$

Composite Section at Interior Support

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	55.95		-69.7			13,183
Reinf. Steel 14 No. 5	4.34	23.34	101.3	2,364	—	2,364

$$d_r = \frac{31.6}{60.29} = 0.52 \text{ in.}$$

$$60.29 \text{ in.}^2$$

$$31.6 \text{ in.}^3$$

$$-0.52 \times 31.6 = -16$$

$$I_{N.A.} = \frac{15,547}{15,531 \text{ in.}^4}$$

$$d_{\text{Reinf.}} = 23.34 - 0.52 = 22.82 \text{ in.}$$

$$Z_{\text{Reinf.}} = \frac{15,531}{22.82} = 681 \text{ in.}^3$$

$$d_{\text{Top of steel}} = 17.78 + 0.62 - 0.52 = 17.88 \text{ in.}$$

$$d_{\text{Bot. of steel}} = 17.78 + 1.00 + 0.52 = 19.30 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{15,531}{17.88} = 869 \text{ in.}^3$$

$$Z_{\text{Bot. of steel}} = \frac{15,531}{19.30} = 805 \text{ in.}^3$$

Bending Moments (Constant I)

	DL ₁	DL ₂	LL + I
M, kip-ft	-551	-101	-601

Steel Stresses for Maximum Negative Moment

Top of Stringer (Tension)

Bottom of Stringer (Compression)

$$DL_1: f_b = \frac{551 \times 12}{666} = 9.93$$

$$f_b = \frac{551 \times 12}{747} = 8.85$$

$$DL_2: f_b = \frac{101 \times 12}{869} = 1.39$$

$$f_b = \frac{101 \times 12}{805} = 1.51$$

$$LL + I: f_b = \frac{601 \times 12}{869} = \frac{8.30}{19.62 \text{ ksi} < 20}$$

$$f_b = \frac{601 \times 12}{805} = \frac{8.96}{19.32 \text{ ksi}}$$

The stress in the reinforcing bars is

$$f_b = \frac{(101 + 601)12}{681} = 12.37 \text{ ksi}$$

Allowable Compressive Stress

As for Design III, length L for lateral buckling is 17.5 ft, the distance from interior support to dead-load inflection point.

$$F_b = 20,000 - 7.5 \left(\frac{L}{b} \right)^2 = 20,000 - 7.5 \left(\frac{17.5 \times 12}{12} \right)^2 = 17.70 \text{ ksi}$$

Because of continuity, the allowable stress at the interior support may be increased 20%, up to 20 ksi.

$$F'_b = 17.70 \times 1.20 = 21.2 \text{ ksi. Use 20 ksi.}$$

Since the bending stresses are less than 20 ksi, the assumed section is satisfactory.

Fatigue Stress in Top Flange

Since shear connectors are welded to the tension flange of the stringer, fatigue must be considered. The allowable fatigue stress for base metal adjacent to welded-stud shear connectors is calculated and found not to govern.

Minimum Cover-Plate Stress

$$DL_1: f_b = 9.93$$

$$DL_2: f_b = \frac{1.39}{11.32} \text{ ksi}$$

Ratio of minimum to maximum stress in the cover plate is

$$R = \frac{11.32}{19.62} = 0.577$$

The allowable fatigue stress in tension is

$$F_r = \frac{16.5}{1 - 0.65(0.577)} = 26.40 \text{ ksi} > 20 \text{ ksi}$$

LOCATION OF COVER-PLATE CUTOFFS

For positive moment, the same section is required as for Design III—a W36 × 135 with a 10 × 3/8-in. bottom cover plate. Near the inflection point, bottom-flange stresses and the variation in these stresses are nearly the same for Designs III and IV. As was shown in Design III, the 10 × 3/8-in. bottom cover plate cannot be cut off, because of fatigue restrictions, but must be extended through the inflection point and butt welded to the 10 × 1-in. bottom cover plate required at the interior support. Calculations show that this change in cover-plate thickness can be made 5 ft from the interior support and that the top cover plate can be cut off there.

Steel Section 5 Ft from Interior Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 135	39.70				7,796	7,796
Bottom Plate 10 × 3/8	3.75	-17.96	-67.35	1,210		1,210

$$d_s = \frac{-67.35}{43.45} = -1.55$$

$$43.45 \text{ in.}^2 \quad -67.35 \text{ in.}^3 \quad 9,006$$

$$-1.55 \times 67.35 = -104$$

$$I_{NA} = \frac{8,902}{8,902 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 17.78 + 1.55 = 19.33 \text{ in.} \quad d_{\text{Bot. of steel}} = 17.78 + 0.38 - 1.55 = 16.61 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{8,902}{19.33} = 460 \text{ in.}^3 \quad Z_{\text{Bot. of steel}} = \frac{8,902}{16.61} = 536 \text{ in.}^3$$

Composite Section 5 Ft from Interior Support

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	43.45		67.35			9,006
Reinf. Steel 14 No. 5	4.34	23.34	101.30	2,364		2,364

$$d_c = \frac{33.95}{47.79} = 0.71 \text{ in.}$$

$$47.79 \text{ in.}^2 \quad 33.95 \text{ in.}^3 \quad 11,370$$

$$-0.71 \times 33.95 = -\frac{24}{11,346 \text{ in.}^4}$$

$$d_{\text{Top of steel}} = 17.78 - 0.71 = 17.07 \text{ in.} \quad d_{\text{Bot. of steel}} = 17.78 + 0.38 + 0.71 = 18.87 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{11,346}{17.07} = 665 \text{ in.}^3 \quad Z_{\text{Bot. of steel}} = \frac{11,346}{18.87} = 601 \text{ in.}^3$$

Maximum Moments 5 Ft from Interior Support, Kip-Ft

With Positive Live-Load Moment

$$DL_1: M = -360$$

$$DL_2: M = -67$$

$$LL+I: M = +65$$

With Negative Live-Load Moment

$$DL_1: M = -360$$

$$DL_2: M = -67$$

$$LL+I: M = -412$$

Stresses at Bottom of Steel

Minimum (Compression)

$$DL_1: f_b = \frac{360 \times 12}{536} = 8.06$$

$$DL_2 + LL + I: f_b = \frac{2 \times 12}{601} = 0.04$$

$$8.10 \text{ ksi}$$

Maximum (Compression)

$$f_b = \frac{360 \times 12}{536} = 8.06$$

$$f_b = \frac{479 \times 12}{601} = \frac{9.56}{17.62 \text{ ksi} < 17.70}$$

Ratio of minimum to maximum stress in bottom cover plate is

$$R = \frac{-8.10}{-17.62} = 0.460$$

The allowable fatigue stress in compression is

$$F_r = \frac{19.8}{1 - \left[\left(\frac{19.8}{10.6} - 1 \right) 0.461 \right]} = 32.96 \text{ ksi} > 17.70$$

Lateral buckling governs.

CUTOFF OF TOP COVER PLATE

Since stresses due to DL_2 and $LL+I$ are resisted by a composite section including the reinforcing bars, top-flange stresses and the variation in these stresses are sufficiently small that it is possible to cut off the top cover plate near the interior support and not exceed allowable fatigue stresses in base metal at the end of the cover plate.

The theoretical cutoff point for the 10 x 5/8-in. top cover plate is 3.5 ft from the interior support. Fatigue is not critical. The plate may be ended 5 ft from the support. Thus, when composite action is developed in the negative-moment region, the top cover plate is decreased in thickness over the interior support and can be ended a few feet from the interior support.

Steel Section 3.5 Ft from Interior Support

Material	A	d	Ad	Ad ²	I _o	I
W36 × 135	39.70				7,796	7,796
Bottom Plate 10 × 1	10.00	-18.28	-182.80	3,342		3,342

$$d_s = \frac{182.80}{49.70} = -3.68 \text{ in.}$$

$$49.70 \text{ in.}^2 \quad -182.80 \text{ in.}^3 \quad 11,138$$

$$-3.68 \times 182.80 = -673$$

$$I_{NA} = 10,465 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 17.78 + 3.68 = 21.46 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{10,465}{21.46} = 488 \text{ in.}^3$$

Composite Section 3.5 Ft from Interior Support

Material	A	d	Ad	Ad ²	I _o	I
Steel Section	49.70		-182.80			11,138
Reinf. Steel 14 No. 5	4.34	23.34	101.30	2,364		2,364

$$d_c = \frac{-81.50}{54.04} = -1.51 \text{ in.}$$

$$54.04 \text{ in.}^2 \quad -81.50 \text{ in.}^3 \quad 13,502$$

$$-1.51 \times 81.50 = -123$$

$$I_{NA} = 13,379 \text{ in.}^4$$

$$d_{\text{Top of steel}} = 17.78 + 1.51 = 19.29 \text{ in.}$$

$$Z_{\text{Top of steel}} = \frac{13,379}{19.29} = 694 \text{ in.}^3$$

Bending Moments 3.5 Ft from Interior Support

	DL ₁	DL ₂	LL + I
M, kip-ft	-415	-75	-465

Stresses at Top of Steel (Tension)

$$DL_1: f_b = \frac{415 \times 12}{488} = 10.20$$

$$DL_2 + LL + I: f_b = \frac{540 \times 12}{694} = \frac{9.34}{19.54 \text{ ksi} < 20}$$

Fatigue Check 5 Ft from Interior Support

A theoretical cutoff of the top cover plate 3.5 ft from the interior support is satisfactory; but for fatigue considerations, the plate must be extended at least $1.5 \times 10 = 15$ in. For convenience, the plate is ended 5 ft from the support. Computations show that allowable fatigue stress does not govern there.

Maximum Moments 5 Ft from Interior Support, Kip-Ft

With Positive Live-Load Moment

$$DL_1: M = -360$$

$$DL_2: M = -67$$

$$LL + I: M = +65$$

With Negative Live-Load Moment

$$DL_1: M = -360$$

$$DL_2: M = -67$$

$$LL + I: M = -412$$

Stresses at Top of Steel

Minimum (Tension)

$$DL_1: f_b = \frac{360 \times 12}{460} = 9.39$$

$$DL_2 + LL + I: f_b = \frac{2 \times 12}{665} = 0.04$$

9.43 ksi

Maximum (Tension)

$$f_b = \frac{360 \times 12}{460} = 9.39$$

$$f_b = \frac{479 \times 12}{665} = \frac{8.64}{18.03 \text{ ksi}}$$

Ratio of minimum to maximum stress in the top flange of the W36 × 135 is

$$R = \frac{9.43}{18.03} = 0.523$$

The allowable fatigue stress is

$$F_r = \frac{12.0}{1 - 0.523} = 25.16 \text{ ksi} > 18.03$$

Weld at End of Top Cover Plate

Size of the weld required to develop the 10 × 5/8-in. top cover plate at the theoretical cutoff point, 3.5 ft from the interior support, is determined. The section is the same as that at the interior support. A 1/4-in. fillet weld 46 in. long would be adequate for strength. The 13/16-in. thickness of the W36 × 135 flange, however, requires a 5/16-in. weld.

Steel Stresses 3.5 Ft from Interior Support

Top of Steel (Tension)

$$DL_1: f_b = \frac{415 \times 12}{666} = 7.48$$

$$DL_2 + LL + I: f_b = \frac{540 \times 12}{869} = \frac{7.46}{14.94 \text{ ksi}}$$

$$\text{Force in cover plate} = 10 \times \frac{5}{8} \times 14.94 = 9.34 \text{ kips}$$

Since the cutoff is close to the support, where the stress range is small, fatigue does not control. The allowable load on the fillet weld is $12.4 \times 0.707 = 8.76$ kips per in.

$$\text{Weld size required} = \frac{9.34}{8.76 \times 46} = 0.23 \text{ in., say } \frac{1}{4} \text{ in.}$$

Use 5/16-in. fillet weld, required for flange thickness.

SHEAR-CONNECTOR SPACING

Shear connectors are required the full length of the stringer. Sample calculations for shear-connector spacing with three studs per space, at the interior support are shown.

The range of shear for live load at the interior support is $V_r = 62.1$ kips. The statical moment of the reinforcing steel (area = 4.34 in.²) about the neutral axis is

$$Q = 4.34 \times 22.82 = 99.0 \text{ in.}^3$$

$$S_r = \frac{V_r Q}{I} = \frac{62.1 \times 99.0}{15,531} = 0.40 \text{ kips per in.}$$

$$\text{Spacing required (3 studs)} = \frac{3 \times 8.11}{0.40} = 60.8 \text{ in.}$$

Use the maximum allowable spacing = 24 in.

The tensile force in the reinforcing bars is

$$H_3 = A_r F_y = 4.34 \times 40.0 = 173.6 \text{ kips}$$

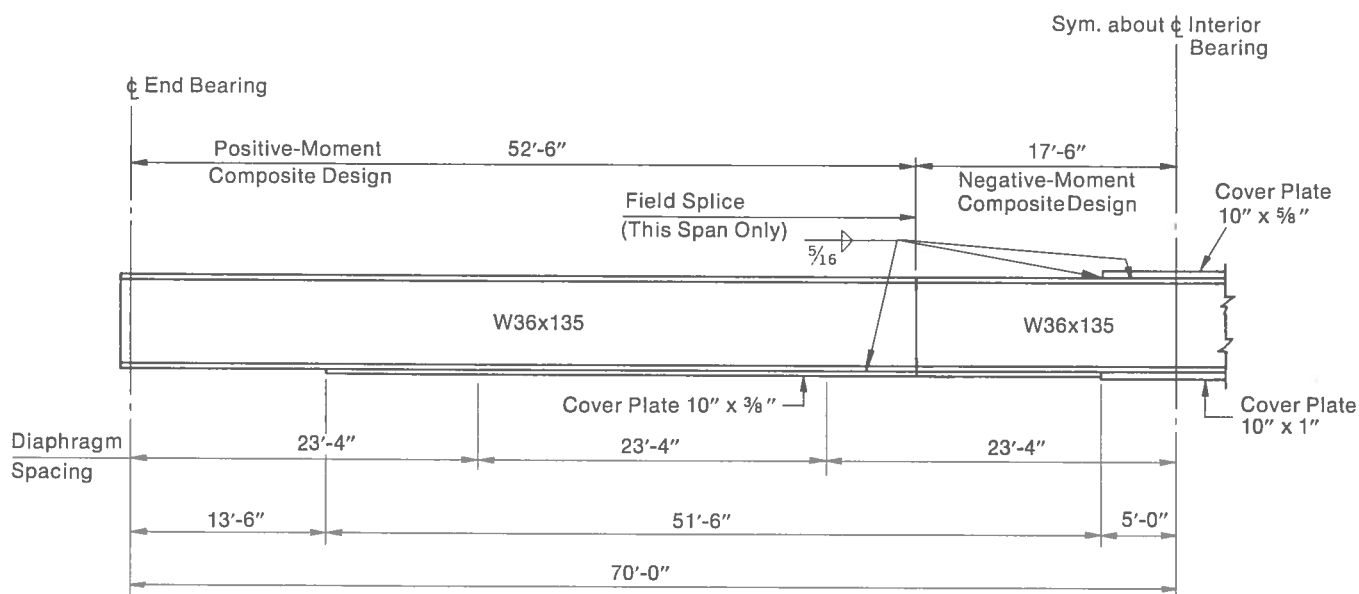
The number of studs required between point of maximum negative moment and dead-load inflection point is

$$N = \frac{H_3}{\phi Q_n} = \frac{173.6}{0.85 \times 45.0} = 4.5$$

Use the maximum allowable spacing of 24 in. throughout the negative-moment region except over the support. There, use 48 in. spacing. This spacing provides more than enough studs to satisfy strength requirements.

FINAL DESIGN

An elevation of the two-span beam, composite for positive and negative moment, is shown below.



BEAM ELEVATION—DESIGN IV

Design V—Four-Span Continuous Beam (70-90-90-70 Ft) Composite For Positive Moment Only

This design is included to show the advantages of multi-span continuous construction. The design procedure is similar to that for the two-span continuous beam of Design III.

When the end spans are held at 70 ft, as in the two-span design, and the interior spans are increased to 90 ft, the same section—a W36 × 135 with a 10 × 3/8-in. bottom cover plate—suffices for maximum positive moment in both end and interior spans. Negative moments over the first interior support and the center support are slightly higher than the negative moment for Design III. Maximum economy is obtained by extending the W36 × 135, with the maximum permissible cover plate (10 × 1 1/8-in.), over the first interior support and then introducing a short length of W36 × 150 with 10 × 1 1/4-in. cover plates over the center support.

Cover-plate cutoff locations are controlled by fatigue limitations. The bottom cover plates cannot be ended near inflection points, because allowable fatigue stresses are lower than bending stresses. Hence, the bottom cover plate must extend along the stringer, except for a short length near each end support. Although the bottom cover-plate transition, from 10 × 1 1/8-in. to 10 × 3/8-in., in the second span near the first

interior support can be made at 14'-0" from the support, the thicker plate is continued to the splice point. This is done to avoid the additional butt-welded splice which would be required in the bottom cover plate. The bottom cover-plate transition over the center support is made at 19 ft from the center support.

Investigation shows that the top cover plates may be cut off 14 ft from each interior support, if enough shear connectors to develop the force in the slab are placed on the cover plates adjacent to the cutoffs, to insure composite action of steel section and concrete slab. Six studs in two rows are sufficient to develop this force.

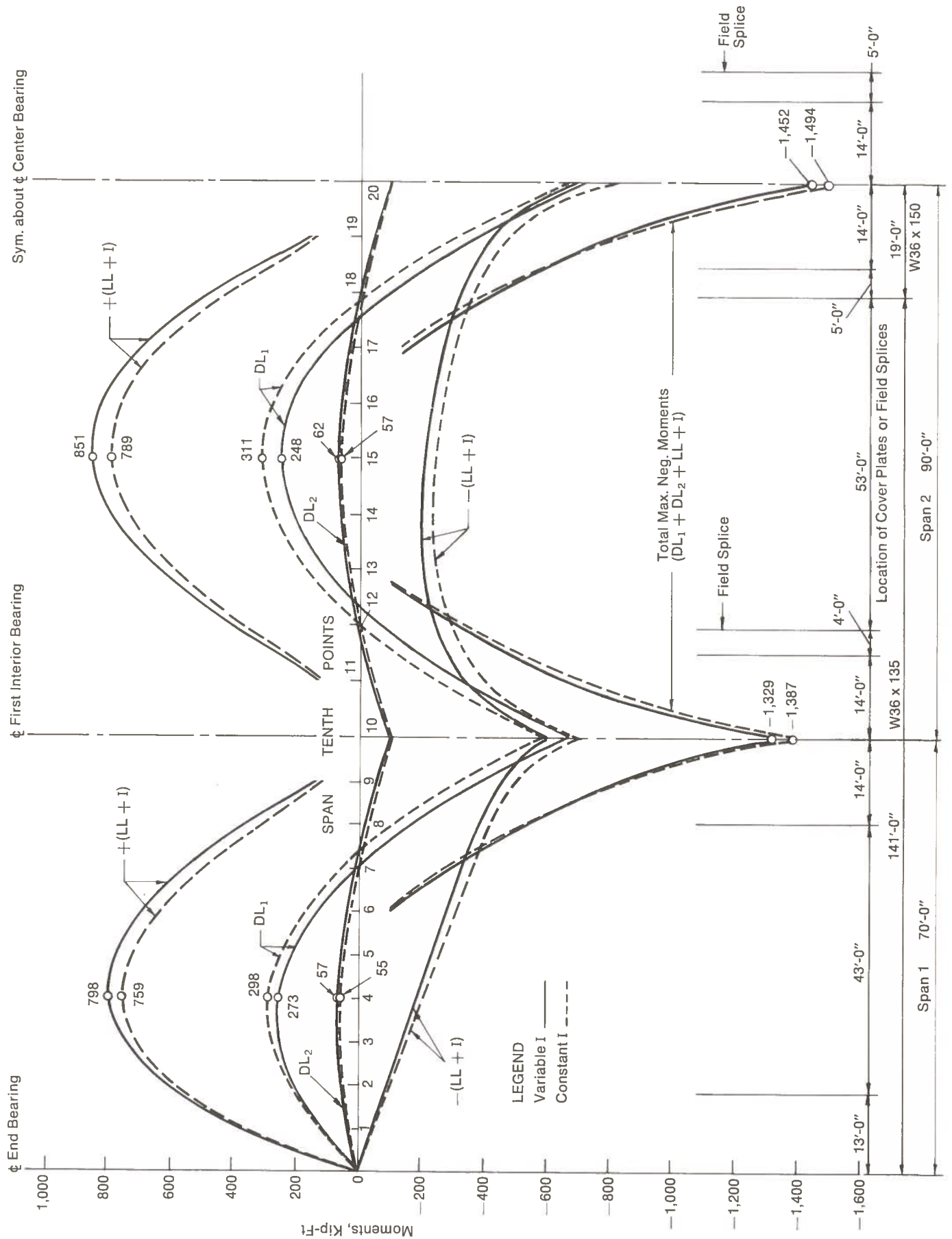
Field splices, which are generally located at dead-load inflection points, should be kept to a minimum, consistent with the longest practical shipping lengths. For this design example field splices are made 18 ft right of the first interior support and 19 ft right of the center support.

For the spans considered, the weight of stringers and framing details per square foot of roadway are nearly the same for two-span and four-span construction. Thus, three- or four-span continuous construction permits the use of longer interior spans at about the same steel weight per square foot of roadway. Four-span construction also is more economical because fewer bearing assemblies and expansion joints are required. These are expensive to fabricate. Furthermore, fewer expansion joints result in a better riding roadway.

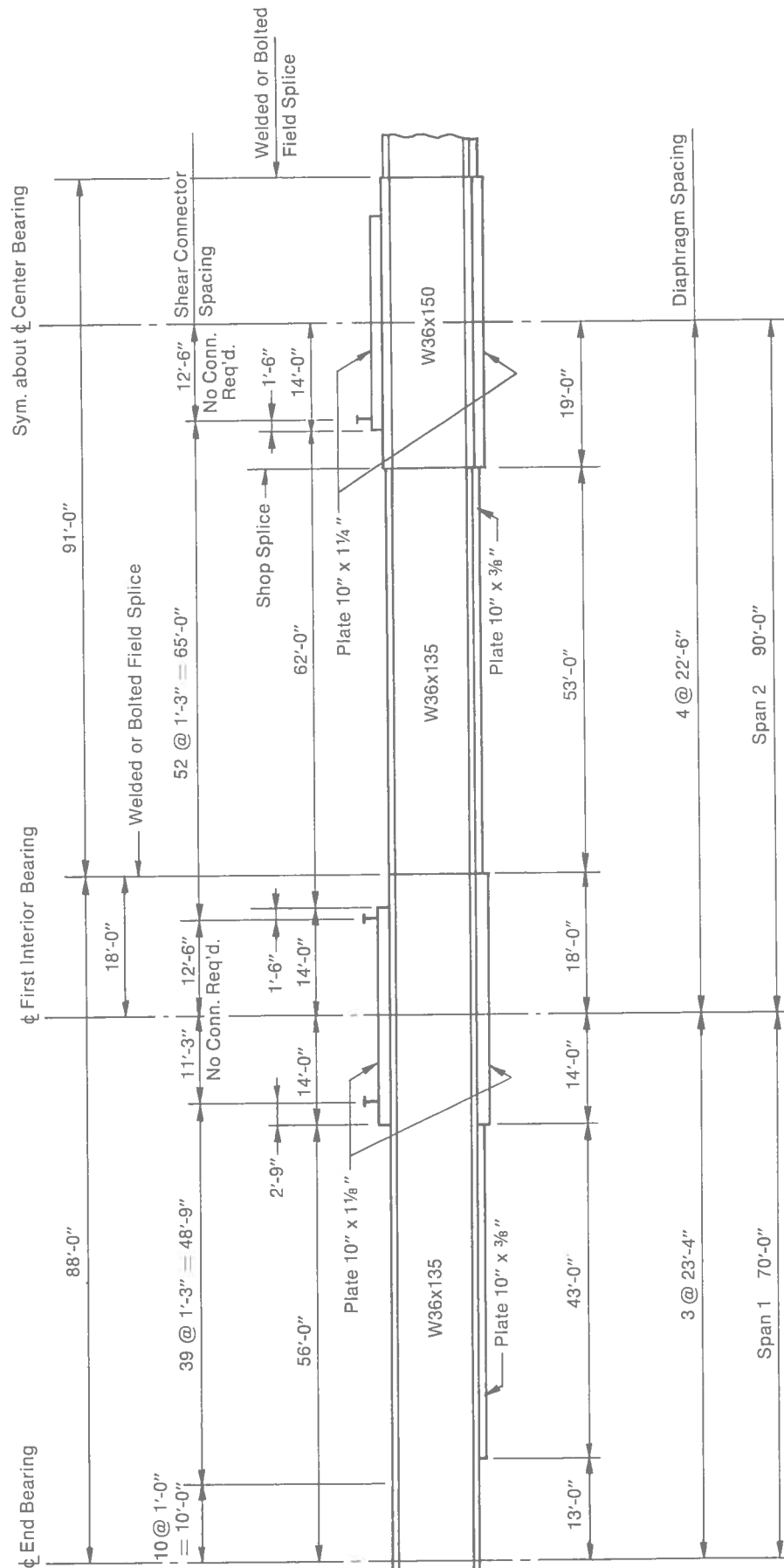
Maximum-moment curves and an elevation of the four-span stringer, with either welded or bolted field splices, are shown on the following pages.

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MAXIMUM-MOMENT CURVES—DESIGN V



BEAM ELEVATION—WELDED OR BOLTED FIELD SPLICE—DESIGN V







